Selection, Market Size and International Integration: Do Vertical Linkages Play a Role?*

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Comments are welcome.

Abstract

We analyze how increases in the market size and in the level of international integration interact with the process of selection among firms with heterogeneous productivity levels when they are interconnected by vertical linkages. We show that larger economies do not always exhibit higher productivity levels and higher welfare levels. Specifically, when vertical linkages among firms are allowed, and they are relatively weak (strong), an increase in the market size softens (toughens) the competition facing firms in this market and more firms of a lower (higher) efficiency survive, increasing (decreasing) the welfare level. Moreover, when costly trade occurs between two symmetric countries, an increase in the level of economic integration softens competition only for intermediate vertical linkages, worsening the welfare level only for strong linkages.

Keywords: firm selection, vertical linkages, market size, international integration.
J.E.L. Classification: F12, F14, F15, R12, R13

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1 Introduction

This paper introduces vertical linkages in a model of trade with heterogeneous firms producing in a differentiated manufacturing good sector in order to take into account the rising importance of 'vertical' trade worldwide, and it analyzes the interfirm reallocations and the welfare effects that occur in response to changes in the size and in the level of international economic integration.

There is plenty of empirical evidence on the fact that firms producing in the world economy are heterogeneous in their productivity levels, and an important stream of the theoretical literature on trade describes the role that international integration plays in reallocating resources from less to more productive firms (i.e. Montagna (2001), Melitz (2003) and Melitz and Ottaviano (2008)).[1] The relationship between firm heterogeneity and exports has been extensively analyzed, and, given that is well known that imports play a key role in the global economy,[2] a recent empirical literature on firm heterogeneity and international trade, spurred by the availability of new detailed transaction data, has started to explore the relationship between import behavior and firm's characteristics combining information on both the import and export sides.[3]

As a matter of fact, it has been shown that a better access to imports can improve domestic manufacturing, because international trade provides domestic firms access to cheaper and previously unavailable intermediate inputs (Amiti and Konings (2007)), while part of the productivity premium of exporting firms can be explained by the fact that they are also importing some of their inputs (Altomonte and Bekes (2008)).[4] Moreover, Castellani et al. (2009) show not only evidence in favour of recent theories on firm heterogeneity and international trade, but also highlight some new stylized facts that describe the role of imports in the global economy, finding, for instance, that firms engaged in both import and export activities often outperform firms involved in importing only.[5]

In this paper, we argue that, given that firms use intermediate goods, some of which are imported, and given that they exports not only final goods, but also intermediate inputs that can be used for further processing abroad, we should consider vertical linkages among firms and analyze the role that they play in determining the selection process among heterogeneous firms generated by international integration. Hence, we assume that *imperfectly competitive* firms

[1] A large body of literature has originated that extends Melitz seminal contribution. In a different class of models, e.g. Bernard et al. (2003) and Eaton and Kortum, stochastic firm productivity are introduced into a multi-country Ricardian framework, with firms using different technology to produce the same good in the presence of market segmentation.

[2] For instance, Hummels et al. (2001) show that around 20% of total exports of the 10 OECD and the four emerging market countries analyzed (three-fifths of world trade) are due to intermediate inputs being used for further processing.

[3] See, for instance, Castellani et al. (2009)

[4] This literature appears to be particularly relevant for developing countries because imports can be as useful to developing countries as exports are. Goldberg et al. (2008, 2009), for instance, find that for India the access to new input varieties from abroad enabled the creation of new varieties in the domestic market and that India's trade liberalization relaxed the technological constraints faced by Indian firms under import substitution policies.

[5] Castellani et al. (2009) focus their analysis on Italian firms that trade goods.

are interconnected by backward and forward linkages and we analyze how this assumption affects the selection process and the changes in the welfare level produced by trade liberalization episodes in the model proposed by Melitz (2003). More precisely, we start from the Baldwin and Forslid (2004) analytical version of the Melitz' model, and - following Krugman and Venables (1995) and Fujita, Krugman and Venables (1999) - we modify it by introducing vertical linkages among firms producing in the differentiated manufacturing good sector.[6]

Specifically, we assume that the goods firms produce in the manufacturing sector can be employed not only as final consumption goods, but also as intermediates to produce manufactured goods. In so doing we try to understand how the explicit consideration of the fact that firms import intermediate goods - and that they can also, eventually, export goods that can be used as input by other firms abroad - may alter the results of the process of selection among heterogeneous firms and the changes in the welfare level determined by international integration, either in the case in which it simply consists in an enlargement of the size of the economy - because of the transition from autarky to free trade - or when it reduces the traditional measure of iceberg trade costs used to represent the obstacles to trade that exist between two countries. In this way, we investigate if we are able to uncover some new insights from the theory that can be either empirically tested or, eventually, used to explain some empirical puzzles already highlighted in the literature, such as that presented in the following paragraph.

Tybout (2006, p. 932), commenting the findings by Bernard, Jensen and Schott (2006), points out that "[i]n contrast with the predictions of the heterogeneous-firm models, changes in industry-level trade costs are uncorrelated with changes in plant-level domestic market share in all specifications".[7] Moreover, he writes that (p. 931) "[t]he absence of a substantial response of domestic market share by U.S. firms to falling trade costs suggests a role for other forces and perhaps a need for models exhibiting a richer set of predictions about the response of domestic output to international trade." Hence, Tybout (2006, p. 941) himself suggests that "[o]ne interpretation is that exporters, and perhaps other high productivity firms, tend to import their intermediate goods. Thus when trade costs fall, these producers enjoy lower marginal production costs and they adjust their domestic sales accordingly." We argue that investigating the role that vertical linkages play in the process of selection among heterogeneous firms can explain part of this puzzle by showing that when trade costs fall it can become easier for less productive firms to survive. The reason is that, in this case, they experience an increase in their demand, that partly comes from other firms employing their output as intermediates, and, at the same time, they benefit from a reduction in their marginal costs, which is due to the larger availability of

[6] Let us recall that, starting from the seminal work by Venables (1996), the New Economic Geography literature has shown that intermediates and vertical linkages among firms play a relevant role in determining the space distribution of firms. For instance, vertical linkages contribute to strengthen agglomeration forces when they are sufficiently strong.

[7] This is specifically the second puzzle that Tybout (2006) highlights in the findings by Bernard, Jensen and Schott (2006).

cheaper intermediate varieties. With our model we are also able to replicate the empirical finding that a sort of hierarchy emerges not only between domestic and exporting firms, but also among traders given that, as Castellani et al. show (2009), firms engaged in both import and export activity often outperform firms involved in importing only.[8]

Furthermore, by introducing vertical linkages among firms producing in the differentiated monopolistic sector, we show that market size gains a role in determining the equilibrium distribution of firms that is not played in the original framework proposed by Melitz (2003), where all firm level variables (the productivity cut-off, the average productivity, profit and revenue) are independent of the country size. In contrast to these predictions, "Campbell and Hopenhayn (2005) present empirical evidence that retail establishments in larger markets have higher sales and employment. Similarly, Syverson (2004) examines the concrete industry as an example of a good with high transport costs, and finds that larger markets have both higher average plant size and higher average productivity" (Redding (2010), p. 15). As Melitz (2003) writes in a note, a key factor determining his results is the assumption of an exogenously constant elasticity of substitution between varieties that once dropped, as in Krugman (1979), could make the presence of heterogeneity of firms relevant in determining the impact of trade on firm level variables even when trade costs are equal to zero. In this work, we show that *size* plays a role in determining the equilibrium distribution of firms when vertical linkages are considered, and this even without relying on alternative assumptions on preferences, such as those suggested by Melitz (2003) himself in his note or by Melitz and Ottaviano (2008), who consider a quasilinear utility function, sacrificing income effects for variable markups, and find that an increase in the size of the economy toughens competition in the market.[9]

We show that we are able not only to reproduce the same type of selection effect determined by international integration in Melitz and Ottaviano (2008) for different reasons, but also that we can shed light on the conditions that can lead to the *opposite effect* because - when we consider vertical linkages between upstream and downstream firms - *an increase in the size of the economy can also soften competition in the market allowing less efficient firms to survive*. Indeed, we show that with vertical linkages the market size is able to affect the productivity distribution of firms in equilibrium, even under the standard assumption of constant elasticity of substitution because different levels of de-

[8] We recall that, in their work on italian trading firms, Castellani et al. (2009) find that firms involved in both importing and exporting (two-way traders) are the best performers, while firms involved only in importing activities perform better than those involved only in exporting. Moreover, Kasahara and Lapham (2008) find that two-way traders are the best performers, but, on the contrary, that firms involved only in exporting have a relatively high productivity, while those involved only in importing have a relatively low productivity. Here we focus our attention on the relationship between two-way traders and importing firms that use both domestic and foreign intermediates and we show that firms engaged in both import and export activity outperform firms involved in importing only.

[9] In Melitz and Ottaviano (2008), consumers in larger markets face lower average prices and enjoy higher welfare, because of both higher average productivity and lower average mark-ups.

mand have a different impact on the cost of intermediates, and, thus, pecuniary externalities might arise benefitting all firms. Specifically, an increase in the size of the economy softens competition when the strength of vertical linkages is below a threshold value making it easier for less productive firms to survive, both because of the larger demand that comes from other firms employing their output as intermediate and because of the reduction in the price of intermediates due to the larger availability of varieties. When, on the contrary, vertical linkages are sufficiently strong, a larger economy shows a stronger selection because the price of intermediates increases toughening competition in the market, as it happens in the Melitz and Ottaviano (2008) model for different reasons.[10] Our findings can be explained by the action of three forces at work in our model, which, following the New Economic Geography literature, can be identified as the 'market access' effect, the 'cost of living/producing' (or price index) effect and the 'market crowding' effect. Particularly, according to these three effects, *an increase in the size of the economy*, respectively, tends to: (i) increase the sales of firms and, in turn, their operating profits, softening competition, allowing less efficient firms to survive, and increasing the number of producing firms ('market access' effect); (ii) make more varieties available to be used as intermediates (and consumption) goods and, therefore, reduce the marginal cost of production softening, in this way, competition and favouring the increase in the number of firms ('cost of living/producing' effect); (iii) increase the demand for intermediates pushing upward the price of intermediates, and, consequently, toughening competition and reducing the number of producing firms ('market crowding' effect). In summary, while the first two channels tend to soften competition and to promote the increase in the number of producing firms, the third one acts in the opposite direction, with the final prevailing effects depending on the strength of vertical linkages related to the other parameters of the model.

In the second part of the paper, we investigate the effects produced by the inclusion of vertical linkages on the selection process when trade is costly, and our findings suggest that: (i) it is the *strength of vertical linkages* that determines whether less or more efficient firms can survive in the domestic market for any given level of the market size or of trade cost in comparison with the case in which vertical linkages are absent; (ii) a higher level of international integration between two economies *decreases* the level of efficiency required to produce for the domestic market when vertical linkages assume "intermediate" values. Therefore, we find that the traditional result that the level of efficiency required to produce for the domestic market decreases when the level of economic integration increases is valid only for certain levels of the parameters that expresses the strength of vertical linkages, the elasticity of substitution between varieties and the shape parameter that characterizes the probability distribution of productivities. Moreover, our findings are consistent with those established in the literature that a larger level of economic integration can allow less produc-

[10] In Melitz and Ottaviano (2008), as Redding writes (2010, p. 17), "comparing a large to a small market, some low productivity firms have marginal costs above the choke price in the larger market and exit, while surviving firms price on a more elastic segment of their demand curve, and hence charge lower mark-ups than in the smaller market."

tive firms to export because they can acquire their intermediate inputs at lower prices. Again, the final outcomes of a process of economic integration are due to the interactions of different types of effects that are at work, which mirror those already described for the closed economy when its size increases. These effects are, respectively, given by: (i) the 'market access' effect, which shows that when the level of economic integration increases it produces an increase in the sales of firms and in their operating profits that can reduce the competitive pressures faced by firms producing for both the domestic and the foreign market, allowing also the less efficient ones to survive and increasing the number of firms producing for both markets; (ii) the 'cost of living/producing' effect, which shows that when the level of economic integration increases more varieties become available to be used as intermediates (and consumption) goods and, therefore, the price index decreases softening, in turn, competition in both the domestic and the foreign market and favouring the increase of the number of firms producing for both markets; (iii) the 'market crowding' effect, which arises when the level of economic integration increases because the demand for intermediates rises pushing upward their prices. This last effect toughens competition pushing upward the cutoff levels of productivity required to survive for both the domestic and the foreign market and reduces the number of firms producing for both markets. In summary, while the first two channels so far described tend to soften competition and to promote the survival of less productive firms, the third one acts in the opposite direction, with the final prevailing effects depending on the strength of vertical linkages related to the other parameters of the model.

Let us finally recall that, related to the present work is that by Kasahara and Lapham (2008) that considers the relationship between productivity and the decision to import and export of firms. The model they present is rich in its predictions, but is different from ours. This because they introduce a fixed cost of importing, and they do not have vertical linkages of the type proposed by Venables (1996) with *monopolistically competitive firms* producing the intermediate goods.[11] Indeed, Kasahara and Lapham (2008) assume *perfect competition* in the sector of intermediate goods produced in a *finite* measure of varieties - and we think that this is an important departure from the assumptions of imperfectly competitive markets in many models in the international trade literature.[12] Moreover, even if in their setup firms that produce the final good are assumed to use intermediates, they do not sell their production as inputs for other firms. Hence, Kasahara and Lapham (2008) show that opening trade in either final goods or intermediates or both causes firms with lower inherent

[11] In particular, by introducing fixed costs of importing intermediates in the Melitz (2003) framework, they find that firms can be divided among four groups, that is: i) firms with relatively low productivity and low fixed cost of importing that choose to import but not export; ii) firms with relatively low productivity and higher fixed cost of importing that choose to neither import nor export; iii) firms with relatively high productivity and high fixed cost of importing that choose to export but not import; iv) firms with relatively high productivity that choose to both import and export.

[12] Kasahara and Lapham (2008) also assume that intermediate goods can be imported in one country after paying both a fixed cost and iceberg costs of importing. As usual, exporting the final consumption goods entails both a fixed cost of export an iceberg cost.

productivity to exit - with even more exit than in Melitz (2003) with no importing of intermediate. As we have already mentioned, also in our framework the reduction of trade costs can potentially make firms with lower productivity exit the market. However, we investigate the role played by the strength of vertical linkages among heterogeneous firms in determining this type of effect and its contrary. Moreover, we have different results in the case in which the economy moves from autarky to full trade, given that Kasahara and Lapham (2008, p. 15) always finds that "market shares are shifted away from firms which do not engage in trade (low productivity firms) to firms which both export and import (high productivity firms). [...] This effect was identified by Melitz (2003) in the economy with no importing of intermediates. If the economy also opens to intermediates imports this effect is strengthened because of additional resource reallocation and a direct increase in productivity from the use of additional intermediates." As we pointed out before, in our case moving from autarky to full trade can also softens competition in the domestic market, and we are able to unveil a new role that the strength of vertical linkages may play in affecting the selection processes among heterogeneous firms generated by international integration.[13]

The remaining of the paper is organized as follows. Section 2 presents the structure of the closed economy model, which is based on the open economy framework by Baldwin and Forslid (2004) modified in order to introduce vertical linkages in the production of the differentiated varieties of the manufacturing good.[14] Section 3 highlights how, by considering vertical linkages, the size of the economy can affect the selection process and the equilibrium values of firm level variables, showing that an increase in the size of the economy not always produces an increase in the welfare level. Section 4 describes the open economy case with costly trade, and shows how the effects of a trade liberalization process on the selection effects and on the changes of the welfare level crucially depend on the presence and on the strength of backward and forward linkages. Section 5 concludes.

[13] Finally, Kugler and Verhoogen (2009), using information from the Colombian manufacturing census, show that more productive plants select into import market and purchase higher quality inputs with quality differences between imported and domestic inputs, suggesting that imported inputs are of higher quality than domestic inputs. However, they do not build a formal model, and simply refer to their previous work, Kugler and Verhoogen (2008), where the model by Melitz is accomodated to introduce the hypothesis that input quality and plant productivity are complementary in generating output quality. However, in this latter paper, intermediates are only domestic goods produced in a perfectly competitive sector with constant returns to scale employing only (domestic) labor as input, with no international trade of inputs.

[14] Baldwin and Forslid (2004) presents a slight variant of Melitz (2003) that is in the spirit of Helpman, Melitz and Yeaple (2004). We notice that a modified version of Baldwin and Forslid (2004) has been published with the same title. However, the new version, that is Baldwin and Forslid (2010), differs from that published in 2004 because it includes elements from Baldwin (2005) and it considers countries that can differ in their size. In our case, we assume that the two countries are symmetric in the open economy section because this allows us to make more direct comparaison with Melitz (2003) that considers symmetric countries, and because in this way we get closed form solutions for the variables in the model.

2 The closed economy: vertical linkages and the selection effect of market size changes

The economy we consider is populated by L individuals, each supplying one unit of labor used to produce two kinds of goods in two sectors: an homogeneous competitive good and a differentiated manufactured good composed by different varieties produced in a standard Dixit-Stiglitz monopolistic competition sector with increasing returns. Firms in the monopolistic sector are heterogeneous in their productivity levels and, to produce, each manufacturing firm incurs in: two types of fixed sunk costs - which are common to all firms and are the fixed cost, f_I, required to develop a new variety, and the fixed production cost required to produce and introduce the new variety into the market;[15] and in a constant marginal production cost that differs across firms. Both the variable production cost and the fixed production cost are incurred in term of a composite of labor and intermediate goods produced in the monopolistic sector. Thus, following Krugman and Venables (1995) and others, we assume that the varieties produced in the differentiated good sector serve both as intermediate goods and final goods. We recall that, in this case, the upward and downward sectors are collapsed in one sector and that this specification has been widely used in New Economic Geography models showing that vertical linkages tend to reinforce centripetal forces leading to more agglomeration (i.e. Venables (1996), Krugman and Venables (1995), Puga (1999) and Nocco (2005)).[16] Finally, the outcome of the initial R&D activity is uncertain and firms learn about their actual production cost levels only after making the irreversible investment required for entry. Given that the blueprints employed in this innovation process are freely available, the innovating cost only consists in the wage paid to employ f_I units of labour to develop a new variety.[17]

The representative consumer has preferences described by a two-tier utility

[15] As Baldwin and Forslid (2004) and Melitz (2003), we focus on steady states. Moreover, following Baldwin and Forslid (2004), when we use fixed costs, they correspond to the flow equivalent of total fixed costs. Specifically, it is possible to ignore discounting by assuming that firms die according to a Poisson process with a hazard rate of δ. Hence, given that the expected life of a variety is $1/\delta$, the flow equivalent of the fixed innovation cost F_I is $f_I = \delta F_I$. The same reasoning applies for the other fixed costs in the model.

[16] Given that the functional forms of these models are similar to those in Krugman (1991), they are often classified as "core-periphery vertical-linkage" models. Alternative ways to introduce vertical linkages in New Economic Geography models are those suggested by Robert-Nicoud (2002) in a "footloose capital" model and by Ottaviano (2002) in a "footloose entrepreneurs" model. Ottaviano and Robert-Nicoud (2006) later show that the models by Krugman and Venables (1995) and by Ottaviano (2002) are isomorphic and can be encompassed in a more general model with vertical linkages.

[17] It is straigthforward to notice that, as in Melitz (2003), the innovation process is not modeled. Moreover, in our case, we know that some units of labour are devoted to the development of new varieties and that blueprints of the available varieties could be used as free goods to develops the new ones in a static model.

function of the following type

$$U(C_T, C_M) = \frac{C_T^{1-\mu} C_M^{\mu}}{(1-\mu)^{1-\mu}\,\mu^{\mu}};\; C_M \equiv \left(\int_0^N C_\varepsilon(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},\; 0 < \mu < 1 < \sigma \tag{1}$$

where C_T and C_M are, respectively, the individual consumption of the homogeneous good T and of the composite of all differentiated varieties i consumed in quantity $C_\varepsilon(i)$; N is the mass of produced varieties available for consumption, μ is the expenditure share on manufacturing goods, and σ is the elasticity of substitution between any pair of manufactured varieties. Utility maximization of (1) generates the familiar demand function for variety i

$$C(i) = \mu \frac{p(i)^{-\sigma}}{P_M^{1-\sigma}} I \tag{2}$$

where I is the aggregate consumer income, $p(i)$ is the price of variety i and P_M is the standard CES price index of all manufactured varieties with

$$P_M = \left(\int_0^N p(i)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{3}$$

On the production side, the homogeneous agricultural good is characterized by perfect competition, constant return to scale and is chosen as the numeraire of the model. Thus, given that one unit of labour is required to produce one unit of the agricultural good, the wage w is equal to one.

In the monopolistic sector, the outcome of the initial R&D activity is uncertain and firms learn about their actual production cost levels only after making the irreversible investment required for entry. The sunk investment delivers a new horizontally differentiated variety with a random unit Cobb-Douglas composite requirement of intermediate and labour, $a(i)$, drawn from a cumulative distribution, $G[a]$. As a result, R&D generates a distribution of entrants across marginal costs, with a firm i that produces in the economy facing the marginal cost of production $w^{1-\alpha} P_M^{\alpha} a(i)$, where α is the intermediate share with $0 < \alpha < 1$. Following the standard practice in the literature, we assume that a is distributed according to a Pareto probability distribution that has a higher bound a_M and shape parameter $\kappa > 0$, that is

$$g(a) = \kappa \frac{a^{\kappa-1}}{a_M^{\kappa}}, \quad 0 \le a \le a_M \tag{4}$$

Let us recall that when $\kappa = 1$, the as are uniformly distributed and that larger values of κ implies that the relative number of firms with a higher value of a increases, making the distribution of a more concentrated at higher levels.

In general, producing variety i requires a fixed cost of f_D units of a Cobb-Douglas composite of intermediate and labour, and $a(i)$ units of the same composite per unit of output. This implies that the total cost function of producing quantity $q(i)$ of variety i is

$$TC(i) = P_M^{\alpha} \left[a(i)q(i) + f_D \right] \tag{5}$$

Applying the Shephard's lemma to previous function, we find that the demand of variety i used as intermediate good by the firm producing variety j, $B_j(i)$, is

$$B_j(i) = \frac{p(i)^{-\sigma}}{P_M^{1-\sigma}} \alpha P_M^{\alpha} \left[a(j)q(j) + f_D \right] = \frac{p(i)^{-\sigma}}{P_M^{1-\sigma}} \alpha TC(j) \tag{6}$$

Moreover, the total demand function for the firm producing variety i is given by the sum of the total final demand, $C(i)$, and by the total intermediate demand, $B(i) \equiv \int\limits_0^N B_j(i)dj$, for variety i, that is

$$q(i) \equiv C(i) + B(i) \tag{7}$$

Making use of (2) and (6), we can rewrite the demand function (7) as follows

$$q(i) = \frac{p(i)^{-\sigma}}{P_M^{1-\sigma}} \left(\mu I + \alpha \int\limits_0^N TC(j)dj \right) \tag{8}$$

The optimal pricing rule for the firm producing variety i implies that

$$p(i) = \frac{\sigma}{\sigma - 1} P_M^{\alpha} a(i) \tag{9}$$

Using (4) and (9), we can rewrite (3) as

$$P_M^{(1-\alpha)(1-\sigma)} = \left(\frac{\sigma}{\sigma - 1} a_D \right)^{1-\sigma} \left(\frac{\beta}{\beta - 1} \right) N$$

where $\beta \equiv \frac{\kappa}{\sigma - 1}$ and $\beta > 1$ is required to have the price index P_M converging to a positive value.[18]

It can be easily shown that operating profits of the firm producing variety i, that is $\pi(i) = \left[p(i) - w^{1-\alpha} P_M^{\alpha} a(i) \right] q(i)$, with $w = 1$ representing the unit wage of workers, can be rewritten as follows[19]

$$\pi(i) = \frac{p(i)^{1-\sigma}}{\sigma P_M^{1-\sigma}} \left(\mu I + N \alpha P_M^{\alpha} f_D + \alpha P_M^{\alpha} \int\limits_0^N a(j)q(j)dj \right) \tag{10}$$

[18] See Baldwin and Forslid (2004)

[19] We also observe that it can be readily verified that operating profits of firm i in a market are $1/\sigma$ times the revenues $r(i)$ in the same market, that is $\pi(i) = \frac{r(i)}{\sigma}$. In this case, revenus are given by the price $p(i)$ multiplied by the total demand for firm i, $q(i)$, that is given by expression (8).

As usual, we can identify a threshold, or cut-off, level of technical efficiency at which a firm will be indifferent between staying in the market or exiting, which we shall denote by a_D. Firms with a level of $a(i) = a_D$ will just break even. Therefore, a_D denotes the upper limit of the range of a of firms actually producing in the economy. More productive entrants with a value of $a(i) \leq a_D$ will start producing, while entrants with a value of $a(i) > a_D$ will exit the market. Thus, the cut-off level, a_D, is defined by the following equivalent zero profit condition

$$a_D = \sup \left\{ a : \pi(a) = P_M^{\alpha} f_D \right\}, \tag{11}$$

which describes the indifference condition of marginal firms (i.e. the firms that are just able to cover their costs of production).

Given that in the long run, the number of produced varieties is endogenously determined to eliminate expected pure profits, ex ante expected operating profits of a winner must be equal to his expected fixed cost $\bar{F}$, that is

$$\frac{\int_0^N \pi(i)di}{N} = \bar{F} \tag{12}$$

Moreover, given that free entry drives pure profits to zero, aggregate workers income is $I = L$.

Following the variant of Melitz (2003) by Baldwin and Forslid (2004), $\bar{F}$ can be written for our closed economy analysis with vertical linkages as follows

$$\bar{F} = P_M^{\alpha} f_D + \frac{f_I}{G[a_D]} \tag{13}$$

where $G[a_D]$ is the cumulative density function corresponding to $g(a)$. Hence, the (ex-ante) expected operating profit of a winning variety must be equal to the expected fixed cost of a winner, which is given by the fixed cost of production $P_M^{\alpha} f_D$ (for all active producers, that is winners), plus the expected development cost of getting a winner, that is $f_I / G[a_D]$.

In Appendix A we show how it is possible to obtain from previous expressions a system of three equations (40), (41) and (42) in three unknowns: P_M, N and a_D. Solving this system, we find that the cut-off level a_D is given by

$$a_D = \left[\frac{\mu L \left(\frac{\sigma}{\sigma - 1} \right)^{(1-\sigma)} \kappa}{\delta f_D} \right]^{\frac{\alpha}{\gamma}} \left[(\beta - 1) \frac{f_I}{f_D} a_M^{\kappa} \right]^{\frac{(\sigma - 1 - \alpha\sigma)}{\gamma}} \tag{14}$$

where $\delta = \alpha (\sigma - 1) + \kappa\sigma (1 - \alpha) > 0$ and $\gamma = (\sigma - 1)(\alpha + \kappa) - \alpha\kappa\sigma$. The first term in square brackets disappears when $\alpha = 0$, so that L becomes irrelevant in determining a_D when vertical linkages are not considered.[20]

[20] If $\alpha = 0$ we fall back to the results in Baldwin and Forslid (2004) where, as Melitz (2003, p. 1705) writes, "all the firm level variables are independent from the country size".

Let us define the threshold value $\alpha_1 \equiv \frac{\kappa}{\beta\sigma-1} < 1$ for the parameter determining the strenght of vertical linkages α. It can then be readily shown that γ is positive if $\alpha \in (0, \alpha_1)$, while it is negative if $\alpha \in (\alpha_1, 1)$. Therefore, the cut-off a_D increases with the size of the economy (because $\frac{\partial a_D}{\partial L} > 0$) when the strength of vertical linkages is relatively small, that is when $\alpha \in (0, \alpha_1)$, while it decreases with L ($\frac{\partial a_D}{\partial L} < 0$) when vertical linkages are relatively strong, that is when $\alpha \in (\alpha_1, 1)$. The explanation of these results has to take into account the effects produced by the increase in the size of the economy also on other endogenous variables of the model, such as the price of intermediates, that affects the marginal cost of production, and the number of produced varieties, to which we will turn in the following paragraphs. However, anticipating part of these results, we point out that an increase in the size of the economy softens competition when the strength of vertical linkages is below the threshold value α_1 making it easier for less productive firms to survive, both because of the larger demand that comes from other firms employing their output as intermediate and because of the reduction in the price of intermediates due to the larger availability of varieties. When, on the contrary, vertical linkages are sufficiently strong with α above α_1, a larger economy shows a stronger selection because the price of intermediates increases toughening competition in the market and requiring a larger level of efficiency to survive.

The threshold value α_1 increases with σ and decreases with κ, and in Figure 1 we show the value of α_1 and the sign of the derivative $\frac{\partial a_D}{\partial L}$ for the different admissible values of the parameters α (that is, $0 \leq \alpha < 1$) and σ (that is, $1 < \sigma < \kappa + 1$). This graphic shows that, for any given level of σ and κ, an increase in the size of the economy increases (decreases) the cut-off level a_D when the parameter that indicates the strength of the vertical linkages, α, is relatively small (large) and $\gamma > 0$ ($\gamma < 0$). Moreover, the range of α for which we find a positive sign of $\frac{\partial a_D}{\partial L}$ increases for a larger elasticity of substitution between varieties, σ, when varieties become stronger substitutes, and a lower shape parameter, κ, when the relative number of low-cost firms increases.

Insert Figure 1 about here

Moreover, we are now able to compare the cut-off a_D when vertical linkages are considered (with $\alpha \neq 0$) with that observed in the case in which they are absent (with $\alpha = 0$), for any given value of the size of the economy, L. In particular, when $\alpha < \alpha_1$ and $\gamma > 0$, we can assess that vertical linkages make it more (less) difficult to survive for less productive firms producing in the economy, with respect to the case in which we have no vertical linkages, only when the size of the economy is relatively small (large). If, however, the size of the economy increases (for instance because of transition from autarky to free trade) firms experience a reduction in competitive pressures they face in the markets and less productive firms become able to produce (with a_D increasing) because of the increased demand that comes from other firms that use their products as intermediates, and because of the reduction in the price of intermediates. Hence, if the size of the economy is smaller (larger) than a threshold value, the cut-off a_D with vertical linkages is smaller (larger) than that found when $\alpha = 0$.[21] This is shown in Figure 2 in panel a. The opposite takes place when vertical linkages are strong, that is when $\alpha > \alpha_1$ and $\gamma < 0$, as it is shown in panel b in Figure 2.

Insert Figure 2 about here

In addition, in our case also the share of income devoted to the consumption of manufactured goods, μ, becomes relevant for determining the cut-off level a_D, and the effects of changes in μ on the cut-off level a_D depends on the parameters in a similar way to that so far described for the effects produced by changes in the size of the economy L. We recall that, on the contrary, μ had no effect on a_D in Baldwin and Forslid (2004), and thus had also no effect on firm level variables, while it affected only aggregate variables such as N and P_M.

Finally, we notice that the sign of the derivative $\frac{\partial a_D}{\partial f_I}$ is not anymore positive as in the absence of vertical linkages, but it depends on the sign of the exponent $(\sigma - 1 - \alpha\sigma)/\gamma$. In particular, it can be shown that this sign is positive if $\alpha \in (0, \alpha_0)$ and $\alpha \in (\alpha_1, 1)$, while it is negative for intermediate values of α, that is if $\alpha \in (\alpha_0, \alpha_1)$ with $\alpha_0 \equiv \frac{\sigma-1}{\sigma} < \alpha_1$. Thus, reducing the cost of innovation does not always result in a larger cut-off value, but it can also result in a smaller cut-off a_D for intermediate values of vertical linkages.

The expression found in equilibrium for the price index is the following

$$P_M = \left[\frac{\mu L \left(\frac{\sigma}{\sigma-1} \right)^{(1-\sigma)} \kappa}{\delta f_D} \right]^{-\frac{\kappa}{\gamma}} \left[(\beta - 1) \frac{f_I}{f_D} a_M^{\kappa} \right]^{\frac{\sigma-1}{\gamma}}$$

where the value of α, which determines the sign of γ, is relevant also in defining the value of P_M. In particular, γ is positive, as in the traditional case (that is with $\alpha = 0$) when vertical linkages are weak, that is when $\alpha \in (0, \alpha_1)$, and in this case the price index decreases with the size of the economy. On the contrary, when vertical linkages are strong, that is when $\alpha \in (\alpha_1, 1)$, γ is negative, and the price index increases with L, because the increase in the demand coming from the increase in the size of the economy pushes the price index upward when vertical linkages are very strong. And this requires to understand also how the number of firms producing in the economy is affected by the presence of backward and forward linkages among firms. Before moving to this question, let us observe that also the effect of changes in the fixed cost of innovation on the price index depends on the value of α. Indeed, if $\alpha \in (0, \alpha_1)$, the price index rises when the fixed cost of innovation increases. The opposite happens when $\alpha \in (\alpha_1, 1)$. The effects of changes in f_I will be commented more extensively at the end of the Section.

[21] This threshold value can be found by equating the value of a_D in (14) to that obtained when $\alpha = 0$.

Finally, the number of firms producing and selling their products in the economy is given by

$$N = \frac{\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\left(\frac{\beta-1}{\beta}\right)\left[\frac{\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)}\kappa}{\delta f_D}\mu L\right]^{\frac{(\sigma-1)[\kappa(1-\alpha)+\alpha]}{\gamma}}}{\left[\frac{f_I}{f_D}\left(\beta-1\right)a_M^{\kappa}\right]^{\alpha\frac{(\sigma-1)}{\gamma}}}$$

Again, the effects of changes in the size of the economy, L, (or in the share of consumption expenditure devoted to manufactures, μ) depend on the sign of γ, and therefore on the size of α. When vertical linkages are strong (that is when $\alpha \in (\alpha_1, 1)$) a larger value of L decreases the number of firms producing in the economy, while the opposite happens when vertical linkages are weak (that is when $\alpha \in (0, \alpha_1)$). This result is more complex than that obtained by Melitz and Ottaviano (2008) where an increase in the size of the economy unambiguously increases the number of firms. This can be explained in our case by the fact that if the size of the economy increases, demand pressures increases relatively more (less) in the case of strong (weak) vertical linkages and this results, as we have already seen, in an increase (decrease) in the price index of the manufactured goods and, consequently, on the cost of production of firms, that therefore experience more (less) exit. This allows us to underline how the 'cost-of-producing' effect can influence the number of firms producing in the market: the fact that firms use the products of other firms as intermediates implies that increases in the cost of production of firms reduce the number of firms producing in the country and make survival of less efficient firms more difficult. On the other side, if vertical linkages are not that strong with $\alpha \in (0, \alpha_1)$, an increase in the size of the economy produces an increase in the number of firms competing in the market and reduces the cost of production.

Commenting our results at the light of the terminology used by the New Economic Geography literature and introduced in the first Section, *an increase in the size of the economy* will, respectively, tend to: (i) increase the sales of firms and, in turn, their operating profits, softening competition and allowing less efficient firm to survive, with a larger value of the cutoff a_D, producing an increase in the number of active firms, N, ('market access' effect); (ii) make more varieties available to be used as intermediates (and consumption) goods and, therefore, reduce the price index, P_M, in this way softening competition (that is, increasing a_D) and favouring the increase of the number of producing firms ('cost of living/producing' effect); (iii) increase the demand for intermediates, pushing upward their prices, and, consequently, fostering the reduction in the number of firms and toughening competition (that is, reducing a_D) pushing upward the productivity cutoff level required to survive ('market crowding' effect). Table 1.a summarizes the effects of changes in L on the endogenous variables when trade is costless, showing how the interactions of the three effects are able to affect the relevant variables. Specifically, when vertical linkages are weak, that is when $\alpha \in (0, \alpha_1)$, the market access effect and the cost of living/producing effect dominate on the market crowding effect.

It is also very important to underline that in this model increases in L have no univocal effects on welfare. These effects, instead, depend on the size of the parameter that denotes the relevance of vertical linkages, that is α. More precisely, given that the welfare level of the representative consumer/worker associated to the utility function in (1) is $W = 1/P_M^{\mu}$, increases in the size of the population L increases the welfare level only if vertical linkages are weak (that is, if $\alpha \in (0, \alpha_1)$) because in this case we observe a reduction in the price index P_M. On the contrary, if vertical linkages are strong, that is if $\alpha \in (\alpha_1, 1)$, the welfare level decreases with L because the price index increases with L.

Insert Table 1.a about here

Finally, it can also be noticed that the term in the denominator for the solution for N is equal to 1 when $\alpha = 0$ so that f_I becomes irrelevant in determining N when vertical linkages are not considered. In other words, while f_I in Baldwin and Forslid (2004) affects only the values of the cut-off a_D and the price index P_M, in the presence of vertical linkages it is also able to affect the number of firms producing in the economy N, as it happens in Melitz and Ottaviano (2008), where a different structure of preferences is employed. Moreover, we observe that the finding by Melitz and Ottaviano (2008) that an increase in the fixed cost of innovation f_I reduces the number of firms selling in the economy is present in our model only in the case in which γ is positive (that is when vertical linkages are not too strong with $\alpha \in (0, \alpha_1)$). In other words, we are able to describe a new effect given that if vertical linkages are sufficiently strong (that is γ is negative because $\alpha \in (\alpha_1, 1)$) an increase in the fixed cost of innovation results in an increase in the number of sellers. The explanation of this result should rely on the fact that increases in f_I imply that more workers are required in the innovative process reducing the number of workers that can be employed in the production of goods; if the share of total production costs, α, devoted to intermediate goods is small, the number of firms producing in the economy has to decrease, while it can increase when α is large and firms producing in the differentiated good sector employ more of the composite input produced in the same sector by all firms.

3 The open economy: vertical linkages and the selection effect of market size changes and trade liberalization

In the previous Section we have shown that introducing vertical linkages among heterogeneous firms influences the effects produced by the transition from autarky to free trade on consumers' welfare and on the selection process among heterogeneous firms in a way that crucially depends on the *strength of vertical linkages among firms.*

In this Section we extend the model presented above to consider two regions/countries, H and F, that are symmetric in terms of tastes, technology, openness to trade and size. While trade for the homogeneous good is frictionless, the two markets for the differentiated manufactured varieties are *segmented*, because firms in this sector face iceberg trade costs and a fixed cost, f_X, to produce and introduce the new variety into the export market. Firms producing for the domestic and the foreign markets will endogenously be selected. All firms producing in a country employ intermediates that are not only locally produced, but also imported from the foreign country. In other words, while it is not true that all firms produce for both the domestic and the foreign markets, it is always true that firms use as intermediates all the available varieties sold in their country. Thus, all firms use both domestic and foreign intermediate manufactured goods as input, and, therefore, all firms imports when the two economies are not completely closed.

In particular, each firm producing variety i in a country requires $a(i)$ units of the Cobb-Douglas composite of intermediate goods and labour per unit of output, plus f_D units of the same composite to produce and sell in the domestic market and f_X units of this composite input to export. In principle, we can have two of the three following types of firms producing in a country (and in the other, given the assumption of symmetry): firms producing only for the domestic market, firms producing for both markets and firms producing only for the foreign market. Given the assumption on the distribution of the values of a, we will always have firms producing for both markets, while firms producing for only one of the two markets will be engaged only in the production for the domestic market when $f_D < f_X$, or in the production for exports when $f_D > f_X$.[22]

Consumers in the two countries share the same preferences described in the previous section, and given that the numeraire good is freely traded and produced with the same technology in both countries, the unit wage is equal to one in both of them. The pricing rule for monopolistic firms is the same as (9) for the price set for the domestic market, $p_D(i)$, and it becomes

$$p_X(i) = \frac{\sigma}{\sigma-1}\tau P_M^{\alpha} a(i) \tag{15}$$

for the price set for the foreign market, because iceberg trade cost $\tau \geq 1$ increase the marginal cost of production. Then, the CES price index for the differentiated varieties for the open economies written in terms of the cost parameter a is

$$P_M = \left(N_D \int_0^{a_D} (\tfrac{\sigma}{\sigma-1} P_M^{\alpha} a)^{1-\sigma} \kappa \tfrac{a^{\kappa-1}}{a_D^{\kappa}} da + N_X \int_0^{a_X} (\tfrac{\sigma}{\sigma-1} \tau P_M^{\alpha} a)^{1-\sigma} \kappa \tfrac{a^{\kappa-1}}{a_X^{\kappa}} da \right)^{\frac{1}{1-\sigma}} \tag{16}$$

[22] As we will state later on in the paper, we will write the free entry condition for the monopolistic sector focusing on the case in which $f_X \geq f_D$. This assumption relies on the consideration that fixed costs of production are usually larger when a firm has to produce for two markets and/or has to keep active two plants, or two production lines within a plant, one for the domestic market and the other for exports. The reason for the same type of assumption by Baldwin and Forslid (2004) on the value of fixed costs is justified by the fact that they reflect informational asymmetries or protectionism.

where N_D and N_X are, respectively, the number of firms that sell to the domestic market and the number of firms that export to the foreign market; while a_D and a_X are the two cut-off levels that identify the upper values of a for firm producing, respectively, for the domestic market and for the foreign market. Expression (16) can be rewritten to write explicitly the value of P_M as follows

$$P_M = (\frac{\sigma}{\sigma - 1} a_D)^{\frac{1}{1-\alpha}} (\frac{\kappa}{\kappa - \sigma + 1})^{\frac{1}{(1-\alpha)(1-\sigma)}} \left[N_D + \phi N_X (\frac{a_X}{a_D})^{1-\sigma} \right]^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{17}$$

with $\phi = \tau^{1-\sigma} \in [0, 1]$ denoting the usual measure of the freeness of trade, with ϕ equal to zero when trade costs are infinite, to one when they are null, and with ϕ increasing when trade costs decrease. Notice that the following condition $\beta \equiv \frac{\kappa}{\sigma - 1} > 1$ is required to have a positive value for the price index P_M.[23]

Let us now turn to the demand facing each firm. If firm i produces for both markets, its final production $q(i)$ is given by the sum of the production addressed to satisfy the domestic demand, $q_D(i)$, and the foreign demand, $q_X(i)$, both respectively obtained aggregating consumers' demand, $C(i)$, and firms' demand for intermediates. In particular, each exporting firm i faces the following demands: (1) the local consumers' demand, $C_D(i)$; (2) the foreign consumers' demand, $C_X(i)$; (3) the intermediate demand by firms producing in the same country, H, for the domestic market, $B_{HD}(i)$, and for the foreign market, $B_{HX}(i)$; (4) the intermediate demand by firms producing in the foreign country, F, for their domestic market, $B_{FD}(i)$, and for exports, $B_{FX}(i)$. Hence, the *local demand* faced by firm i in country H is

$$q_D(i) = C_D(i) + B_{HD}(i) + B_{HX}(i) \tag{18}$$

while its production for the foreign country, F, is given by τ times the *foreign demand*, that is

$$q_X(i) = \tau \left[C_X(i) + B_{FD}(i) + B_{FX}(i) \right] \tag{19}$$

Then, let us define $B_{jvs}(i)$ as the intermediate demand function of variety i by firm j producing in country $v = H, F$ to satisfy either the local demand (when $s = D$) or the foreign demand (when $s = X$). The intermediate demand $B_{jvs}(i)$ is obtained by applying the Shepard's lemma to the total cost of production of firm j, that is

$$TC_{vs}(j) = P_M^{\alpha} \left(f_s + a(j) q_s(j) \right)$$

This gives the following intermediate demand

$$B_{jvs}(i) = \frac{\partial TC_{vs}(j)}{\partial p_s(i)} = \frac{p_s(i)^{-\sigma} \alpha P_M^{\alpha}}{P_M^{1-\sigma}} \left(f_s + a(j) q_s(j) \right) \tag{20}$$

Moreover, we define the *aggregate intermediate demand* $B_{vs}(i)$ for production of firm i by firms located in country, v, for the prodution for market s, as follows

$$B_{vs}(i) = \int_0^{N_s} B_{jvs}(i) dj \tag{21}$$

[23] Cfr. Baldwin and Forslid (2004).

with $v = H, F$ and $s = D, X$ (where, as usual, N_D stands for number of firms producing for the domestic market, and N_X for the export market).

The value of the total cost of production incurred by all firms located in country H (and symmetrically F) is

$$TC = P_M^{\alpha}\left(N_D f_D + N_X f_X + \int_0^{N_D} a(j) q_D(j) dj + \int_0^{N_X} a(j) q_X(j) dj\right) \tag{22}$$

Then, the total value of the domestic expenditure of country H in the differentiated manufactured varieties can be defined as the sum of the share of consumers' income, μI, and of the share of the total cost of production in the same country, αTC, spent on intermediates, that is

$$E \equiv \mu I + \alpha TC \tag{23}$$

Making use of (20)-(23), in equilibrium we can rewrite the production of firm i for the local market in (18) as follows

$$q_D(i) = \frac{p(i)^{-\sigma}}{P_M^{1-\sigma}} E, \tag{24}$$

and its production for the foreign market, in the case in which it will export, in (19) as follows

$$q_X(i) = \frac{p(i)^{-\sigma}}{P_M^{1-\sigma}} \phi E \tag{25}$$

Firms characterized by a input requirement level $a(i)$ produce for the local market D if, and only if, operating profits $\pi_D(i)$ from domestic sales are not smaller than the fixed cost $P_M^{\alpha} f_D$, that is only if

$$\pi_D(i) = \left[p_D(i) - P_M^{\alpha} a(i)\right] q_D(i) \geq P_M^{\alpha} f_D, \tag{26}$$

Moreover, they export if, and only if, operating profits $\pi_X(i)$ from exports are not smaller than the fixed cost $P_M^{\alpha} f_X$, that is only if

$$\pi_X(i) = \left[p_X(i) - \tau P_M^{\alpha} a(i)\right] \left[C_X(i) + B_{FD}(i) + B_{FX}(i)\right] \geq P_M^{\alpha} f_X \tag{27}$$

It then follows that firms would be forced to leave if their profits were negative, and thus the cut-off levels for firms that sell in the domestic market and for firms that export are defined respectively by:

$$\begin{aligned} a_D &= \sup\left\{a : \pi_D(a_D) = P_M^{\alpha} f_D\right\}, \\ a_X &= \sup\left\{a : \pi_X(a_X) = P_M^{\alpha} f_X\right\} \end{aligned} \tag{28}$$

Operating profits in (26) and (27) can be rewritten as

$$\pi_D(i) = \frac{1}{\sigma} \frac{p_D(i)^{1-\sigma}}{P_M^{1-\sigma}} E \quad \text{and} \quad \pi_X(i) = \frac{1}{\sigma} \frac{p_X(i)^{1-\sigma}}{P_M^{1-\sigma}} E \tag{29}$$

where E is equal for both countries given the assumption of symmetry. Marginal firms have respectively the following operating profits

$$\pi_D(a_D) = \frac{1}{\sigma}\frac{(a_D \frac{\sigma}{\sigma-1} P_M^{\alpha})^{1-\sigma}}{P_M^{1-\sigma}} E \quad \text{and} \quad \pi_X(a_X) = \frac{1}{\sigma}\frac{\left[a_X \tau \left(P_M\right)^{\alpha} \frac{\sigma}{\sigma-1}\right]^{1-\sigma}}{P_M^{1-\sigma}} E \tag{30}$$

Making use of (11) and the ratio between the marginal profits realized in the domestic and export markets by marginal firms and given in (30), we find the ratio between the input requirements a of the marginal firms, that is

$$\frac{a_X}{a_D} = \left(\phi \frac{f_D}{f_X}\right)^{\frac{1}{\sigma-1}} \tag{31}$$

Then, we notice that, because of the assumption of a Pareto distribution, the relationship between the number of firms producing for the domestic market and the number of firms exporting is given by the following expression

$$\frac{N_X}{N_D} = \left(\frac{a_X}{a_D}\right)^{\kappa} = \left(\phi \frac{f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}} \tag{32}$$

Following Baldwin and Forslid (2004), we write the free entry condition for the monopolistic sector focusing on the case in which $f_X \geq f_D$. In this particular case, we know from (31) and (32) that $a_D \geq a_X$ and that $N_D \geq N_X$ (with N_D equal to the active mass of firms in a country). The (ex-ante) expected operating profit of a winning variety must be equal to the expected fixed cost of a winner, which is given by the fixed cost of the sum of $P_M^{\alpha} f_D$ (for all active producers, that is winners), plus $P_M^{\alpha} f_X$ times the probability of being an exporter (conditional on it being a winner), plus the expected development cost of getting a winner, that is $f_I/G[a_D]$. Thus, the free entry condition is

$$\frac{\int_0^{N_D} \pi_D(i)di + \int_0^{N_X} \pi_X(i)di}{N_D} = P_M^{\alpha}\left(f_D + \frac{G[a_X] f_X}{G[a_D]}\right) + \frac{f_I}{G[a_D]} \tag{33}$$

with total operating profits given, as usual, by the total expenditure on manufactures E over σ, that is

$$\int_0^{N_D} \pi_D(i)di + \int_0^{N_X} \pi_X(i)di = \frac{E}{\sigma} \tag{34}$$

In Appendix B we show how we can derive a_D, P_M, N_D, a_X and N_X.

The cut-off level for the open economy is given by

$$a_D = \left[\frac{\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)} \kappa}{\delta f_D} \mu L\right]^{\frac{\alpha}{\gamma}} \left\{ \frac{f_I}{f_D} \frac{(\beta - 1)}{\left[1 + \phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}\right]} a_M^{\kappa} \right\}^{\frac{(\sigma-1-\alpha\sigma)}{\gamma}} \tag{35}$$

Hence, when we consider vertical linkages ($\alpha \neq 0$), both the size of the economy, L, and the share of consumption devoted to manufactured goods, μ, become relevant in determining the result of the process of selection among heterogeneous firms also in the case in which trade is costly. Moreover, the sign of the derivative of a_D with respect to L (or with respect to μ) depends on that of γ, in exactly the same way described in previous Section and summarized by Figure 1. We also find that Figure 2 can be applied to the case of costly trade because when, for instance, vertical linkages are weak, that is when $\alpha \in (0, \alpha_1)$ and, thus, $\gamma > 0$, the cut-off a_D is smaller (larger) than that found when vertical linkages are absent, that is $\alpha = 0$, when the size of the economy is smaller (larger) than a threshold value. Thus, in this case, vertical linkages make it more (less) difficult to survive firms producing for the domestic market if the size of the economy is relatively small (large), while increasing the size of the economy reduces the competitive pressures for less productive firms that become able to produce, given the increased demand that comes from other firms for their products that are used as intermediates (even if they are not exporting because $a > a_X$).

The results of changes in the size L of the two economies on a_D, P_M and N_D when trade in manufactures is costly are equivalent to those summarized in Table 1.a and reported in Table 1.b, which is enriched to consider the effects of changes in L on a_X and N_X. Table 2, instead, summarizes the effects of changes in the level in the freeness of trade ϕ on all the relevant variables for the open economy case, changes that will be discussed below.

Insert Table 2 about here

What is important to notice is that increasing the level of economic integration, ϕ, between the two countries has not always the same effect on the cut-off a_D, but this depends on the strength of vertical linkages, α, on the elasticity of substitution between varieties, σ, and on the shape parameter κ of the Pareto distribution. Specifically, we observe that the sign of the exponent in a_D of the term in curly brackets, that is $(\sigma - 1 - \alpha\sigma)/\gamma$, which determines whether the cut-off increases (if it is positive) or decreases (if it is negative) with ϕ, is positive only when vertical linkages are relatively weak, that is if $\alpha \in [0, \alpha_0)$, or when they are relatively strong, that is when $\alpha \in (\alpha_1, 1]$, with $\alpha_0 \equiv (\sigma - 1)/\sigma < \alpha_1$.[24] Otherwise, this exponent is negative for intermediate vertical linkages, that is when $\alpha \in (\alpha_0, \alpha_1]$. In Figure 3 we summarize how the values of α_0 and α_1 depends on those of the elasticity of substitution σ and of the shape parameter κ (with $\alpha_0 \leq \alpha_1$ when $\sigma \geq 1$): for any given level of κ, increases in σ enlarge both the ranges $\alpha \in [0, \alpha_0)$ and $\alpha \in (\alpha_0, \alpha_1]$, and shrink the range $\alpha \in (\alpha_1, 1]$; for any given level of σ, increases in κ increase the range $\alpha \in (\alpha_1, 1]$, and shrink the range $\alpha \in (\alpha_0, \alpha_1]$ making it possible to have solutions for a wider range of σ.

The reasons because we have these different effects for different values of α on the cut-off a_D when the level of international economic integration, ϕ, changes, can be well understood only if we look at the changes that take place

[24] More precisely, $\alpha_0 < \alpha_1$ when $\sigma > 1$, and $\alpha_0 = \alpha_1 = 0$ when $\sigma > 1$. See Figure 3.

in the other relevant variables, such as the price index, P_M, and the number of producing firms, N_D.

Insert Figure 3 about here

Thus, we turn our attention to the price index that, substituting a_D from (35) into expression (48) in Appendix B, is given by

$$P_M = \left[\frac{\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\mu L\kappa}{\delta f_D}\right]^{-\frac{\kappa}{\gamma}} \left\{\frac{(\beta-1)}{\left[1+\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}\right]}\frac{f_I}{f_D}a_M^{\kappa}\right\}^{\frac{\sigma-1}{\gamma}}$$

Moreover, substituting a_D from (35) into expression (49) in Appendix B, we obtain the number of firms producing in the domestic market N_D, that is

$$N_D = \frac{\left(\frac{\beta-1}{\beta}\right)\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\left[\frac{\left(\frac{\sigma}{\sigma-1}\right)^{(1-\sigma)}\kappa\mu L}{\delta f_D}\right]^{\frac{(\sigma-1)[\kappa(1-\alpha)+\alpha]}{\gamma}}}{\left[\frac{f_I}{f_D}(\beta-1)a_M^{\kappa}\right]^{\frac{\alpha(\sigma-1)}{\gamma}}\left[1+\phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}\right]^{\kappa\frac{\sigma-1-\alpha\sigma}{\gamma}}} \tag{36}$$

Hence, the ranges of α that are relevant in determining the sign of the derivative of a_D with respect to ϕ, are equal to those used to establish the sign of $\frac{\partial N_D}{\partial \phi}$, while the sign of $\frac{\partial P_M}{\partial \phi}$ depends on the sign of γ. Specifically, as it is summarized in Table 2, an increase in the level of economic integration between the two economies that increases ϕ results in a decrease in P_M and in the number of producing firms in each country N_D when vertical linkages are low (that is, when $\alpha \in (0, \alpha_0)$). The column in Table 2 with $\alpha \in (0, \alpha_0)$ shows that *only in this case*, and only provided that $f_X > f_{X_0}$, we have the same effects on the variables found in the case in which $\alpha = 0$, that is in Baldwin and Forslid (2004) that reinterpret Melitz (2003). For instance, we find that an increase in ϕ both reduces the total number of varieties sold in the economy (the so-called 'anti-variety' effect) and produces the so-called 'MacDonaldisation' effect (that is N_X decreases) only when $\alpha \in (0, \alpha_0)$ provided that $f_X > f_{X_0}$. In all other cases, we have different results. For instance, if $f_X < f_{X_0}$ when $\alpha \in (0, \alpha_0)$, we find the Krugman's 'variety' effect with the number of consumed varieties increasing in ϕ, which is also present in Melitz and Ottaviano (2008). Moreover, intermediate linkages (with $\alpha \in (\alpha_0, \alpha_1)$) make the price index P_M decrease and the number of firms N_D increase when the freeness of trade increases. Instead, when vertical linkages are strong, that is when $\alpha \in (\alpha_1, 1]$, an increase in ϕ produces an increase in the price index and a decrease in the number of firm producing in each country. Therefore, even tough the price index P_M decreases when ϕ increases for low and intermediate linkages (that is when $\alpha \in (0, \alpha_1)$), this reduction in the cost of production allows the number of producing firms, and the cut-off level a_D, to increase only if vertical linkages are sufficiently high (that is at intermediate values with $\alpha \in (\alpha_0, \alpha_1)$), because in this case the demand coming for intermediates from other firms is sufficiently large. Otherwise, when vertical linkages are weak ($\alpha \in (0, \alpha_1)$), the stronger competition that must be faced by domestic firms from firms exporting from the other country, will reduce the cut-off level a_D and the number of producing firms N_D. When linkages among firms are strong (that is when $\alpha \in (\alpha_1, 1])$, increases in ϕ are associated with increases in the price index P_M, which result in increases in the cost of intermediates reducing, therefore, both the range of cost parameter α for which firms are able to survive in the domestic market (that is, a_D decreases) and the number of firms producing in the domestic market N_D.

Let us now complete our analysis by looking at the characteristics of the exporting firms. The cut-off a_X for exporting firms can be obtained by substituting a_D from (35) into (31), and it can be readily shown that the cut-off a_X depends on the size of the economy, L, and on the share of consumption devoted to manufactured, μ, only if there are vertical linkages at work (that is, only if $\alpha > 0$). Changes in the level of market integration represented by changes in ϕ affect the cut-off a_X. Specifically, we compute the elasticity of the cut-off a_X with respect to the freeness of trade ϕ, which is equal to

$$\frac{\partial a_X}{\partial \phi}\frac{\phi}{a_X} = \frac{\gamma + \alpha\Omega\,(\sigma - 1)}{(\Omega + 1)\,(\sigma - 1)\,\gamma}$$

where $\Omega \equiv \phi^{\beta}\left(\frac{f_X}{f_D}\right)^{1-\beta}$ is a measure of the openness to trade of the economy bounded between zero and unity – with openness Ω rising from zero when the economy is perfectly closed, to 1 when the economy is perfectly open. The elasticity $\frac{\partial a_X}{\partial \phi}\frac{\phi}{a_X}$ is always positive if $\alpha \in (0, \alpha_1)$ because $\gamma > 0$, while if $\alpha \in (\alpha_1, 1)$ – that is if $\gamma < 0$ – it is positive (negative) only if $\Omega < \Omega_0$ $(\Omega > \Omega_0)$, with $\Omega_0 \equiv \frac{-\gamma}{\alpha(\sigma-1)} < 1$.

Moreover, substituting N_D from (36) in (32), we derive the number of exporting firms N_X, and, hence, the elasticity of N_X with respect to the freeness of trade ϕ, which is given by

$$\frac{\partial N_X}{\partial \phi}\frac{\phi}{N_X} = \frac{\kappa\left[\gamma + \alpha\Omega\,(\sigma - 1)\right]}{(\Omega + 1)\,(\sigma - 1)\,\gamma} = \kappa\frac{\partial a_X}{\partial \phi}\frac{\phi}{a_X}$$

As for the previous case, we notice that the elasticity of N_X with respect to ϕ is always positive when $\alpha \in (0, \alpha_1)$, while when if $\alpha \in (\alpha_1, 1)$ it is positive (negative) if $\Omega < \Omega_0$ $(\Omega > \Omega_0)$.

We also point out that the welfare level of individuals living in these open economies crucially depends on the level of economic integration. In particular, when integration takes place reducing trade cost levels, welfare increases only if vertical linkages are not too strong (that is $\alpha \in (0, \alpha_1)$) because in this case the price index P_M decreases, otherwise, a larger level of economic integration associated to strong linkages ($\alpha \in (\alpha_1, 1)$) results in a lower welfare level, given that the price index P_M increases. Hence, the overall impact of a freer trade in the presence of vertical linkages *is not* unambiguously positive.

To complete our analysis, we compute the number of varieties sold in each economy that, making use of $N_X = \left(\phi\frac{f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}} N_D$, is given by $N = N_D + N_X =$

$\left[1+\left(\phi\frac{f_D}{f_X}\right)^{\frac{\kappa}{\sigma-1}}\right]N_D$. It can, then, be readily seen that if L changes, N increases (decreases) when N_D increases (decreases). Moreover, using the solution for N_D, we compute the elasticity of the number of varieties sold in each country with respect to the freeness of trade, that is

$$\frac{\partial N}{\partial \phi}\frac{\phi}{N}=\frac{f_X\kappa\left(\alpha\sigma-\sigma+1\right)+f_D\left[\gamma+\alpha\Omega\left(\sigma-1\right)\right]}{\gamma}\frac{\beta\Omega}{\left(f_X+\Omega f_D\right)\left(\Omega+1\right)}$$

Hence, this elasticity is unambiguously positive if $\alpha\in(\alpha_{0,}\alpha_1]$, and negative if $\alpha\in(\alpha_1,1]$. Otherwise, if $\alpha\in(0,\alpha_0)$, its sign depends on the value of f_X: more precisely, the elasticity is positive if $f_X<f_{X_0}$, and negative if $f_X>f_{X_0}$.

In summary, let us observe that the final effects produced by changes in the size of the economy, L, or in the level of economic integration, ϕ, can be explained by the interplay of substantially three forces that are at work in this model, which, following the New Economic Geography literature, we have identified as the 'market access' effect, the 'cost of living/producing' (or price index) effect and the 'market crowding' effect. Specifically, according to these three effects, *an increase in the size of the economy or in the level of economic integration* will, respectively, tend to: (i) increase the sales of firms and, in turn, their operating profits, fostering an increase in both the number of firms surviving in the domestic market, N_D, and of those exporting, N_X, and softening competition for firms producing for both markets, that is increasing both a_D and a_X ('market access' effect); (ii) make more varieties available to be used as intermediates (and consumption) goods and, therefore, reduce the price index, in this way softening competition in both markets and favouring the increase of the number of firms producing for both markets ('cost of living/producing' effect); (iii) increase the demand for intermediates, pushing upward their prices, and, consequently, fostering the reduction in the number of surviving firms and toughening competition pushing down the two cutoff levels a_D and a_X ('market crowding' effect). Hence, we notice that while the first two channels tend to soften competition and to promote the survival of firms, the third one acts in the opposite direction, and that, in our setup, the final prevailing effects depend on the strength of vertical linkages related to the other parameters of the model.

Finally, let us notice that the graphics plotted in Figure 2 used in previous Section can also be used to describe the effects of changes in L (and eventually in ϕ) on a_D in the case in which trade is costly and explain the puzzle in the work by Bernard, Jensen and Schott (2006) highlighted by Tybout (2006). If increases in the level of economic integration takes place in a range of L or of ϕ for which we observe that $a_D=a_M$, we can explain the absence of a substantial response of domestic market share by firms to falling trade costs.

4 Conclusions

In this work we have been able to highlight a *new role that forces generated by vertical linkages among firms play in international trade models with monopolis-*

tic competition: that is, backward and forward linkages not only contribute to determine the spatial distribution of firms resulting from the interaction among agglomeration and dispersion forces in New Economic Geography models, but *they also alter the competitive pressures in the economy when firms are heterogeneous in their productivity levels, therefore affecting the process of selection among them caused by international integration, which has not univocal effects on the welfare level*.

In particular, when trade costs are negligible we show that: (i) relatively weak (strong) vertical linkages soften (toughens) competition within the economy, with respect to the case in which they are absent when the economy's size is relatively large, while they toughens (soften) competition when the economy is relatively small; (ii) increases in the size of the economy when vertical linkages are relatively weak (strong) soften (intensifies) competition and make it less difficult (more difficult) to survive for less productive firms. These types of findings are confirmed when trade in manufactures is costly. Moreover, in this latter case we find that also changes in the level of economic integration have relevant selection effects. Specifically, vertical linkages can either strengthen or soften competition according on their size: when, for given values of the elasticity of demand between varieties and the shape parameter of the Pareto distribution, they are relatively weak or strong, increases in the level of economic integration will make it more difficult to survive for less productive firms in the domestic market, while a smaller level of efficiency will be required to export to the foreign market (provided that the overall measure of trade openness is not too large, because, otherwise, the opposite will happen for exporting firms). On the contrary, when vertical linkages are intermediate, increases in the freeness of trade will make it easier to survive for less productive firms in both the domestic and the foreign market. We have also shown that these ranges for the parameter that represents the strength of vertical linkages depend on the values of the elasticity of substitution between varieties and on the shape parameter that influences the relative number of high cost firms in the Pareto distribution.

Thus, we can say that larger and/or more integrated markets experience two kinds of effects at work with vertical linkages: one acting trough the demand linkage and the other through the cost linkage. Specifically, the demand (backward) linkage makes, when the economies become more integrated, the demand increase for all firms producing in an economy: that is, the demand rises not only for firms that are more efficient and exports, but also for those that are less efficient and produce only for domestic consumers and firms, therefore tending to make it easier to survive. On the other side, the existence of a cost (forward) linkage captures the fact that, if the effect of a larger demand is to make the price index of manufactured goods increase, all firms will experience an increase in the cost of production, because of the increase of the price of intermediates: this does not allow less efficient firms to survive in the domestic market, while relatively less efficient firms could find it easier to export, but only provided that the level of overall openness of the economy is sufficiently small. In any case, we have shown that the final outcomes of the selection process and the changes in the welfare levels determined by an increase in the level of international integration depend on how vertical linkages interact with all other relevant factors considered in the model.

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Appendix A

Using (10) and (13), expression (12) becomes

$$\frac{\mu L + N\alpha P_M^\alpha f_D + \alpha P_M^\alpha \int\limits_0^N a(j)q(j)dj}{\sigma N} = P_M^\alpha f_D + f_I \frac{a_M^\kappa}{a_D^\kappa} \tag{37}$$

It can be readily verified from (9), (8) and (10), that $a(j)q(j) = \frac{\sigma-1}{P_M^\alpha}\pi(j)$. Thus, the free entry condition (37) can be rewritten as follows

$$\frac{\mu L + N\alpha P_M^\alpha f_D + \alpha\left(\sigma-1\right)N\int_0^{a_D}\pi(a)dG_D(a)}{\sigma N} = P_M^\alpha f_D + f_I \frac{a_M^\kappa}{a_D^\kappa} \tag{38}$$

Making use of (11), (9) and of (10) evaluated both at the cut-off level a_D and in general at a, we can rewrite $\pi(a)$ as

$$\pi(a) = (\frac{a}{a_D})^{1-\sigma} P_M^\alpha f_D \tag{39}$$

and substitute this expression for $\pi(a)$ into (38) to rewrite the free entry condition as follows

$$\frac{\mu L}{N} + \alpha P_M^\alpha f_D + \alpha P_M^\alpha f_D \frac{\kappa\left(\sigma-1\right)}{\kappa-\sigma+1} = \sigma\left(P_M^\alpha f_D + f_I\frac{a_M^\kappa}{a_D^\kappa}\right) \tag{40}$$

Moreover, using (10) evaluated at the cut-off a_D together with (11) and (9), we obtain that

$$\frac{\left(P_M^\alpha\right)^\sigma f_D \sigma P_M^{1-\sigma}}{(a_D\frac{\sigma}{\sigma-1})^{1-\sigma}N} = \frac{\mu L}{N} + \alpha P_M^\alpha f_D + \alpha P_M^\alpha f_D \frac{\kappa\left(\sigma-1\right)}{\kappa-\sigma+1} \tag{41}$$

Finally, substituting (9) into the price index $P_M = N^{\frac{1}{1-\sigma}}\left(\int_0^{a_D} p(a)^{1-\sigma}dG_D(a)\right)^{\frac{1}{1-\sigma}}$, we get

$$P_M^{1-\alpha} = \frac{\sigma}{\sigma-1}N^{\frac{1}{1-\sigma}}a_D\left(\frac{\kappa}{\kappa-\sigma+1}\right)^{\frac{1}{1-\sigma}} \tag{42}$$

Hence, we have a system of three equations (40), (41) and (42) in three unknowns, P_M, N and a_D.

Appendix B

Substituting (34) and making use of (23), we can rewrite expression (33) as follows

$$\frac{\mu I+\alpha P_M^\alpha\left(N_D f_D+N_X f_X+\int\limits_0^{N_D} a(j)q_D(j)dj+\int\limits_0^{N_X} a(j)q_X(j)dj\right)}{\sigma N_D}=P_M^\alpha\left(f_D+\frac{G[a_X]f_X}{G[a_D]}\right)+\frac{f_I}{G[a_D]} \tag{43}$$

Moreover, it can be readily verified from (9), (24) and (29), that $a(j)q(j)=\frac{\sigma-1}{P_M^\alpha}\pi_D(j)$; and from (15), (25) and (29), that $a(j)q_X(j)=\frac{\sigma-1}{P_M^\alpha}\pi_X(j)$. Substituting these into (43), the free entry condition becomes

$$\frac{\mu I+\alpha P_M^\alpha(N_D f_D+N_X f_X)+\alpha(\sigma-1)\left(\int\limits_0^{N_D}\pi_D(j)dj+\int\limits_0^{N_X}\pi_X(j)dj\right)}{\sigma N_D}=P_M^\alpha\left(f_D+\frac{G[a_X]f_X}{G[a_D]}\right)+\frac{f_I}{G[a_D]} \tag{44}$$

It can be shown from (29) and (30) that operating profits are such that

$$\pi_D(j)=\left[\frac{a(j)}{a_D}\right]^{1-\sigma}P_M^\alpha f_D \quad \text{and} \quad \pi_X(j)=\left[\frac{a(j)}{a_X}\right]^{1-\sigma}P_M^\alpha f_X$$

Hence, total operating profits from domestic sales are

$$\int\limits_0^{N_D}\pi_D(j)dj=N_D\int_0^{a_D}\pi_D(a)dG_D(a)=P_M^\alpha f_D N_D\frac{\kappa}{\kappa-\sigma+1} \tag{45}$$

while total operating profits from exports are

$$\int\limits_0^{N_X}\pi_X(j)dj=N_X\int_0^{a_X}\pi_X(a)dG_X(a)=N_X P_M^\alpha f_X\frac{\kappa}{\kappa-\sigma+1} \tag{46}$$

Substituting (45) and (46) into (44), and using the Pareto distribution, we rewrite the free entry condition as follows

$$\frac{\mu I+\alpha P_M^\alpha\frac{\beta\sigma-1}{\beta-1}(N_X f_X+N_D f_D)}{\sigma N_D}=P_M^\alpha\left(f_D+\frac{a_X^\kappa}{a_D^\kappa}f_X\right)+\frac{a_M^\kappa}{a_D^\kappa}f_I \tag{47}$$

with the condition on $\beta\equiv\frac{\kappa}{\sigma-1}>1$ required to have a positive value for the price index P_M (cfr. Baldwin and Forslid, 2004). Hence, we use the free entry condition (47), the two cut off conditions derived substituting $\pi_D(a_D) = P_M^\alpha f_D$ and $\pi_X(a_X) = P_M^\alpha f_X$ into (30), the price index (17), (31) and (32), to find a_D, P_M, N_D, a_X and N_X.[25]

The value of the cut-off a_D is that reported in the text by expression (35), while the price index, P_M, and the number of firms producing for the domestic market, N_D, can be, respectively, expressed as a function of this cut-off as follows

$$P_M = \left[\frac{1}{1 - \alpha \frac{\sigma\beta - 1}{\sigma\beta}} \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{\mu L}{\sigma f_D} \right]^{\frac{1}{\alpha\sigma - \sigma + 1}} a_D^{\frac{1-\sigma}{\alpha\sigma - \sigma + 1}} \tag{48}$$

and

$$N_D = \frac{\left(\frac{1}{1-\alpha \frac{\sigma\beta-1}{\sigma\beta}} \frac{\mu L}{\sigma f_D} \right)^{\frac{(\sigma-1)(1-\alpha)}{(\sigma-1)(1-\alpha)-\alpha}}}{\frac{\beta}{\beta-1} \left(1 + \phi^\beta \left(\frac{f_X}{f_D} \right)^{1-\beta} \right)} \left(a_D \frac{\sigma}{\sigma - 1} \right)^{\alpha \frac{\sigma-1}{\alpha\sigma - \sigma + 1}} \tag{49}$$

[25] Let us notice that when vertical linkages are absent, that is when $\alpha = 0$, we fall back in the setup described by Baldwin and Forslid (2004), with $a_D = a_M \left[\frac{(\beta-1)(f_I/f_D)}{1+\Omega} \right]^{\frac{1}{\kappa}}$, $N_D = \frac{\mu L(\beta-1)}{\beta\sigma f_D} \frac{1}{(1+\Omega)}$ and $P_M = a_D \frac{\sigma}{\sigma-1} \left(\frac{\sigma f_D}{\mu L} \right)^{\frac{1}{\sigma-1}}$ with $\Omega = \phi^\beta \left(\frac{f_X}{f_D} \right)^{1-\beta}$.

	$1 < \alpha < \alpha_1$	$\alpha_1 < \alpha < 1$
If $L \uparrow$	$P_M \downarrow$	$P_M \uparrow$
	$a_D \uparrow$	$a_D \downarrow$
	$N \uparrow$	$N \downarrow$
	$W \uparrow$	$W \downarrow$

Table 1.*a*. Effects produced by changes in L when trade is costless

	$1 < \alpha < \alpha_1$	$\alpha_1 < \alpha < 1$
If $L \uparrow$	$P_M \downarrow$	$P_M \uparrow$
	$a_D \uparrow$	$a_D \downarrow$
	$N_D \uparrow$	$N_D \downarrow$
	$W \uparrow$	$W \downarrow$
	$a_X \uparrow$	$a_X \downarrow$
	$N_X \uparrow$	$N_X \downarrow$
	$N \uparrow$	$N \downarrow$

Table 1.*b*. Effects produced by changes in L when trade in manufactures is costly

	$0 < \alpha < \alpha_0$	$\alpha_0 < \alpha < \alpha_1$	$\alpha_1 < \alpha < 1$
If $\phi \uparrow$	$P_M \downarrow$	$P_M \downarrow$	$P_M \uparrow$
	$a_D \downarrow$	$a_D \uparrow$	$a_D \downarrow$
	$N_D \downarrow$	$N_D \uparrow$	$N_D \downarrow$
	$W \uparrow$	$W \uparrow$	$W \downarrow$
	$a_X \uparrow$	$a_X \uparrow$	$a_X \begin{cases} \uparrow \text{ if } \Omega < \Omega_0 \\ \downarrow \text{ if } \Omega > \Omega_0 \end{cases}$
	$N_X \uparrow$	$N_X \uparrow$	$N_X \begin{cases} \uparrow \text{ if } \Omega < \Omega_0 \\ \downarrow \text{ if } \Omega > \Omega_0 \end{cases}$
	$N \begin{cases} \uparrow \text{ if } f_X < f_{X_0} \\ \downarrow \text{ if } f_X > f_{X_0} \end{cases}$	$N \uparrow$	$N \downarrow$

Table 2. Effects produced by changes in ϕ

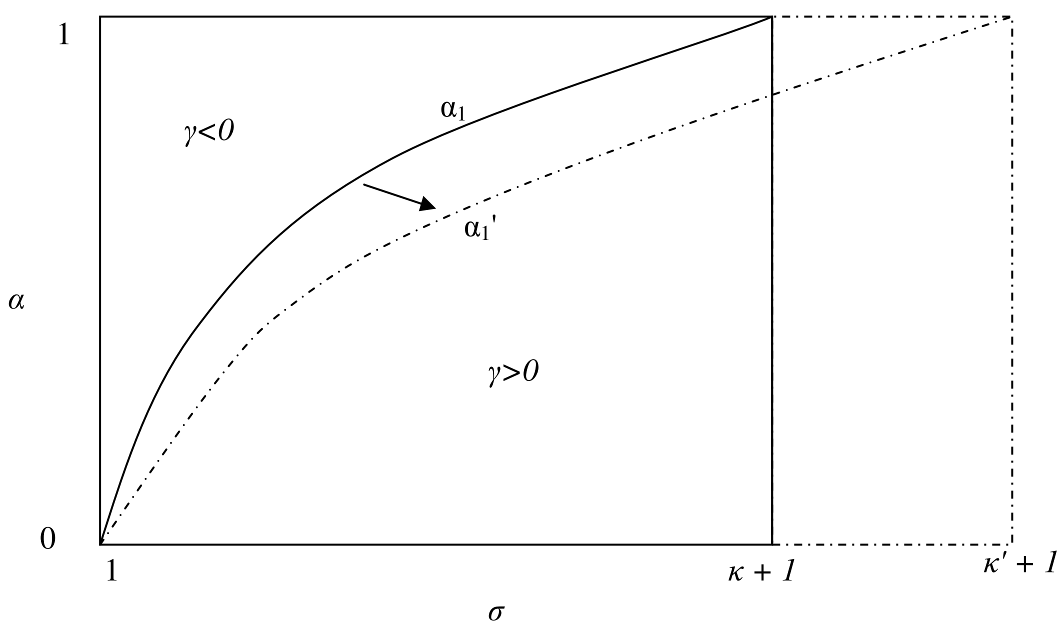

Figure 1. The sign of γ and changes in α_1 if $\kappa' > \kappa$.

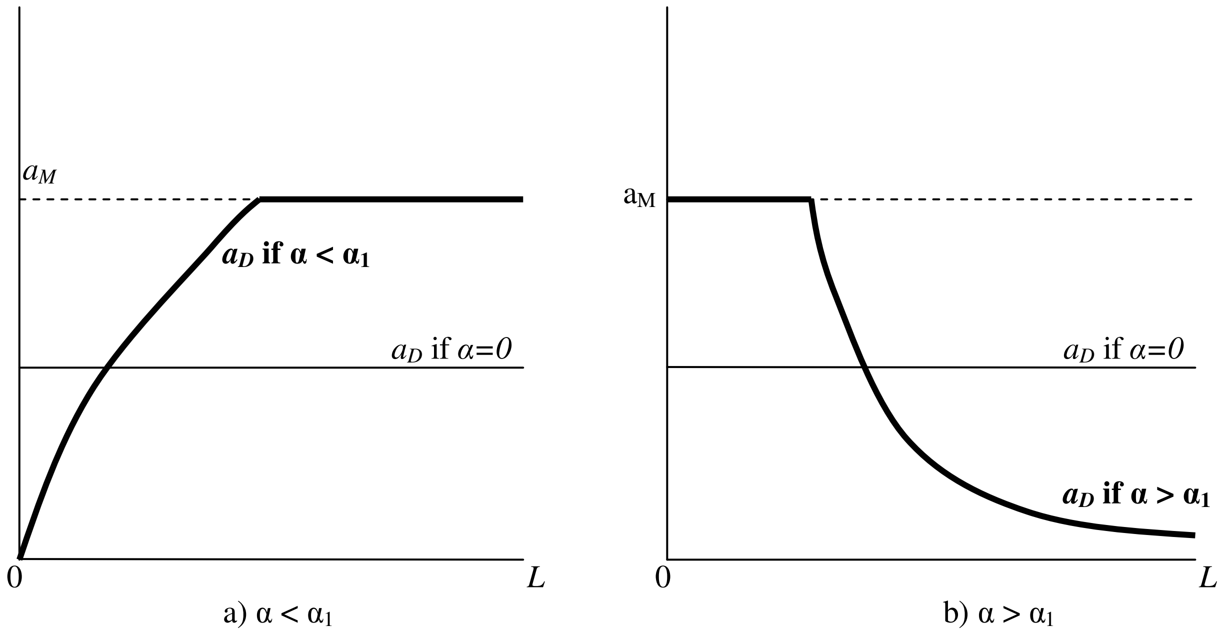

Figure 2. The cut-off a_D as a function of the size of the economy L.

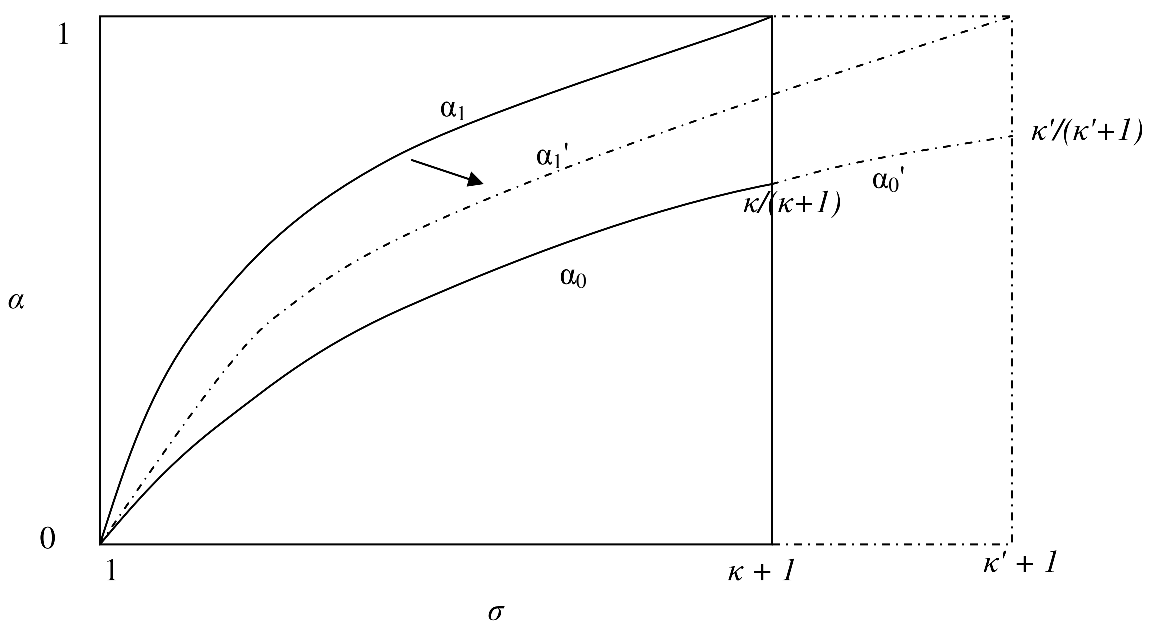

Figure 3. The values of α_0 and α_1 and their changes (discontinuous curves) if $\kappa' > \kappa$.