The behavior of others acts as a reference point: An application of prospect theory*

Francesco Bogliacino† Pietro Ortoleva‡

September 2011

Abstract

A vast literature has been devoted to analyzing the consequences of reference-dependence on economic models. However, most of the literature has focused only on two special cases of reference point: the one in which it is the agent's past behaviour, and the one in which it is her expectations. In this paper we consider a third possibility: the case in which the reference point is the behavior of the other members of the society. In particular we study an overlapping generations model in which a continuum of consumers is reference-dependent, modeled using prospect-theory. However, in our model the agent's reference point is the average choice of the society in that period. We show that in any of the equilibria of the economy, in finite time the wealth distribution will become, and remain either missing class (a particular form of polarization between rich and poor) or of perfect equality. We then study growth rates and show that, if we look at the equilibria with the highest growth, then the society that grows the most is the one that starts with perfect equality. If we look at the equilibria with the lowest growth for each economy, however, then the society with a small amount of initial inequality is the one that grows (strictly) the most, while a society with perfect equality is the one that grows the least. All of these growth rates are weakly higher than the growth rate of a corresponding economy without reference-dependence.

JEL: D11; D91; O11; O12
Keywords: Prospect Theory, Reference-dependence, Aspirations, Inequality and Growth

*We would like to thank Federico Echenique, Efe Ok, Debraj Ray, and Giuseppe Veltri for comments and suggestions. We greatly appreciated the comments from the participants to the XVI World Congress of the International Economic Association in Beijing, to the Aspirations and Poverty Workshop at the University of Warwick and at various seminars at the California Institute of Technology, Universidad EAFIT, Universidad Externado y Universidad de Los Andes.

†AIAS-University of Amsterdam, and RISE Group (Universidad EAFIT)
‡California Institute of Technology.

1. Introduction

1.1 Basic Idea

It is now an established finding in behavioral economics and psychology that the utility that subjects associate to the different available alternatives often depends on a *reference point* – some amount, or object, that subjects compare the available options with in order to make a choice. That is, the preferences of the individual do not appear to be a fixed ordering irrespective of the environment in which she operates, as suggested by traditional economic modeling, but rather depend on some 'point of comparison' used to guide choice.[1] These findings have spurred the development of a large literature aimed at identifying the correct functional form to represent such reference-dependence, giving raise to a variety of models – most prominently Prospect Theory of Kahneman and Tversky (1979), Tversky and Kahneman (1991).[2] This model has seen a large number of applications in various branches of economics, generating a large literature. With few exceptions however, the entire such literature has focused only on two very specific types of reference-point: the case in which the reference point is the agent's past consumption, or her status quo – as in almost all the applications of Prospect Theory; and the case in which it is the agent's expectations – mostly following Köszegi and Rabin (2006) and Koszegi (2010).

While both these elements seem to be important for the formation of a reference point, a third additional element seem to be playing an important role: *the behavior of others*. To wit, one could argue that a consumer who receives 10 is 'happier' about this if this happens while everybody else receives zero, than she would be if she received 10 while everybody else received 1000. This seems to take place exactly for reference-dependence reasons – but here the reference point is the consumption of *others*. The goal of this paper is then to study exactly this case: we study a model in which the reference point of this agent is the average consumption of the rest of the society during that period. In particular, we consider a standard overlapping generations model in which we have a continuum of agents who are reference-dependent, modeled using standard Prospect Theory. In particular, we assume that the agent's reference-point is simply the average consumption of the rest of the society. We then analyze the consequences that such reference-dependence has on the dynamics of wealth distribution and on growth.

1.2 Our results

We study a warm-glow overlapping generations model in which the utility that households derive in the second period of their lives depends on the average wealth of the

[1] In this vast literature see, amongst many, Tversky and Kahneman (1974), Kahneman and Tversky (1979), Tversky and Kahneman (1981), Samuelson and Zeckhauser (1988), Camerer (1995).

[2] Other models, some of which are strongly related to Prospect Theory, appear in Köszegi and Rabin (2006), Koszegi (2010), as well as Chateauneuf and Wakker (1999), Masatlioglu and Ok (2005, 2008), Diecidue and Van de Ven (2008), Ortoleva (2010), Wakker (2010), and Ok et al. (2011).

other member of the society during that period, denoted $\bar{x}_t$. We model such reference dependence following prospect theory: we assume that the utility function is *convex* in an interval that ends with $\bar{x}_t$, and that it has a kink (it is not differentiable) at $\bar{x}_t$, where the left-derivative is greater than the right-derivative. Everything else in the model is entirely standard.

We obtain the following results. After noticing that this economy admits multiple equilibria, we show that all equilibria have a common feature: there is a finite period T after which the wealth distribution of the economy is either of perfect equality or it is such that is admits *a missing class* – the distribution has a positive mass above and at the average, positive mass below some value y which is strictly below the average, but has 'a gap,' i.e. no density, between x and the average, the missing class. Will show that the wealth distribution will maintain one of these forms for all subsequent periods, that no other distribution is possible in the long run, and that one of these distributions must be reached in *finite* time (as opposed to asymptotically, or in steady state). Furthermore, we show that whether the economy converges to one type of distribution or to the other depends both on the initial distribution, and on the equilibrium that we are looking at. On the one hand, every society with a non-degenerate initial distribution has an equilibrium in which the wealth distribution will admit a missing class in the long run. On the other hand, a society that has an initial distribution that is 'not too disperse' (the difference between the average and the poorest household is 'small') will *also* have an equilibrium in which the wealth distribution converges to perfect equality in finite time.

We then study how the initial distribution of endowments affects the relative growth of societies with reference-dependence. First, we show that if we focus on the equilibria with the highest growth for each economy, for a given average initial endowment, the economy that grows the most is the one with an initial distribution of perfect equality. However, things change considerably If we look at the equilibria with the *lowest* growth for each society: in this case, the society that grows the most is the one with *some* initial inequality, but that admits an equilibrium of perfect equality. In particular, this economy will grow *strictly more* than an economy that starts from perfect equality. The latter will grow just like an economy without reference-dependence in this equilibrium. This means that if we focus on the equilibria with the lowest growth, then 'a little bit' of initial inequality is good for growth.

Finally, we show that if the utility exhibits Constant Relative Risk Aversion, then any society with reference-dependence grows (weakly) more than a standard economy with the same initial endowment but without reference-dependence. Put differently, we show that the presence of reference-dependence, in the form that we study, cannot be bad for growth. Intuitively, this happens because reference-dependence allows agents to 'push' each other, generating growth.

1.3 Outline and Related Literature

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model, the notion of equilibrium that we use, and describes the optimal choice of the households in this environment. Section 3 presents the consequences of this reference-dependence on the wealth distribution of the society in the long run, while Section 4 analyses those on growth. Section 5 concludes. The proofs appear in the appendix.

Our model is directly related to papers that apply Prospect-Theory to standard economic models. This constitutes a vast literature that we could not survey here. See Spiegler (2011) for applications to Industrial Organization. Application to Law and Economics are discussed in Sunstein (2003). Amongst the applications to growth, Koszegi and Rabin (2009) and Bowman et al. (1999) analyze the consequence of reference-dependent behavior in a consumption-saving model. As opposed to our work, however, their focus is on the case in which the reference point is the agent's expectations of future consumption. Foellmi et al. (2011) use instead past consumption as reference points in a standard Ramsey growth model which incorporates prospect theory. Loewenstein and Prelec (1992) apply it (together with behavioral discount theory) to intertemporal choice. Kyle et al. (2006) use prospect theory to model liquidation decision by an investor.

Camerer (2000) reviews a significant amount of studies where prospect theory is able to account for empirical puzzles of standard theories.

Rabin (2002) models one of the heuristics associated with prospect theory (Kahneman and Tversky (1979)) and applies it to finance. Other application to finance are reviewed in Barberis and Thaler (2003) and Thaler (1993).

In addition, our model is related to the literature in development economics that study how the behavior of others might affect one agent's 'aspirations' about what that they would like to accomplish. Amongst them, Appadurai (2005) and Ray (2006) suggest how aspirations could play both a positive and a negative role in the development of a country: they might induce subjects to 'work hard' to reach their aspirations; at the same time, individuals who are too far from reaching them might simply 'give up.' The latter paper in particular was an important source of inspiration for our work. In addition, formal models that study the role of aspirations appear in Banerjee (1990), Genicot and Ray (2009), Dalton et al. (2010), and Mookherjee et al. (2010). Mookherjee et al. (2010) studies the case in which agents look at their 'neighbors' to form their aspirations – i.e. they focus on a 'local' origin of aspirations, instead of a society-wide origin like we do. (Furthermore, they use a functional form very different from prospect theory.) Genicot and Ray (2009) studies an OLG model in which subjects live one period and are reference-dependent, with a utility function, differentiable everywhere, which is convex in an interval right before the aspirations level. They first show that there exists an inverse U effect of aspirations on accumulation decision, proving the conjecture in Ray (2006). Then, focusing on the case in which aspirations are the average income of the society, they show that the support of the income distribution in steady state is generically finite, and that, under the condition that aspirations are, loosely speaking, 'important enough,' a distribution of perfect equality cannot exist. In addition, they also consider the case of 'upward looking' aspirations, i.e. the case in which each individual's aspirations are the average income of those *above* her in the income distribution, and show that in this case continuous income distributions can exist in a steady state if and only if they are Pareto distributions. By contrast, in our paper we focus only on the case in which the reference-point is the average endowment of the society, but consider a utility function which not only has an area of convexity, but also admits a kink at a reference point – directly following prospect theory and the notion of loss aversion. This feature allows us obtain our different results on the long run distribution of endowment: that the distribution becomes, and remains, either of perfect equality or has a missing class, therefore also admitting the case of continuous support. (In our model these features are reached in finite time, as opposed to in the steady state.) Furthermore, it also allows us to characterize the effects of the initial distribution on growth. [3]

Finally, our work is also more broadly connected with the vast literature in macroeconomics usually referred to as *keeping up with the Joneses*, that models agents whose utility for consumption depends also on the relative position of the agent in the wealth distribution (their status). (An analysis closer to the one in our work appears in Konrad (1992), Fershtman et al. (1996), Rauscher (1997), Corneo and Jeanne (2001), Cooper et al. (2001), Stark (2006), Hopkins and Kornienko (2006).) Our approach is different for two reasons. First, these models assume that individuals care about their status using some additive or multiplicative functional form, which renders the model very different from assuming that subjects care about how their income relate to the average one using a prospect-theory form. From this point of view, one case loosely see our work as exploring a similar idea, albeit using a functional form which has been derived in behavioral economics and psychology analyzing behavior. Second, most of these works focus more on the properties of the accumulation path than on the implications on the wealth distribution in the long run – as evident from the fact that most use a representative agent. By contrast, the development of the wealth distribution, and its effects on growth, are the essential focus of our study.

2. The Model

2.1 Formal Setup

We study an economy with overlapping generations and warm glow preferences. The economy is populated by a continuum of size L of agents who live for two periods, make choices in the first period, generate an offspring exogenously in the second period, and

[3] Apart from the literature on aspirations, the relationship between saving decisions and long run distribution was the object of a couple of very well known contributions, such as Stiglitz (1969) and Bourguignon (1981). However, they do not have microfoundation for the saving function and their model is an exogenous growth model.

have a bequest motive directly in the utility function.[4] Every agent is endowed with e_{it} of human capital and k_{it} of physical capital, which are determined by the decision taken by the previous generation. We assume constant return to scale in the accumulation of human capital, i.e. by investing e_{it+1} households obtain Ae_{it+1}, and without loss of generality we put $A = 1$. $x_{i,t+1}$ is the market value of the assets of the agent, i.e. the wealth of each agent in our model. Since our interest is mostly related to developing countries, we impose that the access to credit market is restricted: individual consumers cannot borrow. (We will later argue that this assumption does not seem to be what is driving our results. See Remark 2.) By $\beta \leq 1$ we denote the discount factor; μ_t stands for the distribution of endowments in each period; r_t and w_t represent the interest rate and the wage. Finally, in the economy we have a representative firm with a production function $F(K, H)$.

The individual household maximizes:

$$\begin{array}{c} \max_{c_{i,t}, x_{i,t+1}, e_{i,t+1}, k_{i,t+1}} u(c_{it}) + \beta v(x_{i,t+1}, \bar{x}_{i,t+1}) \\ \text{such that} \\ c_{i,t} + e_{i,t+1} + k_{i,t+1} \leq w_t e_{i,t} + (1 + r_t) k_{i,t} \\ x_{i,t+1} = w_t e_{i,t+1} + (1 + r_t) k_{i,t+1} \end{array} \tag{1}$$

As opposed to standard OLG problems, the agent's utility for the second period, v, can be different from the one of the first period, u. In particular, the former also depends on an additional term, $\bar{x}_{i,t}$, which we will interpret as the agent's *reference-point* for period t.

We start by imposing standard restrictions.

Assumption 1. $u(\cdot)$ is increasing and concave at satisfies the Inada conditions.

Assumption 2. The production function $F(K, H)$ is increasing, concave, homogeneous of degree one, and satisfies the Inada conditions.

Both requirement above are standard. In particular, agents have a standard utility function in the first period of their lives. (In Section 4 we will also consider the more specific case in which u exhibits Constant Relative Risk Aversion, CRRA.) The main feature of our model is the shape of v.

Assumption 3. There exist a $H > 0$ such that for every $\bar{x} \in \mathbb{R}_+$, the following holds:

1. *v is continuous and monotone*: $v(\cdot, \bar{x})$ is strictly monotone and continuos; $v(x, \cdot)$ is continuous for every $x \in \mathbb{R}_+$;

2. *$v(\cdot, \bar{x})$ moves like u when not close to the reference-point*: for all $x \notin [\bar{c} - H, \bar{c}]$, $v(x, \bar{x})$ is twice differentiable and $\frac{\mathrm{d}v(x,\bar{x})}{\mathrm{d}x} = \frac{\mathrm{d}u(x)}{\mathrm{d}x}$;

[4] The presence of an uncountable number of agents is inessential for our results. It is routine to show that everything we prove would hold true with finitely many agents.

3. *v has a prospect-theory form*:

   (a) v is strictly convex on $(\bar{x} - H, \bar{x})$;

   (b) $\lim_{x \to \bar{x}^-} v_1(x, \bar{x}) > \lim_{x \to \bar{x}^+} v_1(x, \bar{x})$.[5] We define $v_1(x,\ x')$ as the first derivative with respect of the first argument

The idea behind Assumption 3 is that our agents have a reference-point which affects their preferences. This reference point affects the second period utility v in two ways, both of which are the fundamental features of prospect theory (part (3) of Assumption 3):

1. *v is steeper for losses than for gains, and in particular it is not differentiable at $v(\bar{x}, \bar{x})$*: this generates the well-known effect called 'loss-aversion,' and is motivated by the different approach that subjects have to losses as opposed to gains with respect to the reference point.

2. *v is convex below the reference-point, and concave above it*: this feature, which leads to the so-called 'diminishing sensitivity,' is motivated by the fact that the marginal change in gain-loss sensations is greater the closer we get to the reference point. (Notice that v is strictly concave after $\bar{x}$, since there it coincides with u.)

(See, among others, Kahneman and Tversky (1979), Tversky and Kahneman (1991, 1992) for more discussion on these properties.) While reference-dependent, we assume that v remains 'well-behaved:' the presence of a reference point does not render the function discontinuous or not-monotone (part (1)). Moreover, as opposed to what usually assumed in prospect theory, we posit that the reference point affects the agents' utility in the area 'close to' the aspiration level, but not if 'far below:' we posit that there exists a positive H such that v is not subject to reference-effects (it coincides with u) for levels of x below $\bar{x} - H$ (part (2)). The rationale of this restriction is that reference effects are the strongest in the area immediately preceding the reference point, but then they tend to fade, all the way to disappearance, as we get further below. At the same time, we posit that when the reference point is reached subjects should go back to their standard behavior: v becomes concave and behaves like u (part (2)).

Because v depends on the reference-point, but u does not, then with Assumption 1 and 3 we are imposing that households are reference dependent *only in the second period of their life* – the reference point affects how they value the income later in their life, and their bequest. Conversely, they are not reference dependent when they are young. This approach is motivated by the observation that individuals tend to set objectives for themselves to be accomplished when they have reached a certain age, or for their offsprings to reach, as opposed to for their young age. That is, that the accomplishment of life goals tends to be evaluated only in the later part of life. (A similar approach appears in Mookherjee et al. (2010) and Genicot and Ray (2009).)

[5] By $\lim_{x \to \bar{x}^-} v(x, \bar{x})$ we understand the limit of $v(x, \bar{x})$ as x approaches $\bar{x}$ from *below*. (The limit from above is defined analogously.)

Thus far in our analysis we have not imposed any restriction on the origin of agents' reference-point. In this paper we choose to focus on a very specific source: we study the case in which the reference-point in period t is the *average* endowment of the society *as a whole* in period t. (Denote by μ_t the distribution of endowments at time t.)

Assumption 4. For $i \in I$, $t \in T$, $\bar{x}_{i,t} = \int_I x_{i,t} \mu_t(x) \mathrm{d}i$.

With Assumption 4 we are ruling out the case in which reference-points are affected by the income of the parents, *à la* Koszegi and Rabin (2009), or of the neighbors, *à la* Mookherjee, Napel, and Ray (2010). We focus on this special case because we are interested in understanding the effect of this society-wide reference-dependence.

2.2 Notion and multiplicity of equilibria

One of the features of the economy described above is that each agent's utility depends on the behavior of the other members of the economy. Since the different agents decides *simultaneously*, the expectations of each agent on the behavior of others will play an essential role: agents are best reacting to what they expect others to do. In line with this, we focus our attention on the following (standard) kind of equilibria.

Definition 1. An equilibrium of the economy is a sequence of consumption decisions, bequest decisions, and factor prices

$$e = \{\{c_{it}\}_{i \in L}, \{x_{it+1}\}_{i \in L}, w_t, r_t\}_{t=0}^{\infty}$$

such that factor markets clear and (c_{it}, x_{it+1}) solves (1) for all $i \in L$ and for all t.

Definition 1 is a standard notion of equilibrium. Notice that by using this definition together with Assumption 4, we are implicitly assuming that subjects can correctly forecast the behavior of others: in fact, reference-points are defined as the average of the true income distribution in the second period of the agents' life, but affect their behavior already in the first – which implicitly implies that households can correctly forecast this distribution.[6]

It is standard practice to show that one such equilibrium exists for any economy that satisfies Assumptions 1-4. As opposed to the case without reference-points, however, the economy described in Section 2.1 will not admit a unique equilibrium. While indeed the presence of multiple equilibria is not surprising in general, in this economy this is mostly due the presence of reference-dependence: by rendering the agent's utility dependent on the behavior of others we introduce obvious coordination issues. To see why, consider the simplest possible case: an economy in which all agents start with the same endowment in the first period. In this simple case, all agents have the same reference-point, and face

[6]Alternatively, we could have defined the reference-point as the expectations of the distribution in the future period, and added the requirement that in equilibrium such expectations are correct and degenerate. The two approaches are clearly identical.

the same problem: they will then choose the same consumption plan.[7] And since all households have the same consumption, they will all reach their reference-point – they will all invest exactly what needed to reach the average. The problem is, however, that how high this average will be depends on the future behavior of the other households. Consider for example some equilibrium in which all households choose some $a > 0$ in the second period (and each of them knows that this will happen). Then, consider some alternative equilibrium in which, instead, each household knows that all others agents will choose $a + \epsilon$ (where ϵ is small and positive). In this latter case, if one household were to consume only a, it would fall short of its reference-point. Then, as long as ϵ is small, it might choose instead to save a little more in the first period so that she could reach the rest of the society – and so its reference-point. And since this can be true for all agents, then everybody will in fact choose $a + \epsilon$, guaranteeing that this is an equilibrium. Naturally this is true only as long as ϵ is 'small': there will be some $\bar{\epsilon}$ large enough such that, even if every agent knew that everyone else will consume $a + \bar{\epsilon}$, they'd rather fail to reach their reference-point and consume only a. And since this will be true for everybody, then the average consumption will be only a. Thus there cannot be an equilibrium with second-period consumption $a + \bar{\epsilon}$ for $\bar{\epsilon}$ arbitrarily large. In the analysis that follows we will therefore analyze the features of the *set* of possible equilibria of the economy.

Before we proceed, let us briefly add a small remark about the role of credit constraints for our results.

Remark 1. Since there exists two assets (human and physical capital), there should be equality of returns among them. Given the assumption of competitive markets and the assumption on the differentiability of the production function, at every t we must have $w_t = 1 + r_t$. Then the household problem simplifies to the choice of I_{t+1} to maximize:

$$\max_{I_{t+1}} u(x_{it} - I_{t+1}) + \beta v(w_t \; I_{t+1}) \tag{2}$$

As customary define $K_t = \int_L k_{it} di$, $H_t = \int_L e_{it} di$, $\lambda_t = \frac{K_t}{H_t}$ and $f(\lambda_t) = F(K_t, H_t)/H_t$. (Notice that by Assumption 2 there are constant returns to scale.) By the arbitrage condition above, and since factor rental prices are equal to the marginal productivity at every t, we must have

$$f'(\lambda_t) = f(\lambda_t) - f'(\lambda_t)\lambda_t. \tag{3}$$

By Assumption 2 there exists a unique λ which satisfies (3), since the LHS is always decreasing and the right hand side always increasing.

As a result, the economy is in balanced growth path since period zero and the growth rate of the economy is equal to the growth rate of investment. From now on, we use $w^* = f(\lambda^*) - f'(\lambda^*)\lambda^*$ to indicate the wage rate at every t.

[7] This requires that subjects cannot make different choices when they are indifferent. This is proved in Lemma 1 in the Appendix.

Remark 2. Since we find our analysis particularly interesting in the case of developing countries, where credit constraints are binding, we have chosen to focus on the case in which access to credit markets is restricted. However, this restriction does not seem to be what is driving our result. In fact, the most important point in our setup is the possibility for households to fail to reach their reference-point, i.e. to have a second-period consumption below the average. We will now show that even if we remove any restriction in the access to credit, households might still fail to reach their reference-point – leading to results qualitatively similar to the ones we discuss.

Assume an economy as described above, but with no credit constraints, and notice that the households problem (2) can be re-written as (define g to be the return on bonds, b_{it})

$$\max_{I_{t+1},b_t} \; u(x_{it} - I_{t+1} + b_{it}) + \beta v(w_t I_{t+1} - g_t b_t)). \tag{4}$$

If we compute the FOCs with respect to I_{t+1} and b_t, we get:

$$\begin{aligned} u'(x_{it} - I_{t+1} + b_{it}) = \beta w_t v'(w_t I_{t+1} - g_t b_t)) \\ u'(x_{it} - I_{t+1} + b_{it}) = \beta g v'(w_t I_{t+1} - g_t b_t)) \end{aligned} \tag{5}$$

which implies $g = w_t = w^*$. The problem is then determined in the difference $I - b$. It is then easy to see that, if $w^*(I - b) = \bar{x}_{it+1}$ and if

$$u'(x_{it} - \bar{x}_{it+1}/w^*) + \lim_{x' \to \bar{x}_{it+1}^-} \beta w^* v'(x') > 0 \tag{6}$$

then the optimal choice of the household is to fail to meet its reference-point, as sought.

2.3 Properties of the optimal behavior

Because of the convexity and non-differentiability of the second-period utility function, the behavior of households in this economy will be different from that in the standard model. To better express its features, let us define the optimal choice of an household in the second period as a function of the initial endowment and aspiration level:[8] define $\phi : \mathbb{R}_{++} \times \mathbb{R}_{++} \to \mathbb{R}_{++}$ such that $x_{i,t+1} = \phi(e, \bar{x})$ is an optimal solution of (1) for an household i with initial endowment e and reference-point $\bar{x}$. Then, the following must hold.

Proposition 1. *The following holds for all $\bar{x}$:*

1. *there exists $\gamma \in \mathbb{R}$, $\gamma > 0$ such that for all $e \in \mathbb{R}_{++}$, $\phi(e, \bar{x}) \notin (\bar{x}_t - \gamma; \bar{x}_t)$;*
2. *there exist some $\underline{e}, \bar{e} \in \mathbb{R}_{++}$, $\underline{e} < \bar{e}$ such that $\phi(e, \bar{x}) = \bar{x}$ for all $e \in (\underline{e}, \bar{e})$.*

[8] To be precise, this map should also depend on the other elements of the economy, like r_t and w_t. We omit them from the notation for simplicity.

Proposition 1 shows two features of the optimal solution of the household problem. Part (1) shows that there is an interval (of positive measure) of second period wealth, right before the reference-point $\bar{x}$, that will *not* be chosen by any household with reference-point $\bar{x}$, irrespectively of their initial endowment. Put differently, no household will choose a second-period wealth too close below the reference-point: either they reach their reference-point, or they fall short of it of a non-trivial amount. This is naturally due to the fact that v is convex in an interval that ends in $\bar{x}$. As we shall see, this has important consequences on the aggregate behavior of the economy.

Part (2) of Proposition 1 shows another features of the optimal behavior: there is a non-zero interval of initial endowments such that the optimal choice for households with those endowments is the same and equal to the reference-point. That is, many different households, with many different initial endowments, will choose to consume *exactly* same amount in the second period, equal to the reference-point.

3. Distribution in the Long Run

3.1 Distribution in the long run

We now turn to study the distribution of our economy in the long run. To this end, we will define two kinds of wealth distribution. The first kind is standard: we say that a wealth distribution is of *perfect equality* if it is degenerate.

Definition 2. A distribution μ on $\mathbb{R}_+$ is of perfect equality if $\mu(x) = 1$ for some $x \in \mathbb{R}_+$.

Next, we introduce the notion of *missing class.* We say that an interval (x_1, x_2) is a missing class for a wealth distribution μ if this distribution has zero density on (x_1, x_2), but it admits a strictly positive density both above and below this interval. The idea is that μ includes households on both below x_1 and above x_2, but has 'a gap' between them – the missing class.

Definition 3. For any (x_1, x_2) with $x_1 < x_2$, we say that (x_1, x_2) is a *missing class* for a wealth distribution μ if:

1. $\mu([0, x_1)) > 0$ and $\mu([x_2, +\infty)) > 0$;
2. $\mu(x) = 0$ for all $x \in [x_1, x_2]$.

A special case of a wealth distribution that admits a missing class is the one in which average of this distribution, $\mathbb{E}[\mu_t]$, is the *upper* limit of the missing class. In this case, we say that the *wealth distribution admits a missing class below the mean.*

Definition 4. A wealth distribution μ *admits a missing class below the mean* if there exits $x \in \mathbb{R}_{++}$ with $x_1 < \mathbb{E}[\mu_t]$ such that $(x, \mathbb{E}[\mu_t])$ is a missing class for μ.

Any such distribution has a 'gap,' which right below the average, that divides the agents with a wealth below the average, from those above or at the average. Any such distribution cannot be degenerate. Rather, it must be, in some sense, *polarized*, because it divides the rich (above or at the mean) from the poor (below the mean).

It turns out that, in the presence of reference-dependence, these two types of distributions are enough to describe the long run behavior of any the economy that satisfy our assumptions. (By μ_t^e we denote the distribution of endowment in equilibrium e and period t.)

Theorem 1. *Consider an economy as described above that satisfies Assumptions 1-4, and some equilibrium e of this economy. Then there exists some $T \in \mathbb{N}$ such that for all $t > T$, one of the following must hold:*

(a) μ_t^e is of perfect equality;

(b) μ_t^e admits a missing class below the mean.

Theorem 1 shows that the long run distribution of any equilibrium of any economy that satisfy our assumptions can be of only two kinds: either it is of perfect equality, or it admit a missing class below the mean. No other distribution is possible – in particular, no continuous and non-degenerate distributions are possible in the long run. Notice, moreover, that this is not an asymptotic result, or a property of the steady state. Theorem 1 shows that one of these two types of distribution will be reached in finite time (at time T), and will be stable: after that time the economy will be, and forever remain, either of perfect equality or with a missing class.

To analyze the implication of the Theorem 1, recall that in the standard case in which u and v are identical and concave, an economy that started with a continuous and non-degenerate distribution of initial endowments could easily have an equilibrium in which the distribution is continuous and non-degenerate in every period: for example, if u exhibits constant relative risk aversion (CRRA), then any *distribution is possible in the long run* if $u = v$ and both functions are strictly concave.[9] By contrast, Theorem 1 shows that the the presence of reference-dependence has a strong impact on the type of paths that are attainable in equilibrium: no matter what the initial distribution is, and no matter how small the impact of the reference-point on utilities is, a continuous non-degenerate distribution is no longer possible in the long run. Either the poor get "left behind," failing to attain their reference-point: in this case we have a society with a missing class below the mean, which is polarized between those that attain their reference-point, and those that fail to. Or, everybody catches up with their reference-point – but this can happen only if that the distribution of endowments is of perfect equality.

[9] To see why, consider an economy like ours but in which: $u = v$; both are strictly concave; both exhibit CRRA. Then, notice that for such economy the distribution of endowments is constant over time. (This is proved in Lemma 2 in the Appendix.) But of course this implies that the wealth initial distribution is also the long run distribution – which means that any long run distribution is possible.

3.2 Origin of the long run distribution

Theorem 1 shows that in any equilibrium the long run distribution of endowments can be of only two kinds, but it does not provide conditions as to when each of these two cases take place. This is the content of the following proposition.

Proposition 2. *Consider an economy as described above that satisfies Assumptions 1-4 with an initial distribution of endowment μ_0. Then each of the following must hold:*

1. *if μ_0 is not degenerate, then there exists some equilibrium e and some $T \in \mathbb{N}$, such that for all $t > T$, μ_t^e admits a missing class below the mean;*
2. *there exists some $\delta \in \mathbb{R}$, $\delta > 0$, such that, if $\frac{\inf_{x \in \text{supp}(\mu_0)} x}{\mathbb{E}(\mu_0)} \geq \delta$, then there exists an equilibrium e, and some $T \in \mathbb{N}$, such that for all $t > T$, μ_t^e is of perfect equality.*

Proposition 2 shows that the distribution that the economy will assume in the long run depends both on the initial conditions (μ_0), and also on the equilibrium we are looking at. In particular, part (1) shows that any economy admits a long run equilibrium that evolves into a distribution with a missing class below the mean, no matter what the initial distribution is (as long as it is not degenerate). To get an intuition of why this is the case, notice that we can always construct an equilibrium in which every member of the society with an endowment equal or above the mean is expected to increase her consumption by at least some b, where b is defined as the maximum amount that the household with an endowment exactly equal to the mean is willing to increase her consumption to if she knew that the mean will increase by b. In this equilibrium everyone with an endowment above or equal to the mean will "keep up," while the rest will not, generating polarization, and a missing class below the mean. If this is true in every period, then there will always be a missing class, leading to part (1) of the proposition.

On the other hand, part (2) of Proposition 2 shows that, if the initial distribution is not "too disperse," i.e. the ratio between the minimum and the average income is high ($\frac{\inf_{x \in \text{supp}(\mu_0)} x}{\mathbb{E}(\mu_0)} \geq \delta$), then this economy will *also* admit an equilibrium which evolves into perfect equality in the long run. The reason is, again, very simple: if the poor are not too much behind the average, as time goes by they will catch up – leading to perfect equality.

4. Growth

4.1 Comparative Growth

We now turn to investigate how society-dependence reference effects affect growth. Indeed their impact will depend on the wealth distribution, and it is well-known that the relation between inequality and growth is in general rather complicated even with standard preferences (see Aghion et al. (1999)). In our simple setup, however, if we remove any reference-effect (i.e. we posit $u = v$ and that both are concave), and we focus on

CRRA utility, we get a unique growth rate regardless of the initial distribution.[10] In this section we show that in our model with reference-dependence the initial distribution will always affect the growth of the economy, even if the utility is CRRA. In particular, we compare how the average endowment changes over time depending on the initial distribution of the society and on the equilibrium that we are looking at.

In this section we compare economies, where an economy is defined by an initial distribution of endowments and a certain u and v function, with given average initial wealth.

Comparing economies with difference preferences raises the problem of defining a criterion of evaluation. We should emphasize that the endogeneity of the utility does not allow standard Pareto ranking among equilibria. For this reason, in this section we focus on a more objective parameter – the growth rate.

To better express these comparisons, let us introduce a few simple notions of comparison. We begin by saying that an equilibrium *dominates* another if the average endowment of the first is above the average endowment of the other for every period starting from some period T: that is, if there is a point in which the average income of the first surpasses, and stays above, that of the latter. We shall use the symbol $\succeq$ ($\succ$) to express this (strict) relation.

Definition 5. For any equilibrium e of some economy E, and equilibrium e' of some economy E', we say $e \succeq e'$ if there exists some $T \in \mathbb{N}$ such that for all $t \geq T$ we have

$$\int_x x\mu_t^e(x)\mathrm{d}x \geq \int_x x\mu_t^e(x)\mathrm{d}x.$$

We say that $e \succ e'$ if there exists some $T \in \mathbb{N}$ such that the inequality above is strict for all $t \geq T$.

Notice that the comparison $\succeq$ could also be seen also as a comparison of the average growth rate of two economies that start from the same initial average endowment. (If we have two economies E and E' with the same average endowment, and equilibria e of E and e' of E' such that $e \succ e'$, then in the long run not only the endowment, but also the average growth rate of e is above that of e'.)

While $\succeq$ compares two specific equilibria, potentially of two different economies, we now introduce a comparative notion for *all* the equilibria of two economies. We define three notions: $\rhd$, $\rhd_{\max}$, $\rhd_{\min}$, to represent, respectively, full dominance, higher "best" equilibrium, and higher "worst" equilibrium. We start with "full dominance."

Definition 6. For any economies E and E' we say that $E \rhd E'$ if for all equilibria e of E, and e' of E', we have $e \succ e'$. We say $E \trianglerighteq E'$ if for all equilibria e of E, and e' of E', we have $e \succeq e'$.

[10]See Lemma 2 in the Appendix. While this holds for CRRA utility, this is not necessarily true for other specifications (see Lemma 2 again).

The idea behind Definition 6 is that if one economy E "dominates" another economy E' ($E \rhd E'$), then it means that every equilibrium of E dominates (in the sense of $\succ$ above) every equilibrium of E'. This is a very demanding notion, since it requires that even the worst of the equilibria of E strictly dominates the best of the equilibria of E'. Two weaker notions would compare the 'best' equilibrium of each economy, or the 'worst' one. This is what we do in the definition of $\rhd_{\max}$ and $\rhd_{\min}$ below.

Definition 7. For any economies E and E' we say that $E \rhd_{\max} E'$ if there exists some e of E such that $e \succ e'$ for all e' of E'. We say $E \unrhd_{\max} E'$ if there exists some e of E such that $e \succeq e'$ for all e' of E'.

Definition 8. For any economies E and E' we say that $E \rhd_{\min} E'$ if there exists some e' of E' such that $e \succ e'$ for all e of E. We say that $E \unrhd_{\min} E'$ if there exists some e' of E' such that $e \succeq e'$ for all e of E.

We say that an economy $\rhd_{\max}$-dominates another if its best equilibrium dominates (in the sense of $\succ$) *any* equilibrium of the other economy. Conversely, we that an economy $\rhd_{\min}$-dominates another if even its worst equilibrium dominates (again in the sense of $\succ$) *some* equilibrium of the other.

Finally, in order to better express our results on growth, we compare the behavior of four different types of economies, each of which is assumed to have the same average initial endowment. Recall that μ_0 denotes the initial distribution.

Definition 9. Denote by $\mathcal{E}_O$ the set of economies that satisfy Assumptions 1, 2, $v = u$ and such that μ_o is of perfect equality. $\mathcal{E}_1$ is the set of economies that satisfy Assumption 1-4 and such that μ_o is of perfect equality. $\mathcal{E}_2$ is the set of economies that satisfy Assumptions 1-4, in which μ_0 is not of perfect equality, but that admit an equilibrium e and some $T \in \mathbb{N}$ such that for all $t \geq T$, μ_t^e is of perfect equality. $\mathcal{E}_3$ is the set of economies that satisfy Assumptions 1-4, in which initial distribution is not of perfect equality, and that admits no equilibrium e' and some $T \in \mathbb{N}$ such that for all $t \geq T$, $\mu_t^{e'}$ is of perfect equality.

Economies in $\mathcal{E}_0$ is a standard economy with a representative agent who has a standard concave utility function with no reference-effects. Economies in $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_3$, instead, have reference-dependent households as modeled in Assumptions 1-4. They differ from each other because of the initial distribution: economies in $\mathcal{E}_1$ have an initial distribution with perfect equality; those in $\mathcal{E}_2$ do not, but admits at least one equilibrium in which the wealth distribution converges to perfect equality (in finite time); those in $\mathcal{E}_3$, instead starts with a distribution with some inequality, and admit no equilibria in which the inequality disappears. From Proposition 2 we know that we can think of economies in $\mathcal{E}_2$ as those in which we have some initial inequality, but in which the wealth distribution is not 'too disperse' – no household has an initial endowment too far below the mean endowment. By contrast, an economy in $\mathcal{E}_3$ would be an economy in which the initial distribution has a more acute inequality: there are households with initial endowments far from the mean.

4.2 Growth in the 'best' equilibria

We are now ready for our first result on growth. Let us start by looking at the 'best' equilibria – those in which growth is the highest.

Theorem 2. *Consider four economies $E_0 \in \mathcal{E}_O$, $E_1 \in \mathcal{E}_1, E_{2,3} \in \mathcal{E}_2 \cup \mathcal{E}_3$ that have the same initial average endowment. Then:*

$$E_1 \rhd_{\max} E_{2,3} \quad \text{and} \quad E_1 \rhd_{\max} E_0.$$

If u exhibit CRRA, then:

$$E_1 \rhd_{\max} E_{2,3} \trianglerighteq_{\max} E_0.$$

Theorem 2 shows that the presence of reference-dependence could have a strong impact on growth depending on the initial distribution. First of all, it shows that there exists an equilibrium of E_1 that dominates *all* other equilibria of *all* other economies: the equilibrium with the (strictly) highest growth amongst all is one of an economy with reference-dependence and with an initial distribution of perfect equality. The intuition is that with reference-dependence and perfect equality, agents could "push" each other into consuming more, and since the distribution is of perfect equality, then by doing this there is no subject that is "left behind" – leading to the highest growth. Without reference-dependence we loose the mechanism of "pushing each other," rendering the growth of the best equilibrium of E_1 higher than that of any equilibrium of E_0. At the same time, this growth will also be higher than the one of any equilibrium of a society with reference-dependence and initial inequality: in this case, the wealthy cannot expect the society to increase the income too much, since they know that the poor would not follow, and therefore they won't increase their own income as much as they do in E_1.

If we further assume that u exhibits constant relative risk aversion, then we obtain the full, transitive rank: $E_1 \rhd_{\max} E_{2,3} \trianglerighteq_{\max} E_0$. That is, if we look at the best equilibria, we have that the highest growth is found with reference-dependence and perfect equality, then with reference-dependence and initial inequality, and finally the economy with no reference-dependence. This should be compared with what happens when there are no reference-dependence and both u and v are identical, concave, and CRRA: it is well known (we show it again in Lemma 2 in the Appendix) that in this case the growth rate is constant and independent of the initial distribution. (Notice that this implies that in the case of CRRA utility the ranking above is true also for some E_0' which, like E_0, has no reference-dependence, but that has an initial distribution which is not necessarily of perfect equality.) This means that: the presence of reference-dependence is always positive for growth, and it renders growth dependent of the initial distribution, where the presence of initial inequality is actually harmful for growth.

This discussion shows a feature of society-influenced reference-dependence. On the one hand, it induces households to 'push each other,' generating growth. On the other hand, this mechanism works better when households are not 'too far' from each other. As we shall see below, however, while this is true to the equilibria with the highest growth, this is not necessarily true in other equilibria.

4.3 Comparative growth in all equilibria

Theorem 2 analyzes the ranking only for the 'best' equilibria. It turns out that things can be quite different in other equilibria.

Theorem 3. *Consider four economies $E_0 \in \mathcal{E}_O$, $E_1 \in \mathcal{E}_1, E_2 \in \mathcal{E}_2$, and $E_3 \in \mathcal{E}_3$ that have the same average initial endowment. Then the following holds:*

1. $E_1 \trianglerighteq E_0$;

2. *for any equilibrium e of E_0, there exists some equilibrium e' of E_1 such that $\mu_t^e = \mu_t^{e'}$.*

Moreover, if u exhibits CRRA, then:

4. $E_2 \triangleright E_0$;

5. $E_2 \triangleright_{\min} E_1$;

6. $E_3 \trianglerighteq E_0$;

7. $E_3 \trianglerighteq_{\min} E_1$.

Theorem 3 considers other equilibria besides those with the highest growth. First, it shows that the presence of reference-dependence never reduces growth: every equilibrium of E_1 must grow at least as much as any equilibrium of E_0 ($E_1 \trianglerighteq E_0$); and if u is CRRA, then also any equilibria of E_2 and E_3 grow more than that E_0 ($E_2 \triangleright E_0$ and $E_3 \trianglerighteq E_0$). In fact, equilibria in E_2 *strictly dominate* all equilibria in E_0. At the same time, however, the presence of reference-dependence is not sufficient to have a higher growth: for example, there are equilibria of E_1 in which the wealth distribution coincides *in every period* with that of the (unique) equilibrium of E_0.

Also in the comparison between E_1, E_2, and E_3, the ranking in Theorem 3 is quite different from the one in Theorem 2. While in the latter we have seen that the economy with the highest growth in the best equilibrium is E_1, Theorem 3 shows that, if u is CRRA, then a society with a small amount of initial inequality (E_2) has a lowest-growth equilibrium where it grows *strictly* more than the lowest-growth equilibrium of a society with initial perfect equality (E_1): we have $E_2 \triangleright_{\min} E_1$. Moreover, we also have that $E_2 \triangleright E_0$: the presence of a little bit of inequality renders minimal growth *strictly higher* also of the case with no reference-effects. The intuition is that, with reference-dependence, the households that are right below the average income might choose to 'push up,' and reach their reference-point – generating growth. And since this cannot happen with perfect equality or with no reference-dependence, then we have that the minimal growth of an economy of type E_2 must lie strictly above that of an economy of type E_2 or E_0. A similar argument suggests why we have $E_3 \trianglerighteq_{\min} E_0$, and $E_3 \trianglerighteq_{min} E_1$.

(Here the inequality is weak, $\trianglerighteq$, since there could be no subjects who 'push up' of the kind described above.[11])

The results of Theorems 2 and 3 can then be summarized as follows: the presence of reference-dependence can only increase the growth rate with respect to the case of no reference-dependence. Depending on the initial distribution of the economy, the growth rate will be strictly higher (as is the case if the initial range is small enough), or identical at least for some equilibria (as is the case for initial perfect equality). If we compare the growth rates between economies with reference-dependence but different initial distributions, we find that the results depend on the equilibrium we are looking at: economies that have an initial distribution with perfect equality have equilibria with a growth rate strictly higher than any other economy; at the same time, they also have minimal equilibria that are worse than the minimal equilibria of any other economy – strictly worse than the minimal equilibria of E_2. A graphical intuition of the results appears in Figure 4.3, which represent the set of average growth rates for all equilibria of each type of economy. (Notice that both the highest and the lowest growth rate for E_3 could be either above or below those of E_2, albeit always (weakly) above that of E_0. To represent this possibility, Figure 4.3 contains the lighter shade area for E_3.).

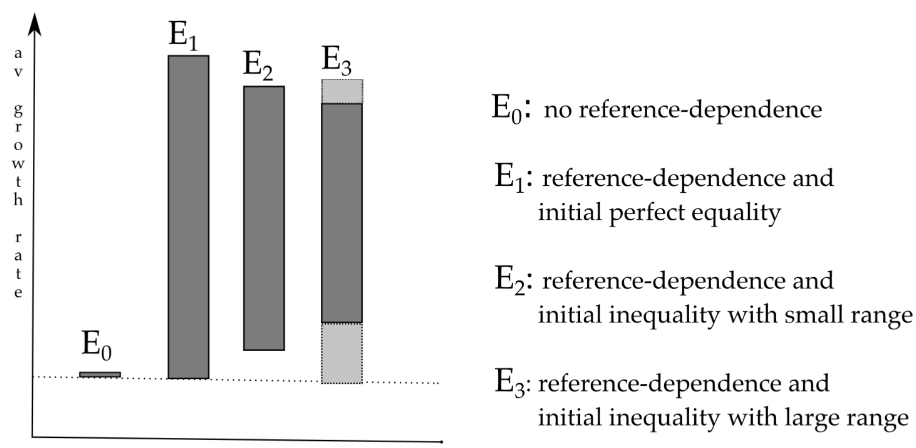

FIGURE 1 Average growth rates for different equilibria of economies E_0, E_1, E_2, E_3 when u is CRRA

5. Conclusion

In this paper we study a warm glow OLG model in which agents' utility on bequest depends on the the average behavior of the rest of society, which acts as a reference-point. In line with this interpretation, we model this reference-dependence using the standard prospect theory form. We notice that this leads to multiple equilibria. We show that in any of these equilibria the wealth distribution will converge (in finite time)

[11]That is, there could be no household with an endowment below but close to the average one.

to be either of perfect equality or to have a missing class. We then turn to study the growth rate of different societies, and notice that comparisons strongly depend on the equilibrium that we are looking at. If we look at the equilibrium with the highest growth for each society, then a society that starts from perfect equality is the one that grows the most. At the same time, if we look at equilibria with the lowest growth, then the society that grows the most is the one with a small amount of initial inequality. Finally, we show that any society with reference-dependence, no matter what the initial distribution is, grows (weakly) more than any society without reference-dependence.

Appendix

A. Preliminary Results

Lemma 1. *For all equilibria e, if μ^e_T is of perfect equality, then μ^e_t is of perfect equality $\forall t \geq T$.*

Proof. As in Remark 1, define $w^* = f(\lambda^*) - f'(\lambda^*)\lambda^*$, with $\lambda_t = \frac{K_t}{H_t}$. Assume that μ_T is of perfect equality. We claim that $x_{i,T+1} = \mathbb{E}[\mu_{T+1}]$ for all $i \in L$. By means of contradiction, assume instead that there exists $i \in L$ such that $x_{i,T+1} \neq \mathbb{E}[\mu_{T+1}]$. If we have $x_{i,T+1} > \mathbb{E}[\mu_{T+1}]$, there must also exist some $j \in I$ such that $\mathbb{E}[\mu_{T+1}] > x_{j,T+1}$, by definition of $\mathbb{E}[\mu_{T+1}]$. Assume then WLOG that we have $x_{i,T+1} > \mathbb{E}[\mu_{T+1}] > x_{j,T+1}$ for some $i, j \in L$. Notice that for both i and j we must have that the optimal solution meets the FOCS, i.e. $u'(x_T - x_{i,T+1}/w^*) = \beta w^* v'(x_{i,T+1})$ and $u'(x_T - x_{j,T+1}/w^*) = \beta w^* v'(x_{j,T+1})$. By Assumption 1 and 3, we know that outside $(\bar{x} - H,\ \bar{x})$ the $v(\cdot,\ \bar{x})$ behaves like the $u(\cdot)$, which implies that, since the $u'(\cdot)$ is everywhere decreasing, $v_1(\bar{x} - H, \bar{x}) > \lim_{x \to \bar{x}^-} v_1(x, \bar{x})$. For $x_{i,T+1} < x_{j,T+1}$ we have that $\beta v'(x_{j,T+1}) < \beta w^* v'(x_{i,T+1})$ which implies that $u'(\cdot)$ is increasing between $x_T - x_{j,T+1}$ and $x_T - x_{i,T+1}$ which violates Assumption 1, which is a contradiction. □

Lemma 2. *If $u = v$ and u is CRRA, the growth rate of the economy is unique and invariant. At the same time, there exists a strictly concave u such that this is not true.*

Proof. Consider the first order condition for (2) imposing $u(\cdot) = v(\cdot)$ (internal solution is guarantee by Assumption 1) and substitute $x_{it+1} = \psi_{it} x_{it}$, we get:

$$-u'\left(x_{it}\left(1 - \frac{\psi_{it}}{w^*}\right)\right) + \beta w^* u'(x_{it}\psi_{it}) = 0 \tag{7}$$

applying the implicit function theorem we get:

$$\frac{d\psi_{it}}{dx_{it}} = -\frac{-\left(1 - \frac{\psi_{it}}{w^*}\right) u''\left(x_{it}\left(1 - \frac{\psi_{it}}{w^*}\right)\right) + \beta w^* u''(x_{it}\psi_{it})\psi_{it}}{\left(\frac{x_{it}}{w^*}\right) u''\left(x_{it}\left(1 - \frac{\psi_{it}}{w^*}\right)\right) + \beta w^* u''(x_{it}\psi_{it}) x_{it}} \tag{8}$$

where the sign is not *a priori* guaranteed and is clearly dependent on the expression at numerator.

Substituting for $u(x) = \frac{c^{1-\theta}}{1-\theta}$ it can be easily checked that the growth rate is unique and invariant to the distribution.

□

B. Proofs

Proof of Proposition 1

To prove both points consider the household problem 1, and notice that the FOC are

$$\lim_{x' \to \bar{x}_{it+1}^-} \beta w^* v'(x') \geq u'\left(x_{it} - \frac{x_{it+1}}{w^*}\right) \geq \lim_{x' \to \bar{x}_{it+1}^+} \beta w^* v'(x')$$

.

Notice that if no $x \in (0, \bar{x}_{it+1})$ satisfy them, the claim is trivially true. Otherwise, define x^α the highest $x \in (0, \bar{x}_{it+1})$ that satisfies the FOC. (The existence of x^α is guaranteed by standard arguments.) By Assumption 3, for all $\epsilon > 0$ we must have $-u'(x - x^\alpha/w^* - \epsilon/w^*) + \beta w^* v'(x^\alpha + \epsilon) > 0$ and $-u'(x - x^\alpha/w^* + \epsilon/w^*) + \beta w^* v'(x^\alpha - \epsilon) < 0$, which implies that x^α cannot be a maximum since it fails the second order conditions (recall that since $x < \bar{x}_{it+1}$, the function is differentiable at x^α). In turns, this implies that $\bar{x}_{it+1}$ is an optimal solution for all initial endowments x such that $u'(x - \bar{x}_{it+1}/w^*) \in [\lim_{x' \to \bar{x}_{it+1}^+} \beta w^* v'(x'),\ \lim_{I' \to \bar{I}^-} \beta w^* v'(x')]$.

Q.E.D.

Proof of Theorem 1

We start by noticing that, from period 1 onwards, the distribution of endowments must be either of perfect equality, or admit a missing class below the mean. To see why, consider any initial distribution μ_0, and notice that by Proposition 1 we know that there exists some $\gamma > 0$ such that $\mu((\mathbb{E}[\mu_1]_1 - \gamma, \mathbb{E}[\mu_1]_1)) = 0$. This implies that, if $\mu_1([0, \mathbb{E}[\mu_1])) > 0$, then $\mu_1([0, \mathbb{E}[\mu_1] - \gamma)) > 0$. Notice also that, by construction, we must have that $\mu_1([\mathbb{E}[\mu_1], +\infty)) > 0$. Therefore, if $\mu_1([0, \mathbb{E}[\mu_1])) > 0$, then the distribution admits a missing class below the mean. Conversely, if $\mu_1([0, \mathbb{E}[\mu_1])) = 0$, then we must have $\mu_1(\{\mathbb{E}[\mu_1]\}) = 1$ (every distribution with a support above its average must be degenerate). We have therefore proved that μ_1 can be either of perfect equality, or admit a missing class below the mean. An identical argument shows that the same would hold true for all μ_t for all t.

We are only left to show that there exists some $\bar{T}$ from which the distribution is either of perfect equality, or with a missing class below the mean. Notice that, if μ_t is of perfect equality, so will be μ_{t+1}. Therefore if there exists some T such that μ_T is of perfect equality, then we can set $\bar{T} = T$. Otherwise, if such T does not exist, we can set $\bar{T} = 1$, since we have proved that μ_1 has a missing class below the mean if it is not of perfect equality. *Q.E.D.*

Proof of Proposition 2

1) Consider some non-degenerate μ_t, and notice that if μ_t is not continuous at its mean, then it must have a missing class below the mean. Otherwise, consider some μ_0 which is continuous around its mean , and consider the equilibrium in which each i at time t chooses $\max_{I'_{t+1}} \left\{ I'_{t+1} \in \arg\max_{I_{t+1}} u(x_0 - I_{t+1}) + \beta v(w_t \; I_{t+1} | w_t \; I^*_{t+1}) \right\}$. Then there exists some x_{it} such that $\lim_{I' \to \bar{I}^-} \beta w^* v'(w^* I', w^* I') = u'(x_{it} - \bar{I})$. Notice that we must have that x_{it} is in the interior of the support of μ_t, otherwise this would violate Assumption 4 (reference-points are equal the average income).

This x_{it} should not stay on the lower bound of the support because it will violates Assumption 4. But then $\forall x_{jt} < x_{it}$, household j will not reach her reference-point, and by Proposition 1 the distribution will have a missing class below the mean. Since t has been chosen arbitrarily, this proves the first part of the proposition.

2) By Proposition 1, we know that there exists an interval $S \subseteq \mathbb{R}_+$ such that $\forall x' \in S$, $\bar{x} = \phi(x', \bar{x})$. This clearly implies that if $\text{supp}(\mu_0) \subseteq S$, then μ_1 is of perfect equality. By Lemma 1 we also know that it will remain in perfect equality for all t. We can define $\delta > 0$ implicitly as any $\delta > 0$ such that

$$\frac{\min\limits_{x \in \text{supp}(\mu_0)} x}{\mathbb{E}(\mu_0)} \geq \delta \quad \Rightarrow \quad \text{Supp}(\mu_0) \subseteq S.$$

Q.E.D.

Proof of Theorem 2 and 3

Consider four economies $E_0 \in \mathcal{E}_O$, $E_1 \in \mathcal{E}_1, E_2 \in \mathcal{E}_2$, and $E_3 \in \mathcal{E}_3$ that have the same average initial endowment. Call e_0 the unique equilibrium of E_0, and $x_t^{e_0}$, ..., the average endowments in e_0 at period t. Then, the following holds.

Claim 1. For all $t > 0$, there exists a compact, convex, and positive-measure set S_t such that for all equilibria e of E_1, $\mathbb{E}[\mu_t^e] \in S_t$. Moreover, for all $t > 0$ we have $x_t^{e_0} \in S_t$ and $x_t^{e_0} \leq x'_t, \; \forall \; x' \; \in \; S_t$.

Proof. Notice first that in any equilibrium of E_1 the distribution must remain of perfect equality in all periods by Lemma 1. Then, in every period every household must choose a consumption exactly equal to her reference-point, which implies that in all equilibria we must have $x_{i,t} = \phi(x_{i,t-1}, x_{i,t})$ for all $i \in L$, for all t. In turns, this means that we have $\bar{x}_{t+1}$ such that $\lim_{x' \to \bar{x}_{t+1}^-} \beta w^* v'(x') \geq u'(x - \bar{x}_{t+1}/w^*) \geq \lim_{x' \to \bar{x}_{t+1}^+} \beta w^* v'(x')$. Since the two limits are finite, by Assumption 1 E_1 has a set of equilibrium solutions such that at every $t > 0$ the optimal x_t^* belong to a compact convex set S_t. In turns, this implies that S_t is a set of positive measure for all t.

Notice that $x_t^{e_0}$ satisfies first order conditions as an interior point by Assumption 1. Consider now $x'_t \; \in \; S_t$. Assume that $x'_t < x_t^{e_0}$. By Assumption 4, the reference-point should be equal to x'_t. Since by Assumption 3 the representative agents in E^0 and E^1

have the same utility outside an interval the highest point of which is the reference-point, then between x'_t and $x_t^{e_0}$ the $u'(\cdot)$ must be either constant or increasing and decreasing. But this contradicts Assumption 1.

We are left to show that $x_t^{e_0} \in S_t$. Assume by contradiction that the $x_t^{\min}$ defined as $u'(x_{t-1} - x_t^{\min}/w^*) = \lim_{x \to x_t^{\min -}} \beta w^* v'(x)$ is such that $x_t^{\min} > x_t^{e_0^*}$. This implies $u'(x_{t-1} - x_t^{e_0^*}/w^*) < \beta w^* v'(x_t^{e_0^*})$ but since $u' = v'$ above the aspiration level, $x_t^{\min}$ is a stationary point of the problem without reference-dependence, which implies a violation of Assumption 1. □

Notice that Claim 1 implies $E_1 \trianglerighteq_{\max} E_0$. This, together with the observation above that S_t has positive measure for all t, implies that $E_1 \triangleright_{\max} E_0$. (The reason is, E_1 has multiple equilibria with different growth, and all of them have a growth weakly above E_0; but then, there must exist an equilibrium with a growth strictly above E_0.) In turns, Claim 1 also proves points (1) and (2) of Theorem 3.

We will now show that $E_1 \triangleright_{\max} E$ for all $E \in \mathcal{E}_2 \cup \mathcal{E}_3$. Denote by e_1 and e the equilibria of maximum expansion of E_1 and E, respectively. Also, denote e_t the equilibrium of maximum expansion of an economy which in period t has the same average endowment as e, but has perfect equality at time t, i.e. an economy of type $\mathcal{E}_1$ such that $\mu_t^{e_t}(\mathbb{E}[\mu_t^e]) = 1$. We will first of all show that e_t has an average consumption that grows strictly more than e between time t and time $t+1$ for all t such that μ_t^e is not of perfect equality. (If μ_t^e is of perfect equality then e and e_t coincide.). To see why, consider first the case in which μ_{t+1}^e is of perfect equality. By definition of maximum rate of expansion, $\mathbb{E}[\mu_{t+1}^{e_t}] = \max_{x'}\{x' \in \arg\max_y u(\mathbb{E}[\mu_t^{e_t}] - y/w^*) + \beta v(y,y)\}$. We claim that $\mathbb{E}[\mu_{t+1}^e] \leq \max_{x'}\{x' \in \arg\max_y u(E[\mu_t^e] - y/w^*) + \beta v(y,y)\}$ where the inequality is strict if μ_t^e is not of perfect equality. If this were not the case, there would exist $i \in L$ such that $x_{i,t+1}^e = \max_{x'}\{x' \in \arg\max_y u(x_{i,t} - y/w^*) + \beta v(y,y)\}$. This, however, would mean that then the distribution is not of perfect equality, which is a contradiction. Then $x_{t+1}^e < \max_{x'}\{x' \in \arg\max_y u(E[\mu_t^e] - y/w^*) + \beta v(y,y)\}$, proving that if μ_{t+1}^e is of perfect equality then e_t has an average consumption that grows strictly more than e.

Consider now the case in which μ_{t+1}^e is not of perfect equality.

Fix some $t \geq 0$, and consider $\bar{x} \in \mathbb{R}_{++}$ such that $\mathbb{E}[\mu_{t+1}^e] = \max_{x'}\{x' \in \arg\max_y u(\bar{x} - y/w^*) + \beta v(y,y)\}$. This $\bar{x}$ should be strictly greater than the lower bound of the support and strictly less than the higher bound in any distribution with a missing class below the mean.

Claim 2. $\bar{x} < \mathbb{E}[\mu_t^e]$

Proof. By contradiction, assume $\bar{x} \geq \mathbb{E}[\mu_t^e]$. Then by definition of $\bar{x}$, all the income strictly lower should determine failure to reach their reference-point, i.e. $\forall j \in L$ such that $x_{jt} < \bar{x}$ we have $x_{j,t+1} < \mathbb{E}[\mu_{t+1}^e]$. Consider now $k \in L$ such that $x_{kt} > \bar{x}$, and notice that either $x_{k,t+1} = \mathbb{E}[\mu_{t+1}^e]$, or $x_{k,t+1} = \gamma(x_{kt})x_{kt} > \mathbb{E}[\mu_{t+1}^e]$, where the growth rate $\gamma(x_{kt})$ is defined by $-u'\left(x - \frac{x\gamma(x)}{w^*}\right) + \beta w^* v'(x\gamma(x)|x\gamma') = 0$. (Notice that this last expression is twice differentiable.) Then, notice that we must have

$$\frac{d\gamma(x)}{dx} = \frac{\beta w^* \frac{dv'(x\gamma(x)|x\gamma')}{dx\gamma'}\gamma'}{-\left[-u''\left(x - \frac{x\gamma(x)}{w^*}\right)\frac{x}{w^*} + \beta w^* v'(x\gamma(x)|x\gamma')x\right]} \leq 0. \tag{9}$$

Call $\gamma^* = \frac{E[\mu^e_{t+1}]}{\bar{x}}$, we have:

$$\mathbb{E}[\mu^e_{t+1}] = \int_{j\in L \ | \ x_{jt}<\bar{x}} x_{jt+1} d\mu_{t+1} \ + \ \int_{k\in L \ | \ x_{kt}\geq\bar{x}} x_{kt+1} d\mu_{t+1} =$$

$$< \gamma^* \int_{j\in L \ | \ x_{jt}<\bar{x}} x_{jt} d\mu_t + \ \gamma^* \int_{k\in L \ | \ x_{kt}\geq\bar{x}} x_{kt} d\mu_t = \gamma^* \mathbb{E}[\mu^e_t]$$

where the inequality is determined by Proposition 1 and Equation (9). But then $\mathbb{E}[\mu^e_{t+1}] < \gamma^* E[\mu^e_t]$ and $\mathbb{E}[\mu^e_{t+1}] = \gamma^* \bar{x}$ which is a contradiction since $\bar{x} \geq \mathbb{E}[\mu^e_t]$. This completes the proof. □

Since $\bar{x} < \mathbb{E}[\mu^e_t]$, then we have that $\mathbb{E}[\mu^e_{t+1}] = \max_{x'}\{x' \in \arg\max_y u(\bar{x} - y/w^*) + \beta v(y, y)\} < \max_{x'}\{x' \in \arg\max_y u(\mathbb{E}[\mu^e_t] - y/w^*) + \beta v(y, y)\} = \mathbb{E}[\mu^{e_t}_t]$, proving that also if μ^e_{t+1} is not of perfect equality, then e_t has an average consumption that grows strictly more than e.

We have therefore showed that e_t has an average consumption that grows strictly more than e between time t and time $t+1$ for all t such that μ^e_t is not of perfect equality. Observe also that if we take two economies $E'_1, E''_1 \in \mathcal{E}_1$ such that the initial endowment of E'_1 is strictly higher than that of E'_2, then we have $E'_1 \rhd_{\max} E''_1$. These two observations jointly imply $E_1 \rhd_{\max} E$ for all $E \in \mathcal{E}_2 \cup \mathcal{E}_3$.

We are left to analyze the case in which u is CRRA.

Claim 3. If u satisfies CRRA, then $E_3 \trianglerighteq E_0$ and $E_2 \rhd E_0$.

Proof. Notice first of all that E_0 has a unique equilibrium E_0, and that if u exhibits CRRA, then the growth rate of E_0 is constant, and use λ to define it. We need to prove that e never grows less than λ. Call λ^* the growth rate of E_0 the first period, and divide the population L into four groups:

- A: Subjects whose initial endowments is above or equal the average endowment;
- B: Subjects whose initial endowments is below the average endowment, but who meet their reference-point in the second period, i.e. $\phi(x_{i,0}, x_0(1+\hat{\lambda})) = x_0(1+\hat{\lambda})$;
- C: Subjects whose initial endowments is below the average endowment, and who do not meet their reference-point in the second period but have a second period consumption in a point where v is convex, i.e. $\phi(x_{i,0}, x_0(1+\hat{\lambda})) \in [x_0(1+\hat{\lambda} - \gamma, x_0(1+\hat{\lambda})$;

D: Subjects whose initial endowments is below the average endowment, and who do not meet their reference-point in the second period but have a second period consumption in a point where v is concave, i.e. $\phi(x_{i,0}, x_0(1+\hat{\lambda})) < x_0(1+\hat{\lambda}) - \gamma$;

(Some of the groups above might be empty.) Define by λ_i^* the growth rate of the average consumption of each of the groups above, for $i = A, B, C, D$. Notice first of all that we must have $\lambda_A^* \geq \lambda$ Notice that every subjects in either group would increase her consumption of exactly λ if she had $v = u$ instead of being reference dependent. Now, if $\lambda^* \leq \lambda$, then if subjects of group (a) increased their consumption of λ, they would remain strictly above their reference-point: but since above the reference-point $u' = v'$, increasing the consumption of λ must remain optimal for them. Therefore, $\lambda_A^* \geq \lambda$. Consider now subjects in B, and notice that they have an initial endowment below the average endowment, but a second period consumption exactly equal to the mean. This means that subjects of group B must have $\lambda_B^* > \lambda^*$. Consider now subjects in group C: were they not reference-dependent, they would increase their consumption of λ. And, their second-period consumption must lead is in an area in which the second period utility v is convex. But exactly since v is convex, they consume more in the second period then they would have if they were not reference-dependent – v coincides with u until a point after which is goes *above* u and becomes convex. Therefore, we must have $\lambda_C^* > \lambda$. Finally, consider the subjects in group D. Notice that, among them, we cannot have subjects such that, if they increased their consumption of λ, they would have a second period consumption in a point where v is convex, i.e. $x_{i,0}(1+\lambda) \in [x_0(1+\hat{\lambda}-\gamma, x_0(1+\hat{\lambda})$, but that instead increase it less, so that $\phi(x_{i,0}, x_0(1+\hat{\lambda})) < x_0(1+\hat{\lambda}) - \gamma$. The reason, just like subjects in group C, if increasing the consumption of λ were optimal with no reference-dependence, it is even more so now, with reference-dependence, since v is strictly above u from $x_0(1+\hat{\lambda}) - \gamma$ on. Therefore, the only subjects in group D must be those for whom $x_{i,0}(1+\lambda) < x_0(1+\hat{\lambda} - \gamma$. But since v coincides with u before $x_0(1+\hat{\lambda} - \gamma$, then these subjects must increase their consumption of at least λ.

We have just proved that: if $\lambda^* \leq \lambda$ then $\lambda_A^* \geq \lambda$; $\lambda_B^* > \lambda^*$; $\lambda_C^* \geq \lambda$; $\lambda_D^* \geq \lambda$. Clearly this implies that we cannot have $\lambda^* < \lambda$, hence $\lambda^* \geq \lambda$. Notice, moreover, that if group B were not empty, this would imply $\lambda^* > \lambda$.

The argument above must hold true for all periods, i.e. the growth rate of e must be above λ for all periods. This means $E_3 \trianglerighteq E_0$ and $E_2 \trianglerighteq E_0$. We are left to show that $E_2 \triangleright E_0$. To see why, notice that E_2 is characterized by the fact that, at some period t, group B above must be non-empty – the distribution must become of perfect equality, which implies that is a period in which some subjects 'jump' from being below to being at the average consumption. We have already argued that this implies that the growth rate must then be strictly above λ, proving the claim. □

Finally, notice that Claim 3 together with (2) imply (4) and (6) of Theorem 3. *Q.E.D.*

References

Aghion, P., E. Caroli, and C. García-Peñalosa (1999): "Inequality and Economic Growth: The Perspective of the New Growth Theories," *Journal of Economic Literature*, 37, 1615–1660.

Appadurai, A. (2005): "The Capacity to Aspire," in *Culture and Public Action*, ed. by V. Rao and M. Walton, The World Bank. Washigton DC.

Banerjee, A. (1990): "Envy," in *Economic Theory and Policy: Essays in Honour of Dipak Banerjee*, ed. by B. Dutta, S. Gangopadhyay, D. Mukherjee, and D. Ray, Bombay: Oxford University Bombay: Oxford University Press.

Barberis, N. and R. Thaler (2003): "A survey of behavioral finance," in *Handbook of the Economics of Finance*, ed. by G. M. Constantinides, M. Harris, and R. Stultz, Elsevier BV.

Bourguignon, F. (1981): "Pareto Superiority of Unegalitarian Equilibria in Stiglitz' Model of Wealth Distribution with Convex Saving Function," *Econometrica*, 49, 1469–1475.

Bowman, D., D. Minehart, and M. Rabin (1999): "Loss aversion in a consumption–savings model," *Journal of Economic Behavior and Organization*, 38, 155–178.

Camerer, C. (1995): "Individual decision making," in *Handbook of Experimental Economics*, ed. by A. R. J. Kagel, Princeton Unviersity Press, Princeton, vol. 2.

——— (2000): "Prospect Theory into the WIld: Evidence from the Field," in *Choices, values and frames*, ed. by D. Kahneman and A. Tversky, Cambridge University Press.

Chateauneuf, A. and P. Wakker (1999): "An Axiomatization of Cumulative Prospect Theory for Decision Under Risk," *Journal of Risk and Uncertainty*, 18, 137–145.

Cooper, B., C. García-Penalosa, and P. Funk (2001): "Status effects and negative utility growth," *Economic Journal*, 642–665.

Corneo, G. and O. Jeanne (2001): "Status, the distribution of wealth, and growth," *The Scandinavian Journal of Economics*, 283–293.

Dalton, P., S. Ghosal, and A. Mani (2010): "Poverty and Aspirations Failure," CAGE Online Working Paper Series 22.

Diecidue, E. and J. Van de Ven (2008): "Aspiration level, probability of success and failure, and expected utility," *International Economic Review*, 49, 683–700.

Fershtman, C., K. Murphy, and Y. Weiss (1996): "Social status, education, and growth," *Journal of Political Economy*, 108–132.

Foellmi, R., R. Rosenblatt-Wisch, and K. Schenk-Hoppé (2011): "Consumption paths under prospect utility in an optimal growth model," *Journal of Economic Dynamics and Control*, 35, 273–281.

Genicot, G. and D. Ray (2009): "Aspirations, Inequality, Investment and Mobility," *Georgetown University and New York University, mimeo.*

Hopkins, E. and T. Kornienko (2006): "Inequality and Growth in the Presence of Competition for Status," *Economics Letters*, 93, 291–296.

Kahneman, D. and A. Tversky (1979): "Prospect theory: an analysis of choice under risk," *Econometrica*, 47, 263–291.

Konrad, K. (1992): "Wealth seeking reconsidered," *Journal of Economic Behavior and Organization*, 18, 215–27.

Koszegi, B. (2010): "Utility from Anticipation and Personal Equilibrium," *Economic Theory*, 44, 415–444.

Köszegi, B. and M. Rabin (2006): "A Model of Reference-Dependent Preferences," *Quarterly Journal of Economics*, 121, 1133–1165.

Koszegi, B. and M. Rabin (2009): "Reference-dependent consumption plans," *American Economic Review*, 99, 909–936.

Kyle, A. S., H. Ou-Yang, and W. Xiong (2006): "Prospect Theory and Liquidation Decisions," *Journal of Economic Theory*, 129, 273–288.

Loewenstein, G. and D. Prelec (1992): "Anomalies in Intertemporal Choice: Evidence and Interpretation," *Quarterly Journal of Economics*, 107, 573–597.

Masatlioglu, Y. and E. A. Ok (2005): "Rational Choice with status quo bias," *Journal of Economic Theory*, 121, 1–29.

——— (2008): "A Canonical Choice Model with Initial Endowment," Mimeo, New York University.

Mookherjee, D., S. Napel, and D. Ray (2010): "Aspirations, Segregation and Occupational Choice," *Journal of the European Economic Association*, 8, 139–168.

Ok, E. A., P. Ortoleva, and G. Riella (2011): "Revealed (P)Reference Theory," Mimeo, New York University.

Ortoleva, P. (2010): "Status Quo Bias, Multiple Priors and Uncertainty Aversion," *Games and Economic Behavior*, 69, 411–424.

Rabin, M. (2002): "Inference by believers in the law of small numbers," *Quarterly Journal of Economics*, 117, 775–816.

Rauscher, M. (1997): "Conspicuous consumption, economic growth, and taxation," *Journal of Economics*, 66, 35–42.

Ray, D. (2006): "Aspirations, Poverty and Economic Change," *In Understanding Poverty (eds. A. Banerjee, R. Bénabou and D. Mookherjee)*, New York: Oxford University Press.

Samuelson, W. and R. Zeckhauser (1988): "Status quo bias in decision making," *Journal of Risk and Uncertainty*, 1, 7–59.

Spiegler, R. (2011): *Bounded rationality and industrial organization*, Oxford University Press.

Stark, O. (2006): "Status Aspirations, Wealth Inequality, and Economic Growth," *Review of Development Economics*, 10, 171–176.

Stiglitz, J. (1969): "Distribution of Income and Wealth Among Individuals," *Econometrica*, 37, 382–397.

Sunstein, C., ed. (2003): *Behavioral Law and Economics*, Cambridge University Press.

Thaler, R., ed. (1993): *Advances in behavioral finance*, Russell Sage Foundation.

Tversky, A. and D. Kahneman (1974): "Judgment under uncertainty: Heuristics and biases," *Science*, 185, 1124–1131.

——— (1981): "The framing of decisions and the psychology of choice," *Science*, 211, 453–458.

——— (1991): "Loss Aversion in Riskless Choice: A Reference-Dependent Model," *Quarterly Journal of Economics*, 106, 1039–1061.

——— (1992): "Advances in prospect theory: cumulative representation of uncertainty," *Journal of Risk and Uncertainty*, 5, 297–323.

Wakker, P. (2010): *Prospect Theory for Risk and Ambiguity*, Cambridge University Press.