A growth model with corruption in public procurement: equilibria and policy implications.

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Abstract

We study the relationship between corruption in public procurement and economic growth, within the Solow framework in discrete time, while assuming that the public good is an input in the productive process and that the State fixes a monitoring level on corruption depending on the tax revenues. The resulting model is a two-dimensional, continuous and piecewise smooth map describing the evolution of the capital per capita and that of the corruption level. We study the model from the analytical point of view: we determine its fixed points, we study their local stability and, finally, we find conditions on parameters such that multiple equilibria co-exist. We also present numerical simulations useful to explain the role of parameters in the long run path of the model and to analyze the structure of the basins of attraction when multiple equilibria emerge. Our study aims at demonstrating that stable equilibria with positive corruption may exist (according to empirical evidences), even though the State may reduce corruption by increasing the wage of the bureaucrat or by increasing the amount of tax revenues used to monitor corruption.

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JEL classification codes: H57, O04, C61, C63, E27

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1 Introduction

Since the pioneering paper of Rose-Ackerman (1975) which deals with the economic analysis of corruption, many works have appeared in the economic literature and much attention has been paid to the relationship between corruption and economic growth. There are several ways in which corruption may reduce economic growth. Rose-Ackerman (1978), Murphy, Shleifer, and Vishny (1991,1993), and Shleifer and Vishny (1993) provide theoretical arguments that corruption deteriorates economic growth through the misallocation of talent and other resources. The empirical literature using cross-country data to estimate how corruption affects growth is mixed, reflecting the different theoretical implications which corruption might have. Mauro (1995) produced the point of reference study for empirical investigation of the impact of corruption on growth for a wide cross section of countries. He found that higher levels of corruption significantly decrease both investment and economic growth. Brunetti, Kisunko and Weder (1997), Brunetti and Weder (1998), Campos, Lien and Pradhan (1999), and Wei (2000), all also found that corruption had a negative impact on investment. Poirson (1998) and Leite and Weidmann (1999) found that corruption has a negative effect on growth. In addition, as Mauro (2002) stresses: "Despite a fairly clear understanding of the causes and consequences of corruption, and renewed attention on the part of policymakers, countries relative degree of corruption has proved to be remarkably persistent. Some countries appear to be stuck in a bad equilibrium characterized by pervasive corruption with no sign of improvement. Interestingly, other countries experience corruption to an extent that seems to be much lower, and persistently so"[1]. Therefore, a good model on corruption should not only provide a good explanation of how corruption can influence economic growth negatively, but also of its persistence over time. This represents the main goal of the present work.
More precisely, in our model, the channel which transmits negative effects of corruption on economic growth, is identified in a corrupt management of public procurement. In fact, public procurement affects life in many different ways and represents a large share of national budgets. The Organization for Economic Cooperation and Development (OECD) has estimated the value of government procurement markets worldwide to be US 2 trillion annually[2]. The risk of corruption is high when amounts of money change hands on this scale.

[1] See Mauro (2002) p. 3.

[2] OECD, The size of Government Procurement Markets (Paris, France: OECD, 2002). www.oecd.org/dataoecd/34/14/1845927.pdf. This amount corresponds to potentially contestable government procurement markets, i.e. markets where competitive pricing exists, where there is actual and effective competition, or there is potential competition due to the existence of low barriers to entry to the market.

In fact, corruption has been widespread in public procurement, affecting public finance, damaging public services, such as the building of schools and the provision and quality of medical care; finally, it hinders efforts to reduce poverty. Globally, Transparency International estimates that at least 400 billion a year is lost to bribery and corruption in public procurement, increasing government costs by about $20-25$ percent (Transparency International 2006). Public procurement is very open to corruption because so much public money is spent on public procurement programs. Unlike the other major areas of a government's public expenditure, public procurement usually implies a relatively small number of high-value auctions (generally a few hundred annual procurement procedures, which may involve millions or even billions of dollars). By contrast, most current area expenditure in the public sector invariably involves a much higher number of low-value transactions, making them less worthwhile for potentially corrupt public officials. The pervasiveness of corruption in public procurement is further reinforced by the relatively high degree of discretion that public officials and politicians typically hold over public procurement programs when compared to other areas of public expenditure. D'Souza and Kaufmann (2010), in order to study the determinants of procurement bribery, utilize survey data from over 11,000 firms operating in 125 countries. They find that procurement bribery is widespread: many managers worldwide acknowledge that firms like theirs pay illicit payments in order to secure government contracts. On average, approximately 32 percent of managers report that firms like theirs do bribe to secure a government contract; this percentage ranges from 13 percent of firms reporting bribery in high–income OECD countries, to 32 percent in middle–income countries and 50 percent in low–income countries.

Although the issue of the effect of corrupt public procurement on economic growth is very relevant, it has only recently attracted a lot of attention. To be more precise, the existing literature has been concerned with the effect of corruption on public procurement (e.g. Celentani and Ganuza, 2002) or with the effect of the presence of public goods on economic growth, as an input to private production (e.g. Barro, 1990). Unlike previous works, we combine these two research lines into a single model which analyzes the role of corruption in public procurement and its effects on growth via a reduction in the quality of public infrastructure and services supplied to the private sector. We analyze a discrete-time Solow growth model[3], considering that corruption, in lowering the quality of the public good, can reduce economic growth.

In order to be more precise, in our model widespread corruption reduces economic growth through two channels: one direct and one indirect. In

[3]For a static analysis on the relationship between corruption and production see Coppier and Michetti (2006).

fact, more corruption involves greater provision of low–quality public goods and therefore lower growth; on the other hand, higher corruption implies, via lower production, lower tax revenues, lower resources for the control of corruption and, therefore, more corruption.

Our model deals with strategic complementarity, whereby if one agent does something it becomes more profitable for another agent to do the same thing. Like models involving strategic complementarities, also our model leads to multiple equilibria[4] : a good equilibrium with low corruption, and a bad equilibrium with widespread corruption. As in Mauro's model (2002), we show that will be a good equilibrium characterized by low corruption and high investment and growth; and a bad equilibrium characterized by widespread corruption and low investment and growth. But the link between corruption and growth, in our model, through a corrupt management of public procurement while in Mauro (2002) the link is represented by misallocation of time versus unproductive transfer of resources[5]. In our model, the strategic complementarity is identifiable with the fact that greater corruption implies a lower growth rate, lower tax revenues and, therefore, a lower monitoring level of corruption which incentivizes greater corruption. Like Del Monte and Papagni (2001), we introduce a public input into the production function, assuming that the supply of public input is affected by corruption, which harms the efficiency of public expenditure.

While Del Monte and Papagni (2001) fix the amount of corruption exogenously – in the sense that they consider that the private sector can count only on a share of public good production while corrupt agents take the rest – we endogenize the level of corruption assuming that firms which produce the public good differ with respect to their "reputational cost", i.e. determining the fraction of firms which produce the low–quality public good by solving a one-shot game via the backward induction method.

Following more recent contributions to the literature (e.g. Bajari and Tadelis, 2001), we consider that the ex–ante quality of a public good is the private information of firms, and only after checks by a controller is the quality verifiable. Then, the State, in order to weed out or reduce corruption, monitors bureaucrats' behavior through controllers, and fixes the monitoring level as a fraction of tax revenues. In fact, in contrast with Brianzoni et al. (2011), we consider that all private firms must pay taxes and a part of the tax revenues is used for future monitoring of the entrepreneur in order to reduce corruption (so that the resources to fight corruption increase with

[4] Aids et al. (2008) offer a model with multiple equilibria corruption–economic growth, depending on the quality of political institutions. In Blackburn et al. (2006) the strategic interaction –the incentive for the bureaucrat to be corrupt depends on the number of the other corrupt bureaucrats– produces multiple equilibria in a context of fiscal corruption (evasion).

[5] This model draws on a strategic complementarity similar to that analyzed by Murphy et al. (1993).

the rate of growth). This new formulation of the model allows us to analyze the role of wages paid to the bureaucrats (that represent a cost for the corrupt bureaucrat) and the role of different propensity of the State to fight corruption, i.e. the amount of tax revenue dedicated to monitoring activity.

The resulting model is a two-dimensional, continuous and piecewise-smooth map describing the evolution of the capital per capita and that of the corruption level. We study the model from the analytical point of view: we determine the fixed points, we study their local stability and, finally, we find parameter conditions which allow multiple equilibria to co-exist. Moreover, our model, consistent with the empirical evidence, explains the persistence of corruption over time and the existence of multiple equilibria. We also present numerical simulations to explain the role of parameters in the long run path of the model and to analyze the structure of the basins of attractor when multiple equilibria emerge. Our study aims at demonstrating that equilibria with positive corruption may exist, even though the State may reduce corruption by increasing the wage of the bureaucrat or by increasing the percentage of tax revenues used to monitor corruption. Our results are finally interpreted in terms of economic policy.

The paper is organized as follows. In section 2, we present our framework and we obtain the final model describing the evolution of the capital per capita and that of the corruption level. In section 3, we determine the fixed points of the system and we discuss some properties related to the parameters of the model. In section 4, we study the local stability of fixed points and we find conditions on parameters for multiple equilibria to be owned by the model. Section 5 is devoted to the economic interpretation of the obtained results also supported by numerical simulations. Section 6 concludes.

2 The model

Following Brianzoni et al. (2001), we analyze an economy composed of three types of players: the State, bureaucrats and private firms.[6] We consider two types of private firms: the one –(j-type)– producing a private good and the other –(i-type)– producing a public good. In order to provide the public good for the private j-type firms, the State must buy the public good from the private i-type firms. We assume that at any time $t = 1, 2, ...$ the State procures a unit of public good from each private i-type firm in order to provide it free to j-type firms.

Like in Brianzoni et al. (2011), the public good can be produced at different quality levels (low–quality public good and high–quality public good)[7]. We assume that the public good's price, at any time $t = 1, 2, ...$

[6]We assume that all economic agents are risk-neutral.

[7]For the nature of public good, e.g. infrastructure, we assume that it is not possible

is constant and given by $p_t = p > 0, \forall t$, and let i-type firms compete over the good's quality: the higher the quality offered, the lower the profit for i-type firms and the higher the welfare of the community. Following Bose et al. (2008), the constant cost of production for an i-type firm is such that if the public good's quality is high the unit cost c^h is also high, while if the public good's quality is low, the unit cost c^l is too, that is $c^h > c^l > 0$. Furthermore, the production of public goods is assumed to be profitable, i.e. $p > c^h$. Each i-type firm produces one unit of public good. We assume that the loss of reputation incurred by the i-type firm detected in a corrupt transaction may affect her/his business. More precisely, it is common knowledge[8] that the i-type firm incurs a specific value $m^i \in [0, 1]$ to the loss of reputation derived from being caught in a corrupt transaction. In addition, we assume that i-type firms are uniformly distributed with respect to their "reputational costs", hence m^i represents the fraction of firms with "reputational costs" lesser or equal to m^i. The bureaucrat receives a salary $w > 0$.[9] The bureaucrats organize a reverse auction for the procurement of the public good and the provision of the good is awarded to the firm which offers the best quality good in the sealed bid. Only the bureaucrat observes the firms' sealed bids. As a general rule, the firm which offers the highest quality wins the auction. The corrupt bureaucrat can, when proclaiming the winner, lie about the quality of the public good in exchange for a bribe b. Let b^d be the bribe demanded by the bureaucrat. Then, the firm can refuse to pay the bribe, or agree to pay and start negotiating the bribe with the bureaucrat. The State, in order to weed out or reduce corruption, monitors bureaucrats: in fact, at any time t there is an endogenous probability $q_t \in [0, 1]$ of being monitored according to the control level fixed by the State and, then, of being reported.

The firm, if detected, must supply the high–quality public good, and suffer the "reputational cost", but it is refunded the cost of the bribe[10] while the bureaucrat loses her/his job. The loss of salary can be regarded as a punishment for the bureaucrat caught in a corrupt transaction. The role of punishment has been highlighted by the literature (e.g D'Souza

any kind of arbitrage on the purchased inputs.

[8]The i-type firms experience different reputation costs when they are detected in corrupt transactions. The differences derive from the economic relevance of entrepreneurial activity. Hence, it is possible to provide a specific value to the i-type firms' reputation costs, and such values belong to the bureaucrats' information set.

[9]It is assumed that no arbitrage is possible between the public and the private sector and that therefore there is no possibility for the bureaucrats to become entrepreneurs, even if their salaries w were lower than the entrepreneur's net return. This happens because the bureaucrat individuals in the population have no access to capital markets, but only a job, and therefore may not become entrepreneurs.

[10]This assumption can be more easily understood when, rather than corruption, there is extortion by the bureaucrat, even though, in many countries, the relevant provisions or laws, stipulate that the bribe shall in any case, be returned to the entrepreneur, and that combined minor punishment, (penal and/or pecuniary), be inflicted on him/her.

and Kaufmann (2010) and C. W. Abramo (2003)). In fact, D'Souza and Kaufmann (2010) stress that only increases in costs and/or decreases in benefits from bribing are bound to change the firms cost-benefit calculus sufficiently for it to cease being a briber and compete for contracts through productive efficiency (rather than rent-seeking). In addition, Abramo (2003) analyzes the factors that may be responsible for corruption in procurement, among them the penalties applicable to bribers found guilty of bribery is particularly important.

2.1 Game description and solution

As in Brianzoni et. al (2010), the economic problem can be formalized with a game tree which shows the interaction between the bureaucrat and the i-th firm which produces one unit of public good. In what follows, we refer to the bureaucrat payoff by a superscript (1) and to the i-th firm payoff by a superscript (2): they represent respectively the first and the second element of the payoff vector $\underline{\pi}_{n,t}, n = 1, 2, 3$ at time t.

The timing of the game is as follows.

In the first stage of the game, the bureaucrat decides[11] the amount to ask for as a bribe b_t^d to award the bid. If the bureaucrat decides not to ask for a bribe ($b_t^d = 0$) to award the bid, then the game ends and the payoff vector for bureaucrat and entrepreneur at time t is: $\underline{\pi}_{1,t} = (\pi_{1,t}^{(1)}, \pi_{1,t}^{(2)}) = (w, p - c^h)$. If the bureaucrat decides to ask for a bribe ($b_t^d > 0$), the game continues to stage two, where the entrepreneur should decide whether to negotiate the bribe to be paid to the bureaucrat or to refuse to pay the bribe.

Should s/he decide to carry out a negotiation with the bureaucrat, the two parties will find the bribe corresponding to the Nash solution to a bargaining game (b_t^{NB}) and the game ends.

At time t, the payoffs will depend on whether the bureaucrat and the entrepreneur are monitored (with probability q_t) or not (with probability $1 - q_t$). If the entrepreneur refuses the bribe, then the payoff vector, at time t is given by $\underline{\pi}_{2,t} = (\pi_{2,t}^{(1)}, \pi_{2,t}^{(2)}) = (w, p - c^h)$. Then the game ends.

Otherwise the negotiation starts. Let b_t^{NB} be the final equilibrium bribe associated to the Nash solution to a bargaining game. Then, at time t, given the probability level q_t of being detected, the expected payoff vector is:

$$\underline{\pi}_{3,t} = (\pi_{3,t}^{(1)}, \pi_{3,t}^{(2)}) = \big((1 - q_t)w + (1 - q_t)b_t^{NB},\\ p - (1 - q_t)c^l - (1 - q_t)b_t^{NB} - q_t c^h - q_t m^i\big) .$$

The game ends.

The one-shot game previously described, may be solved by backward induction, starting from the last stage of the game. At any time t, the

[11] The bureaucrats, if indifferent whether to ask for a bribe or not, will prefer to be honest.

bribe resulting as the Nash solution to a bargaining game in the last subgame should be determined. This bribe is the outcome of a negotiation between the bureaucrat and the entrepreneur. In the following proposition we determine the equilibrium bribe b_t^{NB}.

Proposition 2.1. *Let $q_t \neq 1$.*[12] *Then there exists a unique bribe (b_t^{NB}), as the Nash solution to a bargaining game, given by:*

$$b_t^{NB} = \frac{\varepsilon}{\lambda+\varepsilon}\left[(c^h - c^l) - \frac{q_t}{(1-q_t)}m^i\right] + \frac{\lambda}{\lambda+\varepsilon}\frac{q_t w}{(1-q_t)}. \tag{1}$$

where ε and λ are the parameters that can be interpreted as measures of bargaining strength, of the firm and the bureaucrat respectively.

Proof. Let $\underline{\pi}_{\Delta,t} = \underline{\pi}_{3,t} - \underline{\pi}_{2,t} = (\pi_{\Delta,t}^{(1)}, \pi_{\Delta,t}^{(2)})$ be the vector of the differences in the payoffs between the case of agreement and disagreement about the bribe, between bureaucrat and entrepreneur. In accordance with generalized Nash bargaining theory, the division between two agents will solve:

$$\max_{b_t \in \Re^+} \left([\pi_{\Delta,t}^{(1)}]^{\varepsilon} \cdot [\pi_{\Delta,t}^{(2)}]^{\lambda}\right) \tag{2}$$

in formula

$$\begin{aligned} &\max_{b_t \in \Re^+} ([(1-q_t)b_t - q_t w]^{\varepsilon} * \\ &[-(1-q_t)c^l - (1-q_t)b_t - q_t c^h - q_t m^i + c^h]^{\lambda}) \end{aligned} \tag{3}$$

that is the maximum of the product between the elements of $\underline{\pi}_{\Delta,t}$ and where $[w, p - c^h]$ is the point of disagreement, i.e. the payoffs that the entrepreneur and the bureaucrat respectively would obtain if they did not come to an agreement. The parameters $0 < \varepsilon \leq 1$ and $0 < \lambda \leq 1$ can be interpreted as measures of bargaining strength. It is now easy to check that the bureaucrat gets a share $\frac{\varepsilon}{\lambda+\varepsilon}$ of the surplus π, i.e. the bribe is $b = \frac{\varepsilon}{\lambda+\varepsilon}\pi$. Then the bribe b_t^{NB} is an asymmetric (or generalized) Nash bargaining solution and is given by:

$$b_t^{NB} = \frac{\varepsilon}{\lambda+\varepsilon}\left[(c^h - c^l) - \frac{q_t}{(1-q_t)}m^i\right] + \frac{\lambda}{\lambda+\varepsilon}\frac{q_t w}{(1-q_t)}$$

that is the unique equilibrium bribe in the last subgame, $\forall q_t \neq 1$. □

As a consequence of the model, let us assume that the bureaucrat and the firm share the surplus on an equal basis. This is the standard Nash case, when $\lambda = \varepsilon = 1$ and bureaucrat and firm get equal shares. In this case, the bribe is:

$$b_t^{NB} = \frac{1}{2}\left[(c^h - c^l) - \frac{q_t}{(1-q_t)}m^i + \frac{q_t}{(1-q_t)}w\right]. \tag{4}$$

[12] If $q_t = 1$ the last stage of the game is never reached.

In other words, the bribe represents 50 percent of the surplus.
Hence, the payoff vector at time t is given by:

$$\begin{aligned} \underline{\pi}_{3,t} = \Big(& w + \tfrac{(c^h - c^l)(1-q_t)}{2} - \tfrac{q_t}{2} m^i - \tfrac{q_t}{2} w, \\ & p - \tfrac{(1-q_t)c^l}{2} - \tfrac{(1+q_t)c^h}{2} - \tfrac{q_t}{2}(m^i + w) \Big). \end{aligned} \tag{5}$$

By solving the static game, we can prove the following proposition.

Proposition 2.2. *Let* $m_t = \frac{(1-q_t)(c^h - c^l)}{q_t} - w$*. Then,*

(a) if $m_t \leq 0$*, that is* $q_t \geq \frac{(c^h - c^l)}{(c^h - c^l) + w}$*, all the private firms produce a high–quality public good;*

(b) if $0 < m_t < 1$*, that is* $\frac{(c^h - c^l)}{1 + (c^h - c^l) + w} < q_t < \frac{(c^h - c^l)}{(c^h - c^l) + w}$*, then* m_t $(1 - m_t)$ *private firms produce a low (high) quality public good;*

(c) if $m_t \geq 1$*, that is* $q_t \leq \frac{(c^h - c^l)}{1 + (c^h - c^l) + w}$*, all the private firms produce a low–quality public good;*

Proof. Backward induction method. The static game is solved with the backward induction method, which allows identification of the equilibria. Starting from stage 3, the entrepreneur needs to decide whether to negotiate with the bureaucrat or not. Both payoffs are then compared, because the bureaucrat asked for a bribe.

(2) At stage two, the entrepreneur negotiates the bribe if, and only if

$$\underline{\pi}^{(2)}_{3,t} \geq \underline{\pi}^{(2)}_{2,t} \Rightarrow \tag{6}$$

$$\left(p - \frac{(1-q_t)c^l}{2} - \frac{(1+q_t)c^h}{2} - \frac{q_t m^i}{2} - \frac{q_t w}{2} \right) > p - c^h \Rightarrow$$

$$m^i < \frac{(c^h - c^l)(1-q_t)}{q_t} - w = m_t \tag{7}$$

(1) Going up the decision-making tree, at stage one, the bureaucrat decides whether to ask for a positive bribe. If $m_t \leq 0$ then $m^i \geq m_t$ hence the bureaucrat knows that if s/he asks for a positive bribe, the entrepreneur will not accept the negotiation. In such a case, the game ends in the equilibrium without corruption and the i-th entrepreneur produces high–quality goods. Let $m_t \geq 0$ then two cases may occur.

• If $m^i < m_t$ then the bureaucrat knows that if s/he asks for a positive bribe, the entrepreneur will accept the negotiation and the final bribe will be b_t^{NB}. Then, the bureaucrat asks for a bribe if, and only if

$$\underline{\pi}_{3,t}^{(1)} > \underline{\pi}_{1,t}^{(1)} \Rightarrow$$

$$w + \frac{(c^h - c^l)(1-q_t)}{2} - \frac{q_t m^i}{2} - \frac{q_t w}{2} > w \tag{8}$$

that is the bureaucrat's payoff. Observe that being $m^i < m_t$, then (8) is always verified. Hence, the bureaucrat asks for the bribe b_t^{NB}, which the entrepreneur will accept.

The game ends in the equilibrium with corruption and the i-th entrepreneur produces low–quality goods.

• If $m^i \geq m_t$ then the bureaucrat knows that the entrepreneur will not accept any possible bribe, so s/he will be honest and the firm must sell the product at a high level of quality.

The game ends in the equilibrium with no corruption.

Trivially, if $m_t \geq 1$ then $m^i < m_t$, $\forall i$, hence all private firms produce the low quality public good. □

According to our previous result, if $m_t \in (0, 1)$ then the entrepreneurs with "reputational costs" $m^i \leq m_t$ are corrupt, while the entrepreneurs with "reputational costs" $m^i > m_t$ are honest. Since we assume that i–type firms are uniformly distributed in $[0, 1]$ with respect to their "reputational costs", then m_t represents the fraction of corrupt entrepreneurs. On the other hand, if $m_t \leq 0$ ($m_t \geq 1$) then all the entrepreneurs are honest (corrupt).

2.2 Economic growth and taxation

Consider now the j-type firms producing the private good and normalize their number to one. Let $m_t \in (0, 1)$, then a fraction m_t (1-m_t) of public good available to j-type firms to produce the private good is of low (high) quality. We can consistently assume that at any time, j-type firms use a fraction m_t of low–quality public input and a fraction (1-m_t) of high–quality public input to produce the final private good. In order to study the effect of corruption in procurement on economic growth, we consider the Solow neoclassical growth model in discrete time (see Solow 1956 and Swan 1956). Hence, let $y_t^l = \phi^l(k_t)$ ($y_t^h = \phi^h(k_t)$) be the production function to produce a private good by using a low (high) quality public good as an input, where $y_t = Y_t/L_t$ is the output per worker, while $k_t = K_t/L_t$ is the capital-labor ratio (i.e. capital per capita). Obviously, $\forall k_t$ we have that $y_t^l < y_t^h$, since the use of high–quality inputs implies greater production. Hence, the final output per capita is given by:

$$y_t = m_t\phi^l(k_t) + (1 - m_t)\phi^h(k_t), \tag{9}$$

We capture these quality differences through differences in the total productivity factor, so that the total productivity, in the case of the high–quality public good used, (A_h) is higher than in the case of the low–quality public good (A_l). In particular, using the Cobb–Douglas production function, we obtain $\phi^l(k_t) = A_l k_t^\rho$ and $\phi^h(k_t) = A_h k_t^\rho$ with $A_h > A_l > 0$ and $\rho \in (0, 1)$. By substituting the production function in (9) we obtain:

$$y_t = m_t A_l(k_t)^\rho + (1 - m_t)A_h(k_t)^\rho, \quad \rho \in (0, 1). \tag{10}$$

In order to specify the monitoring level fixed by the State on corruption we consider that the profit of j-type firms is given by: $\eta_t = \nu y_t - \varphi$ where ν is the market price of the private good and φ are the costs of producing private good. They are both assumed to be constant and equal for each group of firms. Without loss of generality, we can assume $\nu = 1$ and $\varphi = 0$, hence: $\eta_t = y_t$[13]. The j-type firm's income is taxed by the State with a tax rate equal to $\tau \in (0, 1)$. Each year, the j-type firm invests the fraction of its net profits which remains after consumption, i.e. saving adds to the capital stock (saving is equal to investment). Following the Solow framework, we consider the capital accumulation as given by the following formula:

$$k_{t+1} = \frac{1}{1+n}[sy_t(1 - \tau) + (1 - \delta)k_t], \tag{11}$$

where $n > 0$ is the exogenous population growth rate while $s \in (0, 1)$ is the constant saving ratio and $\delta \in [0, 1]$ is the depreciation rate of capital. Let $\Delta A = A_h - A_l$, then by substituting (10) in (11) we obtain the following equation describing the evolution of the capital per capita in our model with corruption in public procurement:

$$k_{t+1} = \frac{1}{1+n}[s(1 - \tau)k_t^\rho(A_h - m_t\Delta A) + (1 - \delta)k_t].$$

Observe that if $m_t \leq 0$ (that is no corruption takes place), then the public good is of high level so that we obtain:

$$k_{t+1} = \frac{1}{1+n}[s(1 - \tau)k_t^\rho A_h + (1 - \delta)k_t],$$

while if $m_t \geq 1$ (that is all the i-type firms are corrupt) then the public good is of low level and we obtain:

$$k_{t+1} = \frac{1}{1+n}[s(1 - \tau)k_t^\rho A_l + (1 - \delta)k_t].$$

[13] Observe that the same results are obtained while assuming positive and constant costs.

In order to reduce corruption, the State checks the public procurement and fixes the monitoring level depending on the tax revenues. In fact, the State in order to obtain the resources for monitoring activity, taxes the private sector firm's profit. We assume that for the monitoring activity today on procurement and corruption, the State uses a fraction of the tax revenues derived from taxation of private firms.

Define tax revenues R_t as the taxes paid by the i-type firms plus the taxes paid by the j-type firms.[14] We also assume that the corrupt i-type firm will not report real income because officially it is providing a high quality good; therefore, it declares an income corresponding to a c^h cost.

The State uses the tax revenues to finance the monitoring of corruption. According to such an hypothesis, we can assume that the monitoring level is such that

$$q_{t+1} = \omega(R_t) \tag{12}$$

where $\omega' > 0$ $\forall t$ and such that $\lim_{R_t \to 0} \omega(\cdot) = 0$ and $\lim_{R_t \to \infty} \omega(\cdot) = 1$.

Consistently with our hypothesis, we assume that

$$q_{t+1} = \frac{\alpha R_t}{\alpha R_t + 1} \tag{13}$$

where parameter $\alpha > 0$ represents the fraction of tax revenues which the State wants to use for monitoring corruption, i.e. α is the willingness on the part of the State to fight corruption. As we shall see, our model allows us to perform an adequate and very interesting analysis of the role of α.

Let $m_t \in (0, 1)$, then the tax revenues will given by:

$$R_t = \tau[p - c^h] + \tau[m_t A_l (k_t)^\rho + (1 - m_t) A_h (k_t)^\rho]. \tag{14}$$

As regard the evolution of the fraction m_t of corrupt firms, by taking into account Proposition 2.2, we have that $q_{t+1} = \frac{\Delta c}{\Delta c + m_{t+1} + w}$, being $\Delta c = c^h - c^l$, then, while considering equation (13) and after some algebra we obtain

$$m_{t+1} = \frac{\Delta c}{\alpha\tau[p - c^h + k_t^\rho (A_h - m_t \Delta A)]} - w$$

describing the evolution of corrupt firms in our model. Trivially, if $m_t \leq 0$ we reach

$$m_{t+1} = \frac{\Delta c}{\alpha\tau[p - c^h + k_t^\rho A_h]} - w,$$

while if $m_t \geq 1$ we have

$$m_{t+1} = \frac{\Delta c}{\alpha\tau[p - c^h + k_t^\rho A_l]} - w.$$

[14] We do not consider taxes on wages as being a constant of our model, they do not add much to the analysis.

2.3 The dynamical system

According to the previous results, the final dynamic system describing the evolution of the capital per capita and of the corrupted firms in our model, is given by $T = T_1 \cup T_2 \cup T_3$, where:

$$T_1 = \begin{cases} m_{t+1} = f_1(k_t) = \frac{\Delta c}{\alpha\tau[p-c^h+k_t^\rho A_h]} - w \\ k_{t+1} = h_1(k_t) = \frac{1}{1+n}[s(1-\tau)k_t^\rho A_h + (1-\delta)k_t] \end{cases} \text{if } \mathrm{m_t} \leq 0, \quad (15)$$

$$T_2 = \begin{cases} m_{t+1} = f_2(m_t, k_t) = \frac{\Delta c}{\alpha\tau[p-c^h+k_t^\rho (A_h - m_t\Delta A)]} - w \\ k_{t+1} = h_2(m_t, k_t) = \frac{1}{1+n}[s(1-\tau)k_t^\rho (A_h - m_t\Delta A) + (1-\delta)k_t] \end{cases} \text{if } 0 < \mathrm{m_t} < 1, \quad (16)$$

and

$$T_3 = \begin{cases} m_{t+1} = f_3(k_t) = \frac{\Delta c}{\alpha\tau[p-c^h+k_t^\rho A_l]} - w \\ k_{t+1} = h_3(k_t) = \frac{1}{1+n}[s(1-\tau)k_t^\rho A_l + (1-\delta)k_t] \end{cases} \text{if } \mathrm{m_t} \geq 1. \quad (17)$$

Observe that system T is two-dimensional, continuous and piecewise smooth. More precisely, we can define sets $D_1 = \{(m_t, k_t) \in R^2 : m_t \leq 0 \cap k_t \geq 0\}$, $D_2 = \{(m_t, k_t) \in R^2 : 0 < m_t < 1 \cap k_t \geq 0\}$ and $D_3 = \{(m_t, k_t) \in R^2 : m_t > 1 \cap k_t \geq 0\}$. Then system T is define in set $D = D_1 \cup D_2 \cup D_3$, it is continuous on D and the borders are $d_{12} = \{(m_t, k_t) \in R^2 : m_t = 0 \cap k_t \geq 0\}$ and $d_{23} = \{(m_t, k_t) \in R^2 : m_t = 1 \cap k_t \geq 0\}$, these lines separate the domain into three regions characterized by different corruption levels: no corruption (set D_1), total corruption (set D_3) and intermediate corruption (set D_2). It is important to stress the economic meaning of the three systems: T_1 describes the evolution of the capital when there is widespread corruption. In fact, in the system T_1 the reputational costs of entrepreneurs are so high that all entrepreneurs find it worthwhile to be honest. In contrast, the T_3 system describes the evolution of capital and corruption when the second is rampant, i.e. all entrepreneurs find it worthwhile to be corrupt. Finally, intermediate situation is represented by T_2 system which describes the evolution of capital and corruption when the population of entrepreneurs is divided between corrupt and honest.

3 Steady states

In order to determine the fixed points owned by system T, we first consider system T_1. By solving $h_1(k) = k$, we obtain the following two equilibrium values for the capital per-capita

$$k_{11} = 0 \text{ and } k_{12} = \left[\frac{s(1-\tau)A_h}{n+\delta}\right]^{\frac{1}{1-\rho}} > 0$$

where the correspondent equilibrium values for the corruption level are given by $m_{11} = f_1(k_{11})$ and $m_{12} = f_1(k_{12})$ that is

$$m_{11} = \frac{\Delta c}{\alpha\tau(p - c^h)} - w \text{ and } m_{12} = \frac{\Delta c}{\alpha\tau\left[p - c^h + \left(\frac{s(1-\tau)}{n+\delta}\right)^{\frac{\rho}{1-\rho}} A_h^{\frac{1}{1-\rho}}\right]} - w.$$

According to the previous considerations, the following proposition holds.

Proposition 3.1. *Define $E_{11} = (m_{11}, k_{11})$ and $E_{12} = (m_{12}, k_{12})$.*

(11) If $w \geq \frac{\Delta c}{\alpha\tau(p-c^h)}$ then E_{11} and E_{12} are fixed points of system T;

(12) if $w \geq \frac{\Delta c}{\alpha\tau\left[p-c^h+\left(\frac{s(1-\tau)}{n+\delta}\right)^{\frac{\rho}{1-\rho}} A_h^{\frac{1}{1-\rho}}\right]} = w_1$ then E_{12} is a fixed point of system T.

Proof. E_{11} is a fixed point of system T if $E_{11} \in D_1$ that is condition (11) holds. Point E_{12} is a fixed point of system T if $E_{12} \in D_1$ that is condition (12) holds. Finally, if condition (11) holds then also condition (12) holds. □

Recall that T_1 describes our growth model in the case with zero corruption (i.e. $m_t \leq 0$) hence, according to Proposition 3.1, our model may have up to two fixed points characterized by zero corruption. In such a case, the equilibrium E_{11} is characterized by zero capital per-capita so that it has no economic meaning, while the equilibrium E_{12} is with positive capital per capita and no corruption. Furthermore, according to conditions (11) and (12) of Proposition 3.1, if w is sufficiently high, then E_{12} is a steady state with positive capital per capita and zero corruption. So, in keeping with what has been shown in the literature, the punishment inflicted on the bureaucrat caught in a corrupt transaction is an effective instrument at the complete disposal of policy makers to eliminate or reduce corruption by making it economically viable. Clearly, since the punishment is represented by wages and wages are a cost for the public budget, the best strategy is to fix a salary equal to w_1 to eliminate corruption.

Consider now system T_3. By solving $h_3(k) = k$ we obtain the following two equilibrium values for the capital per-capita

$$k_{31} = 0 \text{ and } k_{32} = \left[\frac{s(1-\tau)A_l}{n+\delta}\right]^{\frac{1}{1-\rho}} > 0$$

where the correspondent equilibrium values for the corruption level are given by

$$m_{31} = \frac{\Delta c}{\alpha\tau(p - c^h)} - w \text{ and } m_{32} = \frac{\Delta c}{\alpha\tau\left[p - c^h + \left(\frac{s(1-\tau)}{n+\delta}\right)^{\frac{\rho}{1-\rho}} A_l^{\frac{1}{1-\rho}}\right]} - w.$$

Considering similar arguments as in Proposition 3.1, the following proposition holds.

Proposition 3.2. *Define* $E_{31} = (m_{31}, k_{31})$ *and* $E_{32} = (m_{32}, k_{32})$.

(31) If $0 \leq w \leq \frac{\Delta c}{\alpha\tau(p-c^h)} - 1$ *then* E_{31} *is a fixed point of system* T*;*

(32) if $0 \leq w \leq \frac{\Delta c}{\alpha\tau\left[p-c^h+\left(\frac{s(1-\tau)}{n+\delta}\right)^{\frac{\rho}{1-\rho}} A_l^{\frac{1}{1-\rho}}\right]} - 1 = w_2$ *then* E_{31} *and* E_{32} *are fixed points of system* T.

According to Proposition 3.2, system T may have up to two fixed points characterized by total corruption (i.e. $m_t \geq 1$). The equilibrium E_{31} has zero capital per-capita and it has no economic interest, while E_{32} has positive capital per capita and total corruption.

A first consideration is that $k_{32} < k_{12}$, being $A_l < A_h$, in other words the equilibrium without corruption is characterized by greater capital per capita w.r.t. the equilibrium with corruption, obviously $m_{32} > m_{12}$. In this regard, it is important to highlight the role of two-way linking the capital to corruption. On the one hand, more corruption means lower quality of public goods and therefore lower production by the $j-type$ firm and, thus, less accumulation of capital. On the other hand, *ceteris paribus*, the lower capital implies lower output and hence lower tax revenues be allocated to the control of corruption, which will therefore be high. It follows that the level of capital in the completely corrupt economy is lower than that which characterizes honest economy.

Secondly, if condition (32) of Proposition 3.2 is not satisfied, then no equilibria with total corruption can emerge in our model (since condition (32) implies condition (31)). Furthermore, condition $w_2 \leq 0$ is sufficient to guarantee that equilibria with total corruption cannot emerge in our model (it can happen, for instance, if the difference between the costs is sufficiently low or α is sufficiently high).

Finally, consider system T_2. In order to determine the fixed points, we first solve $h_2(k, m) = k$ and we obtain two different solutions which are $k_{21} = 0$ and, for any given value $m \in (0, 1)$, a value $k > 0$ does exist such that

$$k^{1-\rho}(n+\delta) - s(1-\tau)(A_h - m\Delta A) = 0. \tag{18}$$

Let $m_{21} = \frac{\Delta c}{\alpha\tau(p-c^h)} - w$, then point $E_{21} = (m_{21}, k_{21})$ is a fixed point of T iff $w \geq 0$ and $\frac{\Delta c}{\alpha\tau(p-c^h)} - 1 < w < \frac{\Delta c}{\alpha\tau(p-c^h)}$.

In order to determine other fixed points of system T_2, we come back to condition (18) that can be re-written as

$$m = \frac{k^{1-\rho}(n+\delta)}{s(1-\tau)\Delta A} - \frac{A_h}{\Delta A} = g_1(k).$$

Now we have to take into account equation $m = f_2(m,k)$. Notice that from condition (18) it follows that

$$A_h - m\Delta A = \frac{k^{1-\rho}(n+\delta)}{s(1-\tau)}; \tag{19}$$

by substituting this last formula in $f_2(m,k)$ we obtain

$$m = \frac{\Delta c}{\alpha\tau\left[p - c^h + k\frac{n+\delta}{s(1-\tau)}\right]} - w = g_2(k).$$

As a consequence, if a $k_{22} > 0$ exists such that $g_1(k_{22}) = g_2(k_{22}) = m_{22} \in (0,1)$, then $E_{22} = (m_{22}, k_{22})$ is a fixed point of system T with corruption being at an intermediate level (i.e. a fraction m_{22} of firms are corrupt).

Hence we proceed to two steps.

1) In order to find conditions such that equation $g_1(k) = g_2(k)$ admits a positive solution consider that:

- $g_1(0) = -\frac{A_h}{\Delta A} < 0$, $g_1(+\infty) = +\infty$, g_1 is strictly increasing for all $k > 0$;
- $g_2(0) = \frac{\Delta c}{\alpha\tau(p-c^h)} - w = m_{11}$ may be positive or negative, in any case $g_2(+\infty) = -w$ and g_2 is strictly decreasing for all $k > 0$.

Then, if $g_2(0) > g_1(0)$ there exists a unique $k_{22} > 0$ such that $g_1(k_{22}) = g_2(k_{22})$. Condition $g_2(0) > g_1(0)$ is given by

$$\frac{\Delta c}{\alpha\tau(p-c^h)} - w > -\frac{A_h}{\Delta A}.$$

.

2) Let $m_{22} = g_2(k_{22})$ then point $E_{22} = (m_{22}, k_{22})$ is a fixed point for system T if $0 < m_{22} < 1$. In order to find conditions such that this last inequality holds, the following remark must be verified.

Remark 3.3. *Let $k_1 > 0$ s.t. $g_1(k_1) = 0$ and $k_2 > k_1$ s.t. $g_1(k_2) = 1$. If $g_2(k_1) > 0$ and $g_2(k_2) < 1$ then $m_{22} \in (0,1)$ and E_{22} is a fixed point for system T.*

Simple computations prove that $k_1 = k_{12}$ and $k_2 = k_{32}$ hence $g_2(k_1) = g_2(k_{12}) = m_{12}$ while $g_2(k_2) = g_2(k_{32}) = m_{32}$. Then conditions $m_{12} > 0$ and $m_{32} < 1$ hold iff $w \geq 0$ and $w_2 < w < w_1$, so that E_{22} is a fixed point of system T.

The previous considerations prove the following proposition.

Proposition 3.4. *Define $E_{21} = (m_{21}, k_{21})$ and $E_{22} = (m_{22}, k_{22})$.*

(31) If $\frac{\Delta c}{\alpha\tau(p-c^h)} - 1 < w < \frac{\Delta c}{\alpha\tau(p-c^h)}$, $w \geq 0$, then E_{21} is a fixed point of system T;

(32) if $w_2 < w < w_1$, $w \geq 0$, then E_{22} is a fixed point of system T.

The results presented in this section prove that multiple equilibria can coexist; furthermore, the equilibrium with higher growth rate has a lower corruption level. This fact is in agreement with most theoretical articles in which it is shown that at the fixed point the growth rate level is decreasing w.r.t. the corruption level.

4 Local stability and multiple equilibria

In the previous section, we have determined the fixed points of T and we have found that, depending on parameter values, steady states may be located in D_1, D_2 or D_3 (so that they are characterized by different corruption regimes) and the capital per capita equilibrium levels may be positive or equal to zero.

In this section, we want to determine, once are fixed the parameter values, how many steady states are owned by T, their properties and their local stability.

We first consider the fixed points of system T with zero capital per capita. Recall Propositions 3.1, 3.2 and 3.4, then the following statement holds.

Proposition 4.1. *Define* $\frac{\Delta c}{\alpha\tau(p-c^h)} - 1 = A - 1$*. If* $A - 1 > 0$ *then*

(i) if $0 \leq w \leq A - 1$*,* E_{31} *is a fixed point of* T*;*

(ii) if $A - 1 < w < A$*,* E_{21} *is a fixed point of* T*;*

(iii) if $w \geq A$*,* E_{11} *is a fixed point of* T*.*

If $A - 1 \leq 0$ *then*

(i) if $0 \leq w < A$*,* E_{21} *is a fixed point of* T*;*

(ii) if $w \geq A$*,* E_{11} *is a fixed point of* T*.*

According to the previous proposition our system always admits a unique fixed point with zero capital per capita depending on the parameter values. Such a point may be characterized by zero corruption (if it belongs to D_1) by total corruption (if it belongs to D_3) or by intermediate corruption (if it belongs to D_2). Furthermore, for any given value of A, as w increases then the corruption level associated to the fixed point decreases, providing that the fine (wage) for the bureaucrat caught in a corrupt transaction, can be used by the State to reduce corruption. Being $k_{j1} = 0$ for $j = 1, 2, 3$, these equilibria are not very interesting from an economic point of view, in any case we study their local stability in order to prove that, in addition, they are locally unstable.

To study the local stability of the fixed point owned by T given by E_{j1}, $j = 1, 2, 3$, we consider the Jacobian matrix (see Lines and Medio (2001)). We distinguish between the three cases presented in Proposition 4.1.

Let $w > A$ (hence the fixed point does not belong to the border d_{12}) then E_{11} is a fixed point of T. The Jacobian matrix of T_1 (denoting the matrix of first partial derivatives) is given by

$$DT_1(m,k) = \begin{pmatrix} 0 & \frac{\partial f_1}{\partial k}(m,k) \\ 0 & \frac{\partial h_1}{\partial k}(m,k) \end{pmatrix}. \tag{20}$$

In order to conclude about the local stability of the fixed point E_{11}, observe that the eigenvalues of $DT_1(m_{11}, k_{11})$, being $k_{11} = 0$, are real and given by

$$\lambda_{11}^a = 0, \;\; \lambda_{11}^b = \frac{\partial h_1}{\partial k}(m_{11}, k_{11}) = +\infty$$

respectively, providing that E_{11}, when it exists, is a saddle point.

In an similar way, we can conclude that, according to Proposition 4.1, when the fixed point E_{21} (or E_{31}) exists, it is still a saddle point. In fact, the Jacobian matrixes of T_2 and T_3 are given by

$$DT_2(m,k) = \begin{pmatrix} \frac{\partial f_2}{\partial m}(m,k) & \frac{\partial f_2}{\partial k}(m,k) \\ \frac{\partial h_2}{\partial m}(m,k) & \frac{\partial h_2}{\partial k}(m,k) \end{pmatrix} \tag{21}$$

and

$$DT_3(m,k) = \begin{pmatrix} 0 & \frac{\partial f_3}{\partial k}(m,k) \\ 0 & \frac{\partial h_3}{\partial k}(m,k) \end{pmatrix}, \tag{22}$$

where

$$DT_2(m_{21},0) = \begin{pmatrix} 0 & \frac{\partial f_2}{\partial k}(m_{21},0) \\ 0 & \frac{\partial h_2}{\partial k}(m_{21},0) \end{pmatrix} \tag{23}$$

as a consequence the eigenvalues associated to E_{21} are

$$\lambda_{21}^a = 0, \;\; \lambda_{21}^b = \frac{\partial h_1}{\partial k}(m_{21}, k_{21}) = +\infty,$$

while those associated to E_{31} are

$$\lambda_{31}^a = 0, \;\; \lambda_{31}^b = \frac{\partial h_1}{\partial k}(m_{31}, k_{31}) = +\infty.$$

Consider now the case in which the fixed point belongs to the border-line d_{12} or d_{23}. For instance, if $w = A$ then $E_{11} = (0, 0)$, while if $w = A - 1$, being $A - 1 \geq 0$, then $E_{31} = (1, 0)$. In such cases, as the eigenvalues of T_1, T_2 and T_3 associated to the fixed point does not change when passing trough d_{12} and d_{23}, we have again a saddle point (about piecewise smooth system see, among others, Nusse and Yorke (1992) and (1995)).

All the previous considerations are resumed in the following proposition.

Proposition 4.2. *System T admits a fixed point located on the invariant set $S = \{(m_t, k_t) \in R^2 : k_t = 0\}$.*[15] *Such a point is a saddle point and it attracts only trajectories starting from initial conditions (m_0, k_0) having $k_0 = 0$.*

Our previous proposition enables us to conclude that a steady state with zero capital per capita will never be reached if, at the initial time, the system starts from a positive level of capital per capita. Since $k_0 = 0$ is an unrealistic hypothesis, in what follows we focus only on the equilibria having positive capital per-capita and the analysis will be conducted starting from initial conditions (m_0, k_0), with $k_0 > 0$.

We now consider the fixed points of T with positive capital per capita. Recall Propositions 3.1, 3.2 and 3.4 then the following statement holds.

Proposition 4.3. *Define* $\frac{\Delta c}{\alpha\tau\left[p-c^h+\left(\frac{s(1-\tau)}{n+\delta}\right)^{\frac{\rho}{1-\rho}} A_h^{\frac{1}{1-\rho}}\right]} = w_1 > 0$ *and* $\frac{\Delta c}{\alpha\tau\left[p-c^h+\left(\frac{s(1-\tau)}{n+\delta}\right)^{\frac{\rho}{1-\rho}} A_l^{\frac{1}{1-\rho}}\right]} - 1 = w_2$.

(a) *Let $w_2 > 0$.*

- *(i) Assume $w_1 = w_2$, if $0 \leq w < w_1$ then E_{32} is a fixed point of T, if $w \geq w_1$ then E_{12} is a fixed point of T;*
- *(ii) assume $w_1 < w_2$, if $0 \leq w < w_1$ then E_{32} is a fixed point of T, if $w_1 \leq w \leq w_2$ then E_{12} and E_{32} are fixed points of T, if $w > w_2$ then E_{12} is a fixed point of T;*
- *(iii) assume $w_2 < w_1$, if $0 \leq w \leq w_2$ then E_{32} is a fixed point of T, if $w_2 < w < w_1$ then E_{22} is a fixed point of T if $w \geq w_1$ then E_{12} is a fixed point of T.*

(b) *Let $w_2 \leq 0$ so that $w_2 < w_1$. If $0 \leq w < w_1$ then E_{22} is a fixed point of T, if $w \geq w_1$ then E_{12} is a fixed point of T.*

Proof. A preliminary consideration is that $w_1 > 0$ while w_2 may be positive or negative and, in addition, it can be less, equal or greater than w_1. As a consequence, by taking into account Propositions 3.1, 3.2 and 3.4, our statement holds. □

As a matter of fact, the previous proposition states that one or two fixed points with positive capital per capita may be owned by our model and that they are characterized by different corruption levels, depending on the parameter values.[16]

[15] We recall that a set K is invariant if $T(K) = K$.

[16] Multiple equilibria characterized by different corruption levels emerge quite naturally in this kind of models (see, for instance, Mauro (2002).

In order to better analyze this fact, recall that $\Delta A = A_h - A_l$, hence, we fix the value of all the parameters of the model and we let ΔA vary. Then $w_1 = F(\Delta A)$, while w_2 is constant (positive or negative), for all $\Delta A > 0$. Then we can represent w_1 and w_2 in the plane $(\Delta A, w)$ in order to obtain curves that separate the plane into different regions characterized by a different number of equilibria with different corruption levels.

In Figure 1 we present such curves for two different values of A_l. Observe that if A_l is sufficiently high, then $w_2 < 0$ (case (b) of Proposition 4.3) and curve w_1 separates the plane into two regions such that the steady state is given by E_{22} or E_{12}. In contrast, as A_l decreases, w_2 becomes positive (but less than $F(0)$) so that, we obtain four different regions such that our system admits fixed points with different corruption levels or, eventually, two different equilibria coexists (this last case will be studied in the following section). Observe that in both cases $\bar{w} > 0$ does exist such that the steady state with no corruption and positive capital per capita is the unique steady state of T, for all $w \geq \bar{w}$ and for all $\Delta A > 0$.

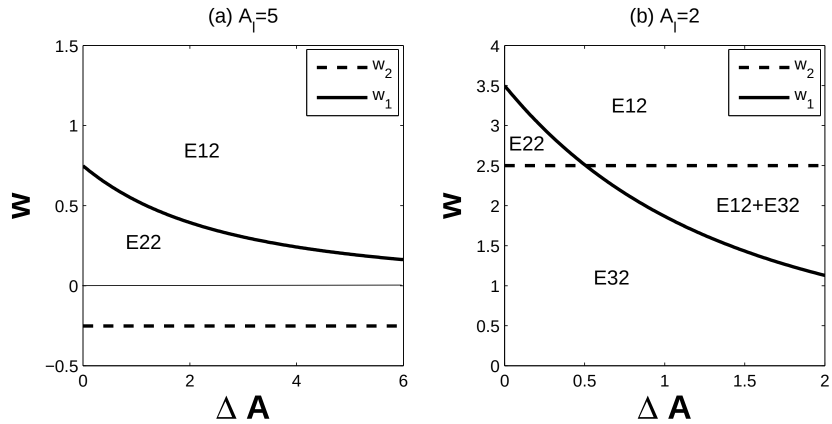

Figure 1: Fixed points owned by system T when varying ΔA and w given the following values of the other parameters: $n = 1$, $s = 0.2$, $\tau = 0.3$, $\rho = 0.5$, $\delta = 0.2$, $\alpha = 1$,$c^h = 0.8$, $c^l = 0.1$ and $p = 1$. In panel (a) $A_l = 5$ while in (b) $A_l = 2$.

In all the cases presented in Proposition 4.3, once the steady state of T is known, we have to conclude about their local stability. We first assume that $w \neq w_1$ and $w \neq w_2$ (if $w_2 \geq 0$) so that the steady states belong to the interior of sets D_1, D_2 or D_3.

Assume that E_{12} is a fixed point of T. The Jacobian matrix is given by (20) where

$$\frac{\partial h_1}{\partial k}(m,k) = \frac{1}{1+n}\left[\frac{\rho s(1-\tau)A_h}{k^{1-\rho}} + (1-\delta)\right]$$

so that

$$\frac{\partial h_1}{\partial k}(E_{12}) = \frac{1}{1+n}\left[\rho(n+\delta) + (1-\delta)\right] \in (0,1).$$

Hence the eigenvalues associated to E_{12} are $\lambda_{12}^a = 0$ and $\lambda_{12}^b \in (0,1)$ providing that E_{12} is a locally stable fixed point.

Assume that E_{32} is a fixed point of T. Then, similarly to what has been previously proved, also E_{32} is a stable node for system T.

Finally, we consider the case in which E_{22} is a fixed point of T. In this last case, we cannot conclude analytically about the local stability of the steady state, in any case, many numerical simulations have proved that also such an equilibrium is locally stable (the eigenvalues are both non-negative and less than one).

Hence we can conclude that all the fixed point having positive capital per capita are locally stable. Furthermore, as the related eigenvalues are always positive, fluctuations around the fixed points cannot emerge.

Note that this fact confirms empirical evidence. In fact, it is possible to state that, from an empirical point of view, fluctuations (periodic or even chaotic) in the corrupton level are very rare: corruption is quite stable in time as it is strongly affected by cultural values.

5 Economic results and numerical simulations

In order to better understand the long run dynamics of our growth model with corruption in public procurement, in this section we present some numerical simulations.

First of all observe that, as proved in the previous section, our system cannot fluctuate. Hence a level of structural corruption may exist and the State can only try to reduce it. More precisely, system T always admits a fixed point with zero capital per-capita, that is locally unstable, and up to two fixed points with positive capital per capita; furthermore, any fixed point characterized by positive capital per capita is locally stable.

Hence two questions have to be investigated.

Firstly, the role of the parameters. As our system will converge to a steady state with positive capital per capita for any initial condition having $k_0 \neq 0$, we want to determine the qualitative properties of such equilibria in terms of corruption level as varying the parameters of the model. This analysis will be helpful in order to determine some political economy instruments to push the system toward the most desirable steady state.

Secondly, the role of the initial conditions. When two locally stable fixed points with positive capital per capita co-exist, we have to investigate their basins of attraction. Also in such a case interesting considerations in terms of economic policy may be conduced.

In what follows, we fix some parameter values: $n = 1$, $s = 0.2$, $\tau = 0.3$, $\rho = 0.5$, $\delta = 0.2$, $c^h = 0.8$, $c^l = 0.1$ and $p = 1$. As it will be shown, the levels of w and α play an important role as a political economy instrument so that we let such parameters vary; furthermore, in order to consider the cases in which multiple equilibria may emerge, we will choose opportune values of A_l and ΔA.

Consider now the first question. Let $k_0 \neq 0$, then our system will converge to one of the following equilibria: E_{12} with high capital per capita and no corruption (first best steady state), E_{22} with intermediate capital per capita and corruption (second best) or E_{32} with low capital per capita and total corruption (third best). A preliminary consideration is that, given the state of the system at the initial time, i.e. (m_0, k_0), the evolution of the two state variables will reach one of such equilibria according to the parameter values. In any case, taking into account the considerations presented in the previous section, the following statement holds.

Proposition 5.1. *Let $k_0 > 0$, then a $\bar{w}$ does exist such that $T(m_0, k_0)$ converges to the first best equilibrium point for any $w > \bar{w}$.*

According to this proposition, given the initial state of the system, the State can fix a sufficiently high wage for the bureaucrat in order to cancel out corruption. In fact, if w is high then, by considering the dynamic game, the bureaucrat will not find it worthwhile to ask for a bribe and the equilibrium with corruption will never be reached. In a similar way, by considering condition (12) of Proposition 3.1, we observe that the State may also fix a sufficiently high level of α in order to avoid corruption. In fact, also the following proposition holds.

Proposition 5.2. *Let $k_0 > 0$, then an $\bar{\alpha}$ does exist such that $T(m_0, k_0)$ converges to the first best equilibrium point for any $\alpha > \bar{\alpha}$.*

Proof. This proposition can be trivially proved by considering that $w_1 \to 0$ as $\alpha \to +\infty$, then $\bar{\alpha}$ does exist such that if $\alpha > \bar{\alpha}$ the the system converges to E_{12} for any positive value of w and initial condition having $k_0 \neq 0$. □

As a consequence, the State can avoid corruption if it uses a high percentage of the fiscal revenue to reduce corruption. In fact, greater control of corruption makes it less attractive, reducing it. The role of both the parameters w and α can be better understood by looking at Figure 2.

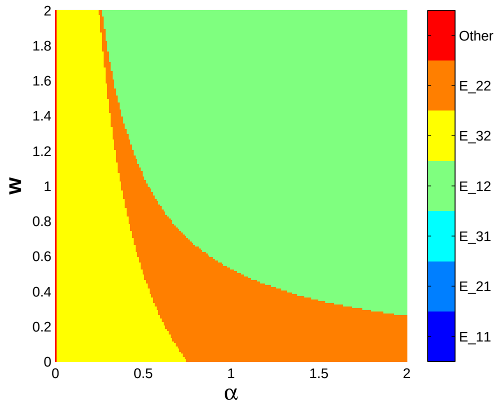

Figure 2: Long run equilibrium points for system T when varying w and α given the following values of the other parameters: $n = 1$, $s = 0.2$, $\tau = 0.3$, $\rho = 0.5$, $\delta = 0.2$, $A_l = 5$, $A_h = 6$, $c^h = 0.8$, $c^l = 0.1$ and $p = 1$. The initial condition is $m_0 = 0.5$ and $k_0 = 0.5$.

Assume that the wage of the bureaucrat w and the fiscal revenue used to reduce corruption α are both very low. Then an economic system starting at a positive level of capital per capita will converge to the third best equilibrium point. Observe that if the State increases w and/or α then, given the other parameters of the model, the steady state will move to the second best and then to the first best equilibrium point (that is with high capital per capita and zero corruption).

In any case, while looking at Figure 3, the final equilibrium can also depend on the initial conditions. To obtain panels (a) and (b) we considered the same parameter values but different initial conditions. Then it is possible to observe the existence of a combination of w and α which can produce dynamics converging to different equilibria depending on the initial conditions.

In fact, as proved in the previous section, for certain parameter values, two different long run equilibria with positive capital per capita may coexist: the first best and the third best steady states.

Then we have to analyze the second question, that is the basins of attraction of such equilibria have to be investigated. Recall Proposition 4.3, then for some parameter values equilibria E_{12} and E_{32} are two co-existing attractors for T. In such a case we have to determine the set of initial conditions generating trajectories converging to E_{12} or to E_{32}. Define such sets B_{12} and B_{32}.

In Figure 4 we consider parameter values such that both E_{12} and E_{32} are fixed points for T.

The two basins B_{12} and B_{32} are connected and divided by a continuous

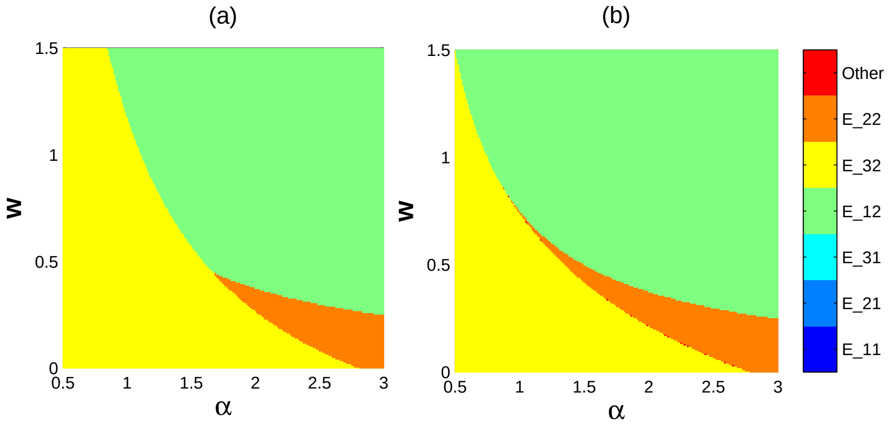

Figure 3: Long run equilibrium points for system T when varying w and α given the following values of the other parameters: $n = 1$, $s = 0.2$, $\tau = 0.3$, $\rho = 0.5$, $\delta = 0.2$, $A_l = 2$, $A_h = 5$, $c^h = 0.8$, $c^l = 0.1$ and $p = 1$. (a) The initial condition is $m_0 = 0.5$ and $k_0 = 0.01$; (b) the initial condition is $m_0 = 0.5$ and $k_0 = 1$.

curve.[17] According to Proposition 4.3, we observe multiple equilibria for intermediate level of w. Obviously Proposition 5.1 applies, so that the State can increase w in order to obtain the convergence to the first best equilibrium point. In any case, with co-exhisting equilibria, this is not the unique way to reduce corruption at the steady state. In fact, the State may try to conduct the system to a new initial state characterized by a greater level of capital per capita (for instance using a policy to push up the investment) in a way such that the new initial state belongs to B_{12}. Obviously, these two different economic policies may be compared in terms of costs.

6 Conclusion

The relationship between corruption and growth is a topic attracting an increasing interest in economic literature. Most theoretical articles find that there are many levels of equilibria and that the equilibrium with higher rate of growth has a low level of corruption. Our paper confirms this result: we present a discrete dynamic model of growth with corruption linked to public

[17]See Abraham et. al (1997).

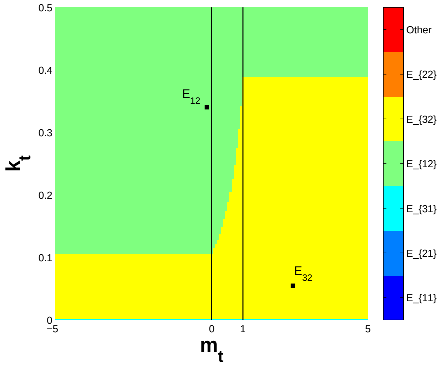

Figure 4: Basins of attraction of fixed points E_{12} and E_{32}, given the following values of the parameters: $n = 1$, $s = 0.2$, $\tau = 0.3$, $\rho = 0.5$, $\delta = 0.2$, $A_l = 2$, $A_h = 5$, $c^h = 0.8$, $c^l = 0.1$, $\alpha = 1$, $w = 0.9$ and $p = 1$.

procurement in which multiple equilibria may co-exist: a good equilibrium (with low corruption and high investment and growth) and a bad one (with high corruption and low growth). The mechanism responsible for the existence and the stability of the bad equilibrium is the following: greater corruption implies a lower growth rate, lower tax revenues and, therefore, a lower monitoring level of corruption which incentivize greater corruption. As a matter of fact, corruption in public procurement is responsible for the reduction in the quality of public infrastructure and services supplied to the private sector and, consequently, it may lower economic growth, according to empirical evidence and to theoretical studies in economic literature. In addition, both the analytical and numerical study of the final system enable us to discuss the strategies the State may adopt to reduce corruption. For instance, the State may reduce corruption by increasing the wage of the bureaucrat (that is a cost for a detected corrupt bureaucrat) or by increasing the percentage of tax revenues used to monitor corruption (that is by increasing the probability to be detected in a corrupt activity).

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