Intercorporate guarantees, leverage and taxes*

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Abstract

This paper characterizes optimal intercorporate guarantees, under the classical trade-off between bankruptcy costs and taxation. Conditional guarantees, allowing the provider to maintain limited liability vis-à-vis the beneficiary, maximize joint value. They indeed achieve the highest tax savings net of default costs. We provide conditions ensuring that - at the optimum - guarantees increase total debt, which bears mostly on the beneficiary. This difference in optimal leverage between the provider and the beneficiary explains why optimal conditional guarantees (i) generate value independently of cash flow correlation (ii) are unilateral rather than mutual, at least for moderate default costs (iii) dominate the unconditional ones, that are embedded in mergers, at least when firms have high cash-flow correlation. We also endogenize the choice of the guarantor, showing that it has higher proportional bankruptcy costs and lower tax rates.

Keywords: debt, taxes, bankruptcy costs, limited liability, capital structure, subsidiary, groups, mergers.

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1 Introduction

Corporations routinely guarantee the debt obligations of their Subsidiaries[1]. Despite being so common, it is not clear why such guarantees exist: while providing gains to the Subsidiary, they generate a cost for the Parent company, so that net value creation might be zero. At most one expects them to increase joint value to the extent that firm cash flows are less than perfectly correlated, because of diversification gains. Even in this case, it remains unclear whether any given firm should both provide and receive support, or specialize in either being a guarantor or a beneficiary. Another set of issues emerges if specialization obtains: which are the characteristics of the firm providing support relative to that receiving it? And how much debt should each one raise?

This paper analyzes intercorporate guarantees, that to our knowledge have not been systematically addressed before, simply assuming - as in Leland (2007) - that debt provides a tax shield but increases the likelihood of bankruptcy, absent any information or incentive problems. Such absence allows to deal with guarantees that are credible from the lenders' point of view, for instance because they are enforceable by law.

Our focus will be on conditional guarantees, that allow the supporting Parent to retain limited liability with respect to the supported Subsidiary's lenders. Corporate limited liability is indeed the norm in major jurisdictions, according to the legal literature[2]. Moreover, companies tend to terminate support to a struggling subsidiary when its needs are large with respect to group equity or when group profits turns negative[3]. Knowing this, rating agencies take intercorporate guarantees into consideration when evaluating the risk that a subsidiary will default (Standard & Poor's (2001)).

Our main result is that such guarantees increase the joint value of Parents and Subsidiaries, irrespective of cash flow correlation**.** Indeed, the Subsidiary is able to increase its debt financing and, as a consequence, tax savings net of default costs. We provide analytical conditions for its debt to be higher than total debt of the two firms in the absence of any guarantee, and for the Parent to be unlevered - so as to increase its ability of providing support.

The guarantee described so far is unilateral, while a priori support could be mutual. We show that any given firm specializes in either providing or receiving support, at least for moderate proportional default costs. The intuition is that unilateral guarantees permit to save on default costs and tax payments, thanks to both rescue and a different leverage in the Parent and its Subsidiary. Mutual guarantees still save on default costs, but create a tension because each firm

[1] See Bodie and Merton (1992) for US firms and Chang and Hong (2000) for Korean chaebols. Deloof and Vershueren (2006) analyze contractual guarantees in Belgian groups, while Khanna and Palepu (2000) observe that Indian group firms assist each other in times of financial distress.

[2] See f.i. Blumberg (1989). The exception to this rule is the "piercing of the corporate veil" that requires to prove both the lack of separate existence of the subsidiary and parent company's conduct "akin to fraud".

[3] See Dewaelheyns and Van Hulle (2006) and Gopalan et al. (2007) for non-financial firms, Herring and Carmassi (2009) for financial groups.

should at the same time increase its debt - since it is guaranteed - and decrease it - in its quality of guarantor. This tension results in lower total debt and tax savings than with unilateral guarantees. If default costs are moderate, tax incentives prevail, making unilateral guarantees preferable.

The above conclusions on the properties of guarantees hold when Parent and Subsidiaries do not differ in default costs and tax rates, as well as when they do. Our next set of results concerns the characterization of Parents and Subsidiaries. The guarantor should be the firm with higher bankruptcy costs (lower tax rates), because its optimal debt - and hence bankruptcy loss (foregone tax savings) - is lower.

So far we have been discussing guarantees between two separately incorporated activities. These, we will argue in section 4, are common in business groups and private equity funds and provide a rationale for their diffusion. In the real world we also observe conglomerate Mergers, in which activities' cash flows are pooled so that they become jointly liable vis-à-vis lenders. We think of this as an unconditional guarantee being provided by each division to the other. Leland (2007) shows that a purely financial Merger destroys value, when the loss of limited liability exceeds gains from higher optimal debt - for instance when the high correlation between activities' cash flows limits diversification opportunities. Below we show that conditional guarantees - of the Parent-Subsidiary type - are value-enhancing relative to the M case in such situations. This result highlights that, contrary to intuition, conditional guarantees work even when cash flow diversification is limited, to the extent that the Parent is free to have lower debt and therefore higher cash-flows available for supporting its levered Subsidiary.

Last but not least, we examine some real-world counterparts of our results. In our stylized set-up, debt of the Parent-Subsidiary structure is higher than total debt of the two firms without guarantee. In reality, group affiliates appear to rely on debt more than comparable Stand Alone firms (Masulis et al. (2008), Deloof et al. (2006)). This is also true in the case of leveraged buy-outs, a situation where our assumption of no agency costs vis-à-vis lenders applies reasonably well. In our model, optimizing the tax-bankruptcy trade-off results in an unlevered Parent and a highly levered Subsidiary. In the real world, the private equity fund is unlevered while portfolio firms - that have moderate leverage when incorporated as (non guaranteed) public companies - display debt levels which dramatically reduce their tax burden (Kaplan (1989)). The interplay betweeen tax savings and guarantees lies at the core of thin capitalization rules that are enforced in several countries[4] (including Australia, China, Germany, Italy, the Netherlands, UK and the US). Since our theoretical findings are

[4] Her Majesty Revenue and Customs explains that "thin capitalisation commonly arises where a company is funded by another company in the same group. It can also arise where funding is provided to a company by a third party, typically a bank, but with guarantees or other forms of comfort provided to the lender by another group company ...(typically the overseas Holding company). The effect of funding a UK company... with excessive intra-group or Parentally- guaranteed debt is, potentially, excessive interest deductions. It is the possibility of excessive deductions for interest which the UK legislation on thin capitalisation seeks to counteract."

broadly consistent with observation, we will argue below that value creation due to intercorporate guarantees is able to explain the diffusion of Parent-Subsidiary structures.

The rest of the paper is organized as follows. Section 2 clarifies our contribution relative to previous theoretical literature. Section 3 lays out the model and analyzes how value and optimal debt change due to guarantees, providing the main results summarized above. In section 4 we discuss our findings on firm scope. Section 5 concludes. The proofs are in the on-line Appendix.

2 Previous literature

Our paper is closely related to the literature on non-synergistic firm combinations.

Lewellen (1971) argues that merging imperfectly correlated Stand-Alone activities has a coinsurance effect that, by reducing the risk of default, increases debt, tax savings and value for the firms. We argue that these effects obtain with an intercorporate conditional guarantee, irrespective of correlation, and we provide analytical conditions for the result.

Leland (2007) highlights that merging two different companies may actually reduce value, because the riskier firm may drag the safer one in distress. This is due to the loss of limited liability and occurs also when firms are unlevered, if operating cash flows can be negative - as in Sarig (1977) and Scott (1985). It follows that splitting a merger may create value, despite the loss of coinsurance, as spinned-off firms are able to increase debt upon gaining back their limited liability.

We take Leland's reasoning one step forward, and argue that firms can exploit the coinsurance effect without giving up their limited liability, thanks to a conditional guarantee. Importantly, firms need neither differ in operating cash flows, tax or default cost rates nor have less than perfectly correlated cash flows in order to exploit coinsurance. This obtains because of debt diversity, i.e. a level of debt which is optimally lower in the guarantor than in the beneficiary, allowing the former to preserve cash flows for rescue. To make these points precise, we generalize Leland's structural model to allow for PS and for cash flows which have any (even non-Gaussian) distribution function.

The relevance of limited liability links our study to previous work emphasizing this trait of business groups, as Cestone and Fumagalli (2005) and Bianco and Nicodano (2006). Both papers focus on incentive issues, instead of a tax-bankruptcy trade-off, and posit exogenous debt needs. Thus, neither paper uncovers the first order effects on debt, tax savings and ultimately firm value that lie at the heart of the current research.

Emphasis on tax benefits connects our paper to a large literature on tax avoidance and corporate finance (see the survey in Graham (2003)), which focusses on arbitrage with unequal tax rates. For instance, multinational groups raise more debt from subsidiaries in high-tax countries (Desai et al. (2004), Huizinga et al. (2008)). In our model, guarantees minimize the tax burden -

net of default costs - even with equal tax rates across firms. Thus, we point out a powerful tax avoidance tool which, to our knowledge, has not been analyzed yet.

Last but not least, our paper owes to Merton (1977), who recognizes that the provision of a guarantee for all the debt of a company - with no corporate limited liability - is akin to the issue of a put option on that company assets, and prices it accordingly. We observe that a conditional guarantee produces savings akin to an option on the Subsidiary's cash flows. We price it accordingly, taking into consideration its effects on optimal capital structure.

3 The model

We consider a no arbitrage environment with two dates $t = \{0, T\}$. An entrepreneur owns two production units, generating a random operating cash flow[5] X_i $(i = 1, 2)$ at time T. The owner can "walk away" from negative cash flows thanks to personal limited liability. He receives only the positive part of X_i, denoted as X_i^+, net of taxes at the rate $0 < \tau < 1$. The unlevered firm value is therefore

$$V_{0i} = (1 - \tau)\phi \mathbb{E} X_i^+, \tag{1}$$

where $\mathbb{E}$ is the risk-neutral expectation operator, ϕ is the discount factor, $\phi \triangleq (1 + r_T)^{-1}$, and r_T is the riskless rate for the time span T.

At time zero the entrepreneur can lever up each firm by issuing a zero-coupon debt, with value D_{0i}. The debt principal, $N_i \geq 0$, is due at T. We assume that there is an incentive to issue debt, as interest is a deductible expense. Taxable income is the operating one net of interests, $X_i - (N_i - D_{0i})$, when positive. The tax rate τ applies when cash flows are greater than the tax shield, X_i^Z:

$$X_i^Z \triangleq N_i - D_{0i}. \tag{2}$$

Operating cash flows, net of tax payments, are therefore equal to:

$$X_i^n = X_i^+ - \tau(X_i - X_i^Z)^+ \tag{3}$$

Issuing debt reduces the tax burden from $\tau\phi\mathbb{E}X_i^+$ to[6]

$$T_i \triangleq \tau\phi\mathbb{E}(X_i - X_i^Z)^+ \tag{4}$$

Issuing debt may however generate costs. Similarly to Merton (1974), default occurs when net operating cash flows are smaller than the principal, namely

[5] X_i is a continuous random variable, endowed with the first two moments ($X_i \in L^2$), that may take both negative and positive values. Denote as F_i its distribution function; this means $0 < F_i(0) < 1$. We also assume that the joint density of the cash flows X_i - denoted as $f(x, y)$ - exists and is positive on the whole plane. This rules out the cases of maximal linear correlation, which can be treated numerically.

[6] The tax burden is a call option on X_i with strike X_i^Z. The call is decreasing in the principal, since the strike is shown in Appendix A to be increasing in it.

$X_i^n < N_i$. This default triggering condition can be restated as $X_i < X_i^d$, where the default threshold X_i^d is defined as:

$$X_i^d \triangleq N_i + \frac{\tau}{1-\tau} D_{0i} = \frac{N_i - \tau X_i^Z}{1-\tau} \tag{5}$$

In the event of default, we assume that a fraction $0 < \alpha < 1$ of positive operating cash flows is lost. Bondholders will receive a fraction $1 - \alpha$ of after-tax operating cash flow, when this is positive. There is then a trade-off between the dissipative default costs, αX_i^+, and the tax savings generated by debt[7].

The entrepreneur chooses the face value of debt in the two activities, N_i, given this tax-bankruptcy trade-off, so as to maximize the time-zero combined value of the two units. He can also allow one or both units to guarantee the lenders of the other activity.

The combined value of the two levered firms, $\nu_{01} + \nu_{02}$, is given by the sum of their equities and debts, $E_{0i} + D_{0i}$, determined as the present value of the future expected payoffs to equity and bond holders, respectively. Tedious algebra shows that, for each and every guarantee, the levered values ν_{0i} coincide with the present value of (positive) cash flows, less taxes T_i and default costs[8] DC_i:

$$\nu_{0i} = \phi \mathbb{E} X_i^+ - T_i - DC_i \tag{6}$$

Since the first term is independent of leverage, maximizing the levered value is equivalent to minimizing the tax burden plus default costs:

$$\min_{N_i \geq 0} \sum_{i=1}^{2} (T_i + DC_i) \tag{7}$$

The key point for understanding this problem - and how it differs across guarantees - is the following. The market value of debt, D_{0i}, depends on the guarantees, for any given principal. Since D_{0i} enters both into the tax shield X_i^Z and default threshold X_i^d, these differ across guarantees, even for the same principal. This makes the tax burden T_i vary across guarantees. Default costs differ both because of the thresholds and because the expression for DC_i is directly affected by guarantees, that we now describe.

3.1 Conditional guarantees

This section models a unilateral guarantee from a Parent (P) to its Subsidiary (S). It studies its properties and assesses its effects on lenders' payoffs, default costs and value using as benchmark the case of no guarantee. Throughout, we posit that the cash flows of P(S) are equal to the ones of firm 1 (2).

[7] We assume (as in Leland) that the firm receives no tax refunds, while companies may carry forward some losses in the real world.

[8] In the absence of guarantees, the proof is given in Leland (2007). In the presence of them, it can be given only once appropriate expressions for DCs are introduced (see later sections).

The key feature of this guarantee is conditionality, which is made possible by corporate limited liability, as we argued in the introduction. P provides support by injecting in S the cash it needs in order to stay solvent, $N_S - X_S^n$, if two conditions hold. First, the cash flows of the defaulting company must be non negative, else the guarantor would bear an operating loss that it can avoid by using its limited liability:

$$0 < X_S < X_S^d \tag{8}$$

Second, the cash flows of the supporting company must be sufficient to honour both its own and the subsidiary's debt obligations[9]:

$$X_P^n - N_P > N_S - X_S^n \tag{9}$$

The second condition can be written as $X_P > h(X_S)$, where

$$h(X_S) \triangleq \begin{cases} X_P^d + \frac{N_S}{1-\tau} - \frac{X_S}{1-\tau} & X_S < X_S^Z \\ X_P^d + X_S^d - X_S & X_S > X_S^Z \end{cases} \tag{10}$$

The transfer from P to S, associated with a conditional guarantee, is therefore equal to:

$$(N_S - X_S^n)\mathbf{1}_{\left\{0<X_S<X_S^d, X_P>h(X_S)\right\}} \tag{11}$$

where $1_{\{\bullet\}}$ is the indicator function and its argument represents the cash-flow combinations which lead to rescue.

Note that we can define the case of no guarantee as $h(Xs) \to +\infty$ instead of (10). In this case, debt holders receive the minimum between after-tax operating profits, net of default costs, and the face value of debt, N_i. The corresponding debt value is equal to the expected present value of these two sums conditional on cash flows respectively exceeding or falling below the default threshold: [10]

$$D_{0i}(N_i) = \phi\mathbb{E}\left[\left[X_i(1-\alpha) - \tau(X_i - X_i^Z)^+\right]\mathbf{1}_{\left\{0<X_i<X_i^d\right\}}\right] + \tag{12}$$
$$+\phi N_i \mathbb{E}\left[\mathbf{1}_{\left\{X_i>X_i^d\right\}}\right] \qquad i = 1, 2$$

With the guarantee, the debt value of S becomes:

$$D_{0S}(N_P, N_S) = \phi\mathbb{E}\left[\left[X_S(1-\alpha) - \tau(X_S - X_S^Z)^+\right]\mathbf{1}_{\left\{0<X_S<X_S^d, X_P<h(X_S)\right\}}\right] + \tag{13}$$
$$+\phi N_S \mathbb{E}\left[\mathbf{1}_{\left\{X_S>X_S^d\right\}} + \mathbf{1}_{\left\{0<X_S<X_S^d,\ X_P>h(X_S)\right\}}\right]$$

The first expectation refers to the case when S defaults and P does not support it because its own cash flow is insufficient. The second expectation takes into account that S is able to reimburse its debt either when it is solvent

[9] We keep the indices $i = 1, 2$ for cash flows without rescue, the indices $i = S, P$ for those with rescue.

[10] Here and below D_{0i} is defined implicitly - as a fixed point - since the tax shield and default thresholds X_i^Z and X_i^d depend on it.

on its own or thanks to P's transfer. Clearly, if $h(X_S)$ diverges the expression for debt collapses into (12).

What is the effect of a guarantee on default costs? Absent it, a firm defaults as soon as its gross cash flows fall short of the default threshold, $X_i < X_i^d$. As a consequence, default costs are equal to[11]:

$$DC_i(N_i) = \alpha\phi\mathbb{E}\left[X_i \ \mathbf{1}_{\left\{0<X_i<X_i^d\right\}}\right] \qquad\qquad i = 1,2 \tag{14}$$

With the guarantee, S pays default costs when rescue does not occur:

$$DC_S(N_P, N_S) = \alpha\phi\mathbb{E}\left[X_S\mathbf{1}_{\ \left\{0<X_S<X_S^d,\ \ X_P<h(X_S)\right\}}\right] \tag{15}$$

For any debt level of P and S, denote with Γ the savings in (expected, discounted) default costs of S with respect to the no-guarantee case:

$$\Gamma(N_P, N_S) \triangleq DC_2(N_S) - DC_S(N_P, N_S) = \alpha\phi\mathbb{E}\left[X_S\mathbf{1}_{\ \left\{0<X_S<X_S^d,\ \ X_P>h(X_S)\right\}}\right] \tag{16}$$

These savings are akin to an option on S's cash flows. The Appendix establishes several properties of Γ. It is non-increasing in P's debt. Indeed, for any joint cash flow distribution and any capital structure, lowering debt in P enlarges its ability to provide support, reducing default costs. Changes in S's debt generate less obvious effects. Raising N_S increases potential default costs, thus making the guarantee more valuable when rescue is successful. On the other hand, it makes rescue less likely. When debt in S diverges, the second effect dominates and the marginal value of savings in default costs is negative. Clearly, the guarantee has value only if S levers up. Under the standard convexity assumptions on the objective of minimization, (7), which we maintain throughout[12], the following holds:

Lemma 3.1 *If $T_P + DC_P + T_S + DC_S$ is convex in the principals N_P, N_S, the Subsidiary is optimally levered: $N_S^* > 0$.*

For any fixed principal N_P of P, this positive leverage of S makes $X_S^d > 0$. This, together with the assumption that the joint density of X_S and X_P is positive on the whole plane, ensures that the probability associated to rescue is non-null and $\Gamma(N_P, N_S^*) > 0$.

The Appendix shows that non-guaranteed firms are also levered, under a convex objective. Note that they coincide with the Stand Alone (SA) firms in Leland (2007).

[11] Default costs are a barrier call option on X_i with zero strike and barriers equal to zero and X_i^d. The call is increasing in the principal, since the upper barrier is increasing in it (see Appendix A).

[12] Numerical simulations of the Gaussian case - starting from Leland's parametrization - show that the assumption holds for principals up to the 95th percentile of cash flows.

3.1.1 Effects on value and leverage

This section first establishes that conditional guarantees create value with respect to no guarantees, absent any assumption on the correlation of the firms' cash flows. It then shows that, at the optimum, P is less levered than before offering the guarantee, since this increases its chances of being able to provide support. Last but not least, it proves that total optimal debt of the two companies is larger than without the guarantee.

In the light of the general expression (6), the combined value of the firms - which we denote as ν_{0PS} - is equal to:

$$\nu_{0PS}(N_P, N_S) = \phi \mathbb{E} X_P^+ - T_P - DC_P + \phi \mathbb{E} X_S^+ - T_S - DC_S \tag{17}$$

We call "value of the guarantee" the difference between this combined value and the sum of the two SA firms' values, $\nu_{0PS}(N_P, N_S) - \nu_{01}(N_1) - \nu_{02}(N_2)$. Theorem 3.1 shows that it is positive[13]:

Theorem 3.1 *Conditional guarantees are value increasing.*

Proof. For any given level of N_P and N_S, recall that $T_P(N_P) = T_1(N_P)$, $DC_P(N_P) = DC_1(N_P)$ and $T_S(N_S) = T_2(N_S)$, while $DC_2 - DC_S = \Gamma$. Substituting in (17) we have

$$\nu_{0PS}(N_P, N_S) = \nu_{01}(N_P) + \nu_{02}(N_S) + \Gamma(N_P, N_S) \tag{18}$$

The value of the guarantee is then equal to

$$\nu_{01}(N_P) + \nu_{02}(N_S) + \Gamma(N_P, N_S) - \nu_{01}(N_1) - \nu_{02}(N_2) \tag{19}$$

Suppose that both P and S maintain the optimal debt levels of non-guaranteed firms ($N_P = N_1^*, N_S = N_2^*$). Since $\nu_{01}(N_P) = \nu_{01}(N_1^*), \nu_{02}(N_S) = \nu_{02}(N_2^*)$, the value of the guarantee reduces to $\Gamma(N_1^*, N_2^*)$, which is positive since N_2^* is. A fortiori it is positive when the principals of P and S are optimized, $N_P = N_P^*, N_S = N_S^*$. ■

To understand this result, we split the optimal value of the guarantee into three components. The first term is the difference in the unlevered firm value, which is the "limited liability effect" defined in Leland (2007). The second one is the reduction in default costs due to rescue, when leverage - and hence the tax burden - is the same as in the SA case, i.e. $N_P = N_1^*, N_S = N_2^*$. The last term measures value creation associated with the possibility of levering up more thanks to the guarantee.

$$\nu_{0PS}\left(N_P^*, N_S^*\right) - \nu_{01}\left(N_1^*\right) - \nu_{02}\left(N_2^*\right) = \tag{20}$$

$$= \underbrace{\nu_{0PS}(0,0) - \nu_{01}(0) - \nu_{02}(0)}_{\text{limited liability effect}} + \underbrace{\Gamma\left(N_1^*, N_2^*\right)}_{\text{rescue effect}} + \underbrace{\nu_{0PS}\left(N_P^*, N_S^*\right) - \nu_{0PS}\left(N_1^*, N_2^*\right)}_{\text{leverage effect}}$$

[13] The theorem holds weakly if convexity is not required.

It is easy to show that the first term is zero, because there is no loss in limited liability when shifting from two SA firms to a PS structure. The proof of the above theorem shows that the second and, a fortiori, the sum of the last two terms, are positive.

The next theorem studies how debt levels in PS differ from their SA counterparts, arguing that the leverage effect is positive.

Theorem 3.2 *Under the convexity assumption, i) the Parent is optimally unlevered ($N_P^* = 0$); ii) the Subsidiary principal - and, a fortiori, the PS one - is higher than in two Stand Alone companies ($N_S = N_S^* + N_P^* > N_1^* + N_2^*$) if and only if the ratio of percentage default costs to the tax rate is bounded above by a constant Q.*

The first part of the theorem states that the guarantor must be unlevered, so as to maximize the effectiveness of the guarantee. Debt increases the tax shield whether it bears on P or S: however, shifting debt from P to S protects P's cash flows from lenders' claims when P is unable to cover S's repayment needs. This reduces P's default costs. As for the second part, tax savings increase in total debt, which optimally coincides with S's debt. This cannot increase too much, though, since a higher N_S reduces the ability of P to provide support, thus boosting default costs. The Q condition ensures that marginal tax gains exceed marginal default costs if S levers as much as two SA companies ($N_S = N_1^* + N_2^*$). As a consequence, S optimally levers up more than two SA firms.[14] Summing up, guarantees not only increase value, but they do so by increasing leverage, as we see in practice[15].

3.1.2 Mutual guarantees

So far we have been addressing the case of only one firm supporting the other. It is however possible for the entrepreneur to establish a mutual, but still conditional, guarantee. This section examines such an arrangement and shows that, with moderate default costs, a one-way guarantee dominates it.

A conditional mutual guarantee is composed of two guarantees: one from P to S - as above; and a second one from S to P - which is triggered by cash flows satisfying $0 < X_P < X_P^d,\ X_S > h(X_P)$. Default costs for S coincide with those obtained with a unilateral guarantee, namely (15). For P they have a symmetric expression.

Denote as Γ_m the overall savings in (expected, discounted) default costs which a mutual conditional guarantee provides with respect to no-guarantee:

$$\Gamma_m(N_P, N_S) = \alpha\phi\mathbb{E}\left[X_S 1_{\left\{0<X_S<X_S^d, X_P>h(X_S)\right\}} + X_P 1_{\left\{0<X_P<X_P^d, X_S>h(X_P)\right\}}\right] \tag{21}$$

[14] This holds in Leland's numerical case.

[15] We can also prove a weaker version of this theorem, showing that P is less levered than in the no-guarantee case, without necessarily being unlevered.

Note that they can be split into the values of the corresponding unilateral savings (from P to S, as in (16), and viceversa). Name the two parts $\Gamma_{12} \triangleq DC_S - DC_2 = \Gamma$ and $\Gamma_{21} \triangleq DC_P - DC_1$.

The expression for S's debt is unchanged with respect to the unilateral guarantee case described above, while debt of P becomes:

$$D_{0P}^{\Gamma_m}(N_P, N_S) = \phi\mathbb{E}\left[\left[X_P(1-\alpha) - \tau(X_P - X_P^Z)^+\right] 1_{\left\{0<X_P<X_P^d, X_S<h(X_P)\right\}}\right] + \\ +\phi P_S \mathbb{E}\left[\mathbf{1}_{\left\{0<X_P<X_P^d,\ X_S>h(X_P)\right\}} + \mathbf{1}_{\left\{X_P>X_P^d\right\}}\right] \tag{22}$$

Given these expressions, the Appendix proves that:

Theorem 3.3 *There exists a default cost level α^* below which unilateral guarantees are the only optimal guarantees.*

Why should the entrepreneur forego one of the two options to save on default costs? The intuition for this result is the following. Unilateral guarantees permit to save on tax payments net of default costs by concentrating debt in the beneficiary. With mutual guarantees each firm should both increase its debt - since it receives support - and decrease it - in its quality of guarantor. This tension results in lower total debt and tax savings. The theorem indicates that it is not profitable to give up the increase in overall debt which unilateral guarantees permit, at least if default costs are moderate: tax incentives - and the asymmetric leverage which optimally exploits them - prevail.

3.2 Unconditional guarantees

The entrepreneur can alternatively make both firms jointly liable with respect to lenders, by establishing an unconditional guarantee on reciprocal cash flows. This section examines this case and clarifies restrictions - on cash flow correlation - under which a conditional guarantee dominates an unconditional one. The rationale is that an unconditional guarantee may generate default costs by obliging the guarantor to rescue, while conditional guarantees provide the option - but not the obligation - to rescue. A conditional guarantee is preferable if tax savings do not offset - due to high cash flow correlation - higher default costs.

In order to model such straightforward intuition we consider the Merger (M) case in Leland as the case of unconditional guarantees. For this reason, in this section we restrict the attention to Gaussian cash flows. Formally, the Merger value obtains when substituting both the merger's cash flow $X_M = X_1 + X_2$ and its debt $N_M \geqslant 0$ into the tax burden and default costs of equations (4) and 14. It follows that the optimal Merger debt is positive, $N_M^* > 0$, under the usual convexity assumption.

Leland points out that merging two firms may reduce their joint value. We now show that, in such situations, it is still possible to create value through a unilateral conditional guarantee.[16]

Theorem 3.4 *Suppose that cash flows are Gaussian. Unilateral conditional guarantees are value increasing with respect to a Merger if either (i) activities' cash flows are equal and perfectly correlated, or (ii) they have $\rho^Q < \rho \leq 1$ and (common) volatility $\sigma > \sigma_L$, where $\sigma_L = \arg\min \nu^*(N_M)$ or (iii) they have $\rho^R < \rho \leq 1$ and distinct volatilities: $\sigma_P \neq \sigma_S$.*

Proof. Available from the authors. ρ^Q and ρ^R are in Leland. ■

Let us comment on these three cases. Leland (2007) shows that a Merger is value decreasing - at least for high volatility, i.e. $\sigma > \sigma_L$ - when diversification gains disappear. Case (i) shows that - with perfect correlation and independently of the level of volatility - gains from the conditional guarantee obtain. Why is it so? P is still able to rescue S because its own debt is optimally lower than the one of S. Debt diversity - i.e., the fact that P and S have distinct principals - preserves the value of the guarantee when diversification opportunities for a Merger vanish. In cases (ii) and (iii), Leland (2007) shows that the loss in limited liability - due to the Merger - is large enough to make it less desirable than separation. A fortiori - due to theorem 3.1 - a Merger is less desirable than a PS structure.

3.3 Which firm provides support?

In the previous sections we assumed that the first company, called the Parent, was providing a guarantee. In this section we explicitly consider different characteristics for firms 1 and 2, and find conditions ensuring that it is optimal for firm 1 to be the guarantor. This is the case if

$$v_{12}(N_{1P}^*, N_{2S}^*) > v_{21}(N_{1S}^*, N_{2P}^*) \tag{23}$$

where the left-hand side (rhs) is defined as total PS value when 1 supports 2 (2 supports 1) and N_{iP}^* (N_{iS}^*) is the optimal debt of firm i when it acts as Parent (Subsidiary).

Observe that the inequality (23) can be written as:

$$TG_1 + DCG_1 < TG_2 + DCG_2 \tag{24}$$

where TG_i and DCG_i are the incremental tax burden and default costs when firm i shifts from providing to receiving support (and optimally levers up):

$$TG_i(N_{iP}^*, N_{iS}^*) \triangleq T_i(N_{iP}^*) - T_i(N_{iS}^*) \tag{25}$$

$$DCG_i(N_{jP}^*, N_{iP}^*, N_{iS}^*) \triangleq DC_i(N_{iP}^*) - DC_S(N_{jP}^*, N_{iS}^*),\ j \neq i \tag{26}$$

Thus, firm 1 should provide support if and only if it has smaller incremental tax burden net of default costs, relative to firm 2. It is now easy to demonstrate that:

[16] Again, the results of the following theorem hold weakly when we do not assume convexity.

Theorem 3.5 *i) If $X_1 = X_2$ (in distribution) and tax rates as well as default costs are equal across the two firms, then each firm can either provide or receive support; (ii) under the convexity assumption, firm 1 supports firm 2 if - all others equal - it has a higher percentage default cost ($\alpha_1 > \alpha_2$) or a lower tax rate ($\tau_1 < \tau_2$).*

The theorem argues that the guarantor is the one that would lever up less even as stand alone, because of higher default costs or lower tax rates.

4 Implications for firm scope

Our results imply that PS should be pervasive, as they embed a conditional guarantee that maximizes firm value. They are indeed the norm in both emerging markets (Khanna and Yafeh (2007)) and continental Europe (De Jong et al. (2009), Barca and Becht, 2001). They are also present in innovative industries in the US and the UK (Sahlman (1990); Mathews and Robinson (2008)), as well as in private equity groups[17]. Risk taking in the banking industry is also related to the presence of intercompany guarantees (Dell'Ariccia and Marquez (2010)), which can be imposed by supervisors such as the Federal Reserve (Herring and Carmassi (2009)).

However, in practice we also observe Mergers, that should not exist if correlation between activities' cash flows is high enough, and SA organizations, that in our set up should not exist. We discuss below some of our simplifying assumptions, such as credible guarantees, no frictions and no regulation, that could be relaxed so as to reflect such empirical occurrences.

Our model posits full credibility of the guarantee, which can be associated to enforceability in court. In practice, alternative jurisdictions ensure different degrees of lenders' protection associated with the same guarantee. Moreover, within a given jurisdiction, the parties may write alternative contracts that make the ensuing guarantee more or less binding. Credibility may also stem from the guarantor's reputation, as in the case of comfort letters. These are legally unenforceable promises of rescue often sent by P to S's lenders (Boot et al. (1993)). Similarly, Herring and Carmassi (2009) cite cases of financial institutions providing additional funds to troubled SIV, despite the absence of legal obligations, so as to protect their reputation. When we embed these less-than-fully-credible guarantees in unreported numerical simulations of our model, the Merger may become an optimal arrangement also for higher cash flow correlations, because the unconditional guarantee is more reliable than the conditional one**.** This reconciles our theoretical conclusions with empirical evidence, in which Parent-Subsidiary structures coexist with Mergers, both under different legislations and under the same one[18].

[17] Conditional guarantees are implicit in private equity. Partners need to periodically raise funds because of the limited temporal commitments of financiers. They succeed only if their reputation concerning participation in restructurings is good. Moreover, the managers of LBO targets receive bonuses only when they repay their debt obligations. See Jensen (2007).

[18] The reader may object that conditional guarantees go hand-in-hand with intercorporate

Financial frictions associated with firm combinations may also explain the coexistence of PS and SA firms. For instance, previous models highlight that internal capital markets, which are present inside both Mergers and PS, may distort allocations (see Inderst and Mueller (2003), Faure-Grimaud and Inderst (2005), Rajan et al. (2000)); and that shareholders' heterogeneity may lead to minority shareholders expropriation (Almeida and Wolfenzon (2006)). Non-financial frictions and diseconomies of scale may similarly explain discrepancies between our model and reality, if they generate a cost of PS which offsets the financial synergies we uncover.

Last but not least, regulation has been targeting complex organizations in certain countries - for instance the US (see Morck (2005)). Regulation is absent in our frictionless model.

5 Conclusions

This paper provides new insights on intercorporate guarantees. Up to our knowledge, it models for the first time the provision of these guarantees, the associated optimal leverage and their impact on tax savings net of default costs. Given the correspondence with conditional guarantees, our model offers a rationale for the diffusion of Parent-Subsidiary structures without relying on internal capital markets and expropriation of minority shareholders. It also explains the observed reliance of Parent-Subsidiary structures on debt and their high tax gains, which appear to be of concern to tax authorities around the world.

Importantly, our model is just a first step towards a better understanding of intercorporate guarantees, as it relies on a simple static setting with two activities and no agency or credibility problems. Developments based on more general settings are postponed to further work.

6 Appendix

6.1 The optimization problem without guarantees

This Appendix studies the maximization of firm value with respect to non-negative debt levels, $N_i \geq 0$, with $i = 1, 2$, through its equivalent problem, namely the minimization of the tax burden plus default costs:

$$\min_{N_i \geq 0} \left[T_i(N_i) + DC_i(N_i) \right]. \tag{27}$$

We first establish some properties of the market value of debt:

ownership, that is absent in our model. Simulations indicate that results are insensitive to it, as long as guarantees are fully credible. With less credible guarantees, S becomes able to pay a dividend thanks to its lower debt service. This makes the value of the conditional guarantee sensitive to ownership.

Lemma 6.1 *Debt is increasing less than proportionally in the face value of debt:*

$$0 \leq dD_{0i}(N_i)/dN_i < 1 \qquad \textit{with} \qquad \lim_{N_i \to 0+} \frac{dD_{0i}(N_i)}{dN_i} > 0$$

Proof. The non-negativity of $dD_{0i}(N_i)/dN_i$ can be proven by contradiction. In order to demonstrate that $dD_{0i}(N_i)/dN_i < 1$ we use the fact that risky debt D_{0i} can be written as the difference between the corresponding riskless debt, $N_i\phi$, and lenders' discounted expected loss and assume instead that $dD_{0i}(N_i)/dN_i \geq 1$. Observe that ϕ, the derivative of riskless debt with respect to the face value of debt, is smaller than one. In order for risky debt to have a derivative not smaller than one, the discounted expected loss should have a derivative smaller than zero, i.e. it should decrease in the face value of debt. This contradicts the minimal requirement that both default probability and expected default costs increase in the face value of debt. As a consequence, again by contradiction, $dD_{0i}(N_i)/dN_i < 1$.

In order to get the limit result, let us on the contrary assume that $\lim_{N_i \to 0+} \dfrac{dD_{0i}(N_i)}{dN_i} \leq 0$. This implies that the discounted expected loss has a derivative, when $N_i \to 0+$, which is positive and not smaller than ϕ. This in turn implies that lenders' expected loss has a derivative greater than one with respect to debt , which is absurd. ■

This Lemma implies that both the tax shield and the default threshold are increasing in the face value of debt:

$$0 < \tfrac{dX_i^Z}{dN_i} = 1 - \tfrac{dD_{0i}(N_i)}{dN_i} < 1, \tag{28}$$

$$\tfrac{dX_i^d}{dN_i} = 1 + \tfrac{\tau}{1-\tau}\tfrac{dD_{0i}(N_i)}{dN_i} \geq 1 \tag{29}$$

The following lemma holds:

Lemma 6.2 *If the tax burden and default costs $T_i + DC_i$ are convex in the principal N_i, the Stand Alone company is optimally levered: $N_i^* > 0, i = 1, 2$.*

Proof. The Kuhn-Tucker (KT) conditions for (27) are:

$$\begin{cases} \tfrac{dT_i(N_i^*)}{dN_i} + \tfrac{dDC_i(N_i^*)}{dN_i} \geq 0 \\ N_i^* \geq 0 \\ \left[\tfrac{dT_i(N_i^*)}{dN_i} + \tfrac{dDC_i(N_i^*)}{dN_i}\right] N_i^* = 0 \end{cases} \tag{30}$$

Conditions (30) are necessary and sufficient if $T_i + DC_i$ is convex in $N_i \geq 0$. The derivative of tax burdens and default costs, appearing on the lhs of (30), is equal to:

$$-\tau(1 - F_i(X_i^Z))\frac{dX_i^Z}{dN_i}\phi + \alpha X_i^d f_i(X_i^d)\frac{dX_i^d}{dN_i}\phi \tag{31}$$

where f_i is the density of X_i. If $\tau > 0$ a minimum at $N_i = 0$ cannot exist, since the first condition in (30) is violated. The optimum is interior, and (31) is set to zero. ■

6.2 Proofs of lemma 3.1 and theorem 3.2.

We first prove a lemma which characterizes savings in default costs Γ, as in (16).

Lemma 6.3 Γ *a) is non increasing in N_P and has a null derivative if and only if $N_S = 0$; b) has a null derivative with respect to N_S at $N_S = 0$; c) is decreasing in N_S when the latter diverges.*

Proof. Part (a) requires $\frac{\partial \Gamma}{\partial N_P} \leq 0$, that is:

$$\frac{\partial \Gamma}{\partial N_P} = -\alpha\phi \times \frac{dX_1^d}{dN_1} \times$$

$$\times \left[\begin{array}{c} \int_0^{X_S^Z} xf\left(x, X_P^d + \frac{N_S}{1-\tau} - \frac{x}{1-\tau}\right) dx + \\ + \int_{X_S^Z}^{X_S^d} xf(x, X_P^d + X_S^d - x) dx \end{array} \right] \leq 0 \tag{32}$$

which is true by (29) and the fact that the integrals are computed for $X_S \geq 0$.

Equality in (32) holds if and only if the third term is zero, that is $X_S^d = X_S^Z = 0$, which in turn happens if and only if $N_S = 0$.

As concerns part (b), we compute:

$$\frac{\partial \Gamma}{\partial N_S} = \frac{\alpha}{(1-\tau)(1+r_T)} \times \left\{ -\int_0^{X_S^Z} xf\left(x, X_P^d + \frac{N_S}{1-\tau} - \frac{x}{1-\tau}\right) dx + \right. \tag{33}$$

$$\left. + \left(1 - \tau + \tau\frac{dD_{02}(N_2)}{dN_2}\right) \left[\begin{array}{c} -\int_{X_S^Z}^{X_S^d} xf(x, X_P^d + X_S^d - x) dx + \\ + \int_{X_P^d}^{+\infty} X_S^d f(X_S^d, y) dy \end{array} \right] \right\}$$

When $N_S = 0$, then $X_S^d = X_S^Z = 0$, all the integrals vanish and the previous derivative is null.

As concerns part (c), namely $\lim_{N_S \to +\infty} \frac{\partial \Gamma}{\partial N_S} < 0$, consider that $\lim_{N_S \to +\infty} X_S^d = +\infty$. For fixed y, $\lim_{x \to +\infty} xf(x, y) = 0$ - since f is a density - implies that, for any sequence x_n which goes to $+\infty$, then $x_n f(x_n, y)$ converges to zero. We suppose that the function $f_n(y)$ satisfies the dominated convergence property. This allows us to exchange integration and limit. As a consequence, the last integral in (33) vanishes when X_S^d diverges. Together with (28) this proves part (c). ■

We are now ready to prove lemma 3.1.

Proof. Let us examine the KT conditions for a minimum of the total tax burden and default costs of P and S, namely $T_{PS} + DC_{PS} \triangleq T_P + DC_P + T_S + DC_S$, with respect to non-negative S debt. Recall that such conditions are necessary and sufficient, under the convexity assumption.

$$\left\{ \begin{array}{c} \frac{\partial T_{PS}(N_P^*, N_S^*)}{\partial N_S} + \frac{\partial DC_{PS}(N_P^*, N_S^*)}{\partial N_S} = \frac{dT_2(N_S^*)}{dN_S} + \frac{dDC_2(N_S^*)}{dN_S} - \frac{\partial \Gamma(N_P^*, N_S^*)}{\partial N_S} \geq 0 \\ N_S^* \geq 0 \\ \left[\frac{dT_2(N_S^*)}{dN_S} + \frac{dDC_2(N_S^*)}{dN_S} - \frac{\partial \Gamma(N_P^*, N_S^*)}{\partial N_S}\right] N_S^* = 0 \end{array} \right. \tag{34}$$

As a consequence of part (b) of lemma (6.3) and of (31), if $N_S^* = 0$, then $X_S^d = X_S^Z = 0$ and the left hand side of the first condition in (34) becomes

$$-\tau(1 - F_2(0)) \left[1 - \frac{dD_{02}}{dN_2} \mid_{N_2=0}\right] \phi \tag{35}$$

which is negative, since $F_2(0) < 1$ and lemma 6.1 holds. The KT conditions are then violated. This concludes the proof. ■

We are now ready for the proof of theorem 3.2

Proof. Consider part i). Under the convexity assumption, the KT conditions for a minimum of $T_{PS} + DC_{PS}$, with respect to non-negative debt for both P and S, under the constraint $N_P^* + N_S^* \geq N_1^* + N_2^* \triangleq K$ are necessary and sufficient. They are equal to:

$$\begin{cases} \frac{dT_1(N_P^*)}{dN_P} + \frac{dDC_1(N_P^*)}{dN_P} - \frac{\partial\Gamma(N_P^*,N_S^*)}{\partial N_P} = \mu_1 + \mu_3 & (i) \\ N_P^* \geq 0 & (ii) \\ \mu_1 N_P^* = 0 & (iii) \\ \frac{dT_2(N_S^*)}{dN_S} + \frac{dDC_2(N_S^*)}{dN_S} - \frac{\partial\Gamma(N_P^*,N_S^*)}{\partial N_S} = \mu_2 + \mu_3 & (iv) \\ N_S^* \geq 0 & (v) \\ \mu_2 N_S^* = 0 & (vi) \\ N_P^* + N_S^* \geq K & (vii) \\ \mu_3(N_P^* + N_S^* - K) = 0 & (viii) \\ \mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0 & (ix) \end{cases} \tag{36}$$

We temporarily ignore constraints (vii) and $(viii)$, and set $\mu_3 = 0$ in $(i), (iv)$ and (ix). We want to demonstrate that there exists a point $(0, N_S^*)$ which solves them. All the conditions but *(iv)* are easy to discuss.

Consider them first. Having $\mu_3 = 0$, the right hand side of condition (i) becomes μ_1. The left-hand side is positive at $N_P = N_1^*$, since the first two derivatives are null and $-\partial\Gamma/\partial N_P > 0$ (by part (a) of lemma 6.3 and lemma 3.1, which rules out $N_S^* = 0$). The first two terms on the left hand side are negative, if $N_P^* < N_1^*$, given convexity of $T_i + DC_i$ for SA. We also know that the third term is still positive if $N_P^* < N_1^*$. When $N_P^* \to 0$, the left-hand side of (i) cannot be negative, since this would contradict the convexity assumption on the objective function. Thus $N_P^* = 0$ and conditions (i, ii, iii) are satisfied by letting μ_1 equal to the (non-negative) difference between $\frac{dT_1(N_P^*)}{dN_P} + \frac{dDC_1(N_P^*)}{dN_P}$ and $\frac{\partial\Gamma(N_P^*,N_S^*)}{\partial N_P}$. If later we choose $N_S^* > 0$, also conditions (v, vi) are satisfied, provided that we select $\mu_2 = 0$. Given that we chose $\mu_1 \geq 0, \mu_2 = \mu_3 = 0$, condition (ix) holds.

Let us turn to condition (iv), which has to provide us with a choice $N_S^* > 0$. In view of the other conditions, the right hand side of (iv) becomes zero. Consider its left-hand side as a function of N_S, denoting it with $\zeta(N_S)$. We know from the limit behavior of Γ (part (b) of lemma 6.3)) and from convexity of SA taxes and default costs ($T_2 + DC_2$), that ζ has a negative limit when S debt tends to zero, and a positive limit (even non finite) when N_S diverges. It follows that there exists a positive debt level which satisfies condition (iv). This proves part i) of the theorem, since all the KT conditions are satisfied.

Let us turn to part ii). We want to demonstrate that there exists a point $(0, N_S^*)$, with $N_S^* > K$, which solves conditions (i) to (ix). As above, we start by considering all the conditions but *(iv),* which requires some caution.

We are interested in a solution for which the constraint *(vii)* is not binding, implying $\mu_3 = 0$ in condition *(viii).* As above, we choose $N_P^* = 0$ and let μ_1 be equal to the (non-negative) difference between $\frac{dT_1(N_P^*)}{dN_P} + \frac{dDC_1(N_P^*)}{dN_P}$ and $\frac{\partial\Gamma(N_P^*, N_S^*)}{\partial N_P}$. Thus N_P^* and $\mu_1 \geq 0$ satisfy conditions (i, ii, iii). If later we also choose $N_S^* > K$, conditions (v, vi, vii) are satisfied as well, provided that we select $\mu_2 = 0$. Given that we chose $\mu_1 \geq 0, \mu_2 = \mu_3 = 0$, both conditions $(viii, ix)$ hold.

Let us turn to condition (iv), which has to provide us with a choice $N_S^* > K$. In view of the other conditions, the right hand side of (iv) becomes null again, with the left hand side denoted as ζ. We are going to show that, under the conditions posited sub (ii), $\zeta(N_1^* + N_2^*) < 0$, which implies $N_S^* > N_1^* + N_2^*$. We have:

$$\begin{aligned}\zeta(N_1^* + N_2^*) = \phi \Bigg\{ & -\tau(1 - F_2(X_S^{Z**}))\frac{dX_S^{Z**}}{dN_S} + \alpha X_S^{d**} f_2(X_S^{d**})\frac{dX_S^{d**}}{dN_S} + \\ & + \frac{\alpha}{(1-\tau)} \int_0^{X_S^{Z**}} x f\left(x, \frac{N_1^* + N_2^*}{1-\tau} - \frac{x}{1-\tau}\right) dx + \\ & -\alpha \frac{dX_S^{d**}}{dN_S} \times \left[-\int_{X_S^{Z**}}^{X_S^{d**}} x f(x, X_S^{d**} - x) dx + \int_0^{+\infty} X_S^{d**} f(X_S^{d**}, y) dy \right] \Bigg\}\end{aligned} \tag{37}$$

where X_S^{d**} and X_S^{Z**} are the default and tax shield thresholds corresponding to $N_S = N_1^* + N_2^*$. Tedious algebra permits to write it as

$$\alpha/\tau < \frac{\Pr(X_S > X_S^{Z**}) \frac{dX_2^{Z**}}{dN_S}}{X_S^{d**} \frac{dX_S^{d**}}{dN_S} \Pr\left(X_S = X_S^{d**}, X_P < 0\right) + \frac{\partial h}{\partial N_S} \int\limits_0^{X_S^{d**}} x f(x, h(x)) dx} \equiv Q \tag{38}$$

which is the condition in the theorem. This proves part ii). ■

6.3 Proof of theorem 3.3 and 3.4

Recall that $\Gamma^m = \Gamma_{12} - \Gamma_{21}$ in (21) and that the derivative of the first component with respect to the guarantor's debt is non-positive (it is null iff the guaranteed party's debt is null, i.e. $N_S = 0$). Its derivative of the first component with respect to the beneficiary's debt can have any sign. However, we know that it is null when the guaranteed party's debt N_S is high enough and when it is null. Similarly for the derivatives of the second component with respect to the guarantor and beneficiary's debt.

Given these properties, let us proceed to the proof of theorem 3.3.

Proof. We demonstrate separately that i) unilateral guarantees maximize the combined value of P and S; ii) there exists a default cost level α^* below which they are the only optimal guarantees.

Recall that maximizing PS value is equivalent to minimizing the tax burden and default costs of the corresponding SA, net of Γ^m:

$$\min_{N_P \geq 0, N_S \geq 0} [T_1 + DC_1 + T_2 + DC_2 - \ \Gamma_{12} - \Gamma_{21}] \tag{39}$$

When one of the principals is null, the conditions for optimality of the previous function collapse into the ones for optimality with Γ_{12} only in place (if $N_P = 0$), or Γ_{21} only in place (if $N_S = 0$).

For case i), we prove that there is an optimum characterized by a unilateral guarantee from 1 to 2 - $\Gamma_{12}(0, N_S^*)$ - where N_S^* is exactly the principal which maximized the conditional one-way guarantee. The existence of an optimum in $\Gamma_{21}(N_P^*, 0)$ can be demonstrated in a perfectly symmetrical way.

The derivatives of the function (39) wrt N_P, N_S are:

$$\begin{cases} \frac{dT_1(N_P)}{dN_P} + \frac{dDC_1(N_P)}{dN_P} - \dfrac{\partial \Gamma_{12}\left(N_P, N_S\right)}{\partial N_P} - \dfrac{\partial \Gamma_{21}\left(N_P, N_S\right)}{\partial N_P}, \\ \frac{dT_2(N_S)}{dN_S} + \frac{dDC_2(N_S)}{dN_S} - \dfrac{\partial \Gamma_{12}\left(N_P, N_S\right)}{\partial N_S} - \dfrac{\partial \Gamma_{21}\left(N_P, N_S\right)}{\partial N_S}, \end{cases} \tag{40}$$

For $N_P^* = 0, N_S > 0$ we have $\dfrac{\partial \Gamma_{21}}{\partial N_S} = \dfrac{\partial \Gamma_{21}}{\partial N_P} = 0$.In this case, we know that (40) admits a positive solution $N_S^* > K$. As a consequence, the group value is maximized by $\Gamma_{12}(0, N_S^*)$, as needed.

We can now proceed to the proof of part ii). We want to provide conditions under which the system of KT conditions for maximality of a mutual guarantee has no solutions other than the corner ones (N_P or $N_S = 0$).

Tedious algebra permits to show that the two derivatives in (40) are null when:

$$\begin{cases} -\frac{dX_1^d}{dN_1} \int_0^{X_S^Z} xf\left(x, h(x)\right) dx - \frac{dX_1^d}{dN_1} \int_{X_S^Z}^{X_S^d} xf(x, h(x))\ dx + \\ -\frac{1}{1-\tau} \int_0^{X_P^Z} yf\left(h(y), y\right) dy - \frac{dX_1^d}{dN_1} \int_{X_P^Z}^{X_P^d} yf(h(y), y) dy + \\ +\frac{dX_1^d}{dN_1} \int_{X_S^d}^{+\infty} X_P^d f(x, X_P^d) dx = \left[\frac{dT_1(N_P)}{dN_P} + \frac{dDC_1(N_P)}{dN_P}\right] \frac{1}{\alpha\phi} \\ -\frac{1}{1-\tau} \int_0^{X_S^Z} xf\left(x, h(x)\right) dx - \frac{dX_2^d}{dN_2} \int_{X_S^Z}^{X_S^d} xf(x, h(x)) dx \\ +\frac{dX_2^d}{dN_2} \int_{X_P^d}^{+\infty} X_S^d f(X_S^d, y) dy - \frac{dX_2^d}{dN_2} \int_0^{X_P^Z} yf\left(h(y), y\right) dy + \\ -\frac{dX_2^d}{dN_2} \int_{X_P^Z}^{X_P^d} yf(h(y), y)\ dy = \left[\frac{dT_2(N_S)}{dN_S} + \frac{dDC_2(N_S)}{dN_S}\right] \frac{1}{\alpha\phi} \end{cases} \tag{41}$$

Consider the behavior of the lhs and rhs when $\alpha \to 0+$. On the lhs, all the default thresholds' derivatives ($\frac{dX_1^d}{dN_1}, \frac{dX_2^d}{dN_2}$) are positive and bounded; the integrals are positive and bounded too, since both the tax shields and the default thresholds keep bounded: $X_S^Z,\ X_P^Z, X_S^d,\ X_P^d < \infty$. In particular, the last integral, which extends over an infinite range, is bounded, since

$$\frac{dX_1^d}{dN_1} X_P^d \int_{X_S^d}^{+\infty} f(x, X_P^d) dx = \frac{dX_1^d}{dN_1} X_P^d \Pr f(X_1 > X_S^d, X_2 = X_P^d) < \infty \tag{42}$$

The rhs diverges when $\alpha \to 0+$, since both for $i = P$ and $i = S$

$$\lim_{\alpha \to 0+} \left[\frac{dT_i(N_i)}{dN_i} + \frac{dDC_i(N_i)}{dN_i} \right] = -\tau(1 - F_i(X_i^Z)) \frac{dX_i^Z}{dN_i} \phi < \infty \tag{43}$$

It follows that there exists an α^* below which the rhs and the lhs cannot be equal. Below α^* there is no maximum with a bilateral instead of a unilateral guarantee. ∎

6.4 Proof of theorem 3.5

Proof. i) With equally distributed cash flows and equal parameters ($\alpha_i = \alpha, \tau_i = \tau$), we have $N_{1S}^* = N_{2S}^*, N_{1P}^* = N_{2P}^*,\ TG_1(N_{1P}^*, N_{1S}^*)+$ $+DCG_1(N_{2P}^*, N_{1P}^*, N_{1S}^*) = TG_2(N_{2P}^*, N_{2S}^*) + DCG_2(N_{1P}^*, N_{2P}^*, N_{2S}^*)$ and the value difference is null. Indifference between firm 1 being a beneficiary or a guarantor follows.

ii) recall that, under the convexity assumption, the guarantor is unlevered: $N_{iP}^* = 0, i = 1, 2$. Its default threshold and tax shield are null: $X_{iP}^Z = X_{iP}^d = 0$. This implies that $T_i(N_{iP}^*) = T_i(0)$, $DC_i(N_{iP}^*) = DC_i(0) = 0$. This means that the default cost savings - modulo a sign change - are the ones incurred by the firm as a beneficiary: $DCG_i(0, 0, N_{iS}^*) = -DC_S(0, N_{iS}^*)$. The value inequality $\nu_{12} > \nu_{21}$ becomes

$$TG_1(0, N_{1S}^*) - DC_S(0, N_{1S}^*) < TG_2(0, N_{2S}^*) - DC_S(0, N_{2S}^*) \tag{44}$$

We provide conditions under which this inequality is satisfied when debt of S does not optimize the guarantee from 1 to 2, but optimizes the one from 2 to 1. To do this, we assume now that $N_{2S} = N_{1S}^*$ (instead of $N_{2S} = N_{2S}^*$). A fortiori, it will be satisfied when S debt in the first guarantee is optimized ($N_{2S} = N_{2S}^*$).

In order to examine the case of different default costs we need to assume $X_1 = X_2$ and $\tau_i = \tau$. If $N_{2S} = N_{1S}^*$, together with $X_1 = X_2$ and $\tau_i = \tau$, then $T_1(N_{1S}^*) = T_2(N_{1S}^*)$. Inequality $\nu_{12} > \nu_{21}$ becomes

$$\alpha_2 \phi \mathbb{E} \left[X_2 1_{\left\{0 < X_2 < X_2^d(N_{2S}), X_1 > h(X_2)\right\}} \right] < \alpha_1 \phi \mathbb{E} \left[X_1 1_{\left\{0 < X_1 < X_1^d(N_{1S}^*), X_2 > h(X_1)\right\}} \right] \tag{45}$$

Notice that - since $X_1 = X_2$ and $N_{2S} = N_{1S}^*$ - the expectations are equal. The previous inequality reduces to

$$(\alpha_2 - \alpha_1) \phi \mathbb{E} \left[X_2 1_{\left\{0 < X_2 < X_2^d(N_{1S}^*), X_1 > h(X_2)\right\}} \right] < 0 \tag{46}$$

which is true if and only if $\alpha_1 > \alpha_2$. The condition is necessary and sufficient when $N_{2S} = N_{1S}^*$, it becomes sufficient when N_{2S} is optimized in the first guarantee.

In order to examine the case of different tax rates we need to assume $X_1 = X_2$ and $\alpha_i = \alpha$. A reasoning similar to the previous one leads first to $DC_1(N^*_{1S}) = DC_2(N_{2S})$ and then to $\nu_{12} > \nu_{21}$being satisfied if and only if $\tau_1 < \tau_2$, when $N_{2S} = N^*_{1S}$. The condition is necessary and sufficient when $N_{2S} = N^*_{1S}$, it becomes sufficient when N_{2S} is optimized in the first guarantee. ■

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