The Commitment Problem of Secured Lending[*]

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Abstract

The paper challenges the argument that collateral boosts debt capacity and provides a new rationale for the use of trade credit. In a setting with uncertainty, two inputs and investment unobservability, we show that a firm-bank secured credit contract is time-inconsistent: Once credit has been granted, the entrepreneur has an ex-post incentive to alter the input combination towards the input with low collateral value, thus jeopardizing total bank revenues. Anticipating the entrepreneur's opportunism, the bank offers a non-collateralized credit contract, thereby reducing the surplus of the venture. One way for the firm to commit to the contract terms is to purchase inputs on credit and pledge them to the supplier in case of default. Observing the input investment and having a positive stake in the bad state, the supplier acts as a guarantor that the input mix specified in the bank contract will be actually purchased and that the entrepreneur will stick to the contract terms. The analysis is extended to consider the case of collusion between entrepreneur and supplier. The paper concludes that trade credit facilitates the access to collateralized bank financing and identifies new testable predictions between the degree of competition in the up-stream market and the firm financing and input choices.

Keywords: collateral, commitment, trade credit, bank financing.
JEL classification: G32, G33, K22, L14.

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Introduction

Collateral is widely used in lending. Berger and Udell (1990) and Harhoff and Korting (1998) document that nearly 70% of commercial industrial loans in the U.S., the U.K., and Germany are secured. The common explanation for that evidence is that collateral boosts firm debt capacity when credit frictions arise. The paper challenges this argument and provides a new rationale for the use of trade credit. More specifically, the paper argues that pledging an asset as collateral to give the lender greater protection against losses in default not necessarily increases external financing. Collateral might indeed introduce in the lending relationship a problem of moral hazard in the form of asset substitution. Trade credit can be used by the entrepreneur to mitigate this problem.

We construct a model where firms produce facing uncertain demand and using two inputs with different degrees of tangibility, and thus different collateral values. Two sources of financing are available: bank and trade credit. Financiers have different capabilities. Being specialized financial intermediaries, banks have a cost advantage in financing the firm. However, banks do not observe the amount of inputs purchased and thus invested. In contrast, suppliers have an information advantage: Being the providers, they observe input purchases.

Firms prefer bank financing because it is cheaper. In particular, they prefer secured bank financing. Collateral gives the lender greater protection against losses in default, thereby increasing the amount of external financing and the total surplus of the lending relationship. However, because of the unobservability of input investment, upon receiving the bank loan the entrepreneur has an incentive to alter the input combination toward the input with higher productivity but lower collateral value. This jeopardizes bank expected revenues, by reducing the liquidation income in the bad state. Anticipating that it will not break even, the bank gives up the secured contract, thus causing an efficiency loss.

One way for the firm to commit to the contract terms is to purchase a fraction of the collateralizable inputs on credit and pledge them to the supplier in case of default. Because trade credit is more costly than bank credit, the entrepreneur only buys minimum possible amount of inputs on credit. Observing the input investment and having a stake in default state, the supplier implicitly guarantees that the quantity of inputs specified in the financial contracts, and thus available for liquidation to creditors, is actually purchased. Through trade credit, the entrepreneur commits to the input combination which is ex-ante efficient. As a consequence, the entrepreneur can pledge his inputs as collateral to the bank, obtaining larger bank financing and higher profits. It follows that, when investment is non-contractible, trade credit facilitates the access to bank loan by making the secured contract available to banks and entrepreneurs.

This analysis relies on the assumption that the entrepreneur is the only contracting party facing a commitment problem. However, upon granting the loan, supplier and entrepreneur could jointly agree to alter the input combination at the expense of the bank. In this case, having the supplier acting as a financier only shifts the commitment problem from the entrepreneur to the supplier. If the cost of such a collusive deal is small enough, a minimum amount of trade credit may be no longer sufficient to overcome the commitment problem. Then, an increasing fraction of the inputs must be pledged to the supplier and financed by him. In the extreme case in which collusion is costless, the only way for the entrepreneur to exploit the pledgeable income of collateralizable assets is to sign a secured contract with the supplier and an unsecured contract with the bank. Suppliers and banks finance the purchase of the tangible input in different proportions. The higher the collateral value of assets, the larger the financing provided by the supplier. However, since trade credit is more expensive, the cost of signing a secured contract with the supplier could be so high to neutralize the benefits, in terms of commitment, of trade credit use. If this is the case, the optimal contract is a combination of an unsecured bank credit contract and a pure input supply contract (the supplier is not a financier).

Finally, assuming that the cost of collusion depends on the number of firm's input providers, we identify new relations between the structure of the up-stream market and the firm's financing decisions, including not only the mix of trade and bank credit, but also the type of contract (collateralized versus uncollateralized). Specifically, we argue that trade credit facilitates the firm's access to collateralized bank loan the higher the degree of competition of the up-stream market. Moreover, while bank credit is increasing in the degree of competition of the up-stream market, trade credit is decreasing in it and therefore larger when the supplier is a monopolist. Since a different mixture of bank and trade credit corresponds to a different input combination, we also identify new relations between input choices and the up-stream market structure. Specifically, firms use technologies more intensive in tangible assets the higher the degree of competition in the up-stream markets.

Our paper is related to two strands of the literature. The first one focuses on the role of collateral in lending relationships. The second one on the determinants of firm's reliance on trade credit. The literature on collateral has identified several theoretical reasons for the popularity of secured lending. The simplest one assumes uncertainty in the firm revenues and argues that collateral reduces losses in case of default (lender's risk reduction). A second strand of the literature highlights the benefits of collateral in mitigating asymmetric information problems. In case of adverse selection, by conveying valuable information to the bank about the borrowers' default risk, collateral can be a signaling instrument (Bester, 1985; Chan and Kanatas, 1985; Besanko and Thakor, 1987 a,b). Collateral also helps solving a variety of moral hazard problems like asset substitution, under-investment and inadequate effort supply (Stulz and Johnson, 1985; Chan and Thakor, 1987; Boot and Thakor, 1994).[1] All these papers point to the idea that borrowing not only against returns but also against assets provides the lender greater protection against losses in the event of default and increases the firm's debt capacity. An extensive literature also investigates which characteristics of an asset affect its ability to raise external financing. Some papers focus on the degree of asset tangibility (Almeida and Campello, 2007), while others relate the asset debt capacity to its redeployability (Williamson, 1988; Shleifer and Vishny, 1992) or to the easiness of shifting its property to creditors in case of distress (Hart and Moore, 1994).[2] Our paper identifies a new characteristic: the investment contractibility. We argue that if the investment in a given asset is not contractible, this asset has no debt capacity when pledged as a collateral, although tangible and highly redeployable. Our result not only challenges the accepted view that collateral boosts the firm's debt capacity through a lender risk reduction. It also shows that the use of collateral itself may introduce in the bank-firm relationship a problem of entrepreneur's opportunism (in the form of ex-post asset substitution) that was absent in the unsecured debt contract. For the investment contractibility to play a role in our story, we crucially need a project with two inputs, one of which used as inside collateral. The assumption of a two-input-technology is novel. The related literature focuses on projects with only one input and outside collateral. In our model, the time-inconsistency of the bank secured contract comes from the entrepreneur having an ex-post incentive to alter the input combination toward the input with low collateral value. With only one input (or with outside collateral), the non-contractibility of investment would be immaterial, as the loan size could be used to infer the input choice.

This discussion raises the question of which type of bank loan better fits our story. In practice, firms largely use secured loans as opposed to financing primarily based on the firm's cash-flow. Different types of secured loans are offered by banks. Real-estate based lending or loans collateralized by movable goods (like cars, trucks, etc.) have characteristics that depart from our theoretical setting.

[1] Related literature (Galai and Masulis, 1976 and Jensen and Meckling, 1976 among others) investigates the risk-shifting problem in a contest of conflicts of interests between shareholders and bondholders and concludes that collateral mitigates this problem. In our model, is the collateral itself that introduces a problem of asset substitution.

[2] Several empirical papers document that asset tangibility and salability increase debt capacity (see, among others, Almeida and Campello, 2007; Campello and Giambona, 2009; Benmelech, 2009; Rampini and Viswanathan, 2010).

First, the problem of investment unobservability is not so relevant in this case as, being registered goods, their actual purchase is certifiable to the bank. Second, the credit is generally granted directly to the seller of the asset or to the notary (for real asset) or to the leasing company (for movable goods). This implies that the entrepreneur does not have the possibility to misuse the bank money. A secured bank loan that better fits our model is Asset Based Lending (ABL). ABL is a short-term financing (typically, three years maturity) used to support working capital needs. In case of ABL, the bank avoids paying screening costs and lends in exchange of some generic collateral. Collateral generally includes accounts receivable, inventories, machineries and equipment (not real estate). Since the collateral value of ABL is clearly affected by input purchases which are not easily observable by the bank, ABL is likely to be sensitive to the commitment problem analyzed in our model.[3]

Our paper is also related to the literature on trade credit. Papers in this literature have sought to explain why agents might prefer to borrow from firms rather than from financial intermediaries. The traditional explanation is that trade credit plays a non-financial role. That is, it reduces transaction costs (Ferris, 1981), allows price discrimination between customers with different creditworthiness (Brennan et al., 1988), fosters long-term relationships with customers (Summers and Wilson, 2002), and even provides a warranty for quality when customers cannot observe product characteristics (Long et al., 1993). Financial theories (Biais and Gollier, 1997; Burkart and Ellingsen, 2004, among others) claim that suppliers are as good as or better financial intermediaries than banks. In Biais and Gollier (1997) and Burkart and Ellingsen (2004) this is due to an information advantage of suppliers over banks. Within a context of limited enforceability, Cuñat (2007) shows that suppliers can enforce debt repayment better than banks by threatening to stop the supply of intermediate goods to their customers. Fabbri and Menichini (2010) show that trade credit can be cheaper than bank credit because of the liquidation advantage of the supplier.

Our paper is mostly related to the financial theories and in particular to Biais and Gollier (1997). Like them, we assume a supplier's information advantage. However, while in Biais and Gollier (1997) such advantage concerns the borrowers' creditworthiness, in our paper it concerns the investment in the collateralized input. The implications of the supplier information advantage are very different between the two papers. In Biais and Gollier (1997), extending trade credit signals to the bank the borrower's quality and induces banks to extend credit to entrepreneurs with profitable projects that would have

[3]In the last two decades in the U.S. there has been a steady increases in ABL: In 1992, there were $90 billion of ABL in the U.S. as opposed to $326 billion (which corresponds to the 22% of the total short term credit) in 2002 and and $590 billions in 2008. See the book by Udell (2005) for more information about the characteristics of ABL.

been rejected otherwise. In our model, by signaling that the investment in the collateralized asset has taken place as expected, trade credit makes the secured bank loan available. Thus, while collateral is crucial in our story, it plays no role in Biais and Gollier (1997).

One prediction of our model is that trade credit facilitates the access to collateralized bank financing, suggesting a complementarity between bank and trade credit, which is in the spirit of some recent empirical evidence.[4] Cook (1999) documents that trade finance raises the likelihood that a Russian firm obtains a bank loan. Giannetti et al. (2008) show that U.S. firms using trade credit can secure financing from relatively uninformed banks. Alphonse et al. (2006) document that the more trade credit U.S. firms use, the more indebted towards banks they are, in particular firms with a short banking relationship. Along the same lines, Gama et al. (2008) find that trade credit allows younger and smaller firms in Spain and Portugal to increase the availability of bank financing. Finally, Garcia-Appendini (2010) documents that small, non financial U.S. firms are more likely to get bank credit if they have been granted trade credit from their suppliers. The evidence provided by the last papers seems to suggest that the complementarity hypothesis is more relevant for young and small firms with a short banking relationship. This is an interesting finding that could be explained within our theoretical framework. Young, small firms with a short banking relationship are more opaque and also might lack incentives to commit to the contract terms in lending relations (or simply they are perceived by banks to lack incentives) since the cost of deviating from the contracts (i.e., loosing reputation) is still relatively small. So, these firms are the ones that benefit most from the use of trade credit. In contrast, larger and older firms, which are more likely to be public and have stronger relationships with banks, care more about their reputation and therefore have less incentive to misbehave.

The paper is organized as follows. Section 1 presents the model. Section 2 describes the commitment problem that plagues an exclusive entrepreneur-bank lending relationship when the project to be financed uses two inputs. In Section 3, we show that trade credit can solve the commitment problem and we characterize the properties of the optimal financing contract. In Section 4, we extend the model to allow for collusion between entrepreneur and supplier. Section 5 delivers new testable predictions on firms decisions, by exploiting cross-sectional differences in input characteristics (liquidity and substitutability) and in market structures (labor and up-stream markets). Section 6

[4] In Biais and Gollier (1997), trade credit allows credit-constrained firms to get (un-collateralized) bank loan. In Burkart and Ellingsen (2004), trade credit also increases the amount of bank credit limit, but this is a second order effect holding only for a selected group of firms.

discusses the robustness of our theoretical setting. Specifically, Subsection 6.1 identifies the conditions that make trade credit beneficial to the lending relationship. Subsection 6.2 focuses on the degree of information sharing between supplier and bank. Subsection 6.3 questions the role of the supplier as an informed lender and provides alternative interpretations. Subsection 6.4 discusses the seniority rule implied by the optimal contract. Finally, Section 7 concludes.

1 Setup and model assumptions

A risk-neutral entrepreneur has an investment project that uses two inputs, called capital (K) and labor (N). Let I_K, I_N denote the amount of investment in capital and labor inputs. The amount of the input invested is converted into a verifiable state-contingent output $Y \in \{y, 0\}$. The good state ($Y = y$) occurs with probability p. Uncertainty affects production through demand (i.e., production is demand-driven). At times of high demand, invested inputs produce output according to a homothetic strictly quasi-concave production function $f(I_K, I_N)$. At times of low demand, there is no output ($Y = 0$), but unused inputs have a scrap value and can be pledged as collateral to creditors. Inputs are substitutes, but a positive amount of each is essential for production.

The entrepreneur is a price-taker both in the input and in the output market. The output price is normalized to 1, and so is the price of the two inputs.[5]

The entrepreneur has no internal wealth, so he needs external funding from competitive banks ($L_B \geq 0$) and/or suppliers ($L_S \geq 0$).

Banks and suppliers play different roles. Banks lend cash. The supplier of labor provides the input, which is fully paid for in cash. The supplier of capital, however, not only sells the input, but can also act as a financier, by delaying the payment of the inputs supplied.

Cost of funds. Banks have an intermediation advantage relative to suppliers as they face a lower cost of raising funds on the market ($r_B < r_S$). This assumption is consistent with the role of banks as specialized financial intermediaries. Moreover, suppliers are likely to be themselves credit constrained.

Collateral value. Inputs have value when repossessed in default. We assume that only capital inputs can be pledged while labor has zero collateral value. We assume that the two financiers are equally good in liquidating the unused capital inputs and their liquidation value in case of default is given by $C = \beta I_K$, with $0 < \beta < 1$.[6]

[5] This normalization is without loss of generality since we use a partial equilibrium setting.

[6] This assumption allows us to highlight the commitment role of trade credit. Giving the supplier a comparative

Information. Banks and suppliers differ in the type of information they possess. Providing the input, suppliers of capital and labor can costlessly observe that an input transaction has taken place. Banks cannot observe any input transaction and the cost of acquiring this information is too high to make observation worthwhile.[7] The information advantage assumption is commonly accepted in the theoretical literature and frequently interpreted as a natural by-product of the selling activity of the supplier. Suppliers are often in the same industries as their clients, and they often visit their customers' premises. In our setting, this assumption is even more reasonable, given that the information asymmetry refers to the input purchases. Extensive anecdotal evidence supports this assumption. The most recent example is the case of Siemens. In 2010, Siemens has created its own bank, Siemens Bank Gm-bH, mainly to provide lines of credit to its more important clients.

Contracts. The entrepreneur-bank contract specifies the loan, L_B, the state-contingent repayment obligation, R_B^σ, $\sigma = \{H, L\}$, and the share of the collateral obtained in case of default, γ. That with the supplier of the tangible input specifies the input purchase I_K, the amount of credit L_S, the state-contingent repayment obligation, R_S^σ, $\sigma = \{H, L\}$, and the share of the collateral obtained in case of default, $(1-\gamma)$. Since there is no output in the low state, limited liability implies that repayments to bank and supplier are also zero ($R_B^L = R_S^L = 0$).[8] To save notation, we drop the state superscript index from our repayment variables and set $R_B^H = R_B$, and $R_S^H = R_S$.

Last, given that the labor is fully paid for when purchased, the contract between entrepreneur and worker specifies the amount of labor, I_N.

Each party is protected by limited liability.

Fig. 1 summarizes the sequence of events: In $t = 1$, banks and suppliers make contract offers specifying the size of the loan L_B, L_S, the high state repayment obligations, $R_B(\cdot)$, $R_S(\cdot)$, the share of the collateral that goes to the bank and the supplier in case of default $\gamma, (1-\gamma)$, the amount of tangible input to be purchased, I_K. More specifically, banks (and suppliers) propose a set of contracts which may range from the fully secured contract, with $\gamma = 1$, to the unsecured one, with $\gamma = 0$, passing through the partially secured one with $0 < \gamma < 1$. In $t = 2$, the entrepreneur chooses among

advantage in liquidating the capital input would not alter our qualitative results, as long as this advantage is not too high, i.e., $\beta_S \leq \frac{(1-p)\beta_B r_S}{(r_B - p r_S)}$. In this latter case, the liquidation advantage would make trade credit cheaper than bank credit and therefore strictly preferred. This case has been analyzed in Fabbri and Menichini (2010).

[7] Full unobservability from the bank and full observability from the suppliers are not crucial in our analysis. We could still get our results by assuming that both banks and suppliers can partially observe the inputs, as long as suppliers have an information advantage over banks.

[8] Banks and suppliers can still get a repayment in the low state by having the right to a share of the scrap value of unused inputs.

contract offers and receives credit from financiers; in $t = 3$ the investment decisions are taken, I_K, I_N; in $t = 4$, uncertainty resolves; and in $t = 5$, repayments are made.

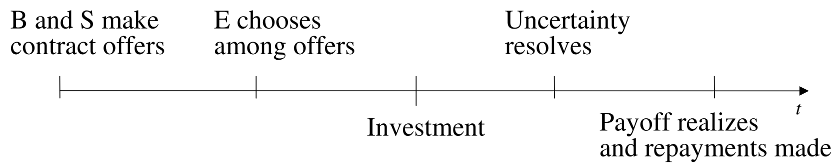

Figure 1: Time-line

2 The firm-bank contract without commitment

In this section, we show that the non-contractibility of the investment to the bank makes any entrepreneur-bank secured contract time-inconsistent and therefore not available to contracting parties. To make this point clear, we first analyze the benchmark case, where investment is observable to the bank and therefore contractible. We derive the well-known result that secured lending is the optimal contract since it increases the surplus of the lending relationship through a risk reduction for the lender. Then we consider the case of non-contractible investment.

Benchmark Case: Contractible Investment. In period $t = 1$, all financiers make contract offers. Since bank credit is cheaper, in period $t = 2$ firms only sign bank contracts and get financing. In period $t = 3$, firms buy and invest the inputs. The amount of inputs and financing solve the following optimization problem ($\mathcal{P}^{\mathcal{FB}}$):

$$\max_{I_K, I_N, L_B, R_B} EP = p\left[f\left(I_K, I_N\right) - R_B\right] \tag{1}$$

$$\text{s.t. } pR_B + (1-p)\,C \geq L_B r_B, \tag{2}$$

$$L_B \geq I_N + I_K. \tag{3}$$

Condition (2) is the bank's participation constraint and states that banks participate to the venture if their expected returns cover at least their opportunity cost of funds. Competition among banks implies that (2) is binding. The resource constraint (3) requires that input purchase cannot exceed available funds. Solving (2) for R_B and using the resource constraint (3), the objective function (1) becomes:

$$\max EP = pf\left(I_K, I_N\right) - r_B\left(I_K + I_N\right) + (1-p)\,\beta I_K. \tag{4}$$

The solution to this problem leads to the following proposition:

Proposition 1 *When investment is contractible, the bank offers a collateralized credit contract with loan $L_B^*\left(I_K^*, I_N^*\right) = I_K^* + I_N^*$, repayments $R_B^*\left(I_N^*, I_K^*\right) = \frac{1}{p}\left\{\left(I_N^* + I_K^*\right) r_B - (1-p)\,\beta I_K^*\right\}$ in the high state, and βI_K^* in the low state, with I_K^*, I_N^* solving the first order conditions (17) and (18).*

Proof. In the Appendix. ■

Figure 2 displays the optimal input combination under the collateralized credit contract (point A). The input mix is stretched toward capital. The collateral value of capital makes its price equal to $r_B - (1-p)\beta$, thus lower than the intangible one, r_B. Notice that in our model, the selling price of each input, which is set equal to one for both inputs by assumption, differs from the actual price. The actual price also includes the cost of credit used to finance the input purchases. This cost might differ between inputs either because of different financiers (bank versus supplier) or because of different contracts. For example, when both inputs are financed with bank credit and a collateralized contract is signed, the cost of credit of the tangible input is lower than the one of the intangible, the difference being the pledgeable income of the collateral. In this case, the two inputs have different actual prices although both the initial selling price and the financier are the same. In contrast, when both inputs are financed through a pure debt contract, both inputs have the same cost of credit, namely r_B, and thus also the same actual price.

Non-contractible investment. The result in Proposition 1 is obtained under the assumption that the entrepreneur can commit to the investment level specified in the bank contract at $t = 1$. However, if investment is unobservable, at $t = 3$, once the loan L_B^* has been granted, the entrepreneur can increase his profits by altering the input combination and worrying only about honoring his repayment obligations in non-defaulting states.[9] Thus, the entrepreneur re-optimizes by solving programme $\mathcal{P}^{\mathcal{D}}$:

$$\max_{I_K, I_N, R_B} \; pf\left(I_K, I_N\right) - pR_B \tag{5}$$

$$\text{s.t. } R_B \geq R_B^*, \tag{6}$$

$$L_B^* \geq I_N + I_K, \tag{7}$$

[9]Because output is verifiable, any return from production will be claimed by creditors and the entrepreneur will get zero return if he does not repay the loan in full.

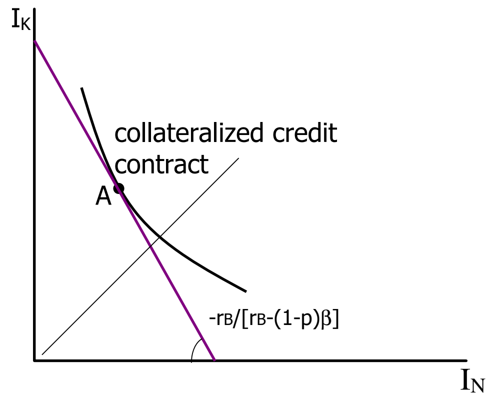

Figure 2: Input combination when investment is contractible. Point A represents the optimal input combination under the collateralized credit contract.

where constraint (6) requires the repayment to the bank in the high state be no less than the one promised in the secured commitment contract (i.e. R_B^* in Prop. 1), while the resource constraint (7) requires that the total input expenditure is no higher than the loan obtained in the secured commitment contract (i.e. L_B^* in Prop. 1).

The solution to the previous problem implies that the entrepreneur buys the same amount of tangible and intangible inputs, namely $\hat{I}_K\left(L_B^*, R_B^*\right) = \hat{I}_N\left(L_B^*, R_B^*\right)$. This input combination is represented by point D in Figure 3. Point D lies to the right of point A on a higher isoquant. Indeed, the new isocost line is flatter than the one going through point A. Since the debt contract used to finance the input purchases of point D is a pure debt bank contract, the cost of credit of the two inputs is the same and equal to r_B. Therefore the relative input price is 1, which is lower than the relative input price implied by the collateralized credit contract, namely $\left(r_B / \left[r_B - (1-p)\,\beta\right]\right)$ used to finance the input purchases of point D. Since at the new input prices it must be still possible to afford the original contract, the new isocost line has to pass through the initial optimum (point A). By the concavity (and smoothness) of the production function, the new input combination lies on a higher isoquant, with larger production and higher expected profits, and involves a decrease in I_K and an increase in I_N.

However, the lower investment in the capital input implies that in case of default the firm cannot meet its repayment obligations. Thus, in point D the bank makes negative expected profits.

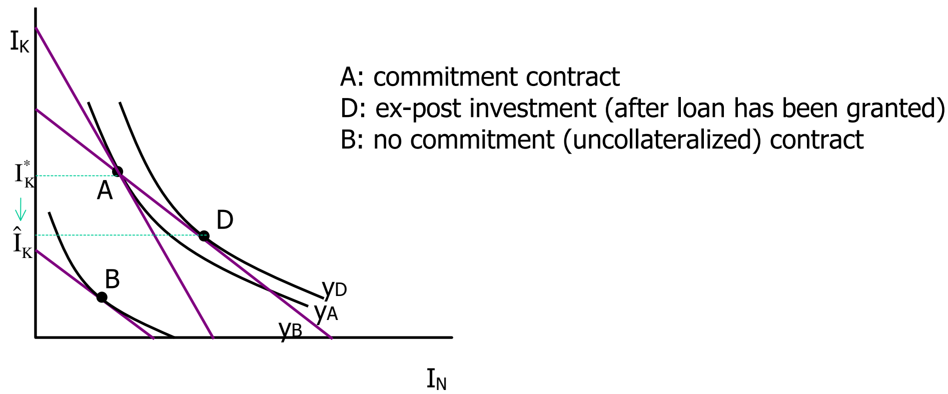

Figure 3: Input combination with and without contractible investment. Point A represents the optimal input combination when the entrepreneur can commit to the investment level implied by the secured credit contract signed with the bank (commitment contract). Point B shows the optimal input combination when the entrepreneur cannot commit to the input combination specified in the bank contract (no commitment contract). Point D is the input combination that the entrepreneur would ex-post choose upon receiving the bank loan. This is not an equilibrium contract since the bank does not break-even.

Anticipating that it will not break even, at $t = 1$ the bank offers an unsecured credit contract with all the repayment obligations paid for in the good state: $R_B = \frac{1}{p} L_B r_B$. Using L_B from the resource constraint (3), the objective function (1) becomes:

$$\max_{I_K, I_N} pf\left(I_K, I_N\right) - \left(I_N + I_K\right) r_B.$$

The solution to this programme is described in the following proposition:

Proposition 2 *When investment is non-contractible, the bank offers an unsecured credit contract lending* $\bar{L}_B = \bar{I}_K\left(p, r_B\right) + \bar{I}_N\left(p, r_B\right)$, *and getting a repayment only in non-defaulting states* $\bar{R}_B = \frac{1}{p} \bar{L}_B r_B$. *The level of investment in the collateralizable input is* $\bar{I}_K\left(p, r_B\right) = \bar{I}_N\left(p, r_B\right)$, *strictly lower than the one obtainable under commitment. There is an efficiency loss due to the inability to pledge inputs as collateral.*

Proof. In the Appendix. ■

Point B in Figure 3 shows the optimal input combination when the investment is not observable to the bank and the entrepreneur cannot commit to the input combination specified in the contract. The bank loan and the investment in capital and labor are lower than in case of investment observability (point A). The new isoquant y_B is below y_A. While the bank is indifferent between points A and B - it gets zero profit in either case - the entrepreneur's profits are strictly lower in point B. This is because the lower debt capacity, implied by the inability to pledge inputs as collateral, reduces the overall investment size.[10] The difference $Y_A - Y_B$, represents the benefits of collateral lost due to investment unobservability. It follows that the entrepreneur would rather commit to the investment level of the collateralized credit contract (point A). Notice that in point B the actual prices of the two inputs are equal, since inputs are financed through a pure debt contract, where the cost of credit for each input is equal to r_B. This explains why the input combination in point B implies an equal amount of tangibles and intangibles (as in point D).

3 The commitment role of trade credit

So far we have shown that when the project needs two inputs and the investment in the pledgeable one is non-contractible, any collateralized debt contract (fully and partially) is plagued by a problem of input substitution which makes the bank loan unprofitable. The only contract that bypasses this problem is the unsecured debt contract ($\gamma = 0$).

In this section, we introduce the supplier of the collateralized input as a second financier. By observing the input transactions, he has a natural information advantage. The entrepreneur can use this advantage to restore his ability to pledge collateral to the bank.[11] In particular, he can sign a partially collateralized credit contract with the supplier. Observing the input transaction and having a stake in the default state, the supplier guarantees that the input investment is carried out as specified in the contract. This in turn induces the bank to offer a partially collateralized credit contract as well, thus mitigating the efficiency loss due to the lack of commitment.

How much trade credit does the entrepreneur has to take to credibly signal the bank his willingness to stick to the collateralized contract? We show that if trade credit is too small amount, the entrepreneur could still cheat at the bank's expenses. More specifically, he could sign a partially collateralized credit contract with the bank, so as to signal its willingness to pledge the remaining collateral to the supplier, but then give up trade credit. Relying only on bank credit, he could then choose the ex-post profit maximizing input combination. This input combination would be different from the one specified in the contract and involve a lower investment in the collateralizable input and

[10] We are assuming here that inputs not pledged to any creditor are valueless to the entrepreneur in case of default.
[11] In our model, the supplier acts as an informed lender. In Section 6, we discuss an alternative interpretation of the informed lender.

thus a lower bank expected return. The benefit from deviation is again the extra profit generated by the change in the input combination. However, unlike the case in which there is no trade credit, the deviation is costly since the entrepreneur has to give up trade credit.[12] The bank can avoid cheating, by reducing its stake in the project financing. This would induce the entrepreneur to rely more on trade credit and make the deviation very costly. Thus, the maximum (minimum) loan provided by the bank (supplier) must be sufficiently low (big) to make deviation unprofitable.

To find the incentive-compatible firm-bank-supplier contract, we proceed in two steps. First, we find the firm-bank-supplier contract for any γ, assuming that deviation is not allowed. Then, we introduce the no deviation condition requiring that the entrepreneur has no incentive to deviate. This condition will allow us to find the optimal γ and therefore to fully characterize the optimal contract.

When bank and supplier can both provide external finance and no deviation is allowed, the optimization problem is the following ($\mathcal{P}^{\gamma}$):

$$\begin{aligned} \max_{L_B,L_S,R_B,R_S I_K,I_N,\gamma} EP &= p\left[f\left(I_K,I_N\right)-R_B-R_S\right], && (8)\\ \text{s.t. } pR_B+(1-p)\,\gamma C &= L_B r_B, && (9)\\ pR_S+(1-p)\,(1-\gamma)\,C &= L_S r_S, && (10)\\ L_B+L_S &\geq I_N+I_K, && (11)\\ R_S &\geq \beta I_K && (12) \end{aligned}$$

where (8) denotes the entrepreneur's expected profits. Conditions (9) and (10) represent the participation constraints of competitive banks and suppliers, respectively. The parameter γ represents the share of the collateral accruing to the bank (and $1-\gamma$ the one accruing to the supplier). Condition (11) is the resource constraint when trade credit is also available. Last, constraint (12) requires the repayments to the supplier be non-decreasing in revenues. This condition prevents the supplier to be used exclusively as a liquidator. More specifically, pledging the tangible inputs to the supplier increases the firm's debt capacity. However, since bank credit is cheaper than trade credit, the entrepreneur would sign a contract where the collateralized inputs are given to the supplier in the bad state in exchange for a payment in the good state. This contract would imply repayments negatively correlated with returns, with no supplier financing taking place. Being interested in the supplier's role as financier, we do not allow for such contracts and require monotonicity of repayments. Proposition 3 describes

[12] Recall the entrepreneur has to give up trade credit to finance an input combination different from the one agreed upon.

the solution to the above maximization problem:

Proposition 3 *The firm-bank-supplier contract has investment* $I_K^*(\gamma), I_L^*(\gamma)$, *and displays the following properties:*

- *the supplier gets a secured contract with flat repayments across states: an amount* $L_S^*(\gamma) = \frac{1}{r_S}(1-\gamma)\beta I_K^*(\gamma)$ *is lent in exchange for the right to a share* $1-\gamma$ *of the collateral value of the unused inputs* ($\beta I_K^*(\gamma)$) *in the default state and to a repayment* $R_S^*(\gamma) = (1-\gamma)\beta I_K^*(\gamma)$ *in the high state;*

- *the bank gets a secured contract with increasing repayments: an amount* $L_B^*(\gamma) = I_K^*(\gamma) + I_N^*(\gamma) - \frac{1}{r_S}(1-\gamma)\beta I_K^*(\gamma)$ *is lent in exchange for the right to a share* γ *of the collateral value of the unused inputs* ($\beta I_K^*(\gamma)$) *in the default state and to a repayment* $R_B^*(\gamma) = \frac{1}{p}\left[\left(\frac{r_B}{\beta} - \frac{r_B}{r_S}(1-\gamma) - (1-p)\gamma\right)\beta I_K^*(\gamma) + I_N^*(\gamma) r_B\right] > \gamma\beta I_K^*(\gamma)$ *in the high state;*

- *expected profits* $\Pi^*(\gamma)$ *are increasing in* γ*;*

- *the input combination* $\frac{I_K^*(\gamma)}{I_N^*(\gamma)}$ *is decreasing in* γ.

Proof. In the Appendix. ■

Prop. 3 derives the optimal contract for any γ, under the assumption that sticking to the contract is optimal for any γ. However, this is not always true. More specifically, the entrepreneur could increase profits by taking the bank loan agreed in the contract ($L_B^*(\gamma)$), give up trade credit and choose an input combination $I_K^{\mathcal{D}}(L_B^*(\gamma)), I_N^{\mathcal{D}}(L_B^*(\gamma))$ different from the one specified in the contract.

Definition 1 *Define by* $\Pi^{\mathcal{D}}(\gamma) \equiv p\left[f\left(I_K^{\mathcal{D}}(L_B^*(\gamma)), I_N^{\mathcal{D}}(L_B^*(\gamma))\right) - R_B^*(\gamma)\right]$ *the entrepreneur's expected profits after deviating from the contract specified in Prop. 3.*

Given all profitable deviations, the optimal contract requires that deviating is less profitable than sticking to the ex-ante efficient contract. This is ensured by adding to programme $\mathcal{P}^{\gamma}$ above the following constraint:

$$\Pi^{\mathcal{D}}(\gamma) \leq \Pi^*(\gamma). \tag{13}$$

Proposition 4 *The set of bank contract offers that generates unprofitable deviation has* $0 \leq \gamma < \gamma^*$, *where* γ^* *satisfies condition 13 with equality. Since firm's profits are increasing in* γ, *the entrepreneur will choose the highest possible* γ , *i.e.,* $\gamma = \gamma^*$.

Proof. In the Appendix. ■

Prop. 4 identifies the optimal share of collateral accruing to the bank as the upper bound of the set of contract offers that are incentive-compatible for the entrepreneur. Using γ^* in Prop. 3, we fully characterize the optimal firm-bank-supplier contract.

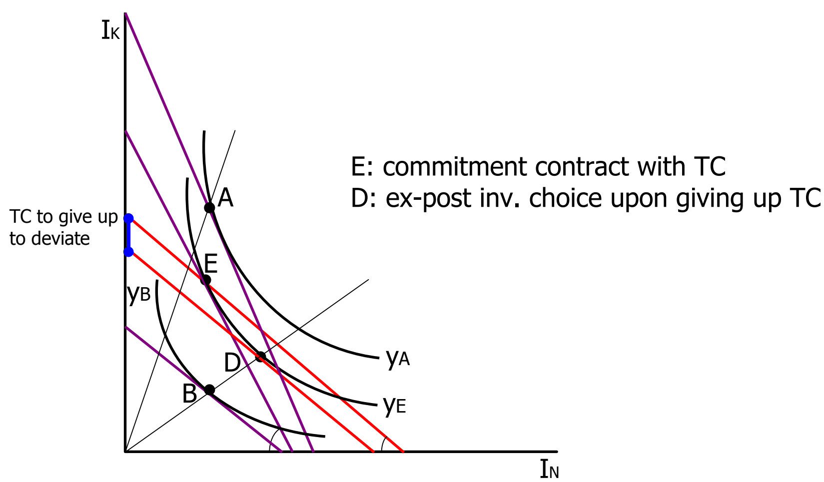

Figure 4: The commitment role of trade credit.

Point E in Fig. 4 represents the equilibrium level of production and the corresponding input combination when trade credit acts as a commitment device. Point E has several properties. First, it is incentive compatible. This is captured by the fact that the level of production obtained under deviation (point D) and under commitment (points E) are the same (D and E lie on the same isoquant). So, by deviating the entrepreneur does no better than in the original contract. Indeed, while it is true that by deviating he can distort the input combination towards the most productive one, this does not necessarily allow him to increase production. This is because the supplier is unwilling to alter the input provision while keeping unaltered the terms of the credit contract, since, if he did, he would not break-even. Thus, in order to deviate, the entrepreneur has to give up trade credit and pay all inputs on account. The bank can thus prevent deviation by reducing the set of contract offers so as to render too costly for the entrepreneur to renounce to trade credit, i.e. in such a way that the amount of trade credit he has to give up implies a reduction in the scale of production that makes him indifferent between sticking to the original contract and deviating. The vertical distance between the two isocosts passing through points D and F represents the amount of trade credit

Point E lies between point A (first-best) and point B (uncollateralized bank contract). The commitment contract cannot replicate the first-best (point A), since the cheaper bank credit is partially substituted with the more expensive trade credit. However, the commitment contract gives higher profits relative to the uncollateralized bank contract (point B). By signaling that the bank loan will be used to purchase the inputs as specified in the bank contract, trade credit makes the collateralized contract available to the parties.[13] Although the bank does not observe the firm-supplier contract, it can foresee the participation of the supplier to the venture and anticipate the commitment effect of trade credit. The commitment role of trade credit arises from the supplier providing a share of the capital inputs on credit and having the right to a share $1-\gamma^*$ of the collateral value of the same input in case of default. Both conditions need to be satisfied for the entrepreneur to have no incentive to ex-post alter the input mix.

Finally, since the commitment effect of trade credit is a larger amount of collateralized external financing, point E has an input combination more intensive in tangible assets than point B.

The previous analysis allows us to get the following predictions:

Prediction 1. *When investment is non-contractible, trade credit facilitates the access to bank financing by making the secured contract available to bank and entrepreneur.*

Prediction 2. *Firms using trade credit and collateralized bank credit invest more intensively in tangible assets than firms using only un-collateralized bank credit.*

4 Firm-supplier collusion

In Section 3 we have argued that taking trade credit allows to overcome the commitment problem between entrepreneur and bank. This is because, upon observing the entreprenur's deviation, the supplier refuses to extend trade credit due to its failure to break even on the new input choice. Anticipating this, the bank reduces its loan sufficiently to make trade credit indispensable and deviation unprofitable. However, even under deviation, it is still possible for the supplier to extend credit while breaking even by renegotiating the terms of the contract. Suppose that entrepreneur, bank and supplier have agreed on the contract terms described in Prop. 3, with $\gamma = \gamma^*$. Once obtained the loan from both financiers, $L_B^*(\gamma^*) + L_S^*(\gamma^*)$, the entrepreneur may then propose the supplier an agreement to alter the input mix at the bank's expense. More specifically, keeping unchanged the total credit received from financiers, entrepreneur and supplier might jointly decide to alter the input combination

[13] Trade credit provides commitment if the entrepreneur cannot resell the inputs bought on credit in a secondary market or, if he can, the transaction costs are high enough to fully compensate the benefits of deviation.

towards the most productive one, thus jeopardizing the bank's repayment in the low state. This collusive agreement is profitable for the supplier as long as he breaks even under the new contract terms, i.e.:

$$pR_S + (1-p)(1-\gamma^*)\beta I_K \geq L_S^*(\gamma^*) r_S. \tag{14}$$

where $L_S^*(\gamma^*)$ is the loan provided by the supplier under the original agreement (see Proposition 3 with $\gamma = \gamma^*$). If agreed, the new arrangement allows to increase overall profits at the expense of the bank. Any collusive rent - the difference between the return under deviation and the return under commitment - is then shared between entrepreneur and supplier. For simplicity, we assume that any collusion rent is seized by the entrepreneur. This is witout loss of generality as alternative distributions would not alter our qualitative results.

Several factors can make it difficult reaching such an agreement. For example, when the firm uses several suppliers, reaching an agreement can be time consuming.[14] Define $\alpha \in [0,1]$ as the fraction of the return from deviation which is lost in reaching an agreement. The next proposition defines the collusion rent and describes its properties.

Definition 2 *Define* $(1-\alpha)\Pi^{\mathcal{C}}(\gamma) - \Pi^*(\gamma)$ *as the collusion rent, where* $\Pi^{\mathcal{C}}$ *is the surplus from collusion for a generic* γ*, and* $\Pi^*(\gamma)$ *is the surplus from sticking to the original contract as defined in Prop. 3 for a generic* γ*.*

Proposition 5 *The collusion rent* $(1-\alpha)\Pi^{\mathcal{C}}(\gamma) - \Pi^*(\gamma)$ *is increasing in* γ *and decreasing in* α*. When* $\alpha = 0$*, the collusion rent is positive for any* $\gamma > 0$*.*

When the collusion rent is positive, supplier and entrepreneur will reach an agreement to change ex-post the input combination causing negative profits to the bank. To avoid this case, the set of contract offers made by the bank will be restricted to those that guarantee a non positive collusion rent, i.e. those satisfying the following condition:

$$(1-\alpha)\Pi^{\mathcal{C}}(\gamma) - \Pi^*(\gamma) \leq 0. \tag{15}$$

Is the commitment contract collusion proof? More specifically, at $\gamma = \gamma^*$ (the one that ensures no incentives to deviate), is constraint (15) satisfied? The answer depends on the cost of collusion, α. We thus define $\alpha(\gamma^*)$ as the threshold level of α at which constraint (15) is satisfied with equality. It

[14] Section 5.2 links the cost of collusion to the number of suppliers.

follows that for any $\alpha \geq \alpha(\gamma^*)$, constraint (15) is slack and the commitment contract is also collusion proof. Conversely, for any $\alpha < \alpha(\gamma^*)$, constraint (15) is violated and the commitment contract is not collusion proof. To prevent collusion, the bank has to reduce γ below γ^* to satisfy (15). For any value of $\alpha < \alpha(\gamma^*)$, there is a $\hat{\gamma}(\alpha)$, such that (15) holds with equality.

It follows that the properties of the optimal contract depend on the cost of collusion. The analysis can be divided in three cases. First, collusion is so costly that it is never profitable: $\alpha(\gamma^*) \leq \alpha \leq 1$. This case corresponds to the one already analyzed in Section 3: The entrepreneur buys commitment from the supplier through trade credit and takes a collateralized loan from the bank.

Second, collusion is still costly but profitable: $0 < \alpha < \alpha(\gamma^*)$. The bank can reduce the scope for collusion by further reducing its participation to the venture. Its stake in the bad state - the maximum share of the collateral to be liquidated - and the bank loan shrink. The lower the cost of collusion, the lower the bank participation and the higher the amount of trade credit necessary to make the contract collusion proof. The properties of the optimal contract are similar to the ones described in Prop. 3, but with $\gamma = \hat{\gamma}(\alpha)$. The new equilibrium point is represented by point C in Fig. 5. This point will be located between point E (commitment contract) and point B (un-collateralized bank contract), depending on the cost of collusion. The higher the cost of collusion, the further from point E and the closer to point B, the new equilibrium will be. A lower bank participation and a larger trade credit will be needed to make the contract collusion-proof.

Lastly, collusion is costless ($\alpha = 0$), so that entrepreneur and supplier can grab the entire surplus from their agreement. In this case, the only way for the bank to break-even is by offering an uncollateralized credit contract. Having no longer a stake in the firm's low state return, the bank is paid only in the high state. This arrangement removes completely the firm-supplier incentives to collude. Trade credit can be still used but only to exploit the liquidation ability of the supplier, not to buy commitment. In this case, the supplier finances and liquidates the full collateral value of the tangible input (not the full input purchases). The new equilibrium point will be still located away from point B and on its left, since the entrepreneur can at least pledge some collateral to the supplier, therefore improving with respect to the case where only the un-collateralized bank credit is available (point B).

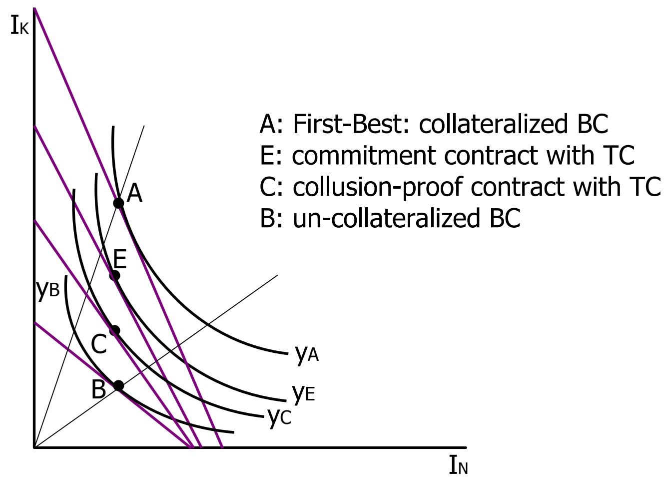

Figure 5: The collusion-proof contract.

5 Testable Predictions

Section 5 delivers new testable predictions on firms decisions (both investment and financing). More specifically, Subsection 5.1 exploits cross-sectional differences in input characteristics like liquidity and substitutability and in labor market organization (efficient versus black labor markets). Subsection 5.2 links the cost of collusion to the number of suppliers and exploits differences in the degree of competition of the up-stream market.

5.1 The commitment problem in the cross-section of firms and markets

In our model, the degree of liquidity of the tangible input is important. Trade credit allows to solve the commitment problem between bank and entrepreneur only if the entrepreneur cannot easily resell the input bought on credit from the supplier. If the tangible input is very liquid (i.e, it is highly redeployable), the entrepreneur, upon receiving credit from bank and supplier and after buying the inputs as agreed in the contract, could resell some of the tangibles to increase the investment in the intangibles. In this case, the entrepreneur would get higher profits but none of the financiers would break-even. Anticipating negative expected profits, the supplier would not be willing to offer trade credit and the bank would refuse to offer any collateralized bank loan. Thus, we get the following prediction:

Prediction 3. *Firms are more likely to use collateralized bank and trade credit, the lower the degree of liquidity of tangible inputs.*

A second important characteristic of the inputs is the degree of substitutability. The commitment problem arises because the entrepreneur has discretion over the choice of the input combination, once the bank has provided its loan, since the bank cannot observe the input purchases before providing the loan. For trade credit to play a role as a commitment device, it is crucial that the project uses inputs that are at least partially substitutes. Moreover, one would then expect that the lower the degree of substitutability between inputs, the less severe the commitment problem between entrepreneur and bank. In the extreme case where inputs are used in fixed proportion, there are no extra-profits to be gained from changing ex-post the input combination. There is no incentive to deviate and the entrepreneur has access to collateralized bank financing without the need to take trade credit. Thus, we get the following prediction:

Prediction 4. *Firms are more likely to use collateralized bank and trade credit, the higher the degree of substitutability between inputs.*

Finally, one could argue that one way to solve the commitment problem could be by conditioning the provision of financing to the number of employees. In a labor market where workers can not easily fired, this would work as a commitment. The bank is willing to offer collateralized bank credit without the need to take any trade credit. However, this practice is more commonly used in the public sector when the government allocates public financing to private companies, than in the banking industry. Moreover, this practice is less likely to work when workers can be hired also in the black market. In this case, the entrepreneur could easily change the input combination specified in the ex-ante contract by hiring the extra workers in the black market. Anticipating that, the bank would refuse any collateralized credit. It follows that with a black market, firms are more likely to take trade credit as a commitemnt device. Thus, we get the following prediction:

Prediction 5. *Firms are more likely to use collateralized bank and trade financing, the easier workers can be hired in the black market.*

5.2 Firm decisions and the up-stream market structure

In section 4, we have shown that the easier is to reach a collusive agreement, the higher the reliance on trade credit necessary to guarantee commitment, the less distorted towards the tangible input is the input choice. In this section, we relate the cost of reaching such a collusive agreement to the number of suppliers and we derive some testable predictions on the relation between the firm's financial structure and the degree of competition of the upstream market.

We argue that if the up-stream market is concentrated, it may be relatively easy to reach a collusive agreement as the firm has to deal with one or few suppliers. This corresponds to the case in which α is small or even zero. Conversely, if the upstream market is very competitive and the firm relies on several suppliers, it may be more difficult to find a collusive agreement given the number of parties involved in it. In this case, α would be relatively large.[15] Under this interpretation, our model delivers new relations not only between number of suppliers and trade credit use, but also between the structure of the up-stream market and both the size of bank financing and the type of bank contract (secured versus unsecured). Finally, since financing decisions affect input choices, we identify new relationships between input choices and the up-stream market structure. Fig. 6 summarizes the use of bank and trade credit and the investment level for different degrees of competition in the up-stream market.

Up-stream market structure and firm decisions

	Competitive mkt $\alpha(\gamma^*) < \alpha \leq 1$	Intermediate mkt $0 < \alpha \leq \alpha(\gamma)^*$	Concentrated mkt $(\alpha = 0)$
Trade credit	Low	Medium Decreasing in α	High TC=βI
Bank credit	Collateralized (γ=γ*); BC high	Collateralized (0<γ(α)<γ*); BC increasing in α	Uncollateralized (γ=0); BC low, BC=(1-β)I
Investment	Second-best	Third-best	Third-best
Tangibility	High	Medium	Low

Figure 6: Firm financing and investment decisions in three different up-stream market structures.

When the up-stream market is very competitive, $\alpha(\gamma^*) \leq \alpha \leq 1$, a small amount of trade credit suffices to generate commitment. Bank credit is highly collateralized and it is the main source of external financing. This situation is represented in the first column of the table in Fig. 6 and corresponds to the case analyzed in Section 3: The economy reaches the second-best and the input combination is highly biased towards tangibles (see point E in Fig. 4). When competition in the upstream market decreases (second column of Fig. 6) , the cost of reaching a collusive agreement decreases as well. To signal a credible commitment, the entrepreneur needs to reduce even more the bank stake and enlarge the use of trade credit, which however reduces the total surplus (third best).

[15] Of course, while it is true that in a concentrated upstream market the firm deals with few suppliers, it is not necessarily true that the firm deals with several suppliers if the upstream market is competitive.

The bank contract is still collateralized but bank financing and collateral are reduced compared to the previous case. The input combination is still biased towards tangibles (as it was in previous case) but to a lower extent. Finally, in the extreme situation where there is only one supplier, i.e. the up-stream market is very concentrated and $\alpha = 0$ (third column in Fig. 6), entrepreneur and supplier will always cheat at the expenses of the bank. Thus, the bank only offers an un-collateralized credit contract. The firm still uses trade credit but just to exploit the liquidation technology of the supplier in case of default, rather than as commitment device. There is still a bias towards tangibles but lower than in the previous two scenarios.

The discussion allows us to get the following predictions:

Prediction 6. *Trade credit facilitates the firm's access to collateralized bank loan the higher the degree of competition of the up-stream market.*

Prediction 7. *The amount of bank credit is increasing in the degree of competition of the up-stream market.*

Prediction 8. *Trade credit is decreasing in the degree of competition of the up-stream market, therefore larger when the supplier is a monopolist.*

Prediction 9. *Firms use technologies more intensive in tangible assets the higher the degree of competition of the up-stream markets.*

6 Discussion

Section 6 discusses the robustness of our theoretical setting. Specifically, Subsection 6.1 identifies the conditions that make trade credit beneficial to the lending relationship. Subsection 6.2 discusses the degree of information sharing between supplier and bank. Subsection 6.3 questions the role of the supplier as an informed lender and discusses alternative interpretations. Subsection 6.4 discusses the seniority rule implied by the optimal contract.

6.1 When take trade credit?

Trade credit has costs and benefits. The main benefit is the commitment effect which brings larger external financing. However, since trade credit is more expensive than bank loans, this benefit comes at a cost. The cost can be relatively small when a small trade credit is needed to create commitment and thus the funding mainly comes from the bank. This case is discussed in Section 3 and represented by point E in Fig. 4. However, when the collusion between entrepreneur and supplier is profitable and not too costly, a larger amount of trade credit is needed to obtain commitment. In such circumstances, the lower the cost of collusion, the more the bank has to reduce its stake in the bad state, and therefore the amount of loan provided, to prevent collusion, the more the entrepreneur has to increase its reliance on trade credit. In the extreme case in which colluding is costless, collusion is always profitable. Since the bank cannot prevent deviation, the only way to break even is by offering an uncollateralized contract. This removes any incentive to collude since no benefits can be gained by altering ex-post the input combination. However, while trade credit is no longer beneficial as a commitment device, it can still be used to exploit the supplier liquidation technology.

The relevant question here is: Whether it is taken for its commitment role ($\alpha > 0$) or for its liquidation role ($\alpha = 0$), are there benefits from taking trade credit always exceeding its costs? Proposition 6 identifies under which condition trade credit use is beneficial.

Proposition 6 *The entrepreneur takes trade credit if and only if:*

$$r_S \leq \frac{r_B}{p} \tag{16}$$

To see why, consider first the commitment case, where both bank and supplier offer a collateralized credit contract. Without any loss of generality and only for convenience, assume that the cost of collusion is high enough that the commitment contract is also collusion-proof. Since trade credit is more expensive than bank credit, the firm takes the minimal amount of trade credit that guarantees commitment, i.e. $(L_S r_S = (1-\gamma^*)\,\beta I_k^*)$. By the non-decreasing repayments condition (12) and the participation constraint (10), the supplier gets equal returns in the two states, $R_S = L_S r_S = (1-\gamma^*)\,\beta I_k^*$, and the price of each unit of trade credit is r_S.

When the firms does not take trade credit, only the un-collateralized bank financing is available. In fact, without trade credit, the firm has incentive to alter the input combination. Anticipating the entrepreneur's deviation, the bank offers an un-collateralized credit contract. Since under this contract, the bank only gets paid in the high state - no collateral can be pledged -, its repayment must be high enough to compensate for the probability of default. Therefore, the cost of each unit of bank credit is r_B/p.

In deciding whether to rely on trade credit, the firm compares the cost of trade credit, r_S, with the cost of un-collateralized bank credit, r_B/p. If $r_S < r_B/p$, having the supplier acting as a financier implies a welfare improvement. The firm uses trade credit at least up to any amount equal to the collateral value of the inputs, i.e. $L_S = \beta I_k / r_S$. For any marginal unit of credit exceeding the collateral value, the firm can repay the supplier only out of the high state return. The cost of financing this extra unit through trade credit becomes r_S/p, thus higher than using un-collateralized bank credit r_B/p. So, the rest of the inputs while be financed trough bank credit.

6.2 How much information sharing between financiers do we need?

So far, we have shown that trade credit can be used by the entrepreneur to signal his willingness to stick to the ex-ante bank efficient contract. Once the bank gets this signal, it offers a collateralized loan. Does our story imply that the bank needs to observe the amount of trade credit taken or even the full properties of the supplier-entrepreneur contract? How much information sharing between bank and supplier is needed to access a collateralized bank loan?

In our model, the bank only needs to know that there are suppliers willing to lend money to their customers. All the other relevant information, including the specific amount of trade credit offered, is inferred by the bank by solving the entrepreneur's optimization problem. The intuition is the following. In period $t = 1$, the bank (as well as the supplier) offers a menu of contracts (see the time-line in Fig. 1). Each contract offer solves the entrepreneur's optimization problem for any given combination of bank and trade credit. Contracts merely satisfying the bank's participation constraint would not be incentive compatible for the entrepreneur and therefore are not offered. The entrepreneur then chooses the contract that minimizes the total costs of external financing. Thus, when the entrepreneur chooses a given bank contract, the bank immediately infers how much trade credit will be taken by the entrepreneur as well as the repayment to the supplier in the good and in the bad state.

Existing evidence shows that banks usually ask for financial information and for balance sheet data (which include data on account payables) when considering a loan application. In addition, banks have access to valuable information about trade credit through credit bureaus. For example, the firm's trade credit payment history is routinely included in the credit reports provided by credit bureaus. This evidence suggests that in practice banks have more information about trade credit use than the one assumed in our model.

Besides questioning the role of information sharing in our model, one could also wonder to what extent the commitment role of trade credit depends on the number of banks and suppliers in the market or on the type of information both financiers have about these two market structures. It turns out that while for banks it is important to know how many other banks the firm is obtaining financing from, information regarding the number and type of financiers is irrelevant for suppliers. To see why, consider that each supplier sets the input provision so as to break-even, when supplying inputs on credit. Since the loan is closely tied to the amount of inputs supplied, the supplier can ignore any information regarding competing financiers. Conversely, each bank needs to know how many other banks concur to provide financing. If not, the total loan size obtained may be larger than the incentive-compatible one, giving the entrepreneur an incentive to deviate. Anticipating this, each bank would offer an uncollateralized contract and trade credit would play no role in providing commitment. Knowing the number of competing banks instead, say n, each bank offers a share $1/n$ of the total incentive-compatible loan and the commitment role of trade credit is preserved. Thus, while the actual number of banks is immaterial, it is crucial that all banks are informed about the exact banking market structure.

6.3 Is the supplier the only informed lender?

In our analysis, we assume the costly informed lender to be the supplier and the cheaper uninformed financier to be the bank. Other interpretations are however possible. We could assume that both financiers are specialized intermediaries but with different sets of information. For example, the costly informed financier is a relationship lender (also called local lender), while the lender providing cheaper funding with no or less information is an arm's-length lender. If we follow this interpretation, our results would predict that while local lenders can offer collateralized credit contracts, arm's-length lenders provide unsecured financing, in line with the existing evidence (Avery *et al.*; 1998; Zucherman, 1996; Frame *et al.*, 2001, 2004).

The reason why we do not follow this interpretation is related to the nature of the unobservability problem. A relationship lender is assumed to have a comparative advantage in collecting customer-specific information through multiple interactions (Boot, 2000). These features are mostly relevant when the unobservability concerns firms' characteristics, such as project quality or entrepreneurial effort. When the unobservability concerns investment, as in our setting, a specialized financial intermediary is not necessary: The relevant information is observed costlessly by the supplier by providing the input, with no need for multiple interactions. For this reason, the bank-supplier interpretation seems more natural in our framework.

Alternatively, we could interpret the informed lender as a lessor that receives a fee from the borrower for the use of the tangible inputs and retains the ownership on those assets. Although appealing, this interpretation is flawed as the lessor fails to solve the unobservability problem. To see why, consider the case analyzed in Section 3, in which the presence of the supplier allows the bank to offer a partially secured contract.

Can the lessor play the same role as the supplier? Consider a contract for the financing of a given amount of the intangible input and several units of the tangible input, say cars. The contract might state that a certain number of cars (say, 20) will be financed through a secured contract with the bank, while the remaining 30 through a leasing contract. Upon receiving the loan from the bank, the entrepreneur has an incentive to reduce the number of cars purchased with bank financing, say from 20 to 10. The lessor will not refrain the entrepreneur from buying a lower number of cars for two reasons: First, he does not observe the entrepreneur's reduction of 10 units of cars purchases. Second, even if he does, he has no incentive to stop the entrepreneur's opportunistic behavior: Being still the owner of his 30 cars, his return in defaulting states is never jeopardized by this reduction. Anticipating the entrepreneur's deviation, the bank will never propose such an agreement. This unilateral deviation from the contracted input mix is not feasible if the informed party is the supplier: He would observe it and prevent it from happening, in order to preserve the return he is promised in defaulting states, which is a fraction of the 50 initial cars.

Thus, while under a leasing contract a profitable unilateral deviation by the entrepreneur is always possible, under a trade credit contract such deviation is never possible unless the entrepreneur colludes with the supplier. This is the very reason why trade credit can provide commitment.

In the extreme case in which reaching a collusive agreement is costless, there is no secured contract the bank is willing to offer (see Section 4). Dealing with an informed party serves only to extract value from unused assets in default (and not to solve the commitment problem). In this case, both a leasing and a trade credit contract may serve this scope. Whether the entrepreneur will then use the supplier or the lessor as a liquidator will depend on the specific provisions of the bankruptcy codes and on the comparison between the liquidation abilities of the two financiers.[16]

6.4 Is there an optimal seniority?

In standard contract theory with multiple financiers, the creditor with a monitoring advantage should be junior so as to have a stronger incentive to monitor the project and gather the relevant information. Applying this finding to our context, one would expect the bank to be senior and the supplier junior.

[16] A liquidation advantage of leasing relative to secured lending has been modeled by Eisfeldt and Rampini (2009).

However, our optimal contract requires both the bank and the supplier to be senior. The reason for this discrepancy is the following. There is no need to give the supplier stronger incentive to monitor the project, given that he becomes informed simply by selling the inputs. Nevertheless he must be given contractual seniority, otherwise his information becomes irrelevant and trade credit fails to generate commitment. Indeed, by making him junior, the supplier has no liquidation rights in the bad state. Being paid only in the good state, the supplier has no longer incentives to stop the entrepreneur from deviating from the efficient contract. Trade credit works as a commitment device only if the supplier has a stake in the bad state.

7 Conclusions

Collateral is largely used in bank lending. This paper investigates how collateral interacts with the use of different financing sources to jointly determine the firm's total financing and its input choice. Specifically, the paper argues that when the investment in a tangible asset is not contractible, pledging this asset as collateral not necessarily increases the firm debt capacity. Collateral might indeed introduce in the lending relationship a problem of moral hazard in the form of asset substitution. This prompts a role for alternative financiers with some information advantage vis-a-vis the bank. We argue that the supplier of the tangible input is the natural candidate, since he observes whether the investment has taken place. Trade credit can be used to mitigate the contract incompleteness with the bank and restores the benefits of bank secured lending. Our paper highlights investment contractibility as a crucial determinant of debt capacity and it helps explaining why some tangible assets have low debt capacity, despite their high resalability, and the existence of a liquid secondary market.

The paper also considers the case of collusion between entrepreneur and supplier at the bank's expenses. By linking the cost of collusion to the number of suppliers used by the firm, the paper delivers new testable relations among financing decisions (including both the amount of trade and bank credit and the type of contract), input choices and the competitive structure of the up-stream market. We leave the empirical verification of these testable predictions to a future work.

Appendix

Proof of Proposition 1. The first-best investment in capital I_K^* and labor I_N^* satisfies the following FOC's:

$$p\frac{\partial f(\cdot,\cdot)}{\partial I_K} = r_B - (1-p)\beta, \quad (17)$$

$$p\frac{\partial f(\cdot,\cdot)}{\partial I_N} = r_B, \quad (18)$$

obtained differentiating the reduced form objective function (4) wrt I_K and I_N. Using I_K^{FB}, I_N^{FB} in constraints (3) and (2) gives the optimal bank loan, L_B^{FB}, and the repayment in the good state, R_B^{FB}, respectively:

$$L_B^{FB} = I_N^{FB} + I_K^{FB}, \quad (19)$$

$$R_B^{FB} = \frac{1}{p}\left\{\left(I_N^{FB} + I_K^{FB}\right) r_B - (1-p)\beta I_K^{FB}\right\}. \quad (20)$$

■

Proof of Proposition 2. The proof proceeds in three steps. The first step consists in showing that under non-contractible investment a secured credit contract is time-inconsistent. In the second step we analyze the input combination chosen after the loan has been granted, showing that it involves a lower investment in the collateralizable input and thus insufficient low state repayments to the bank to break even. In the third step we show that the bank may prevent this by offering an unsecured credit contract with a lower loan and a subsequent efficiency loss.

The first step has been proved in the main text.

To prove the second step, we first need to determine the optimal choice when the firm breaches the terms of the fully collateralized credit contract with contractible investment. Consider programme $\mathcal{P}^{\mathcal{D}}$ faced by the firm which has obtained a loan L_B^{FB}. Solving the resource constraint (7) for $I_N\left(L_B^{FB}, I_K\right) = L_B^{FB} - I_K$ and substituting out in the objective function (5), the firm's problem is:

$$\max_{I_K} \; pf\left(I_K, L_B^{FB} - I_K\right) - pR_B^{FB}$$

whence, differentiating wrt I_K

$$p\left(\frac{\partial f(\cdot,\cdot)}{\partial I_K} + \frac{\partial f(\cdot,\cdot)}{\partial I_N}\frac{dI_N}{dI_K}\right) = 0 \quad (21)$$

From the resource constraint, $\frac{dI_N}{dI_K} = -1$, whence from (21), we get $\hat{I}_K\left(L_B^{FB}, R_B^{FB}\right) = \hat{I}_N\left(L_B^{FB}, R_B^{FB}\right) = L_B^{FB}/2$.

By studying the change in input demand induced by a change in relative input prices while keeping the loan constant,[17] the second step amounts to analyze how the demand for the two inputs changes with an increase in the price of input I_K (and a decrease in the relative price of I_N). By the concavity of the production function, the own-price effect is non-positive, which implies that there is a decrease in the demand for I_K. Because the loan is kept constant and the firm uses only two inputs, the

[17] This implies that at the new input prices it must be possible for the firm to afford the initial input combination.

cross-price effect is non-negative, i.e., the demand for input I_N must increase. The decrease in I_K implies that in default the collateral value of the input is insufficient to repay the bank, which does not break even. The bank offers an unsecured credit contract thereby reducing the loan provided.

The entrepreneur chooses I_K, I_N, R_B to maximize (1) subject to the participation constraint (2) with $C = 0$, and to the resource constraint (3). Solving (2) for R_B and using L_B from the resource constraint (3), the objective function (1) becomes:

$$\max_{I_K, I_N} pf\left(I_K, I_N\right) - \left(I_N + I_K\right) r_B.$$

The optimal input combination must satisfy the following first-order conditions:

$$p\frac{\partial f\left(\cdot,\cdot\right)}{\partial I_K} = 1 \tag{22}$$

$$p\frac{\partial f\left(\cdot,\cdot\right)}{\partial I_N} = 1 \tag{23}$$

which gives $I_K = \bar{I}_K\left(p, r_B\right), I_N = \bar{I}_N\left(p, r_B\right)$, with $\bar{I}_K\left(p, r_B\right) = \bar{I}_N\left(p, r_B\right)$. Using these in (3) and (2) gives $\bar{L}_B = \bar{I}_K\left(p, r_B\right) + \bar{I}_N\left(p, r_B\right)$ and $R_H^B = \frac{1}{p} r_B \bar{L}_B$.

■

Proof of Proposition 3. Consider programme $\mathcal{P}^\gamma$ in the text. Substituting the binding constraints in the objective function gives

$$\max_{I_K, I_N} pf\left(I_K, I_N\right) - \left(I_K + I_N\right) r_B + \beta I_K \frac{r_B}{r_S}\left(1-\gamma\right) + \left(\gamma - p\right)\beta I_K$$

$$\max_{I_K, I_N} pf\left(I_K, I_N\right) - \left(I_N + I_K - \frac{1-\gamma}{r_S}\beta I_K\right) r_B + \left[\gamma\left(1-p\right) - p\left(1-\gamma\right)\right]\beta I_K$$

whence

$$p\frac{\partial f\left(\cdot,\cdot\right)}{\partial I_K} = r_B - \beta\frac{r_B}{r_S}\left(1-\gamma\right) - \left(\gamma - p\right)\beta \tag{24}$$

$$p\frac{\partial f\left(\cdot,\cdot\right)}{\partial I_N} = r_B \tag{25}$$

Using $I_K^*\left(\gamma\right), I_N^*\left(\gamma\right)$ in them in (24) and (25), these are satisfied as identity. Differentiating wrt γ,

$$\begin{aligned} p\frac{\partial^2 f\left(I_K^*\left(\gamma\right), I_N^*\left(\gamma\right)\right)}{\partial I_K^2}\frac{\partial I_K}{\partial\gamma} + p\frac{\partial^2 f\left(I_K^*\left(\gamma\right), I_N^*\left(\gamma\right)\right)}{\partial I_K \partial I_L}\frac{\partial I_L}{\partial\gamma} - \beta\left(\frac{r_B}{r_S} - 1\right) &= 0 \\ p\frac{\partial^2 f\left(I_K^*\left(\gamma\right), I_N^*\left(\gamma\right)\right)}{\partial I_L \partial I_K}\frac{\partial I_K}{\partial\gamma} + p\frac{\partial^2 f\left(I_K^*\left(\gamma\right), I_N^*\left(\gamma\right)\right)}{\partial I_L^2}\frac{\partial I_L}{\partial\gamma} &= 0 \end{aligned}$$

whence

$$\begin{bmatrix} \frac{\partial I_K}{\partial\gamma} \\ \frac{\partial I_L}{\partial\gamma} \end{bmatrix} = p\begin{bmatrix} f_{KK} & f_{KL} \\ f_{LK} & f_{LL} \end{bmatrix}^{-1}\begin{bmatrix} \beta\left(\frac{r_B}{r_S} - 1\right) \\ 0 \end{bmatrix}$$

and, using Cramer's rule, $\dfrac{\partial I_K}{\partial \gamma} = p\dfrac{\begin{vmatrix} \beta\left(\frac{r_B}{r_S}-1\right) & f_{KL} \\ 0 & f_{LL} \end{vmatrix}}{|H|} > 0$, $\dfrac{\partial I_L}{\partial \gamma} = p\dfrac{\begin{vmatrix} f_{KK} & \beta\left(\frac{r_B}{r_S}-1\right) \\ f_{LK} & 0 \end{vmatrix}}{|H|} \lesseqgtr 0$ depending on $f_{LK} \lesseqgtr 0$.

The expected profits are

$$\Pi^*(\gamma) = p\left[f\left(I_K^*(\gamma), I_N^*(\gamma)\right) - R_B^*(\gamma) - R_S^*(\gamma)\right] = \tag{26}$$
$$pf\left(I_K^*(\gamma), I_N^*(\gamma)\right) - \left[I_K^*(\gamma) + I_N^*(\gamma)\right] r_B + \beta I_K^*(\gamma)\frac{r_B}{r_S}(1-\gamma) + (\gamma - p)\beta I_K^*(\gamma).$$

By the envelope theorem, they are increasing in γ:

$$\left(1 - \frac{r_B}{r_S}\right)\beta I_K^*(\gamma) > 0$$

■

Proof of Proposition 4. To prove this, we only need to add to programme $\mathcal{P}^\gamma$ the no deviation condition (13). This is obtained by comparing profits under commitment and a generic γ specified in Proposition 3, $\Pi^*(\gamma)$, with profits under deviation, $\Pi^{\mathcal{D}}(\gamma)$. The latter are obtained by solving the following optimization problem $\mathcal{P}^{\mathcal{D}}$:

$$\max_{I_K, I_N, R_B} \; p\left[f\left(I_K, I_N\right) - R_B\right]$$
$$\text{s.t. } R_B \geq R_B^*(\gamma), \tag{27}$$
$$L_B^*(\gamma) \geq I_N + I_K, \tag{28}$$

where constraint (27) requires the repayment to the bank in the high state be no less than the one promised in the secured commitment contract (i.e. $R_B^*(\gamma)$ in Prop. 3), while the resource constraint (7) requires that the total input expenditure be no higher than the loan obtained in the secured commitment contract (i.e. $L_B^*(\gamma)$ in Prop. 3).

Using $R_B^*(\gamma)$ and $I_N = L_B^*(\gamma) - I_K$, the objective function becomes

$$\max_{I_K} \; p\left[f\left(I_K, L_B^*(\gamma) + I_K\right) - R_B^*(\gamma)\right]$$

whence the FOC

$$p\left(\frac{\partial f}{\partial I_K} + \frac{\partial f}{\partial I_N}\frac{dI_N}{dI_K}\right) = 0$$

Using $\frac{dI_N}{dI_K} = -1$ from the resource constraint, the FOC becomes

$$\frac{\frac{\partial f}{\partial I_K}}{\frac{\partial f}{\partial I_N}} = 1$$

whence $I_K^{\mathcal{D}}\left(L_B^*(\gamma)\right) = I_N^{\mathcal{D}}\left(L_B^*(\gamma)\right) = L_B^*(\gamma)/2$.

Using $I_K^{\mathcal{D}}$, $I_N^{\mathcal{D}}$, $L_B^*(\gamma)$ and $R_B^*(\gamma)$ in the objective function gives $\Pi^{\mathcal{D}}(\gamma) = p\left[f\left(I_K^{\mathcal{D}}\left(L_B^*(\gamma)\right), I_N^{\mathcal{D}}\left(L_B^*(\gamma)\right)\right) - R_B^*(\gamma)\right]$ as defined in Definition 1.

Using in programme $\mathcal{P}^{\gamma}$ the extra constraint that there is no gain from deviation (13), i.e.,

$$\begin{aligned} \Pi^{\mathcal{D}}(\gamma) &\leq \Pi^{*}(\gamma) \\ p\left[f\left(I_K^{\mathcal{D}}\left(L_B^{*}(\gamma)\right), I_N^{\mathcal{D}}\left(L_B^{*}(\gamma)\right)\right) - R_B^{*}(\gamma)\right] &\leq p\left[f\left(I_K^{*}(\gamma), I_N^{*}(\gamma)\right) - R_B^{*}(\gamma) - R_S^{*}(\gamma)\right] \end{aligned}$$

This reduces to:

$$p\left\{f\left(I_K^{\mathcal{D}}\left(L_B^{*}(\gamma)\right), I_N^{\mathcal{D}}\left(L_B^{*}(\gamma)\right)\right) - f\left(I_K^{*}(\gamma), I_N^{*}(\gamma)\right) + \beta I_K^{*}(\gamma)(1-\gamma)\right\}$$

and allows us to determine the maximum γ that guarantees that the entrepreneur will stick to the contract.

Differentiating wrt γ

$$\frac{\partial f}{\partial I_K}\frac{\partial I_K^{\mathcal{D}}}{\partial L_B^{*}}\frac{\partial L_B^{*}}{\partial \gamma} + \frac{\partial f}{\partial I_N}\frac{\partial I_N^{\mathcal{D}}}{\partial L_B^{*}}\frac{\partial L_B^{*}}{\partial \gamma} - \beta I_K^{*}(\gamma)$$

By Proposition ..., $\frac{\partial L_B^{*}}{\partial \gamma} =$

$L_B^{*}(\gamma) = I_K^{*}(\gamma) + I_N^{*}(\gamma) - \frac{1}{r_S}(1-\gamma)\beta I_K^{*}(\gamma)$ ■

Proof of Proposition 5 . To prove this, we need to add to programme $\mathcal{P}^{\gamma}$ the collusion-proofness condition (13). This is obtained by comparing profits under commitment and a generic γ specified in Proposition 3, $\Pi^{*}(\gamma)$, with profits under collusion, $\Pi^{\mathcal{D}-\mathcal{S}}(\gamma)$

1. determination of the collusion rent

The ex-post optimization programme with supplier collusion is given by problem $\mathcal{P}^{\mathcal{D}-\mathcal{S}}$:

$$\max_{I_K, I_N, R_B, R_S} \; p\left[f(I_K, I_N) - R_B - R_S\right]$$

$$\text{s.t. } R_B \geq R_B^{*}(\gamma), \tag{29}$$

$$L_S^{*}(\gamma) + L_B^{*}(\gamma) \geq I_N + I_K, \tag{30}$$

and to constraint (14) ($pR_S + (1-p)(1-\gamma^{*})\beta I_K \geq L_S^{*}(\gamma^{*}) r_S$), where $L_B^{*}(\gamma), L_S^{*}(\gamma), R_B^{*}(\gamma)$ are the commitment values of the loan and the bank repayment as defined in Proposition 3, constraint (29) requires the repayment to the bank in the high state be no less than the one promised in the commitment contract (i.e. $R_B^{*}(\gamma)$), while the resource constraint (30) requires that the total input expenditure be no higher than the loan obtained in the commitment contract.

Using $R_B^{*}(\gamma)$,

Solution to this programme gives the return under collusion, $\Pi^{\mathcal{D}-\mathcal{S}}(\gamma)$, which, compared with the return under commitment defines the collusion rent. This is increasing in γ, as shown in Proposition **??**.

Solving programme $\mathcal{P}^{\mathcal{D}-\mathcal{S}}$, we get $I_K^{\mathcal{D}-\mathcal{S}}\left(\gamma, L_B^{*}(\gamma), L_S^{*}(\gamma), R_B^{*}(\gamma)\right), I_N^{\mathcal{D}-\mathcal{S}}\left(\gamma, L_B^{*}(\gamma), L_S^{*}(\gamma), R_B^{*}(\gamma)\right)$, $R_B = R_B^{*}(\gamma)$, $R_S^{\mathcal{D}-\mathcal{S}} = \frac{1}{p}\left[L_S^{*}(\gamma) r_S - (1-p)(1-\gamma)\beta I_K^{\mathcal{D}-\mathcal{S}}\left(\gamma, L_B^{*}(\gamma), L_S^{*}(\gamma), R_B^{*}(\gamma)\right)\right]$.[18]

The return from deviating is

$$pf\left(I_K^{\mathcal{D}-\mathcal{S}}(\gamma), I_N^{\mathcal{D}-\mathcal{S}}(\gamma)\right) - pR_B^{*}(\gamma) - L_S^{*}(\gamma) r_S + (1-p)(1-\gamma)\beta I_K^{\mathcal{D}-\mathcal{S}}(\gamma)$$

[18]For notational simplicity, when referring to the optimal investment in the two inputs, we will henceforth consider only their dependence on γ.

Notice that $I_K^{\mathcal{D}-\mathcal{S}}(\gamma) < I_K^*(\gamma)$. This implies that the payoff to the bank in case of default $\gamma\beta I_K^{\mathcal{D}-\mathcal{S}}(\gamma)$ falls short of the contracted payoff $\gamma\beta I_K^*(\gamma)$ and the bank fails to break even.
Using from the commitment problem $L_S^*(\gamma) \equiv \frac{1}{r_S}\left[pR_S^*(\gamma) + (1-p)(1-\gamma)\beta I_K^*(\gamma)\right]$, the return from collusion becomes

$$pf\left(I_K^{\mathcal{D}-\mathcal{S}}(\gamma), I_N^{\mathcal{D}-\mathcal{S}}(\gamma)\right) - pR_B^*(\gamma) - pR_S^*(\gamma) + (1-p)(1-\gamma)\beta\left(I_K^{\mathcal{D}-\mathcal{S}}(\gamma) - I_K^*(\gamma)\right) \tag{31}$$

which is increasing in γ.

The benefits from colluding are then given by the difference between the two value functions (the return under deviation (31) and the return under commitment (26)):

$$\begin{aligned}&pf_H\left(I_K^{\mathcal{D}-\mathcal{S}}(\gamma), I_N^{\mathcal{D}-\mathcal{S}}(\gamma)\right) - pR_B^*(\gamma) - pR_S^*(\gamma) + (1-p)(1-\gamma)\beta\left(I_K^{\mathcal{D}-\mathcal{S}}(\gamma) - I_K^*(\gamma)\right) - \\ &-p\left[f^H\left(I_K^*(\gamma), I_N^*(\gamma)\right) - R_B^*(\gamma) - R_S^*(\gamma)\right]\end{aligned}$$

This reduces to

$$p\left[f\left(I_K^{\mathcal{D}-\mathcal{S}}(\gamma), I_N^{\mathcal{D}-\mathcal{S}}(\gamma)\right) - f\left(I_K^*(\gamma), I_N^*(\gamma)\right)\right] + (1-p)(1-\gamma)\beta\left(I_K^{\mathcal{D}-\mathcal{S}}(\gamma) - I_K^*(\gamma)\right) = \Pi^{\mathcal{D}-\mathcal{S}}(\gamma) - \Pi^*(\gamma).$$

By the envelope theorem, the derivative of the collusion rent with respect to γ is:

$$\frac{\partial\left(\Pi^{\mathcal{D}-\mathcal{S}} - \Pi^*\right)}{\partial\gamma} = -(1-p)\beta\underbrace{\left(I_K^{\mathcal{D}-\mathcal{S}}(\gamma) - I_K^*(\gamma)\right)}_{<0} > 0.$$

Programme $\mathcal{P}^{\mathcal{CP}}$ below is the collusion-proof generalization of problem $\mathcal{P}^S$:

$$\begin{aligned}\max_{R_B, R_S L_B, L_S, I_K, I_N, \gamma} EP &= p\left[f\left(I_K, I_N\right) - R_B - R_S\right], \\ \text{s.t. } pR_B + (1-p)\gamma\beta I_K &= L_B r_B, &(32)\\ pR_S + (1-p)(1-\gamma)\beta I_K &= L_S r_S, &(33)\\ L_B + L_S &\geq I_N + I_K, &(34)\\ R_S^H &\geq \beta I_K &(35)\\ p\left[f\left(I_K, I_N\right) - R_B - R_S\right] &\geq \max\left\{(1-\alpha)\Pi^{\mathcal{D}-\mathcal{S}}(\gamma), \Pi^{\mathcal{D}}(\gamma)\right\} &(36)\end{aligned}$$

The constraints have the usual meaning. The only difference relative to problem $\mathcal{P}_{\mathcal{S}}$ is the presence of the collusion-proof constraint (36) that ensures that the terms of the contract are such that the entrepreneur has no incentive to deviate, and in which $\Pi^{\mathcal{D}-\mathcal{S}}(\gamma)$ denotes the return from deviation, as defined in Proposition **??**.

The solution to this problem proceeds in two steps. We first show that the firm-bank-supplier collateralized credit contract is prone to collusion between bank and supplier at the expense of the bank, and work out the optimal deviation. We then derive the collusion-proof contract.

1. The three constraints (14), (29) and (30) are all binding. Substituting them out in the objective function

$$pf_H\left(I_K, \left(L_S^*(\gamma) + L_B^*(\gamma) - I_K\right)\right) - pR_B^*(\gamma) - L_S^*(\gamma)r_S + (1-p)(1-\gamma)\beta I_K$$

and differentiating wrt I_K gives

$$p\left(\frac{\partial f(\cdot,\cdot)}{\partial I_K}+\frac{\partial f(\cdot,\cdot)}{\partial I_N}\frac{\partial I_N}{\partial I_K}\right)+(1-p)(1-\gamma)\beta=0 \tag{37}$$

From the resource constraint $I_N=L_B^*(\gamma)+L_S^*(\gamma)-I_K$, $\frac{\partial I_N}{\partial I_K}=-1$, whence from (37), we get

$$p\frac{\partial f(\cdot,\cdot)}{\partial I_K}+(1-p)(1-\gamma)\beta=p\frac{\partial f(\cdot,\cdot)}{\partial I_N}$$

2. The second part of the proof is the analogue of the one derived for Proposition 3 for a generic γ. In that case the value of γ was determined by the minimum exogenous level of trade credit necessary to generate commitment ($\bar{\gamma}$). In the present case the value of γ is endogenous ($\hat{\gamma}$) and obtains solving (36). This value depends on the cost of collusion α.

Using the properties of the contract described in Proposition 3, $\hat{\gamma}(\alpha)$ solves

$$p\left[f^H(I_K,I_N)-\frac{1}{p}\left(\left(I_N+I_K-\frac{1}{r_S}\left((1-\gamma(1-p))\beta I_K\right)\right)r_B-(1-p)\gamma\beta I_K\right)-\beta I_K\right]\geq(1-\alpha)\Pi^{\mathcal{D}-\mathcal{S}}(\gamma)$$

where $Y^{dev}(\gamma)$ ...

To prove that $\hat{\gamma}$ exists we need to show that $(1-\alpha)Y^{dev}(\gamma)-Y^*$ is increasing in γ. ■

Proof of Proposition 6. Because $\beta_B=\beta_S$ and $r_S>r_B$, constraint (12) binds. Thus $R_H^S=\beta_S C$. Using R_H^S in the participation constraint gives

$$L_S=\frac{\beta_S I_K}{r_S}$$

Using the resource constraint (11) to solve for $L_B=I_K+I_N-\frac{\beta_S I_K}{r_S}$ and the participation constraint) to solve for $R_B=\frac{1}{p}\left(I_K+I_N-\frac{\beta_S I_K}{r_S}\right)r_B$, the objective function (8) reduces to

$$\max_{I_K,I_N} pf(I_K,I_N)-\left(I_K+I_N-\frac{\beta_S I_K}{r_S}\right)r_B-p\beta_S I_K$$

which gives $\tilde{I}_K,\tilde{I}_N$ solving the following FOC's:

$$p\frac{\partial f(\cdot,\cdot)}{\partial I_K}-p\beta_S-r_B\left(1-\frac{\beta_S}{r_S}\right) = 0 \tag{38}$$

$$p\frac{\partial f(\cdot,\cdot)}{\partial I_N}-r_B = 0 \tag{39}$$

Substituting out $\tilde{I}_K$ and $\tilde{I}_N$ in the constraints, we obtain the remaining properties described in the Proposition. ■

Proof of Proposition 7 . In order to determine the threshold level of r_S above which the firm gives up a collateralized credit contract we compare the FOC's under the firm-bank no-commitment contract with those of the firm-bank-supplier collusion-proof contract. In particular using (22) and (38), we look for the value of r_S that makes the expected marginal cost of capital equal across the two cases. This value is equal to $r_S=r_B/p$ (and makes the firm indifferent between a collateralized credit contract, obtained by taking trade credit, and an unsecured credit contract using bank credit only). ■

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