DSGE Model Evaluation and Hybrid Models: A Comparison

Alessia Paccagnini[1]

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[1]Università degli Studi Milano - Bicocca Department of Economics E-mail : alessia.paccagnini@unimib.it. I thank Laurie Anderson, David Barnes, and participants at the Time Series Reading Group at the EUI, especially, Stelios Bekiros, Helmut Lutkepohl, Massimiliano Marcellino, and Tomasz Wozniak.

Abstract

This paper discusses the estimation of Dynamic Stochastic General Equilibrium (DSGE) models using hybrid models. These econometric tools provide the combination of an atheoretical statistical representation and the theoretical features of the DSGE model. A review of hybrid models presents the main aspects of these tools and why they are needed in the recent macroeconometric literature. Some of these models are compared to classical econometrics models (such as Vector Autoregressive (VAR), Factor Augmented VAR and Bayesian VAR) in a forecasting exercise.

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"Dynamic equilibrium theory made a quantum leap between the early 1970s and the late 1990s. In the comparatively brief space of 30 years, macroeconomists went from writing prototype models of rational expectations (think of Lucas, 1972) to handling complex constructions like the economy in Christiano, Eichenbaum, and Evans (2005). It was similar to jumping from the Wright brothers to an Airbus 380 in one generation".

Jesus Fernandez-Villaverde in "The Econometrics of DSGE Models"(2009, pag.2)

"A well-defined statistical model is one whose underlying assumptions are valid for the data chosen"

Aris Spanos in "The Simultaneous-Equations Model Revisited: Statistical Adequacy and Identification" (1990, pag.89)

1 Introduction

Over the last few years, there has been a growing interest in academia and in central banks in using Dynamic Stochastic General Equilibrium Models (DSGE) to explain macroeconomic fluctuations and conduct quantitative policy analysis.

One of the key topics in the recent New Keynesian literature is the estimation and the validation of DSGE models, with a specific focus on the statistical representation of DSGE models.

Despite the popularity of DSGE models, many important challenges remain. For instance, Schorfheide (2010) evidences five main challenges. The first is the fragility of parameter estimates due to lack of identification of the parameters of the model (Canova and Sala (2009)). This problem is particularly relevant since it has an impact on inference and forecasting exercises. Iskrev (2010) and Komunjer and Ng (2009) discuss necessary and sufficient conditions for the identification of the parameters of a DSGE model. Second, in a DSGE model, exogenous disturbances generate macroeconomic fluctuations. Sometimes we cannot be sure whether these shocks capture aggregate uncertainty or misspecification, and formal econometrics is not powerful enough to distinguish between these two interpretations. In this case, we need to be careful in specifying the law of motion for the exogenous shocks to overcome the model misspecification. The third aspect refers to the presence of trends. Many time series show low frequency behavior which makes estimation difficult to implement. The fourth challenge is the statistical fit. If the goal of the empirical analysis is to provide an impulse response function, for example, for an unanticipated change in the target inflation rate, the Vector Autoregressive model (VAR) (Sims (1980)) can be a good estimation instrument (even though the VAR does not provide a coherent economic explanation for the responses). Instead, if the aim is to determine the welfare effect of the change in the inflation target, the VAR is very limited.

The last, but not the least, challenge refers to the reliability of policy predictions. For example, Kocherlakota (2007) explains that while a model with a worse statistical fit may deliver better policy predictions, bad fit is not a guarantee of good policy predictions. In the very recent macro-econometric literature, hybrid or mixture models have become popular for dealing with some of the DSGE model misspecifications. These models are able to solve the trade-off between theoretical coherence and empirical fit. Essentially, two approaches (see Schorfheide (2010)) building empirical models that combine the restrictions of a DSGE model with a pure statistical model exist. These are additive hybrid models and hierarchical hybrid models. These hybrid models provide a complete analysis of the law of motion of the data, capturing the dynamic properties of the DSGE model. But these models are still linear and they do not consider time-variation for parameters. There are several examples of additive hybrid models: the DSGE-AR (Sargent (1989), Altug (1989)), the DSGE-AR à l' Ireland (2004), the DSGE-DFM (Boivin and Giannoni (2006) and Kryshko (2010)), the Augmented DSGE for Trends (Canova (2010)) and the DSGE-Noise (de Antonio Liedo (2010)). In the literature, there are essentially three examples of hierarchical hybrid models: the DSGE-VAR (Del Negro and Schorfheide (2004)), the DSGE-FAVAR (Consolo, Favero and Paccagnini (2009)) and the Augmented (B)VAR (Fernández-de-Córdoba and Torres (2010)).

Unfortunately, the five challenges discussed by Schorfheide (2010) are not the only problems in using DSGE models. There are other important issues in terms of estimation. Sometimes DSGE models exhibit non-linearities, even if the common practice is to solve and estimate a linearized version with Gaussian shocks. For example, during the last 40 years the US economy has shown a very high degree of time variation and strong evidence of time-varying volatility of U.S. aggregate variables. In particular, we can find various periods in the US economy, the so-called *Great Events*. The period identified as the Great Inflation refers to the 70s, the Great Moderation goes from 1984 to 2007 and the Great Contraction is from 2007 onwards, as defined by Kenneth Rogoff, who has described the contraction experienced in the most recent recession preceded by the financial market turmoil. A number of papers provide a measure of the impact of a possible stochastic volatility or of a possible parameter drifts using ad-hoc DSGE models applied to the Great Moderation period (Kim and Nelson (1999), McConnell and Pérez-Quirós (2000), Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004), Canova and Gambetti (2004), Primiceri (2005), Cogley and Sargent (2005), Sims and Zha (2006), Justiniano and Primiceri (2008) and Benati and Surico (2009) among the most cited). But, so far, hybrid models ignores stochastic volatility and parameter drifts, since they only consider linear properties and no time-varying parameters.

In addition, from an econometric point of view, the performance of a DSGE model is often tested against an estimated Vector Autoregressive model (VAR). This procedure requires a Data Generating Process (DGP) that is consistent with the theoretical economic model and has a finite-order VAR representation. However, the statistical representation of a DSGE model is an exact VAR only when all the endogenous variables are observable; otherwise, a more complex Vector Autoregressive Moving Average model (VARMA) is needed. As far as the VARMA representation is concerned, several papers (see Cooley and Dwyer (1998), Chari, Kehoe and McGrattan (2005), Christiano, Eichenbaum and Vigfusson (2006), Ravenna (2007) and Fernandez-Villaverde, Rubio-Ramirez, Sargent and Watson (2007)) have discussed the conditions required to find an infinite-order VAR representation and a finite-order VAR truncation in general terms. Moreover, the VAR is densely parameterized and appears to be misspecified and a Bayesian approach is preferred.

This paper presents a review of the main hybrid models used in the recent macroeconometric literature, combining aspects already discussed in Schorfheide (2010) and in Fernández-de-Córdoba and Torres (2010). After the review, the paper focuses on two of these hybrid models: the DSGE-VAR and the DSGE-FAVAR, two examples of hierarchical models. I choose these two hybrid models since these combinations can be considered modified versions of VAR. Hence, in a comparison exercise with VAR, Factor Augmented VAR and Bayesian VAR, there is a common component.

Two forecasting exercises on US data spanning from 1980:1 to 2010:4 are implemented. The time series for real GDP, Consumer Price Index, and Federal Funds Rate are used in the empirical evidence. A simple three-equation New Keynesian model is the benchmark for the DSGE component of these hybrid models. In the first exercise, a one-step-ahead is implemented using different forecasting samples. The DSGE-VAR outperforms the other models, when the forecasting sample is shorter. In the second exercise, a rolling forecast is implemented from one to five steps-ahead. The DSGE-FAVAR has better forecasting accuracy in the case of real GDP; instead, for the other variables, the hybrid models are outperformed by VAR and BVAR.

The remainder of this paper is organized as follows. Section 2 discusses the estimation of DSGE models, presenting a review of the main approaches in the literature. Section 3 introduces the hybrid or mixture models. Section 4 illustrates a simple DSGE model used as a benchmark in the empirical exercise. Section 5 proposes an empirical comparison of several models used to estimate US economy data. Section 6 closes the article.

2 Model Estimation in the DSGE Approach

In the recent macroeconomic literature, Dynamic Stochastic General Equilibrium Models (DSGE) are widely used in academia and central banks for policy evaluation and to explain macroeconomic fluctuations. DSGE models have the advantage of combining the micro-foundations of both households, and firms ' optimization problems with a large collection of both nominal and real (price/wage) rigidities. Model validation using DSGE models allows the econometrician to establish a link between structural features of the economy and reduced form parameters, something that was not always possible with the usual large-scale macroeconomic models. Improvements in computational power and the development of new econometric methods are crucial to the popularity of the use of DSGE models. The combination of rich structural models, novel solution algorithms and powerful simulation techniques has allowed researchers to develop the so-called "New Macro-econometrics" (Fernandez-Villaverde (2009)).

Despite their success, DSGE models were not considered as forecasting tools until very recent years, when Smets and Wouters (2003, 2004) proposed an interesting study of the forecasting performance of DSGE models compared to alternative non-structural models. Moreover, very few papers discuss the main aspects of validating the DSGE model, despite its widespread use for forecasting (Edge and Gürkaynak (2011)).

The concept of model validation involves selecting a loss function which measures the distance between the set of economic statistics and the set of statistics obtained from the simulated data. Canova (2005) explains that there are essentially four groups:

1. Approaches based on an R^2-type measure. In Watson (1993), the economic model is viewed as an approximation of the stochastic data generating process, considering that in the statistical sense the model is not true. The goodness-of-fit (R^2-type) measure is introduced to provide an evaluation of the approximation. The key ingredient of the measure is the size of the error needed to be added to the data generated by the model for the autocovariance implied by the model plus the error to match the autocovariance of the observed data.

2. Approaches which measure distance by using the sampling variability of the actual data. For example, the GMM-based approach of Christiano and Eichenbaum (1992) or Fève and Langot (1994), the indirect approach of Cecchetti et al. (1993) and the frequency domain approach of Diebold, Ohanian and Berkowitz (1998). This last paper is an extension of Watson (1993), proposing a spectral analysis.

3. Approaches which measure distance by using the sampling variability of the simulated data, such as testing calibration, provide a simple way to judge the distance between population moments and the statistics from a simulated macroeconomic model, as in Gregory and Smith

(1991). This method has been used by Soderlind (1994) and Cogley and Nason (1994) to evaluate their DSGE models. All these examples take the driving forces as stochastic and the parameters as given. In this last context, Canova (1994, 1995) and Maffezzoli (2000) allow parameter uncertainty.

4. Approaches which measure distance by using the sampling variability of both actual and simulated data. It is possible to distinguish approaches which allow for variability in the parameters but not in the exogenous processes, such as DeJong et al. (1996, 2000), Geweke (1999) and Schorfheide (2000), or which allow both to vary.

Canova (1994) explains the main differences are between the two approaches of estimation and calibration. Essentially, the estimation approach tries to answer the question: "Given that the model is true, how false is it?", while the calibration approach tries to answer: "Given that the model is false, how true is it?". In the model testing process, an econometrician takes the model seriously as a DGP (data-generating process) and analyzes whether the features of the specification are at variance with the data. A calibrationist takes the opposite view: the model, as a DGP for the data, is false. As the sample size grows, the data-generated by the model will have more variation from the observed data. Statistical models rely on economic theory so loosely that VAR can fail to uncover parameters that are truly structural. This disadvantage may be crucial in policy evaluation exercises, since VAR can exhibit instability across periods when monetary and fiscal policies change. Calibrated DSGE models are typically too stylized to be taken directly to the data and often yield fragile results, when traditional econometric methods are used for estimation (hypothesis testing, forecasting evaluation) (Smets and Wouters (2003) and Ireland (2004)).

In the recent literature, different attempts at hybrid models have been introduced for solving, estimating and forecasting with the DSGE model: Sargent (1989) and Altug (1989) proposed augmenting a DSGE model with measurement error terms following a first order autoregressive, known as the DSGE-AR approach. Ireland (2004) proposed a method that is similar to the DSGE-AR, but imposing no restriction on the measurement errors, assuming that residuals follow a first-order vector autoregression. This hybrid model can be called DSGE-AR à l'Ireland.

A different approach was proposed by DeJong, Ingram and Whiteman (1993) and Ingram and Whiteman (1994) and further developed by Del Negro and Schorfheide (2004). The main idea behind the DSGE-VAR proposed by the latter is the use of the VAR representation as an econometric tool for empirical validation, combining prior information derived from the DSGE model in estimation. The VAR is a powerful econometric tool for empirical validation of macroeconomic models, since it is essentially an easy statistical model to estimate and, once identification restrictions are imposed, it can be used to evaluate the impact of economic shocks on key variables. However, it has several problems.

One of the main problems in finding a statistical representation for the data, by using a Vector Autoregressive model (VAR) is "overfitting" due to the inclusion of too many lags and too many variables, some of which may be insignificant. The problem of "overfitting" results in multicollinearity and the loss of degrees of freedom, leading to inefficient estimates and large out-of-sample forecasting errors. It is possible to overcome this problem by using what have become well-known as "Minnesota" priors (see Doan, Litterman and Sims (1984)). The main motivation for the use of this methodology is the knowledge that more recent values of a variable are more likely to contain useful information about its future movements than older values. This point is realized by introducing the use of dummy observation priors. The basic idea of dummy observation priors for VAR and regression models in general is the addition of "extra data" to the sample of the real data in order to express prior beliefs about the parameters. The curse of dimensionality is overcome by shrinking of the parameter space and imposing prior beliefs on the parameters. The parameter space can be shrunk by imposing a set of restrictions, for instance obtained from a theoretical structural model, directly on the parameters. Alternatively, one can use techniques where prior distributions are imposed on the parameters to be estimated. This procedure is an example of imposing restrictions on the coefficients by assuming they are near zero by using a normal prior (see also Theil and Goldberg (1961) and Litterman (1981)).

The use of "Minnesota" priors has been proposed to shrink the parameters space. The basic principle behind this procedure is that all equations are centered around a random walk with drift. This idea has been modified by Kadiyala and Karlsson (1997) and Sims and Zha (1998).

In Ingram and Whiteman (1994), a RBC model is used to generate a prior for a reduced form VAR, as a development of the "Minnesota" priors procedure. The key element is that the dimension of the observable vector exceeds the dimension of the state vector. This representation is compared to a structural VAR (SVAR) representation and it holds under the condition that the number of the observable variables exceeding those in the state vector, can provide information only for the first lag of a VAR(p). Theodoridis (2007) explains an important assumption: that the remaining (p-1) blocks of autoregressive parameters are normally distributed with mean zero and a covariance matrix that it is proportional to the covariance matrix of the first lag, with the proportion parameter being an inverse function of the number of lags.

In Ingram and Whiteman (1996, 2000), a prior is placed on the parameters of a simple linearized DSGE, which is then compared with a Bayesian VAR (BVAR) in a forecasting exercise. Smets and Wouters (2003) extend this to medium scale New Keynesian models used in Central Bank and policy analysis. This approach has advantages providing the econometrician with more complete stories about which behavioural mechanisms produce forecast error or policy scenarios. There are, however, some disadvantages. It seems that these models more often than not fail to fit as well as models with little or no behavioural structure.

In Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets and Wouters (2007), a DSGE prior is also developed for a VAR. These papers prepare a way to use a DSGE model to generate a prior distribution for a structural time series model that relaxes the tight theoretical restrictions of the DSGE. The theoretical model is thus treated as a mechanism for generating prior distributions for the parameters in the unrestricted VAR.

Following this idea of combining the DSGE model information and the VAR representation, two alternative econometric tools have been introduced: the DSGE-FAVAR (Consolo, Favero and Paccagnini, 2009) and the Augmented VAR-DSGE model (Fernández-de-Córdoba and Torres, 2010). The main idea behind the DSGE-FAVAR is the use of factors to improve the statistical identification in validating models. Consequently, the VAR representation is replaced by a FAVAR model as statistical benchmark. This relies on the same intuition of expanding the size of the VAR representation by using sequences of artificial non-observed variables generated using the DSGE model.

In Section 3, a more detailed description is provided of some of these hybrids models of estimation.

3 Hybrid or Mixture Models

Econometric modelling faces a trade-off between theoretical coherence and empirical fit. DSGE models do not usually have a good fit with classic time-series objects, such as VAR, and a purely theoretical approach is not good enough to find a match between the theory and the data. A good compromise is the use of a Mixture or Hybrid Model. There exist essentially two approaches (see Schorfheide (2010)) to building empirical models that combine the restrictions of a DSGE model with a pure statistical model: the additive hybrid models and hierarchical hybrid models. These hybrid models provide a complete analysis of the law of motion of the data, capturing the dynamic properties of the DSGE model.

3.1 Additive Hybrid Models

Consider the following linear state-space representation of a DSGE model with no time-varying parameters (θ):

$$\begin{aligned} y_t &= \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_s(\theta)s_t \\ s_t &= \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t \ , \end{aligned} \tag{1}$$

where y_t is a vector of $(k \times 1)$ observables, such as aggregate output, inflation, and interest rates. This vector represents the measurement equation. Instead, the vector s_t $(n \times 1)$ contains the unobserved exogenous shock processes and the potentially unobserved endogenous state variables of the model. The model specification is completed by setting the initial state vector s_0 and making distributional assumptions for the vector of innovations ϵ_t ($E[\epsilon_t] = 0, E\left[\epsilon_t \epsilon_t^{'}\right] = I$ and $E[\epsilon_t \epsilon_{t-j}] = 0$ for $j \neq 0$).

The additive hybrid model augments the state-space model (1) with a latent process z_t that bridges the gap between data and theory:

$$\begin{aligned} y_t &= \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_s(\theta)s_t + \Lambda_0 + \Lambda_1 t + \Lambda_z z_t \\ s_t &= \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t \\ z_t &= \Gamma_1 z_{t-1} + \Gamma_\eta \eta_t. \end{aligned} \tag{2}$$

The process z_t is called measurement error, and blames the collection of data rather than the DSGE model construction for the gap between the theory and the data.

There are several examples of additive hybrid models: the DSGE-AR (Sargent (1989), Altug (1989)), the DSGE-AR à l' Ireland (2004), the DSGE-DFM (Boivin and Giannoni (2006) and Kryshko (2010)), the Augmented DSGE for Trends (Canova (2010)) and the DSGE-Noise (de Antonio Liedo (2010)).

3.1.1 The DSGE-AR method

Sargent (1989) and Altug (1989) proposed an approach to solving DSGE models, by augmenting the model with unobservable errors as described in equation (2).

A matrix Γ_1 governs the persistence of the residuals; the covariance matrix, $E_t \eta_t \eta_t' = V$, is uncorrelated. In this specification the ϵ_t 's generate the comovements between the observables, whereas the elements of z_t pick up idionsyncratic dynamics which are not explained by the structural part of the hybrid model. However, if we set Ψ_0, Ψ_1 and Λ_z to zero, the DSGE model components can be used to describe the fluctuations of y_t around a deterministic trend path, ignoring the common trend restrictions of the structural model. For instance, Smets and Wouters (2003) estimated their model using this pattern with a two-step procedure. In the first step, the deterministic trends are extracted from the data; in the second step, the DSGE model is estimated using linear detrended observations.

Sargent (1989) and Altug (1989) assume that the measurement errors are uncorrelated with the data generated by the model, hence the matrices Γ_1 and V are diagonal and the residuals are uncorrelated across variables:

$$\Gamma_1 = \begin{bmatrix} \gamma_y & 0 & 0 \\ 0 & \gamma_c & 0 \\ 0 & 0 & \gamma_l \end{bmatrix}$$
$$V = \begin{bmatrix} v_y^2 & 0 & 0 \\ 0 & v_c^2 & 0 \\ 0 & 0 & v_l^2 \end{bmatrix}.$$

Essentially, this methodology combines the DSGE model with an AR model for the measurement residuals.

3.1.2 The DSGE-AR à l' Ireland

Ireland (2004) proposed a more general framework for measurement errors, allowing the residuals to follow an unconstrained, first-order vector autoregression. This multivariate approach has the main advantage of imposing no restrictions on the cross-correlation of the measurement errors, allowing it to capture all the movements and co-movements in the data not explained by the DSGE model. The matrices Γ_1 and V are given by:

$$\Gamma_1 = \begin{bmatrix} \gamma_y & \gamma_{yc} & \gamma_{yl} \\ \gamma_{cy} & \gamma_c & \gamma_{cl} \\ \gamma_{ly} & \gamma_{lc} & \gamma_l \end{bmatrix}$$
$$V = \begin{bmatrix} v_y^2 & v_{yc} & v_{yl} \\ v_{cy} & v_c^2 & v_{cl} \\ v_{ly} & v_{lc} & v_l^2 \end{bmatrix}.$$

This multivariate approach is more flexible and general in the treatment of measurement errors, but some empirical evidence (such as Férnandez-de-Cordoba and Torres (2010)) shows the forecast performance of the traditional DSGE-AR outperforms the DSGE-AR à l'Ireland.

3.1.3 The DSGE-DFM

Macroeconomists have access to large cross-sections of aggregate variables that include measures of sectorial economic activities and prices as well as numerous financial variables. Hybrid models can also be used to link DSGE models with aggregate variables that are not explicitly modelled. Using these additional variables in the estimation potentially sharpens inference about latent state variables:

$$y_t = \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_s(\theta)s_t + z_{y,t} \quad (3)$$
$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\varepsilon(\theta)\varepsilon_t \quad (4)$$
$$x_t = \Lambda_0 + \Lambda_1 t + \Lambda_s s_t + z_{x,t}, \quad (5)$$

where y_t is the vector of the observable variables that are described by the DSGE model and x_t is a large vector of non-modelled variables.

Since the structure of this model resembles that of a dynamic factor model (DFM), e.g. Sargent and Sims (1977), Geweke (1977), and Stock and Watson (1989), Schorfheide (2010) refers to the system (3) to (5) as an example of a combination of DSGE and DFM. Roughly speaking, the vector of factors is given by the state variables associated with the DGSE model. The processes $z_{y,t}$ and $z_{x,t}$ are uncorrelated across series and model idiosyncratic but potentially serially correlated movements (or measurement errors) in the observables. Moreover, equation (4) links the variables x_t to the DSGE model. This relation generates comovements between the y_t' s and the x_t' s and allows the computation of impulse responses to the structural shocks ϵ_t. The DSGE-DFM was originally proposed by Boivin and Giannoni (2006) and has been recently used by Kryshko (2010) and Schorfheide, Sill, and Kryshko (2010).

3.1.4 The Augmented DSGE for Trends

One of the most discussed problem in using a DSGE model for estimation is its inability to capture the long-run features of the data. Canova (2010) proposes a way to correct these problems using a hybrid model:

$$\begin{aligned} y_t &= \Psi_0(\theta) + \Psi_s(\theta)s_t + \Lambda_0 + \Lambda_z z_t \\ s_t &= \Phi_1(\theta)s_{t-1} + \Phi_\varepsilon(\theta)\varepsilon_t \\ z_t &= \Gamma_1 z_{t-1} + \Gamma_2 \overline{z}_{t-1} + \Gamma_\eta \eta_t \\ \overline{z}_t &= \overline{z}_{t-1} + v_t. \end{aligned} \quad (6)$$

Depending on the restrictions imposed on the variances of η_t and ν_t, the process z_t is integrated of order one or two and can generate a variety of stochastic trend dynamics.

3.1.5 The DSGE-Noise

According to de Antonio Liedo (2010), estimating the DSGE model along the lines of Smets and Wouters (2007) requires two important assumptions. First, the statistical agency constructs the data report by optimally filtering noisy data with a complete knowledge of the data generating process (Sargent (1989)). Moreover, the statistical agency's signal to noise ratio is assumed to be equal to zero. de Antonio Liedo (2010) on the other hand proposes an estimation approach which is consistent with the statistical agency publishing the data with some noise. This approach is similar to Altug (1989), recently followed by Giannone et al. (2006), who discuss how the solution of a DSGE model can be expressed as a particular parameterization of a dynamic factor model.

The state-space representation of a DSGE model is given as follows:

$$\begin{aligned} y_t^* &= \Psi_0(\theta) + \Psi_1(\theta)t + \Psi_s(\theta)s_t + \Lambda_0 + \Lambda_1 t \\ s_t &= \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t, \end{aligned} \tag{7}$$

where y_t^* contains the endogenous variables of the model. de Antonio Liedo (2010) explains that two alternative assumptions are possible in the estimation stage: i) the benchmark assumption and ii) relaxing the benchmark assumption.

The benchmark assumption consists of considering the y_t^* to correspond exactly to the observables published by the statistical agency, y_t. This assumption allows us to estimate the model expecting both data and prior to provide sufficient information to identify the mode and the shape of the posterior distribution of the model parameters.

Relaxing the benchmark assumption, we can assume that the data has been published with noise ($y_t = y_t^* + \xi_t$, where y_t^* is orthogonal to ξ_t), and the specification of the noise component ξ_t requires further restrictions beyond the implied orthogonality with respect to the structural shocks ϵ_t to distinguish structure from noise.

3.2 Hierarchical Hybrid Models

The second class of hybrid models used for estimating the DSGE model is the hierarchical hybrid.

Consider the following modification of the additive hybrid model:

$$\begin{aligned} y_t &= \Lambda_0 + \Lambda_1 t + \Lambda_s s_t \\ s_t &= \Gamma_1 z_{t-1} + \Gamma_\eta \eta_t, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \Lambda_i &= \Psi_i(\theta) + \eta_i^\Psi, \quad i = 0, 1, s \\ \Gamma_i &= \Phi_i(\theta) + \eta_i^\Phi, \quad i = 1, \epsilon. \end{aligned} \tag{9}$$

In this setup, $\Psi_i(\theta)$ and $\Phi_i(\theta)$ are interpreted as restrictions on the unrestricted state-space matrices Λ_i and Γ_i; instead, the disturbances, η_i^Ψ and η_i^Φ can capture deviations from the restriction functions $\Psi_i(\theta)$ and $\Phi_i(\theta)$. This kind of hybrid model is related to Bayesian econometrics, since the stochastic restrictions (9) correspond to a prior distribution of the unrestricted state-space matrices conditional on the DSGE model parameters θ.

In the literature, there are essentially three examples of hierarchical hybrid models: the DSGE-VAR (Del Negro and Schorfheide (2004)), the DSGE-FAVAR (Consolo, Favero and Paccagnini (2009)) and the Augmented (B)VAR (Fernández-de-Córdoba and Torres (2010)).

3.2.1 The DSGE-VAR

The basic idea of the DSGE-VAR (Del Negro and Schorfheide (2004)) approach is to use the DSGE model to build prior distributions for the VAR. The starting point for the estimation is an unrestricted VAR of order p:

$$\mathbf{Y}_t = \mathbf{\Phi}_0 + \mathbf{\Phi}_1 \mathbf{Y}_{t-1} + ... + \mathbf{\Phi}_p \mathbf{Y}_{t-p} + \mathbf{u}_t. \tag{10}$$

In compact format:

$$Y = X\Phi + U \tag{11}$$

Y is a $(T \times n)$ matrix with rows Y_t', X is a $(T \times k)$ matrix ($k = 1 + np, p =$number of lags) with rows $X_t' = [1, Y_{t-1}', ..., Y_{t-p}']$, U is a $(T \times n)$ matrix with rows u_t' and Φ is a $(k \times n) = [\Phi_0, \Phi_1, ..., \Phi_p]'$.

The one-step-ahead forecast errors u_t have a multivariate normal distribution $N(0, \Sigma_u)$ conditional on past observations of Y.

The log-likelihood function of the data is a function of Φ and Σ_u:

$$L(Y|\Phi, \Sigma_u) \propto |\Sigma_u|^{-\frac{T}{2}} \exp\left\{ -\frac{1}{2} tr \left[\Sigma_u^{-1} \left(Y'Y - \Phi'X'Y - Y'X\Phi + \Phi'X'X\Phi \right) \right] \right\}. \tag{12}$$

The prior distribution for the VAR parameters proposed by Del Negro and Schorfheide (2004) is based on the statistical representation of the DSGE model given by a VAR approximation.

Let Γ_{xx}^*, Γ_{yy}^*, Γ_{xy}^* and Γ_{yx}^* be the theoretical second-order moments of the variables Y and X implied by the DSGE model, where:

$$\begin{aligned} \Phi^* (\theta) &= \Gamma_{xx}^{*-1} (\theta) \Gamma_{xy}^* (\theta) \\ \Sigma^* (\theta) &= \Gamma_{yy}^* (\theta) - \Gamma_{yx}^* (\theta) \Gamma_{xx}^{*-1} (\theta) \Gamma_{xy}^* (\theta) . \end{aligned} \tag{13}$$

The moments are the dummy observation priors used in the mixture model. These vectors can be interpreted as the probability limits of the coefficients in a VAR estimated on the artificial observations generated by the DSGE model.

Conditional on the vector of structural parameters in the DSGE model θ, the prior distributions for the VAR parameters $p(\Phi, \Sigma_u|\theta)$ are of the Inverted-Wishart (IW) and Normal forms:

$$\begin{aligned} \Sigma_u |\theta &\sim IW \left((\lambda T \Sigma_u^* (\theta) , \lambda T - k, n \right) \\ \mathbf{\Phi} |\Sigma_u, \theta &\sim N \left(\mathbf{\Phi}^* (\theta) , \Sigma_u \otimes (\lambda T \Gamma_{XX} (\theta))^{-1} \right), \end{aligned} \tag{14}$$

where the parameter λ controls the degree of model misspecification with respect to the VAR: for small values of λ the discrepancy between the VAR and the DSGE-VAR is large and a sizeable distance is generated between the unrestricted VAR and DSGE estimators. Large values of λ correspond to small model misspecification and for $\lambda = \infty$ beliefs about DSGE misspecification degenerate to a point mass at zero. Bayesian estimation could be interpreted as estimation based on a sample in which data are augmented by a hypothetical sample in which observations are generated by the DSGE model, the so-called dummy prior observations (Theil and Goldberg (1961) and Ingram and Whiteman (1994)). Within this framework λ determines the length of the hypothetical sample.

The posterior distributions of the VAR parameters are also of the Inverted-Wishart and Normal forms. Given the prior distribution, posterior distributions are derived by the Bayes theorem:

$$\Sigma_u \left|\theta, Y \right. \sim IW\left((\lambda+1)\, T\hat{\Sigma}_{u,b}(\theta), (\lambda+1)\, T - k, n \right) \tag{15}$$

$$\mathbf{\Phi} \left|\Sigma_u, \theta, Y \right. \sim N\left(\hat{\mathbf{\Phi}}_b(\theta), \Sigma_u \otimes \left[\lambda T \Gamma_{XX}(\theta) + \mathbf{X}'\mathbf{X}\right]^{-1} \right) \tag{16}$$

$$\begin{aligned} \hat{\mathbf{\Phi}}_b(\theta) &= \left(\lambda T \Gamma_{XX}(\theta) + \mathbf{X}'\mathbf{X}\right)^{-1} \left(\lambda T \Gamma_{XY}(\theta) + \mathbf{X}'\mathbf{Y}\right) \\ \hat{\Sigma}_{u,b}(\theta) &= \frac{1}{(\lambda+1)\, T} \left[\left(\lambda T \Gamma_{YY}(\theta) + \mathbf{Y}'\mathbf{Y}\right) - \left(\lambda T \Gamma_{XY}(\theta) + \mathbf{X}'\mathbf{Y}\right) \hat{\mathbf{\Phi}}_b(\theta) \right], \end{aligned}$$

where the matrices $\hat{\mathbf{\Phi}}_b(\theta)$ and $\hat{\Sigma}_{u,b}(\theta)$ have the interpretation of maximum likelihood estimates of the VAR parameters based on the combined sample of actual observations and artificial observations generated by the DSGE. Equations (15) and (16) show that the smaller λ is, the closer the estimates are to the OLS estimates of an unrestricted VAR. Instead, the higher λ is, the closer the VAR estimates will be tilted towards the parameters in the VAR approximation of the DSGE model ($\hat{\mathbf{\Phi}}_b(\theta)$ and $\hat{\Sigma}_{u,b}(\theta)$).

In order to obtain a non-degenerate prior density (14), which is a necessary condition for the existence of a well-defined Inverse-Wishart distribution and for computing meaningful marginal likelihoods, λ has to be greater than λ_{MIN}:

$$\begin{aligned} \lambda_{MIN} &\geq \frac{n+k}{T}; k = 1 + p \times n \\ p &= \text{lags} \\ n &= \text{endogenous variables.} \end{aligned}$$

Hence, the optimal lambda must be greater than or equal to the minimum lambda $\left(\widehat{\lambda} \geq \lambda_{MIN}\right)$.

Essentially, the DSGE-VAR tool allows the econometrician to draw posterior inferences about the DSGE model parameters θ. Del Negro and Schorfheide (2004) explain that the posterior estimate of θ has the interpretation of a minimum-distance estimator, where the discrepancy between the OLS estimates of the unrestricted VAR parameters and the VAR representation of the DSGE model is a sort of distance function. The estimated posterior of parameter vector θ depends on the hyperparameter λ. When $\lambda \to 0$, in the posterior the parameters are not informative, so the DSGE model is of no use in explaining the data.

Unfortunately, the posteriors (16) and (15) do not have a closed form and we need a numerical method to solve the problem. The posterior simulator used by Del Negro and Schorfheide (2004) is the Markov Chain Monte Carlo Method and the algorithm used is the Metropolis-Hastings acceptance method. This procedure generates a Markov Chain from the posterior distribution of θ and this Markov Chain is used for Monte Carlo simulations. See Del Negro and Schorfheide (2004) for more details.

The optimal λ is given by maximizing the log of the marginal data density:

$$\widehat{\lambda} = \underset{\lambda \geqslant \lambda_{MIN}}{\arg\max} \ln p(Y|\lambda)$$

According to the optimal lambda $\left(\widehat{\lambda}\right)$, a corresponding optimal mixture model is chosen. This hybrid model is called DSGE-VAR$\left(\widehat{\lambda}\right)$ and $\widehat{\lambda}$ is the weight of the priors. It can also be interpreted as the restriction of the theoretical model on the actual data.

Unfortunately, Del Negro and Schorfheide (2004) do not propose any statistical tool to verify the power of their procedure. Moreover, Del Negro, Schorfheide, Smets and Wouters (2007b) explain "*...the goal of our article is not to develop a classical test of the hypothesis that the DSGE model restrictions are satisfied; instead, we stress the Bayesian interpretation of the marginal likelihood function of* $p(\lambda|Y)$*, which does not require any cutoff or critical values. ...* ".

One interesting proposal for assessing the DSGE-VAR could be $\widehat{\lambda} - \lambda_{MIN}$. Adolfson, Laséen, Lindé and Villani (2008) explain that we can check that λ_{MIN} is dependent on the model and sample size by reporting the marginal likelihood as a function of $\widehat{\lambda} - \lambda_{MIN}$.

In this paper, I propose a ratio $\frac{\widehat{\lambda}-\lambda_{MIN}}{\lambda_{MIN}}$ to understand if the DSGE-VAR tends to be well approximated by the DSGE model. If the ratio is high, the distance between $\widehat{\lambda}$ and λ_{MIN} is high over λ_{MIN}, and it means the DSGE model can explain the data well. Consequently, this ratio can be interpreted as how much better the DSGE model explains the actual data than the statistical representation (the VAR) in the hybrid DSGE-VAR.

3.2.2 The DSGE-FAVAR

In the DSGE-FAVAR (Consolo, Favero and Paccagnini (2009)), the statistical representation is a Factor Augmented VAR instead of a VAR model. A FAVAR benchmark for the evaluation of the previous DSGE model will take the following specification:

$$\begin{pmatrix} \mathbf{Y}_t \\ \mathbf{F}_t \end{pmatrix} = \begin{bmatrix} \mathbf{\Phi}_{11}(L) & \mathbf{\Phi}_{12}(L) \\ \mathbf{\Phi}_{21}(L) & \mathbf{\Phi}_{22}(L) \end{bmatrix} \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{F}_{t-1} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_t^Z \\ \mathbf{u}_t^F \end{pmatrix} \tag{17}$$

where Y_t are the observable variables included in the DSGE model and F_t is a small vector of unobserved factors extracted from a large data-set of macroeconomic time series, which capture additional economic information relevant to modelling the dynamics of Y_t. The system reduces to the standard VAR used to evaluate DSGE models if $\Phi_{12}(L) = 0$.

Importantly, and differently from Boivin and Giannoni (2006), the FAVAR is not interpreted as the reduced form of a DSGE model at hand. In fact, in this case the restrictions implied by the DSGE model on a general FAVAR are very difficult to trace and model evaluation becomes even more difficult to implement. A very tightly parameterized theory model can have a very highly parameterized reduced form if one is prepared to accept that the relevant theoretical concepts in the model are a combination of many macroeconomic and financial variables. The remaining part of the procedure is implemented in the same way as the DSGE-VAR.

3.2.3 The Augmented (B)VAR

As in the case of the DSGE-FAVAR, the Augmented (B)VAR is a combination of the unrestricted VAR with the DSGE model and is conducted by increasing the size of the VAR representation. In this methodology, x_t is a vector of observable economic variables assumed to drive the dynamics of the economy. The structural approach assumes that DSGE models contain additional economic information, not fully captured by x_t. The additional information is summarized by using a vector of unobserved variables z_t. Fernández-de-Córdoba and Torres (2010) explain that these non-observed variables can be total factor productivity, marginal productivity, or any other information given by the economic model, but they do not belong to the observed variable set.

The joint dynamics of (x_t, z_t) are given by the following transition equation:

$$\begin{bmatrix} x_t \\ z_t \end{bmatrix} = \Phi(L) \begin{bmatrix} x_{t-1} \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^z \end{bmatrix}$$

This system cannot be estimated directly since z_t are non-observed, but z_t can be obtained using the DSGE model to create a new variable Z_t, which is used to expand the size of the VAR. It is possible to construct a VAR with the following specification:

$$\begin{bmatrix} x_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \phi_{11}(L) & \phi_{12}(L) \\ \phi_{21}(L) & \phi_{22}(L) \end{bmatrix} \begin{bmatrix} x_{t-1} \\ Z_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^z \end{bmatrix}$$

where x_t are the macroeconomic data that the DSGE model seeks to explain and Z_t is a vector derived from the DSGE model. If the model specification is correct, the relation between x_t and Z_t should then capture additional economic information relevant to modelling the dynamics of x_t. A standard unrestricted VAR implies that $\phi_{12}(L) = 0$.

4 The Simple DSGE Model

For the empirical evidence and for the forecasting experiments, in this paper the model is a simple DSGE model with forward-looking features. This model is very simple and it is referred to as a benchmark DSGE model in the literature. For instance, Del Negro and Schorfheide (2004) used this model to introduce the DSGE-VAR, and investigate its predictive ability. Wang (2009) proposes the same model in another forecasting exercise without using the VAR representation of the DSGE model.

The economy is made up of four components. First component is a representative household with habit persistent preferences. This household maximizes an additively separable utility function which is separable into consumption, real money balances and hours worked over an infinite lifetime. The household gains utility from consumption relative to the level of technology, real balances of money, and disutility from hours worked. The household earns interest from holding government bonds and earns real profits from the firms. Moreover, the representative household pays lump-sum taxes to the government.

The second component is a perfectly competitive, representative final goods producer which is assumed to use a continuum of intermediate goods as inputs, and the prices for these inputs are given. The producers of these intermediate goods are monopolistic firms which use labour as the only input. The production technology is the same for all the monopolistic firms. Nominal rigidities are introduced in terms of price adjustment costs for the monopolistic firms. Each firm maximizes its profits over an infinite lifetime by choosing its labour input and its price.

The third component is the government which spends in each period a fraction of the total output, which fluctuates exogenously. The government issues bonds and levies lump-sum taxes, which are the main part of its budget constraint.

The last component is the monetary authority, which follows a Taylor rule regarding the inflation target and the output gap.

There are three economic shocks: an exogenous monetary policy shock (in the monetary policy rule), and two autoregressive processes, AR(1), which model government spending and technology shocks.

To solve the model, optimality conditions are derived for the maximization problems. After linearization around the steady-state, the economy is described by the following system of equations:

$$\widetilde{x_t} = E_t[\widetilde{x_{t+1}}] - \frac{1}{\tau}(\widetilde{R_t} - E_t[\widetilde{\pi_{t+1}}]) + (1-\rho_g)\widetilde{g_t} + \rho_Z \frac{1}{\tau}\widetilde{z_t} \quad (18)$$

$$\widetilde{\pi_t} = \beta E_t[\widetilde{\pi_{t+1}}] + \kappa[\widetilde{x_t} - \widetilde{g_t}] \quad (19)$$

$$\widetilde{R_t} = \rho_R \widetilde{R_{t-1}} + (1-\rho_R)(\psi_1 \widetilde{\pi_t} + \psi_2 \widetilde{x_t}) + \epsilon_{R,t} \quad (20)$$

$$\widetilde{g_t} = \rho_g \widetilde{g_{t-1}} + \epsilon_{g,t} \quad (21)$$

$$\widetilde{z_t} = \rho_z \widetilde{z_{t-1}} + \epsilon_{z,t}, \quad (22)$$

where x is the detrended output (divided by the non-stationary technology process), π is the gross inflation rate, and R is the gross nominal interest rate. The tilde denotes percentage deviations from a steady state or, in the case of output, from a trend path. See details in King (2000) and Woodford (2003).

The measurement equations according to the observable variables, quarterly GDP growth ($\Delta \ln x_t$), quarterly inflation rates ($\Delta \ln P_t$) and annualized nominal interest rates ($\ln R_t^a$), are defined as:

$$\begin{aligned} \Delta \ln x_t &= \ln \gamma + \Delta \widetilde{x}_t + \widetilde{z}_t \\ \Delta \ln P_t &= \ln \pi^* + \widetilde{\pi}_t \\ \ln R_t^a &= 4\left[(\ln r^* + \ln \pi^*) + \widetilde{R_t}\right], \end{aligned}$$

where π^* is the steady-state inflation. Let

$$\theta = [\ln \gamma, \ln \pi^*, \ln r^*, \kappa, \tau, \psi_1, \psi_2, \rho_R, \rho_g, \rho_z, \sigma_R, \sigma_g, \sigma_Z]'$$

denote the vector of the model deep parameters to be estimated.

In this case, the statistical representation of the DSGE model is a finite order VAR(2) (See Ravenna (2007), Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007), and Paccagnini (2010)).

5 Empirical Evidence on US Data

In this Section an empirical exercise is implemented with different specifications using US economy data. The benchmark model is the simple DSGE model explained in Section 4. Three quarterly time series from 1980:1 to 2010:4 are implemented in estimation. The VAR, Factor Augmented VAR, Bayesian VAR, DSGE, DSGE-VAR and DSGE-FAVAR are the models used. After a comparison of the log of the marginal data densities, a forecasting exercise is provided using a simple one-step-ahead and a rolling procedure for h-steps-ahead.

5.1 Data

The data for real output growth come from the Bureau of Economic Analysis (Gross Domestic Product-SAAR, Billions Chained 2005$). The data for inflation come from the Bureau of Labor Statistics (CPI-U: All Items, seasonally adjusted, 1982-1984=100). GDP and CPI are taken in first difference logarithmic transformation. The interest rate series are constructed as in Clarida, Galì and Gertler (2000); for each quarter the interest rate is computed as the average federal funds rate (source: Haver Analytics) during the first month of the quarter, including business days only. These three time series represent the three equations of the DSGE model.

In order to construct the FAVAR I proceed to extract factors from a balanced panel of 109 monthly and quarterly macroeconomic and financial time series, following the dataset built by Stock and Watson (2002). The dataset involves several measures of industrial production, interest rates, various price indices, employment and other important macroeconomic and also financial variables. In this set-up, the number of informational time series N is large (larger than time period T) and must be greater than the number of factors and observed variables in the FAVAR system ($k + M \ll N$). In the panel data used, there are some variables in monthly format, which are transformed into a quarterly data using end-of-period observations. All series have been transformed to induce stationarity. The series are taken the level or transformed into logarithms, first or second difference (in level or logarithms) according to series characteristics (see the Appendix for a description of all series and details of the transformations). Following Bernanke, Boivin and Eliasz (2005), I partition the data into two categories of information variables: slow and fast. Slow-moving variables (for example, wages or spending) do not respond contemporaneously to unanticipated changes in monetary policy impulse-response exercises; while fast-moving variables (for example, asset prices and interest rates) do respond contemporaneously to monetary shocks.

I proceed to extract two factors from the slow variables and one factor from the fast variables, called respectively "slow factors" and "fast factor". The methodology implemented to extract the factors is principal components. Stock-Watson (1998) showed that factors can be consistently estimated by the first r principal components of X, even in the presence of moderate changes in the loading matrix Λ. For this result to hold it is important that the estimated number of factors, k, is larger than or equal to the true number, r. Bai and Ng (2000) propose a set of selection criteria to choose k that are generalizations of the BIC and AIC criteria. As they suggest, I use information criteria to determine the number of factors but, as they are not so decisive, I limit the number of factors to three to strike a balance between the variation in the original series explained by the principal components and the difference in the parameterization of the VAR and the FAVAR. It is also worth noting that the factors are not uniquely identified, but this is not a problem in our context because we will not attempt a structural interpretation of the estimated factors.

5.2 Alternative Econometrics Models

To estimate and to evaluate the forecasting performance of the simple DSGE model I consider alternative models: the unrestricted VAR, the Factor Augmented VAR (FAVAR), the Bayesian VAR, the DSGE-VAR and the DSGE-FAVAR.

5.2.1 Classical VAR

The classical unrestricted VAR, as suggested by Sims (1980), has the following compact format:

$$\mathbf{Y}_t = X_t \Phi + U, \tag{23}$$

where $\mathbf{Y_t}$ is a $(T \times n)$ matrix with rows Y_t', X is a $(T \times k)$ matrix ($k = 1 + np, p =$number of lags) with rows $X_t' = [1, Y_{t-1}', ..., Y_{t-p}']$, U is a $(T \times n)$ matrix with rows u_t' and Φ is a $(k \times n) = [\Phi_0, \Phi_1, ..., \Phi_p]'$.

The one step ahead forecast errors u_t have a multivariate normal distribution $N(0, \Sigma_u)$ conditional on past observations of Y.

5.2.2 Factor Augmented VAR

The FAVAR (Bernanke, Boivin and Eliasz (2005)) can be written as follows:

$$\begin{pmatrix} \mathbf{Y}_t \\ \mathbf{F}_t \end{pmatrix} = \begin{bmatrix} \mathbf{\Phi}_{11}(L) & \mathbf{\Phi}_{12}(L) \\ \mathbf{\Phi}_{21}(L) & \mathbf{\Phi}_{22}(L) \end{bmatrix} \begin{pmatrix} \mathbf{Y}_{t-1} \\ \mathbf{F}_{t-1} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_t^Z \\ \mathbf{u}_t^F \end{pmatrix},$$

where we consider the case in which additional economic information, not fully captured by $\mathbf{Y}_t$, is relevant to modelling the dynamics of inflation output growth and the monetary policy rate. This additional information can be summarized in a (small) $(kx1)$ vector of unobserved factors $\mathbf{F}_t$. Factors are extracted as explained above. The FAVAR is implemented by a two-step estimation (Bernanke, Boivin and Eliasz (2005)). We can assume that the informational time series X_t are related to the unobservable factors F_t by the following observation equation:

$$X_t = \Lambda^f \mathbf{F}_t + e_t \tag{24}$$

where $\mathbf{F}_t$ is a $r \times 1$ vector of common factors, Λ^f is a (Nxk) matrix of factor loadings and the $(Nx1)$ vector of error terms e_t is mean zero and is normal and uncorrelated or with a small cross-correlation. In fact, the estimator we employ allows for some cross-correlation in e_t that must vanish as N goes to infinity. Note that this representation also nests models where X_t depends on lagged values of the factors. See Stock and Watson(2002) for details.

In the first step, factors are obtained from the observation equation by imposing the orthogonality restriction $F'F/T = I$. This implies that $\widehat{F} = \sqrt{T}\widehat{G}$, where the $\widehat{G}$ are the eigenvectors corresponding to the K largest eigenvalues of $XX^{'}$, sorted in descending order.

In the second step, we can estimate the FAVAR equation replacing $\mathbf{F}_t$ by $\widehat{\mathbf{F}_t}$.

5.3 Bayesian VAR

The Bayesian VAR, as described in Litterman (1981), Doan et al. (1984), Todd (1984), Litterman (1986), and Spencer (1993) has become a widely popular approach to overcoming overparameterization. One of main problems in using VAR models is that many parameters need to be estimated, although some of them may be insignificant. This overparameterization problem, resulting in multicollinearity and a loss of degrees of freedom, leads to estimates which are inefficient and large out-of-sample forecasting errors. Instead of eliminating longer lags, the BVAR imposes restrictions on these coefficients by assuming that they are more likely to be near zero than the coefficients on shorter lags. Obviously, if there are strong effects from less important variables, the data can counter this assumption. Usually, the restrictions are imposed by specifying normal prior distributions with zero means and small standard deviations for all coefficients, with a decreasing standard deviation as the lags increase. The only exception is the coefficient on a variable's first lag, which has a mean of unity. Litteman (1981) used a diffuse prior for the constant. The means of the prior are popularly called the "Minnesota Priors" due to the development of the idea at the University of Minnesota and the Federal Reserve Bank at Minneapolis.

Formally speaking, these prior means can be written as follows:

$$\Phi_i \sim N(1, \sigma^2_{\Phi_i}) \text{ and } \Phi_j \sim N(0, \sigma^2_{\Phi_j}),$$

where Φ_i denotes the coefficients associated with the lagged dependent variables in each equation of the VAR, while Φ_j represents any other coefficient. The prior variances $\sigma^2_{\Phi_i}$ and $\sigma^2_{\Phi_j}$ specify the uncertainty of the prior means, $\Phi_i = 1$ and $\Phi_j = 0$, respectively. The specification of the standard deviation of the distribution of the prior imposed on variable j in equation i at lag m, for all i, j and m, denoted by $S(i, j, m)$, is specified as follows:

$$S(i, j, m) = [w \times g(m) \times F(i, j)] \frac{\widehat{\sigma}_i}{\widehat{\sigma}_j},$$

where

$$F(i, j) = \begin{cases} 1 \text{ if } i = j \\ k_{ij} \text{ otherwise, } 0 \leq k_{ij} \leq 1 \end{cases}$$

is the tightness of variable j in equation i relative to variable i,and by increasing the interaction, i.e. it is possible for the value of k_{ij} to loosen the prior (Dua and Ray (1995)).

The ratio $\frac{\widehat{\sigma}_i}{\widehat{\sigma}_j}$ consists of estimated standard errors of the univariate autoregression, for variables i and j. This ratio scales the variables to account for differences in the units of measurement, without taking into account the magnitudes of the variables. The term w measures the standard deviation on the first lag, and also indicates the overall tightness. A decrease in the value of w results in a tighter prior. The function $g(m) = m^{-d}$, $d > 0$ is the measurement of the tightness on lag m relative to lag 1, and is assumed to have a harmonic shape with a decay of d, which tightens the prior on increasing lags. Following the standard Minnesota prior settings, I choose the overall tightness (w) to be equal to 0.1, while the lag decay (d) is 0.5 and the interaction parameter (k_{ij}) is set equal to 0.1. This Bayesian setting is very similar to the model used in Liu, Gupta and Kabundi (2009), Gupta and Kabundi (2010) and Das, Gupta and Kabundi (2011).

5.3.1 DSGE model priors

The two hierarchical hybrid models, implemented in this empirical exercise, are the DSGE-VAR and the DSGE-FAVAR. In both models, the prior distribution for the DSGE model parameters are similar to the priors used by Del Negro and Schorfheide (2004).

TABLE 1. Prior Distribution for DSGE Model Parameters[1]

NAME	RANGE	DENSITY	STARTING VALUE	MEAN	SD
$\ln\gamma$	$\mathbb{R}$	*Normal*	0.500	0.500	0.250
$\ln\pi^*$	$\mathbb{R}$	*Normal*	1.000	1.000	0.500
$\ln r^*$	$\mathbb{R}^+$	*Gamma*	0.500	0.500	0.250
κ	$\mathbb{R}^+$	*Gamma*	0.400	0.300	0.150
τ	$\mathbb{R}^+$	*Gamma*	1.000	2.000	0.500
ψ_1	$\mathbb{R}^+$	*Gamma*	2.500	1.500	0.250
ψ_2	$\mathbb{R}^+$	*Gamma*	0.300	0.125	0.100
ρ_R	$[0,1)$	*Beta*	0.400	0.500	0.200
ρ_G	$[0,1)$	*Beta*	0.800	0.800	0.100
ρ_Z	$[0,1)$	*Beta*	0.200	0.300	0.100
σ_R	$\mathbb{R}^+$	*Inv.Gamma*	0.500	0.251	0.139
σ_G	$\mathbb{R}^+$	*Inv.Gamma*	0.500	0.630	0.323
σ_Z	$\mathbb{R}^+$	*Inv.Gamma*	1.000	0.875	0.430

Using these priors, we can build the hybrid models, taking into account that the statistical representation of the DSGE is given by a restricted VAR(2).

5.4 Estimation results: log of Marginal Data Density

The DSGE-VAR and the DSGE-FAVAR are estimated with a different number of lags on the sample spanning from 1980:1 to 2010:4. The parameter λ is chosen from a grid which is unbounded from above. In my empirical exercise, the log of the marginal data density is computed over a discrete interval, $\ln p(Y|\lambda, M)$. The minimum value, $\lambda_{\min} = \frac{n+k}{T}$, is model dependent and is related to the existence of a well-defined Inverse-Wishart distribution. For completeness, it is worth mentioning that $\lambda = 0$ refers to the VAR and the FAVAR model with no prior and it is not possible to compute the marginal likelihood in this particular case. Therefore, we can show the log of marginal data density for any value of λ larger than $\lambda_{\min}$. Importantly, $\lambda_{\min}$ depends on the degrees of freedom in the VAR or FAVAR and therefore, given estimation on the same number of available observations, $\lambda_{\min}$ for a DSGE-FAVAR will always be larger than $\lambda_{\min}$ for a DSGE-VAR[2].

[1] NOTE: The model parameters $\ln\gamma$, $\ln\pi^*$, $\ln r^*$, σ_R, σ_g, and σ_z are scaled by 100 to convert them into percentages. The Inverse Gamma priors are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, where $v = 4$ and s equals 0.2, 0.5, and 0.7, repsectively. Approximately 1.5% of the prior mass lies in the indeterminancy region of the parameter space. The prior is truncated to restrict it to the determinancy region of the DSGE model (SD is standard deviation).

[2] For the DSGE-VAR, the lambda grid is given by $\Lambda = \left\{ \begin{array}{c} 0,\ 0.06,\ 0.09,\ 0.12,\ 0.14,\ 0.15,\ 0.20,\ 0.25,\ 0.30,\ 0{,}35,\ 0.40, \\ 0.45,\ 0.50,\ 0.55,\ 0.60,\ 0.65,\ 0.70,\ 0.75,\ 0.80,\ 0.85,\ 0.9,\ 1,\ 5,\ 10 \end{array} \right\}$.
For the DSGE-FAVAR, the lambda grid is given by $\Lambda = \left\{ \begin{array}{c} 0,\ 0.09,\ 0.14,\ 0.19,\ 0.24,\ 0.25,\ 0.30,\ 0{,}35,\ 0.40, \\ 0.45,\ 0.50,\ 0.55,\ 0.60,\ 0.65,\ 0.70,\ 0.75,\ 0.80,\ 0.85,\ 0.9,\ 1,\ 5,\ 10 \end{array} \right\}$.
In both lambda intervals, I consider the λ_{MIN} across lags from 1 to 4.

	λ_{MIN}	$\widehat{\lambda}$	$\widehat{\lambda}-\lambda_{MIN}$	$\frac{\widehat{\lambda}-\lambda_{MIN}}{\lambda_{MIN}}$	$\ln p(Y\|\widehat{\lambda},M)$	Bayes Factor vs M_1
DSGEVAR1	0.06	0.09	0.03	0.5	-405.6898	$exp\,[9.5752]$
DSGEVAR2 (M_1)	0.09	0.7	0.61	6.78	-396.1146	1
DSGEVAR3	0.12	0.3	0.18	1.5	-405.217	$exp\,[9.1024]$
DSGEVAR4	0.14	0.7	0.56	4	-375.6336	$exp\,[-20.481]$

TABLE 2: the DSGE-VAR

Table 2 shows the main results related to the DSGE-VAR implemented using a different number of lags (from 1 up to 4). Each minimum λ (λ_{MIN}) is given by the features of the model (number of observations, number of endogenous variables, number of lags), and the optimal lambda ($\widehat{\lambda}$) is calculated using the Markov Chain Monte Carlo with Metropolis Hastings acceptance method (with 10,000 replications). $\ln p(Y|M)$ is the log marginal data density for the DSGE model specifications computed based on Geweke' s (1999) modified harmonic mean estimator. The Bayes factor (ratio of posterior odds to prior odds) (An and Schorfheide (2007))) helps us to understand the improvement of the log marginal data density of a specific model compared to a benchmark model (M), which in this case is the DSGE-VAR (2), since the statistical representation of the DSGE model is given by a VAR(2). According to Table 2, the difference $\widehat{\lambda}-\lambda_{MIN}$ is the greatest in the case of a DSGE-VAR(2), and hence its corresponding ratio $\frac{\widehat{\lambda}-\lambda_{MIN}}{\lambda_{MIN}}$ is the greatest too. Looking at the log of the marginal data densities, we notice that the DSGE-VAR(4) model has the minimum value and the Bayes factor evidences a great difference between the DSGE-VAR(2) (the benchmark model) and the DSGE-VAR(4) ($exp\,[-20.481]$) in favour of the DSGE-VAR with four lags.

The same analysis is repeated for the DSGE-FAVAR with lags from 1 to 4.

	λ_{MIN}	$\widehat{\lambda}$	$\widehat{\lambda}-\lambda_{MIN}$	$\frac{\widehat{\lambda}-\lambda_{MIN}}{\lambda_{MIN}}$	$\ln p(Y\|\widehat{\lambda},M)$	Bayes Factor vs M_2
DSGEFAVAR1	0.09	0.14	0.05	0.56	-366.2225	$exp\,[35.3445]$
DSGEFAVAR2 (M_2)	0.14	0.19	0.05	0.36	-330.878	1
DSGEFAVAR3	0.19	0.45	0.26	1.37	-328.846	$exp\,[-2.032]$
DSGEFAVAR4	0.24	0.5	0.26	1.08	-340.3626	$exp\,[9.4846]$

TABLE 3: the DSGE-FAVAR

As Table 3 shows, the DSGE-FAVAR(3) and DSGE-FAVAR(4) exhibit the greatest difference (0.26), but the ratio $\frac{\widehat{\lambda}-\lambda_{MIN}}{\lambda_{MIN}}$ is greater in the case of the DSGE-FAVAR(3). Looking at the log of the marginal data densities, we notice that the DSGE-FAVAR(3) model has the minimum value and the Bayes factor evidences a small difference between the DSGE-FAVAR(2) (the benchmark model) and the DSGE-FAVAR(3) ($exp\,[-2.032]$) in favour of the DSGE-VAR with three lags.

The log marginal data density has been calculated for the DSGE model estimated and for the VAR and FAVAR using the DSGE-VAR or DSGE-FAVAR model[3]. The minimum lambda is used to compute a well-defined marginal data density ($\lambda_{MIN}=0.09$ for the DSGE-VAR(2) and $\lambda_{MIN}=0.14$ for the DSGE-FAVAR(2,2)). The Bayes factor is calculated versus the VAR(2), which is the benchmark model (M_3) since the DGP of the DSGE is a VAR(2).

[3] The DSGE-FAVAR nest the DSGE, VAR and FAVAR models, so using the same Matlab codes I calculated the log marginal data density for each model, changing the settings opportunetly.

Table 4 shows how the Bayes factor suggests that the benchmark model (VAR(2)) is not the best possible model to estimate the DSGE model. The two hybrid models, the DSGE-VAR and the DSGE-FAVAR, give better performances in terms of log marginal data density, as does the VAR(4) (in this case, the Bayes factor is $\exp[-122]$ in favour of the VAR(4)) and the FAVAR models.

	$lnp(Y\|M)$	Bayes Factor vs M_3
DSGE	-712.398	$\exp[297.777]$
DSGEVAR(2)	-396.115	$\exp[-18.506]$
DSGEFAVAR(2,2)	-330.878	$\exp[-83.743]$
VAR(1)	-431.448	$\exp[16.827]$
VAR(2) (M_3)	-414.621	1
VAR(3)	-406.951	$\exp[-7.67]$
VAR(4)	-292.621	$\exp[-122]$
FAVAR(1)	-412.459	$\exp[-2.162]$
FAVAR(2)	-404.483	$\exp[-10.138]$
FAVAR(3)	-398.783	$\exp[-15.838]$
FAVAR(4)	-379.486	$\exp[-35.135]$

TABLE 4: Log of the Marginal Data Density and Bayes Factor

5.5 Forecasting Performance

Two different forecasting exercises are implemented: a one-step-ahead exercise, changing the estimation sample and the forecasting sample and an h-steps-ahead with a rolling estimation sample. A group of recent papers compare the forecasting performance of a DSGE model and a statistical representation of DSGE models (a VAR, a Factor model, an hybrid model), such as Smets and Wouters (2004), Ireland (2004), Del Negro and Schorfheide (2004), Del Negro et al. (2007), Adolfson et al. (2007), Christoffel et al. (2007), Rubaszek and Skrzypczynski (2008), Ghent (2009), Kolosa et al. (2009), Consolo et al. (2009), Wang (2009), Fernandez-de-Cordoba and Torres (2010), among others. A general result is that the use of a DSGE model or a hybrid improves forecasting performance compared with VAR methods.

In both the two empirical exercises the forecasting performance is evaluated using the root-mean square error (RMSE) which measures the absolute size of errors. The formula is as follows:

$$\begin{aligned} RMSE^y &= \sqrt{\frac{1}{H}\sum_{h=1}^{H}\left(y_{t+h}-\hat{y}_{t+h|t}\right)^2}, \\ y &= \Delta\ln x_t, \Delta\ln P_t, \ln R_t, \end{aligned} \quad (25)$$

where $\hat{y}_{t+h|t}$ is the mean forecast computed as the average across draws, H is the forecasting horizon depending on the exercise, and t depends on the forecasting sample.

5.5.1 The one-step-ahead evidence

I compare the forecasting performance of the VAR, FAVAR, BVAR, DSGE, DSGE-VAR and DSGE-FAVAR models considering three different estimation and forecasting periods. The subsamples are:

1. estimation sample 1980:1 - 2007:4 and forecasting sample 2008:1-2010:4
2. estimation sample 1980:1 - 2005:4 and forecasting sample 2006:1-2010:4
3. estimation sample 1980:1 - 2004:4 and forecasting sample 2005:1-2010:4

In each subsample, I do not consider the period related to the Great Contraction, left to the forecasting sample. For the hybrid models, the DSGE-VAR and the DSGE-FAVAR are evaluated using the $\widehat{\lambda}$ reported in Table 1 and Table 2.

[Table 5 about here]

In Table 5, we see the results for the first subsample (the first three columns). The RMSE suggests that the DSGE-VAR provides the best forecasting performance for CPI and FFR. Instead, the FAVAR has the lowest RMSE for real GDP. However, the results for the DSGE-VAR and DSGE-FAVAR are very similar and only in the case of real GDP do the hybrid models not produce the most accurate forecast. For the second sample (the next three columns), the FAVAR model has the most accurate forecasts for real GDP and for the interest rate. Instead, the DSGE-VAR provides the best forecasting performance for the CPI variable. For the third sample (the last three columns), the FAVAR outperforms the alternative models with the exception of the real GDP, where the BVAR gives the most accurate forecast.

5.5.2 The h-steps-ahead evidence

The second forecasting exercise is performed for horizons ranging from one to five quarters ahead. The forecast accuracy evaluation period is 2001:1- 2009:4 and the estimation sample is initially from 1980:1 to 2000:4. Since I use a range of lags from one up to four quarters, initial four of the sample - 1980:1 to 1980:4 - are used to feed the lags. The forecasting is dynamic and the models are re-estimated each quarter over the forecast horizon to update the estimate of the coefficients, before producing the five-quarters-ahead forecasts. The iterative estimation and five-steps-ahead forecast procedure is carried out for 32 quarters, with the first forecast beginning in 2001:1. This experiment produced a total of 32 one-quarter-ahead forecasts, 32 two-quarters-ahead forecasts, and so on, up to 32 five-steps ahead forecasts. The estimation periods update adding one more observation each time. In the case of the DSGE-VAR and the DSGE-FAVAR, the forecasting is produced considering the $\widehat{\lambda}$ chosen by the procedure for each estimation[4].

[Tables 6, 7, and 8 about here]

The RMSEs for 32 quarters 1 to quarter 5 forecasts for the period 2001:1 to 2009:4 are then calculated and compared for each variables. Tables 6, 7 and 8 display the results. For real GDP, the best forecasting performance is produced by BVAR for one and two steps ahead. Instead, for more than three steps ahead, the DSGE model and the DSGE-FAVAR have the most accurate forecasts.

[4] The new lambda grid for the DSGE-VAR is : $\Lambda = \{0\ 0.25\ 0.30\ 0.35\ 0.40\ 0.45\ 0.50\ 0.55\ 0.60\ 0.65\ 0.70\ 0.75\ 0.80\ 0.85\ 0.90\ 1\ 5\ 10\}$. For the DSGE-FAVAR, it is:
$\Lambda = \{0\ 0.40\ 0.45\ 0.50\ 0.55\ 0.60\ 0.65\ 0.70\ 0.75\ 0.80\ 0.85\ 0.90\ 1\ 5\ 10\}$

6 Concluding Remarks

In the recent literature, DSGE models have become widely popular. Policymakers are particularly interested in DSGE models as tools for estimation and evaluation. Recently, a group of models have been introduced combining statistical representation and features of DSGE models: the hybrid models (the DSGE-AR (Sargent (1989), Altug (1989)), the DSGE-AR à l' Ireland (2004), the DSGE-DFM (Boivin and Giannoni (2006) and Kryshko (2010)), the Augmented DSGE for Trends (Canova (2010)), the DSGE-Noise (de Antonio Liedo (2010)), the DSGE-VAR (Del Negro and Schorfheide (2004)), the DSGE-FAVAR (Consolo, Favero and Paccagnini (2009)), and the Augmented (B)VAR (Fernández-de-Córdoba and Torres (2010))). There are essentially two sets of hybrid models: additive, with the state space representation as statistical support, and hierarchical, with a VAR representation as support for the DSGE model.

Two forecasting exercises have been implemented on US quarterly economy data: the first one considering a simple forecasting one-step-ahead strategy with different samples, and the second a h-steps ahead evaluation with a rolling forecasting estimation. In the first exercise, the last part of the sample from 2007 onwards, which is the Great Contraction period, was not taken into account. The main results show that the hybrid models, such as the DSGE-VAR and the DSGE-FAVAR give the most accurate forecasts for smaller forecasting samples. Instead, a FAVAR model outperforms the other models when we consider a long forecasting sample. The second forecasting experiment uses a rolling sample with the Great Contraction period included. The best forecasting performance is produced by VAR, BVAR and FAVAR, with the exception of real GDP where for h-steps ahead larger than 2 the DSGE-FAVAR outperforms the alternative models.

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Appendix : The data used to extract factors

I describe data used to extract factors in the format adopted by Stock and Watson (2002): series number, long description, short description, transformation code and slow code (The transformation codes are: 1 - no transformation; 2 - first difference; 3 - second difference; 4 - logarithm; 5 - first difference of logarithm and 6 - second difference of logarithm) (The slow codes are: 0 - fast and 1 - slow). The source of the data is the Federal Reserve Economic Data - Federal Reserve Bank of Saint Louis (http://research.stlouisfed.org/fred2/).

Table A1

Date	Long Description	Tcode	SlowCode
PAYEMS	Total Nonfarm Payrolls: All Employees	5	1
DSPIC96	Real Disposable Personal Income	5	1
NAPM	ISM Manufacturing: PMI Composite Index	1	1
UNRATE	Civilian Unemployment Rate	1	1
INDPRO	Industrial Production Index (Index 2007=100)	5	1
PCEPI	Personal Consumption Expenditures: Chain-type Price Index (Index 2005=100)	5	1
PPIACO	Producer Price Index: All Commodities (Index 1982=100)	5	1
FEDFUNDS	Effective Federal Funds Rate	1	0
IPDCONGD	Industrial Production: Durable Consumer Goods (Index 2007=100)	5	1
IPBUSEQ	Industrial Production: Business Equipment (Index 2007=100)	5	1
IPMAT	Industrial Production: Materials (Index 2007=100)	5	1
IPCONGD	Industrial Production: Consumer Goods (Index 2007=100)	5	1
IPNCONGD	Industrial Production: Nondurable Consumer Goods (Index 2007=100)	5	1
IPFINAL	Industrial Production: Final Products (Market Group) (Index 2007=100)	5	1
UNEMPLOY	Unemployed	5	1
EMRATIO	Civilian Employment-Population Ratio (%)	1	1
CE16OV	Civilian Employment	5	1
CLF16OV	Civilian Labor Force	5	1
CIVPART	Civilian Participation Rate (%)	1	1
UEMP27OV	Civilians Unemployed for 27 Weeks and Over	5	1
UEMPLT5	Civilians Unemployed - Less Than 5 Weeks	5	1
UEMP15OV	Civilians Unemployed - 15 Weeks & Over	5	1
UEMP15T26	Civilians Unemployed for 15-26 Weeks	5	1
UEMP5TO14	Civilians Unemployed for 5-14 Weeks	5	1
MANEMP	Employees on Nonfarm Payrolls: Manufacturing	5	1
USPRIV	All Employees: Total Private Industries	5	1
USCONS	All Employees: Construction	5	1
USFIRE	All Employees: Financial Activities	5	1
USTRADE	All Employees: Retail Trade	5	1
DMANEMP	All Employees: Durable Goods Manufacturing	5	1
USGOOD	All Employees: Goods-Producing Industries	5	1
USEHS	All Employees: Education & Health Services	5	1
USLAH	All Employees: Leisure & Hospitality	5	1
SRVPRD	All Employees: Service-Providing Industries	5	1
USINFO	All Employees: Information Services	5	1
USPBS	All Employees: Professional & Business Services	5	1
USTPU	All Employees: Trade, Transportation & Utilities	5	1
NDMANEMP	All Employees: Nondurable Goods Manufacturing	5	1
USMINE	All Employees: Natural Resources & Mining	5	1
USWTRADE	All Employees: Wholesale Trade	5	1
USSERV	All Employees: Other Services	5	1
AHEMAN	Average Hourly Earnings: Manufacturing	5	1
AHECONS	Average Hourly Earnings: Construction (NSA)	5	1
PPIIDC	Producer Price Index: Industrial Commodities (NSA)	5	1

Table A1 (continued)

PPIFGS	Producer Price Index: Finished Goods (Index 1982=100)	5	1
PPICPE	Producer Price Index: Finished Goods: Capital Equipment (Index 1982=100)	5	1
PPICRM	Producer Price Index: Crude Materials for Further Processing (Index 1982=100)	5	1
PPIITM	Producer Price Index: Intermediate Materials: Supplies & Components (Index 1982=100)	5	1
PPIENG	Producer Price Index: Fuels & Related Products & Power (Index 1982=100)	5	1
PPIFCG	Producer Price Index: Finished Consumer Goods (Index 1982=100)	5	1
PFCGEF	Producer Price Index: Finished Consumer Goods Excluding Foods (Index 1982=100)	5	1
CPIAUCSL	Consumer Price Index for All Urban Consumers: All Items (Index 1982=100)	5	1
CPIAUCNS	Consumer Price Index for All Urban Consumers: All Items (Index 1982-84=100)	5	1
CPILFESL	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy (Index 1982-84=100)	5	1
CPILFENS	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy (NSA Index 1982=100)	5	1
CPIUFDNS	Consumer Price Index for All Urban Consumers: Food (NSA Index 1982=100)	5	1
CPIENGNS	Consumer Price Index for All Urban Consumers: Energy (NSA Index 1982=100)	5	1
CPIENGSL	Consumer Price Index for All Urban Consumers: Energy (Index 1982-1984=100)	5	1
CPILEGSL	Consumer Price Index for All Urban Consumers: All Items Less Energy (Index 1982-1984=100)	5	1
CPIMEDSL	Consumer Price Index for All Urban Consumers: Medical Care (Index 1982-1984=100)	5	1
PPIFCF	Producer Price Index: Finished Consumer Foods (Index 1982=100)	5	1
AAA	Moody's Seasoned Aaa Corporate Bond Yield	1	0
BAA	Moody's Seasoned Baa Corporate Bond Yield	1	0
M2SL	M2 Money Stock	6	0
M2NS	M2 Money Stock (NSA)	6	0
M1NS	M1 Money Stock (NSA)	6	0
M3SL	M3 Money Stock (DISCONTINUED SERIES)	6	0
GS5	5-Year Treasury Constant Maturity Rate	1	0
GS10	10-Year Treasury Constant Maturity Rate	1	0
GS1	1-Year Treasury Constant Maturity Rate	1	0
GS3	3-Year Treasury Constant Maturity Rate	1	0
TB3MS	3-Month Treasury Bill: Secondary Market Rate	1	0
TB6MS	6-Month Treasury Bill: Secondary Market Rate	1	0
HOUST	Housing Starts: Total: New Privately Owned Housing Units Started	5	0
PERMIT	New Private Housing Units Authorized by Building Permits	5	0
HOUSTMW	Housing Starts in Midwest Census Region	5	0
HOUSTW	Housing Starts in West Census Region	5	0
HOUSTNE	Housing Starts in Northeast Census Region	5	0
HOUSTS	Housing Starts in South Census Region	5	0
PERMITS	New Private Housing Units Authorized by Building Permits - South	5	0
PERMITMW	New Private Housing Units Authorized by Building Permits - Midwest	5	0
PERMITW	New Private Housing Units Authorized by Building Permits - West	5	0
PERMITNE	New Private Housing Units Authorized by Building Permits - Northeast	5	0
PDI	Personal Dividend Income	5	0
SPREAD1	3mo-FYFF	1	0
SPREAD2	6mo-FYFF	1	0
SPREAD3	1yr-FYFF	1	0
SPREAD4	2yr-FYFF	1	0
SPREAD5	3yr-FYFF	1	0
SPREAD6	5yr-FYFF	1	0
SPREAD7	7yr-FYFF	1	0
SPREAD8	10yr-FYFF	1	0
PCECC96	Real Personal Consumption Expenditures (Billions of Chained 2005 Dollars)	5	1
UNLPNBS	Nonfarm Business Sector: Unit Nonlabor Payments (Index 2005=100)	5	1
IPDNBS	Nonfarm Business Sector: Implicit Price Deflator (Index 2005=100)	5	1
OUTNFB	Nonfarm Business Sector: Output (Index 2005=100)	5	1
HOANBS	Nonfarm Business Sector: Hours of All Persons (Index 2005=100)	5	1
COMPNFB	Nonfarm Business Sector: Compensation Per Hour (Index 2005=100)	5	1
ULCNFB	Nonfarm Business Sector: Unit Labor Cost (Index 2005=100)	5	1
COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour (Index 2005=100)	5	1
OPHNFB	Nonfarm Business Sector: Output Per Hour of All Persons (Index 2005=100)	5	1
OPHPBS	Business Sector: Output Per Hour of All Persons (Index 2005=100)	5	1
ULCBS	Business Sector: Unit Labor Cost (Index 2005=100)	5	1
RCPHBS	Business Sector: Real Compensation Per Hour (Index 2005=100)	5	1
HCOMPBS	Business Sector: Compensation Per Hour (Index 2005=100)	5	1
OUTBS	Business Sector: Output (Ineex 2005=100)	5	1
HOABS	Business Sector: Hours of All Persons (Index 2005=100)	5	1
IPDBS	Business Sector: Implicit Price Deflator (Index 2005=100)	5	1
CP	Corporate Profits After Tax	5	0
SP500	S&P 500 Index	5	0

Appendix : Forecasting Tables

This Section the tables related to the forecasting exercises: Table 5 for the one-step-ahead forecasting; Tables 6, 7 and 8 for the h-steps-ahead forecasting. The measure to compare the forecasting performance is the Root Mean Square Error (RMSE).

I concentrate on the Root Mean Square Error of the forecasting errors from the various models, computed as follows:

$$\begin{aligned} RMSE^{y} &= \sqrt{\frac{1}{H}\sum_{h=1}^{H}\left(y_{t+h}-\hat{y}_{t+h|t}\right)^{2}} \\ y &= \Delta \ln x_t, \Delta \ln P_t, \ln R_t, \end{aligned} \tag{26}$$

where $\hat{y}_{t+h|t}$ is the mean forecast computed as the average across draws, H is the forecasting horizon and depending on the exercise, and t depends on the forecasting sample.

	RMSE of one-step-ahead								
	Sample 1			Sample 2			Sample 3		
	real GDP	CPI	FFR	real GDP	CPI	FFR	real GDP	CPI	FFR
VAR (1)	1.17	1.29	0.94	1.04	1.29	0.47	0.43	0.75	0.42
VAR (2)	1.21	1.29	0.93	1.06	1.30	0.47	0.39	0.71	0.35
VAR (3)	1.1	1.28	0.93	1.04	1.30	0.43	0.36	0.71	0.53
VAR (4)	1.11	1.28	0.99	1.01	1.31	0.41	**0.35**	0.71	0.52
BVAR (1)	1.16	1.36	0.95	1.05	1.31	0.45	0.43	0.72	0.44
BVAR (2)	1.16	1.31	0.98	1.08	1.31	0.45	0.44	0.71	0.30
BVAR (3)	1.16	1.33	0.96	1.08	1.31	0.44	0.46	0.71	0.42
BVAR (4)	1.16	1.32	0.95	1.03	1.30	0.44	**0.35**	0.71	0.45
FAVAR (1,1)	**0.78**	1.27	0.83	**0.83**	1.22	1.39	1.14	0.79	0.34
FAVAR (2,2)	1.78	1.79	1.28	0.99	1.24	**0.32**	0.69	0.71	0.18
FAVAR (3,3)	1.66	1.89	0.79	1.01	1.12	0.40	0.76	0.69	0.29
FAVAR (4,4)	1.36	2.05	0.74	0.85	1.15	0.41	0.70	**0.67**	**0.20**
DSGEVAR(1)	1.07	1.14	0.50	0.99	**1.02**	0.46	0.94	1.03	0.42
DSGEVAR(2)	1.05	1.14	0.50	0.98	**1.02**	0.45	0.94	1.02	0.42
DSGEVAR(3)	1.04	1.14	**0.49**	0.97	**1.02**	0.44	0.92	1.02	0.41
DSGEVAR(4)	1.03	**1.13**	0.50	0.97	**1.02**	0.45	0.89	1.03	0.41
DSGEFAVAR(1,1)	1.07	1.16	0.62	0.98	1.09	0.69	0.93	1.10	0.71
DSGEFAVAR(2,2)	1.06	1.17	0.56	1.01	1.06	0.56	0.97	1.05	0.51
DSGEFAVAR(3,3)	1.06	1.16	0.53	0.98	1.02	0.49	0.92	1.03	0.45
DSGEFAVAR(4,4)	1.05	1.15	0.53	0.99	1.04	0.49	0.91	1.04	0.45
DSGE	1.20	1.33	1.26	1.04	1.07	0.97	0.48	0.71	0.30

TABLE 5: Root Mean Square Error (RMSE) for alternative models for one-step-ahead for each variable, real GDP, Consumer Price Index (CPI) and Federal Funds Rate (FFR), considering three different samples (Sample 1: estimation sample 1980:1 - 2007:4 and forecasting sample 2008:1-2010:4; Sample 2: estimation sample 1980:1 - 2005:4 and forecasting sample 2006:1-2010:4; Sample 3: estimation sample 1980:1 - 2004:4 and forecasting sample 2005:1-2010:4)

	PERIODS AHEAD				
	1	2	3	4	5
VAR (1)	0.88	0.90	0.92	0.91	0.91
VAR (2)	0.88	0.91	0.92	0.92	0.95
VAR (3)	0.88	0.90	0.90	0.90	0.93
VAR (4)	0.88	0.90	0.90	0.91	0.94
BVAR (1)	**0.76**	**0.79**	0.84	0.81	0.83
BVAR (2)	0.80	0.83	0.89	0.86	0.89
BVAR (3)	0.78	0.81	0.86	0.84	0.88
BVAR (4)	**0.76**	0.80	0.85	0.81	0.85
FAVAR (1,1)	1.47	1.39	1.38	1.36	1.36
FAVAR (2,2)	2.70	1.81	1.61	1.56	1.51
FAVAR (3,3)	2.11	1.52	1.35	1.37	1.55
FAVAR (4,4)	2.05	1.46	1.38	1.41	1.86
FAVAR (1,1)	1.54	1.51	1.47	1.45	1.43
FAVAR (1,2)	1	1.09	1.27	1.15	1.06
FAVAR (1,3)	1.14	1.07	1.19	1.05	1.10
FAVAR (1,4)	1.28	1.48	1.17	1.26	1.21
DSGEVAR(1)	0.81	0.85	0.99	0.88	1.01
DSGEVAR(2)	0.82	0.99	0.99	0.96	0.96
DSGEVAR(3)	0.89	0.90	0.99	0.91	0.95
DSGEVAR(4)	0.86	0.88	0.99	0.90	0.96
DSGEFAVAR(1,1)	0.84	0.88	**0.78**	**0.78**	**0.81**
DSGEFAVAR(1,2)	1.28	0.98	1.29	1.16	0.89
DSGEFAVAR(1,3)	1.49	1.07	0.86	1.09	0.95
DSGEFAVAR(1,4)	1.32	0.80	1.08	1.15	0.79
DSGE	0.80	0.98	0.84	0.89	0.98

TABLE 6: Root Mean Square Error (RMSE) for real GDP for h-steps-ahead (from 1 to 5) in a rolling forecasting exercise (from 2001:1 to 2009:4)

	PERIODS AHEAD				
	1	2	3	4	5
VAR (1)	0.98	0.97	0.97	0.97	0.96
VAR (2)	0.98	0.97	0.97	0.97	0.96
VAR (3)	0.98	0.98	0.98	0.97	0.96
VAR (4)	0.97	0.97	0.98	0.97	0.96
BVAR (1)	0.96	0.97	0.97	0.94	**0.93**
BVAR (2)	0.96	0.97	0.97	0.94	0.94
BVAR (3)	0.95	**0.96**	0.97	0.94	0.94
BVAR (4)	0.95	**0.96**	**0.96**	**0.93**	0.94
FAVAR (1,1)	1.29	1.31	1.27	1.23	1.21
FAVAR (2,2)	0.68	1.06	1.03	1.03	1.03
FAVAR (3,3)	**0.60**	0.98	0.99	0.99	1
FAVAR (4,4)	0.78	1.02	1.02	1.06	1.10
FAVAR (1,1)	2.05	2.02	2.21	1.95	1.94
FAVAR (1,2)	1.47	1.50	1.71	1.53	1.36
FAVAR (1,3)	1.71	1.60	1.68	1.81	1.72
FAVAR (1,4)	2.27	2.25	2.11	2.03	2.11
DSGEVAR(1)	0.99	0.99	0.99	0.99	0.97
DSGEVAR(2)	0.99	0.98	0.99	0.98	0.97
DSGEVAR(3)	0.99	0.99	0.99	0.96	0.97
DSGEVAR(4)	0.98	0.99	0.98	0.97	0.97
DSGEFAVAR(1,1)	1.87	1.66	1.62	1.52	1.11
DSGEFAVAR(1,2)	2.17	1.64	1.97	1.82	1.27
DSGEFAVAR(1,3)	2.52	1.90	1.47	1.80	1.72
DSGEFAVAR(1,4)	2.59	1.41	1.73	1.83	1.30
DSGE	0.98	0.98	0.98	0.97	0.97

TABLE 7: Root Mean Square Error (RMSE) for CPI for h-steps-ahead (from 1 to 5) in a rolling forecasting exercise (from 2001:1 to 2009:4)

	PERIODS AHEAD				
	1	2	3	4	5
VAR (1)	0.48	0.46	0.45	0.43	0.42
VAR (2)	0.47	0.43	0.42	0.40	0.40
VAR (3)	0.47	0.44	0.43	0.41	0.40
VAR (4)	**0.46**	**0.42**	**0.41**	**0.39**	**0.37**
BVAR (1)	1.01	0.96	0.89	0.86	0.79
BVAR (2)	1	0.95	0.85	0.74	0.59
BVAR (3)	0.92	0.87	0.81	0.65	0.50
BVAR (4)	0.93	0.84	0.73	0.50	0.70
FAVAR (1,1)	1.07	1.12	1.08	1.06	1.01
FAVAR (2,2)	0.74	0.75	0.68	0.66	0.60
FAVAR (3,3)	0.62	0.63	0.56	0.54	0.50
FAVAR (4,4)	0.63	0.62	0.57	0.55	0.51
FAVAR (1,1)	0.96	0.93	1.06	0.93	0.93
FAVAR (1,2)	0.99	0.97	1.10	0.96	0.97
FAVAR (1,3)	0.97	0.86	0.95	1.01	0.87
FAVAR (1,4)	0.99	0.94	0.91	0.90	0.86
DSGEVAR(1)	0.51	0.52	0.51	0.51	0.50
DSGEVAR(2)	0.51	0.52	0.52	0.48	0.49
DSGEVAR(3)	0.51	0.52	0.52	0.49	0.48
DSGEVAR(4)	0.49	0.49	0.49	0.49	0.5
DSGEFAVAR(1,1)	0.81	0.71	0.68	0.64	0.41
DSGEFAVAR(1,2)	0.97	0.73	0.92	0.82	0.49
DSGEFAVAR(1,3)	1.13	0.91	0.75	0.87	0.65
DSGEFAVAR(1,4)	0.17	0.70	0.84	0.89	0.62
DSGE	1.02	1.04	1.03	1.02	1

TABLE 8: Root Mean Square Error (RMSE) for FFR for h-steps-ahead (from 1 to 5) in a rolling forecasting exercise (from 2001:1 to 2009:4)