Optimal Cartel Output in Two-Sided Markets

Federico Boffa[*] Lapo Filistrucchi[†]

May 16, 2011

Abstract

This paper studies what determines the profit maximizing output of a cartel in two-sided markets. Two differentiated firms catering to two markets linked by external effects engage in a supergame. We find conditions under which above monopoly prices (respectively, below monopoly quantities) may prevail on the market as a means to enhance the sustainability of the cartel.

JEL: L13, L40

1 Introduction and purpose of the paper

This paper studies the optimal collusive pricing in two sided markets.

We analyze a two sided market model that can easily fit the media market. Two firms produce two different goods (we will refer to newspapers in the rest of the paper, but they could be equally thought of as tv channels). They address two different markets: readers and advertizers.

Readers derive utility from reading one of the two newspapers (or possibly both), which they perceive as differentiated products. Their utility is negatively affected by the amount of advertising prevailing in the newspaper.

Advertizers' utility is positively correlated to the number of readers of each of the two newspapers.

Our paper studies how the two-sided interaction between the firms affect the prevailing collusive price in the market.

Indeed, it is well known that, when firms compete in quantities or in price under product differentiation, there may be cases in which, while joint profit maximization cannot be sustained as not incentive-compatible, firms may still settle for a collusive outcome yielding them a higher than competitive price.

In a regular one-sided market, under linear demand, when products are substitute, the Pareto-optimal (from the firms' standpoint) collusive outcome may prescribe an aggregate output above the level resulting from joint profit maximization, both under Cournot and under Bertrand competition. On the

[*]University of Macerata, Department of Legal and Economic Studies.

[†]Tilburg University & University of Florence

other hand, when products are complement, the Pareto-optimal (from the firms' standpoint) collusive outcome may prescribe an aggregate output below the level resulting from joint profit-maximization.

When the analysis turns to two sided markets, the interplay may generate interesting departures from the single side case.

In two sided markets, the two products are substitutes on each side: an increase a firm's number of readers decreases the rival's profit on the reader's side; analogously, an increase in a firm's advertising quantity decreases, ceteris paribus, the rival's advertising quantity.

However, the network externalities between the two sides may generate a cross-side complementarity. For instance, an increase in a firm's readership may end up increasing the rival's profit on the advertizing market.

The combination of direct and network effects shape the collusive optimal price structure, whose analysis is the focus of our paper.

In particular, we find that, under Cournot competition, cartel stability may require, under a condition of particularly strong indirect network externality, producing a lower-than-monopoly newspaper output in the collusive arrangement. This, of course, is particularly detrimental to welfare, even more so as it directly affects readers. Analogously, under Bertrand competition, it may be optimal to price above monopoly on the advertizing side. While the consequences of this on welfare are less clear-cut, it may still be regarded as an interesting result.

2 Literature review

This paper relates to the two strands of literature of collusion and of two sided markets. While the literature extensively analyzes the effects of multi-market contacts on collusive behavior (see, for instance, Bernheim & Whinston, 1990), the analysis of the incentives for collusion with the demand externality typical of two-sided markets has been so far very limited. The only paper we know of on this issue is by Ruhmer (2010), who analyzes how indirect network effects affect the stability of a cartel. She finds that there exists a treshold level of indirect network effects above which collusion becomes harder to sustain as indirect network effects get stronger (while below it, an increase in the network effect increases cartel's stability). This results from the combination of two facts associated to larger indirect network externalities. First, Nash profit declines as competition for each side of the market intensifies, while collusive profits increase due to the internalization of the network effect; second, as the collusive profit increases, the chiselling profit surges as well with larger network effects.

A firm in a two-sided market acts as a platform: sells two products or services to two distinct group of consumers when the demand from one type of consumers depends on the demand from the other type of consumers and vice versa. A firm in a two-sided market is therefore a special case of a multi-product firm in that it faces and recognises the existence of such indirect network effects between demands for its products.

The literature on collusion with differentiated goods (see Deneckere, 1983, Ross, 1992) does not show clear results in terms of how different degrees of product substitutability/complementarity contribute to the stabilization of a cartel.

Although many examples of two-sided markets where the network effects are both positive (e.g. the market for yellow pages or the payment cards' market, see below) give the impression that a two-sided market is not different from a market in which firms sell complementary products, there is in fact a substantial difference: when complement products are sold the buyers of the two products usually buy both, e.g. consider the market for inkjet printers and that for ink cartridges. Unless the consumers are particularly naives, they can be expected to consider the price of both goods as they choose to buy one. They therefore internalize the network effect. As in a two-sided market the buyers buy only one of the two products, the network effect is in fact not internalized by them and is in fact a network externalities.

The sign and size of this indirect network effects, along with the price elasticities, has been shown in the literature to be one of the main determinants of the pricing decisions of firms acting in two-sided markets (see Caillaud & Jullien, 2003; Parker & Van Alstyne,2005; Armstrong, 2006; Rochet & Tirole, 2006). In particular, the literature has distinguished between the price level (i.e. roughly the sum of the prices charged on the two-sides) and the price structure (i.e. roughly the ratio between the two prices). Rochet & Tirole (2006) go as far as defining two-sided markets as markets where not only the price level but also the price structure matters. One can therefore expect that it will be the price structure that will play the key role in explaining the difference between incentives to collude by multi-product firms in general versus two-sided markets in particular.

In addition the literature that focused on competition policy in two-sided markets has pointed out for instance that, unlike the price level, due to the presence of indirect network externalities, the efficient price structure does not reflect the ratio of marginal costs on the two sides of the market and, more generally, that increased competition does not necessarily lead to a more balanced price structure nor to a more efficient one (see Evans,2003, Wright, 2004, and Evans & Schmalensee, 2005),

As it can be envisaged that the two indirect network effects are crucial for the identification of the incentives to collude, we start by comparing the collusive potential under a standard market situation and under two-sided markets assuming the simplest two-sided market, i.e. the one with only one network effect between the demand on the two-sides of the market and the simplest collusive framework, that is, Bertrand with symmetric firms and without capacity constraints.

3 The model

Two firms, i and j, produce two differentiated goods, which we think of as newspapers (although we can regard them as different goods), which are sold to two different groups of customers, R (standing for readers), and A (standing for advertizers). Readers' demand for a newspaper depends negatively on its price, positively on the rival's price; it also depends negatively on its amount of ads, and positively on the rival's amount of ads. Advertizers' demand for a newspaper depends negatively on the newspaper's price for advertizing, and positively on the rival's advertizing price; also, it depends positively on the amount of readers, and negatively on the rival's amount of readers.

Firms engage in a repeated game, and collude whenever it is rational for them to do so. We assume that, after a deviation from the prescribed collusive outcome, firms revert to the Nash equilibrium outcome forever.

The demand system may be written as:

$$\begin{aligned}
q_i^R &= f_i^R\left(p_i^R, p_j^R, q_i^A, q_j^A\right)\\
q_j^R &= f_j^R\left(p_i^R, p_j^R, q_i^A, q_j^A\right)\\
q_i^A &= f_i^A\left(p_i^A, p_j^A, q_i^R, q_j^R\right)\\
q_j^A &= f_j^A\left(p_i^A, p_j^A, q_i^R, q_j^R\right)
\end{aligned}$$

In the basic version of the model, we assume the demand is symmetric.

As indicated in the discussion above, we impose a set of conditions on the demand function reflecting the media market. In particular, we require:

$$\begin{array}{ll}
\frac{\partial q_i^R}{\partial p_i^R} = \frac{\partial q_j^R}{\partial p_j^R} < 0 & \frac{\partial q_i^R}{\partial p_j^R} = \frac{\partial q_j^R}{\partial p_i^R} > 0\\
\frac{\partial q_i^R}{\partial q_i^A} = \frac{\partial q_j^R}{\partial q_j^A} \leq 0 & \frac{\partial q_i^R}{\partial q_j^A} = \frac{\partial q_j^R}{\partial q_i^A} \geq 0\\
\frac{\partial q_i^A}{\partial p_i^A} = \frac{\partial q_j^A}{\partial p_j^A} < 0 & \frac{\partial q_i^A}{\partial p_j^A} = \frac{\partial q_j^A}{\partial p_i^A} > 0\\
\frac{\partial q_i^A}{\partial q_i^R} = \frac{\partial q_j^A}{\partial q_j^R} \geq 0 & \frac{\partial q_i^A}{\partial q_j^R} = \frac{\partial q_j^A}{\partial q_i^R} \leq 0
\end{array}$$

We use a linear version of the demand function. The demand system then becomes:

$$\begin{aligned}
q_i^R &= R - \gamma^R p_i^R + \gamma_C^R p_j^R - \lambda^A q_i^A + \lambda_C^A q_j^A\\
q_j^R &= R - \gamma^R p_j^R + \gamma_C^R p_i^R - \lambda^A q_j^A + \lambda_C^A q_i^A\\
q_i^A &= A - \gamma^A p_i^A + \gamma_C^A p_j^A + \lambda^R q_i^R - \lambda_C^R q_j^R\\
q_j^A &= A - \gamma^A p_j^A + \gamma_C^A p_i^A + \lambda^R q_j^R - \lambda_C^R q_i^R
\end{aligned}$$

where γ denotes the effect of the changes in prices in a given market on output in that same market, and λ denotes the effects of changes in output in a given market on output in the other market. Superscripts R and A refer respectively to the readers and the advertizers markets.

Firms collude whenever it is rational for them to do so, that is, the profit from sticking to the collusive arrangement exceeds the deviation profit. The individual rationality constraint is the following:

$$\frac{\pi_l^{cls}}{1-\delta} \geq \pi_l^D + \frac{\delta}{1-\delta}\pi_l^{NE}$$

for $l = i, j$

where π^{cls} is the collusive profit, π^D is the profit accrued through an optimal one-time deviation from the outcome prescribed in the cartel, and finally π^{NE} is the Nash equilibrium profit.

3.1 Cournot case

Decomposing the IR constraint, we obtain that:

$$\frac{\pi_l^{cls}\left(q_l^{cls,R}, q_l^{cls,A}, q_{-l}^{cls,R}, q_{-l}^{cls,A}\right)}{1-\delta} \geq R_l\left(q_{-l}^{cls,R}, q_{-l}^{cls,A}\right) + \frac{\delta}{1-\delta}\frac{\pi_l^{cour}}{1-\delta}$$

The inverse (linear) demand may be written as:

$$\begin{aligned}
p_i^R &= \frac{R\left(\gamma^R + \gamma_C^R\right) - \gamma^R q_i^R - \gamma_C^R q_j^R - q_i^A\left(\gamma^R\lambda^A - \gamma_C^R\lambda_C^A\right) + q_j^A\left(\gamma^R\lambda_C^A - \gamma_C^R\lambda^A\right)}{\left(\gamma^R\right)^2 - \left(\gamma_C^R\right)^2} \\
p_j^R &= \frac{R\left(\gamma^R + \gamma_C^R\right) - \gamma_C^R q_i^R - \gamma^R q_j^R + q_i^A\left(\gamma^R\lambda_C^A - \gamma_C^R\lambda^A\right) - q_j^A\left(\gamma^R\lambda^A - \gamma_C^R\lambda_C^A\right)}{\left(\gamma^R\right)^2 - \left(\gamma_C^R\right)^2} \\
p_i^A &= \frac{A\left(\gamma^A + \gamma_C^A\right) - \gamma_C^A q_j^A - \gamma^A q_i^A + q_i^R\left(\lambda^R\gamma^A - \lambda_C^R\gamma_C^A\right) - q_j^R\left(\lambda_C^R\gamma^A - \lambda^R\gamma_C^A\right)}{\left(\gamma^A\right)^2 - \left(\gamma_C^A\right)^2} \\
p_j^A &= \frac{A\left(\gamma^A + \gamma_C^A\right) - \gamma^A q_j^A - \gamma_C^A q_i^A - q_i^R\left(\lambda_C^R\gamma^A - \lambda^R\gamma_C^A\right) + q_j^R\left(\lambda^R\gamma^A - \lambda_C^R\gamma_C^A\right)}{\left(\gamma^A\right)^2 - \left(\gamma_C^A\right)^2}
\end{aligned}$$

For expositional clarity, we rewrite the demand function in the following way:

$$\begin{aligned}
p_i^R &= \widehat{R} + \widehat{\gamma}^R q_i^R + \widehat{\gamma}_C^R q_j^R + \widehat{\lambda}^A q_i^A + \widehat{\lambda}_C^A q_j^A \\
p_j^R &= \widehat{R} + \widehat{\gamma}^R q_j^R + \widehat{\gamma}_C^R q_i^R + \widehat{\lambda}^A q_j^A + \widehat{\lambda}_C^A q_i^A \\
p_i^A &= \widehat{A} + \widehat{\gamma}^A q_i^A + \widehat{\gamma}_C^A q_j^A + \widehat{\lambda}^R q_i^R + \widehat{\lambda}_C^R q_j^R \\
p_j^A &= \widehat{A} + \widehat{\gamma}^A q_j^A + \widehat{\gamma}_C^A q_i^A + \widehat{\lambda}^R q_j^R + \widehat{\lambda}_C^R q_i^R
\end{aligned}$$

where the new parameters, with respect to our original parameters, are the following:

$$\widehat{\gamma}^R = \frac{-\gamma^R}{(\gamma^R)^2-\left(\gamma^R_C\right)^2} \qquad \widehat{\gamma}^R_C = \frac{-\gamma^R_C}{(\gamma^R)^2-\left(\gamma^R_C\right)^2}$$

$$\widehat{\gamma}^A_C = \frac{-\gamma^A_C}{(\gamma^A)^2-\left(\gamma^A_C\right)^2} \qquad \widehat{\gamma}^A = \frac{-\gamma^A}{(\gamma^A)^2-\left(\gamma^A_C\right)^2}$$

$$\widehat{\lambda}^A = \frac{-\left(\gamma^R\lambda^A-\gamma^R_C\lambda^A_C\right)}{(\gamma^R)^2-\left(\gamma^R_C\right)^2} \qquad \widehat{\lambda}^A_C = \frac{\left(\gamma^R\lambda^A_C-\gamma^R_C\lambda^A\right)}{(\gamma^R)^2-\left(\gamma^R_C\right)^2}$$

$$\widehat{\lambda}^R = \frac{\left(\lambda^R\gamma^A-\lambda^R_C\gamma^A_C\right)}{(\gamma^A)^2-\left(\gamma^A_C\right)^2} \qquad \widehat{\lambda}^R_C = \frac{-\left(\lambda^R_C\gamma^A-\lambda^R\gamma^A_C\right)}{(\gamma^A)^2-\left(\gamma^A_C\right)^2}$$

Assume that parameter values are such that a cartel is unable to coordinate on the aggregate monopoly output, under which each firm's IR constraint prescribes that it is optimal for the firm to deviate.

We now identify the optimal cartel quantities, that is, the output produced in the cartel that maximizes individual profit subject to the IC constaint holding.

To do this, we first identify the required direction of change of collusive quantities in each of the two markets for the cartel to be sustainable.

A change in the cartel quantities affects the equilibrium outcome in two separate ways: First, it changes the monopoly output; second, it affects the deviation profit. The continuation profit (i.e, the punishment profit accruing in every period after deviation) is unaffected by the collusive outcome, as we have assumed Cournot deviation.

The deviation profit can be thought of as being composed of the deviation profit on the readership's side, and the deviation profit on the advertizing side:

$$\pi^D = \pi^{D,R} + \pi^{D,A}$$

We first analyze how the deviation profit changes as the cartel reader's quantity increases $\frac{\partial \pi^D_l}{\partial q^R_{-l}}$. The derivative can be decomposed:

$$\frac{\partial \pi^D_l}{\partial q^R_{-l}} = \frac{\partial \pi^{D,R}_l}{\partial q^R_{-l}} + \frac{\partial \pi^{D,A}_l}{\partial q^R_{-l}}$$

By the envelope theorem, $\frac{\partial \pi^{D,R}_l}{\partial q^R_{-l}} = \frac{\partial P^R_l\left(q^R_l, q^R_{-l}, q^A_l, q^A_{-l}\right)}{\partial q^R_{-l}} q^R_l$, and $\frac{\partial \pi^{D,A}_l}{\partial q^R_{-l}} = \frac{\partial P^A_l\left(q^R_l, q^R_{-l}, q^A_l, q^A_{-l}\right)}{\partial q^R_{-l}} q^A_l$. The previous equation becomes:

$$\frac{\partial \pi^D_l}{\partial q^R_{-l}} = \frac{\partial P^R_l\left(q^R_l, q^R_{-l}, q^A_l, q^A_{-l}\right)}{\partial q^R_{-l}} q^R_l + \frac{\partial P^A_l\left(q^R_l, q^R_{-l}, q^A_l, q^A_{-l}\right)}{\partial q^R_{-l}} q^A_l$$

This can be rewritten as:

$$\frac{\partial \pi^D_l}{\partial q^R_{-l}} = \widehat{\gamma}^C_R q^R_l + \widehat{\lambda}^C_R q^A_l$$

Observe that $\widehat{\gamma}^C_R = \frac{-\gamma^R_C}{(\gamma^R)^2-\left(\gamma^R_C\right)^2} < 0$.

An increase in the rival's readership has two effects on the firm's profit from readership: First, it directly decreases the willingness to pay for the firm's newspaper, thereby decreasing its value; secondly, it indirectly reduces the value of advertizing in the firm's newspaper, thereby generating a reduction in the firm's advertizing price (given the fixed advertizing quantity). Overall, the effect of the rival's increase in readership on the firm's profit is negative.

On the other hand, $\widehat{\lambda}_R^C = \frac{-\left(\lambda_C^R \gamma^A - \lambda^R \gamma_C^A\right)}{(\gamma^A)^2 - \left(\gamma_C^A\right)^2}$ has an ambigous sign. An increase in the rival's readership may increase or decrease the firm's profit from advertizing. As a direct effect, more people reading the rival's newspaper increases the willingness to pay for the rival's advertizing services, thereby increasing its ad prices. This has an ambigous effect on the firm's profit from advertizing, depending on the term: $-\left(\lambda_C^R \gamma^A - \lambda^R \gamma_C^A\right)$. If this is negative, an increase in the rival's readership decreases the firm's profit from advertizing. If, on the other hand,

$$
\begin{aligned}
-\left(\lambda_C^R \gamma^A - \lambda^R \gamma_C^A\right) &> 0 \\
\frac{\gamma^A}{\gamma_C^A} &< \frac{\lambda^R}{\lambda_C^R}
\end{aligned}
$$

In this case, the differentiation in terms of the (positive) externality of the network effect exceeds the differentiation in terms of other characteristics.

Summing up, the effect of an increase in readership on the deviation profit is:

$$
\frac{\partial \pi_l^D}{\partial q_{-l}^R} = \widehat{\gamma}_R^C q_l^{D,R} + \widehat{\lambda}_R^C q_l^{D,A}
$$

On the other hand, the effect of an increase in the cartel's advertizing quantity on the deviation profit is specular. Indeed,

$$
\frac{\partial \pi_l^D}{\partial q_{-l}^A} = \widehat{\gamma}_A^C q_l^{D,A} + \widehat{\lambda}_A^C q_l^{D,R}
$$

where

$$
\begin{aligned}
\widehat{\gamma}_C^A &= \frac{-\gamma_C^A}{(\gamma^A)^2 - \left(\gamma_C^A\right)^2} \\
\widehat{\lambda}_C^A &= \frac{\left(\gamma^R \lambda_C^A - \gamma_C^R \lambda^A\right)}{(\gamma^R)^2 - \left(\gamma_C^R\right)^2}
\end{aligned}
$$

$\widehat{\lambda}_C^A > 0$ if $\left(\frac{\gamma^R}{\gamma_C^R} > \frac{\lambda^A}{\lambda_C^A}\right)$. In this case, the increase in the cartel advertizing quantity entails a surge in the deviation profit on the reader's side if the differentiation in other dimensions on the reader's side is larger than the differentiation in how advertizing impacts the reader's side.

Proposition 1 *In a quantity supergame, it may be profitable to produce a lower quantity than monopoly in one of the two sides. In particular, it may be profitable to produce below monopoly in the readers market if the deviation profit* *on the advertizing side increases with the collusive quanity of readers, and the advertizers market is sufficiently larger than the readers market. On the other hand, it may be profitable to produce below monopoly in the advertizers' markets if the deviation profit on the readers side increases with the collusive quantity of advertizers, and the readers market is sufficiently larger than the advertizers.*

Proof. The IR constraint may be written as:

$$\frac{\pi_l^{cls}\left(q_l^{cls,R}, q_l^{cls,A}, q_{-l}^{cls,R}, q_{-l}^{cls,A}\right)}{1-\delta} \geq R_l\left(q_{-l}^{cls,R}, q_{-l}^{cls,A}\right) + \frac{\delta}{1-\delta}\frac{\pi_l^{cour}}{1-\delta}$$

This may be decomposed into:

$$\frac{\pi_l^{cls}\left(q_l^{cls,R}, q_l^{cls,A}, q_{-l}^{cls,R}, q_{-l}^{cls,A}\right)}{1-\delta} \geq \pi_l^{D,R}\left(q_l^{cls,R}, q_l^{cls,A}, q_{-l}^{cls,R}, q_{-l}^{cls,A}\right) + \pi_l^{D,A}\left(q_l^{cls,R}, q_l^{cls,A}, q_{-l}^{cls,R}, q_{-l}^{cls,A}\right) + \frac{\delta}{1-}$$

$$\frac{\pi_l^{cls}\left(q_l^{cls,R}, q_l^{cls,A}, q_{-l}^{cls,R}, q_{-l}^{cls,A}\right)}{1-\delta} - \pi_l^{D,R}\left(q_l^{cls,R}, q_l^{cls,A}, q_{-l}^{cls,R}, q_{-l}^{cls,A}\right) - \pi_l^{D,A}\left(q_l^{cls,R}, q_l^{cls,A}, q_{-l}^{cls,R}, q_{-l}^{cls,A}\right) - \frac{\delta}{1-}$$

Suppose that the monopoly output $q_l^{cls,R} = q_{-l}^{cls,R} = \frac{Q^{monopoly,R}}{2}$; $q_{-l}^{cls,A} = q_l^{cls,A} = \frac{Q^{monopoly,A}}{2}$ cannot be sustained as an equilibrium.

By decreasing the individual newspaper sales $q_l^{cls,R}$ below the monopoly level, the constraint becomes:

$$\frac{\partial\left(\frac{\pi_l^{cls}}{1-\delta} - \pi_l^D - \frac{\delta}{1-\delta}\frac{\pi_l^{cour}}{1-\delta}\right)}{\partial q_{-l}^R} = \frac{\widehat{\gamma}_R^C q_l^{cls,R} - \widehat{\lambda}_R^C q_l^{cls,A}}{1-\delta} - \left(\widehat{\gamma}_R^C q_l^{D,R} + \widehat{\lambda}_R^C q_l^{D,A}\right)$$

If $\left(\widehat{\gamma}_R^C q_l^{D,R} + \widehat{\lambda}_R^C q_l^{D,A}\right) > 0$, and larger, in absolute value, than $\frac{\widehat{\gamma}_R^C q_l^{cls,R} - \widehat{\lambda}_R^C q_l^{cls,A}}{1-\delta}$, then $\frac{\partial\left(\frac{\pi_l^{cls}}{1-\delta} - \pi_l^D - \frac{\delta}{1-\delta}\frac{\pi_l^{cour}}{1-\delta}\right)}{\partial q_{-l}^R} < 0$, and a decrease in readership softens the collusive constraint, and therefore helps stabilizing the cartel. Otherwise, the cartel is stabilized by an increase in the collusive number of readers.

Analogously on the other side,

$$\frac{\partial\left(\frac{\pi_l^{cls}}{1-\delta} - \pi_l^D - \frac{\delta}{1-\delta}\frac{\pi_l^{cour}}{1-\delta}\right)}{\partial q_{-l}^A} = \frac{\widehat{\gamma}_A^C q_l^{cls,R} + \widehat{\lambda}_A^C q_l^{cls,A}}{1-\delta} - \left(\widehat{\gamma}_A^C q_l^{D,R} + \widehat{\lambda}_A^C q_l^{D,A}\right)$$

If $\left(\widehat{\gamma}_A^C q_l^{D,R} + \widehat{\lambda}_A^C q_l^{D,A}\right) > 0$ and larger, in absolute value, than $\frac{\widehat{\gamma}_A^C q_l^{cls,R} + \widehat{\lambda}_A^C q_l^{cls,A}}{1-\delta}$, then $\frac{\partial\left(\frac{\pi_l^{cls}}{1-\delta} - \pi_l^D - \frac{\delta}{1-\delta}\frac{\pi_l^{cour}}{1-\delta}\right)}{\partial q_{-l}^A} < 0$, and a decrease in advertizing quantity softens the collusive constraint, and therefore helps stabilizing the cartel. Otherwise, the cartel is stabilized by an increase in the collusive number of advertizements.

■

4 Bertrand case

$$
\begin{aligned}
q_i^A &= f_i^A\left(p_i^A, p_j^A, q_i^R, q_j^R\right) \\
q_j^A &= f_j^A\left(p_i^A, p_j^A, q_i^R, q_j^R\right) \\
q_i^R &= f_i^R\left(p_i^R, p_j^R, q_i^A, q_j^A\right) \\
q_j^R &= f_j^R\left(p_i^R, p_j^R, q_i^A, q_j^A\right)
\end{aligned}
$$

Linearizing the demand functions, we obtain:

$$
\begin{aligned}
q_i^R &= R - \gamma^R p_i^R + \gamma_C^R p_j^R - \lambda^A q_i^A + \lambda_C^A q_j^A \\
q_j^R &= R - \gamma^R p_j^R + \gamma_C^R p_i^R - \lambda^A q_j^A + \lambda_C^A q_i^A \\
q_i^A &= A - \gamma^A p_i^A + \gamma_C^A p_j^A + \lambda^R q_i^R - \lambda_C^R q_j^R \\
q_j^A &= A - \gamma^A p_j^A + \gamma_C^A p_i^A + \lambda^R q_j^R - \lambda_C^R q_i^R
\end{aligned}
$$

We can observe complementarities and substitutaibilities across products if we express the demand in terms of prices, as follows:

$$
\begin{aligned}
q_i^R &= \frac{\left(R + \lambda_C^A A - A\lambda^A + \lambda_C^R \left(\lambda_C^A\right)^2 A - \lambda_C^R A \left(\lambda^A\right)^2 + \lambda^R R \lambda_C^A + \lambda^R R \lambda^A + R\lambda_C^R \lambda_C^A + R\lambda_C^R \lambda^A + \lambda^R \left(\lambda_C^A\right)^2 \cdots\right.}{\cdots} \\
q_j^R &= \frac{\left(R + \lambda_C^A A - A\lambda^A + \lambda_C^R \left(\lambda_C^A\right)^2 A - \lambda_C^R A \left(\lambda^A\right)^2 + \lambda^R R \lambda_C^A + \lambda^R R \lambda^A + R\lambda_C^R \lambda_C^A + R\lambda_C^R \lambda^A + \lambda^R \left(\lambda_C^A\right)^2 \cdots\right.}{\cdots} \\
q_i^A &= \frac{\left(A - R\lambda_C^R + \lambda^R R + \lambda^R \lambda_C^A A + \lambda^R A \lambda^A + \lambda_C^R \lambda_C^A A + \lambda_C^R A \lambda^A + \left(\lambda^R\right)^2 R \lambda_C^A + \left(\lambda^R\right)^2 R \lambda^A - R \left(\lambda_C^R\right)^2 \cdots\right.}{\cdots} \\
q_j^A &= \frac{\left(A - R\lambda_C^R + \lambda^R R + \lambda^R \lambda_C^A A + \lambda^R A \lambda^A + \lambda_C^R \lambda_C^A A + \lambda_C^R A \lambda^A + \left(\lambda^R\right)^2 R \lambda_C^A + \left(\lambda^R\right)^2 R \lambda^A - R \left(\lambda_C^R\right)^2 \cdots\right.}{\cdots}
\end{aligned}
$$

Rewriting, we obtain:

$$
\begin{aligned}
q_i^R &= \widetilde{R} + p_i^R \widetilde{\gamma}^R + p_j^R \widetilde{\gamma}_C^R + p_i^A \widetilde{\lambda}^A + p_j^A \widetilde{\lambda}_C^A \\
q_j^R &= \widetilde{R} + p_j^R \widetilde{\gamma}^R + p_i^R \widetilde{\gamma}_C^R + p_j^A \widetilde{\lambda}^A + p_i^A \widetilde{\lambda}_C^A \\
q_i^A &= \widetilde{A} + p_i^A \widetilde{\gamma}^A + p_j^A \widetilde{\gamma}_C^A + p_i^R \widetilde{\lambda}^R + p_j^R \widetilde{\lambda}_C^R \\
q_j^A &= \widetilde{A} + p_j^A \widetilde{\gamma}^A + p_i^A \widetilde{\gamma}_C^A + p_j^R \widetilde{\lambda}^R + p_i^R \widetilde{\lambda}_C^R
\end{aligned}
$$

where:$\lambda_C^R\gamma^R\lambda_C^A$

$$\begin{aligned}
\widetilde{\gamma}^R &= \frac{\lambda_C^R\gamma_C^R\lambda^A - \lambda_C^R\gamma^R\lambda_C^A - \lambda^R\gamma^R\lambda^A + \lambda^R\gamma_C^R\lambda_C^A - \gamma^R}{D} \\
\widetilde{\gamma}_C^R &= \frac{\left(\gamma_C^R - \lambda^R\gamma^R\lambda_C^A + \lambda^R\gamma_C^R\lambda^A - \gamma^R\lambda_C^R\lambda^A + \lambda_C^R\gamma_C^R\lambda_C^A\right)}{D} \\
\widetilde{\lambda}^A &= \frac{\lambda_C^A\gamma_C^A + \lambda^A\gamma^A + \lambda_C^R\left(\lambda_C^A\right)^2\gamma_C^A - \lambda_C^R\left(\lambda^A\right)^2\gamma_C^A - \lambda^R\left(\lambda_C^A\right)^2\gamma^A + \lambda^R\left(\lambda^A\right)^2\gamma^A}{D} \\
\widetilde{\lambda}_C^A &= \frac{\lambda_C^R\left(\lambda^A\right)^2\gamma^A - \lambda_C^A\gamma^A - \lambda^A\gamma_C^A - \lambda_C^R\left(\lambda_C^A\right)^2\gamma^A + \lambda^R\left(\lambda_C^A\right)^2\gamma_C^A - \lambda^R\left(\lambda^A\right)^2\gamma_C^A}{D} \\
\widetilde{\gamma}^A &= \frac{\left(\lambda^R\lambda_C^A\gamma_C^A - \gamma^A - \lambda^R\lambda^A\gamma^A - \lambda_C^R\lambda_C^A\gamma^A + \lambda_C^R\lambda^A\gamma_C^A\right)}{D} \\
\widetilde{\gamma}_C^A &= \frac{\left(\gamma_C^A - \lambda^R\lambda_C^A\gamma^A + \lambda^R\lambda^A\gamma_C^A + \lambda_C^R\lambda_C^A\gamma_C^A - \lambda_C^R\lambda^A\gamma^A\right)}{D} \\
\widetilde{\lambda}^R &= \frac{\left(\lambda^R\right)^2\gamma_C^R\lambda_C^A - \lambda_C^R\gamma_C^R - \lambda^R\gamma^R - \left(\lambda_C^R\right)^2\gamma_C^R\lambda_C^A - \left(\lambda^R\right)^2\gamma^R\lambda^A + \gamma^R\left(\lambda_C^R\right)^2\lambda^A}{D} \\
\widetilde{\lambda}_C^R &= \frac{\lambda^R\gamma_C^R + \gamma^R\lambda_C^R - \left(\lambda_C^R\right)^2\gamma_C^R\lambda^A - \left(\lambda^R\right)^2\gamma^R\lambda_C^A + \left(\lambda^R\right)^2\gamma_C^R\lambda^A + \gamma^R\left(\lambda_C^R\right)^2\lambda_C^A}{D} \\
D &= \left(\lambda_C^R\right)^2\left(\lambda_C^A\right)^2 - \left(\lambda_C^R\right)^2\left(\lambda^A\right)^2 + 2\lambda_C^R\lambda_C^A - \left(\lambda^R\right)^2\left(\lambda_C^A\right)^2 + \left(\lambda^R\right)^2\left(\lambda^A\right)^2 + 2\lambda^R\lambda^A + 1
\end{aligned}$$

Observe first that:

$$\begin{aligned}
D &= \left(\lambda_C^R\right)^2\left(\lambda_C^A\right)^2 - \left(\lambda_C^R\right)^2\left(\lambda^A\right)^2 + 2\lambda_C^R\lambda_C^A - \left(\lambda^R\right)^2\left(\lambda_C^A\right)^2 + \left(\lambda^R\right)^2\left(\lambda^A\right)^2 + 2\lambda^R\lambda^A + 1 \\
&= \left(\left(\lambda^R\right)^2 - \left(\lambda_C^R\right)^2\right)\left(\left(\lambda^A\right)^2 - \left(\lambda_C^A\right)^2\right) + 2\lambda_C^R\lambda_C^A + 2\lambda^R\lambda^A + 1 > 0
\end{aligned}$$

due to the assumptions that $\left(\lambda^R\right)^2 > \left(\lambda_C^R\right)^2$ and $\left(\lambda^A\right)^2 > \left(\lambda_C^A\right)^2$.

We need to assume that an increase in own price reduces own readership, that is $\widetilde{\gamma}^R < 0$ otherwise, the increase in price unambigously increases revenues from readership, thereby making the problem unbounded. Therefore, we require

$$\lambda_C^R\left(\gamma_C^R\lambda^A - \gamma^R\lambda_C^A\right) + \lambda^R\left(\gamma_C^R\lambda_C^A - \gamma^R\lambda^A\right) < \gamma^R$$

$$\lambda_C^R\gamma^R\lambda_C^A$$

To understand the previous equation, we need to decompose the five possible effect of a change in readership price on the reader's quantity (suppose of an increase in price):

1) the direct effect on readers' quantity $-\gamma^R$

2) the change (reduction) in reader's quantity generates a change (reduction) in the advertizing quantity $-\gamma^R\lambda^R$, which in turns generates a (positive) effect on the reader's quantity $\gamma^R\lambda^R\lambda^A$ DOES NOT MATCH. WHY?

3) the change (reduction) in reader's quantity

The previous equation tells us that the (negative) effect of a change in price on the reader's quantity exceeds the (possibly positive) indirect effects: $\lambda_C^R\gamma_C^R\lambda^A - \gamma^R\lambda_C^R\lambda_C^A - \lambda^R\gamma^R\lambda^A + \lambda^R\gamma_C^R\lambda_C^A$. In particular, these indirect effects are given by:

We first analyze how the deviation profit changes as the cartel reader's price increases $\frac{\partial \pi_l^D}{\partial p_{-l}^R}$. The derivative can be decomposed:

$$\frac{\partial \pi_l^D}{\partial p_{-l}^R} = \frac{\partial \pi_l^{D,R}}{\partial p_{-l}^R} + \frac{\partial \pi_l^{D,A}}{\partial p_{-l}^R}$$

By the envelope theorem, as a result of a similar argument as before, we have:

$$\frac{\partial \pi_l^D}{\partial p_{-l}^R} = q_l^{D,R}\widetilde{\gamma}_C^R + q_l^{D,A}\widetilde{\lambda}_C^R$$

$\widetilde{\gamma}_C^R > 0$, as long as products are substitutes, and the choice variables are strategic complements. IS THIS A NECESSARY CONDITION?

$$\frac{\left(\gamma_C^R + \lambda^R\left(\gamma_C^R\lambda^A - \gamma^R\lambda_C^A\right) + \lambda_C^R\left(\gamma_C^R\lambda_C^A - \gamma^R\lambda^A\right)\right)}{D}$$
$$\frac{\left(\gamma_C^R + \lambda^R\left(\gamma_C^R\lambda^A - \gamma^R\lambda_C^A\right) + \lambda_C^R\left(\gamma_C^R\lambda_C^A - \gamma^R\lambda^A\right)\right)}{D}$$
$$\lambda_C^R\gamma_C^R\lambda^A - \gamma^R\lambda_C^R\lambda_C^A - \lambda^R\gamma^R\lambda^A + \lambda^R\gamma_C^R\lambda_C^A - \gamma^R$$
$$\lambda_C^R\gamma_C^R\lambda^A - (1)\lambda_C^R\lambda_C^A - \lambda^R(1)(2) + \lambda^R(0.5)\lambda_C^A - (1)$$

$\left(\gamma_C^R\lambda^A - \gamma^R\lambda_C^A\right) > 0$ if "c" effect in readers' prices

$$\begin{aligned}\gamma_C^R\left(1 + \lambda^R\lambda^A + \lambda_C^R\lambda_C^A\right) &> \gamma^R\left(\lambda^R\lambda_C^A + \lambda_C^R\lambda^A\right)\\ \gamma_C^R &> \gamma^R\frac{\left(\lambda^R\lambda_C^A + \lambda_C^R\lambda^A\right)}{\left(1 + \lambda^R\lambda^A + \lambda_C^R\lambda_C^A\right)}\end{aligned}$$

PROSEGUIRE QUI : $\lambda^{A+R}\gamma_C^R + \lambda_C^{A+R}\gamma_C^R - \lambda^A\gamma^R\lambda_C^R - \lambda^R\gamma^R\lambda_C^A$

This requires

This can be rewritten as:

$$q_l^{D,R}\frac{\left(\gamma_C^R - \lambda^R\gamma^R\lambda_C^A + \lambda^R\gamma_C^R\lambda^A - \gamma^R\lambda_C^R\lambda^A + \lambda_C^R\gamma_C^R\lambda_C^A\right)}{D} + q_l^{D,A}\frac{\lambda^R\gamma_C^R + \gamma^R\lambda_C^R - \left(\lambda_C^R\right)^2\gamma_C^R\lambda^A - \left(\lambda^R\right)^2\gamma^R\lambda_C^A +}{D}$$

Intuitively, an increase in the rival's reader's cartel price generates an increase in the deviation profit from readership. However, at the same time, it increases the reader's advertizing quantity. This may generate a negative effect on the rival's profit. IS THIS CONSISTENT WITH THE FACT THAT WE ARE AT MONOPOLY?

For the numerator to be negative, one needs:

$$\underbrace{\frac{\gamma_C^R + \lambda^R\left(\gamma_C^R\lambda^A - \gamma^R\lambda_C^A\right) + \lambda_C^R\left(\gamma_C^R\lambda_C^A - \gamma^R\lambda^A\right)}{D}}_{\widehat{\gamma}_C^R} p_i^R +$$

$$+\underbrace{\frac{\lambda^R\gamma_C^R + \lambda_C^R\gamma^R + \left(\left(\lambda^R\right)^2 - \left(\lambda_C^R\right)^2\right)\left(\gamma_C^R\lambda^A - \gamma^R\lambda_C^A\right)}{D}}_{\widehat{\lambda}_C^R} p_i^A$$

The "strange" result here (that $\frac{\partial\pi^{Dev}}{\partial p_j} < 0$) involves:

$$\widehat{\gamma}_C^R p_i^R + \widehat{\lambda}_C^R p_i^A < 0$$

or

$$\widehat{\gamma}_C^R p_i^R < -\widehat{\lambda}_C^R \frac{p_i^A}{p_i^R}$$

.

We now analyze the features of R and A.

1) R is a normal good if $\widetilde{\gamma}^R < 0$, that is, if:

$$\begin{aligned}
\lambda_C^R\gamma_C^R\lambda^A - \gamma^R\lambda_C^R\lambda_C^A - \lambda^R\gamma^R\lambda^A + \lambda^R\gamma_C^R\lambda_C^A - \gamma^R &< 0 \\
\gamma_C^R\left(\lambda_C^R\lambda^A + \lambda^R\lambda_C^A\right) - \gamma^R\left(\lambda_C^R\lambda_C^A + \lambda^R\lambda^A\right) - \gamma^R &< 0 \\
\lambda_C^R\gamma_C^R\lambda^A - \gamma^R\lambda_C^R\lambda_C^A - \lambda^R\gamma^R\lambda^A + \lambda^R\gamma_C^R\lambda_C^A - \gamma^R &< 0
\end{aligned}$$

Observe that, if $\gamma_C^R\left(\lambda_C^R\lambda^A + \lambda^R\lambda_C^A\right) - \gamma^R\left(\lambda_C^R\lambda_C^A + \lambda^R\lambda^A\right) - \gamma^R < 0$, then R is a good whose demand increases with price. Can this be a reasonable assumption? The problem is that this would raise price or quantity to infinity. Therefore, this needs to be imposed by assumption. We can therefore state:

Assumption 1:

$$\gamma_C^R\left(\lambda_C^R\lambda^A + \lambda^R\lambda_C^A\right) - \gamma^R\left(\lambda_C^R\lambda_C^A + \lambda^R\lambda^A\right) - \gamma^R < 0$$

2) A is a normal good (the increase in the advertising price reduces the advertising quantity) if $\widetilde{\gamma}^A < 0$, that is, if:

$$\begin{aligned}
\lambda^R\lambda_C^A\gamma_C^A - \gamma^A - \lambda^R\lambda^A\gamma^A - \lambda_C^R\lambda_C^A\gamma^A + \lambda_C^R\lambda^A\gamma_C^A &< 0 \\
-\lambda^R\left(\lambda^A\gamma^A - \lambda_C^A\gamma_C^A\right) - \gamma^A - \lambda_C^R\left(\lambda_C^A\gamma^A - \lambda^A\gamma_C^A\right) &< 0
\end{aligned}$$

When

BUT THIS HAS TO BE VERIFIED ACCORDING TO COMPUTATIONS FOR THE LIMIT.

2) R is substitute of A - as advertising price increases, people read more newspapers (through the effect on advertising quantity)

$$\frac{\partial q_i^R}{\partial p_i^A} = \widetilde{\lambda}^A > 0$$

5 Numerical examples

$$\begin{aligned}
p_i^R &= \widehat{R} + \widehat{\gamma}^R q_i^R + \widehat{\gamma}_C^R q_j^R + \widehat{\lambda}^A q_i^A + \widehat{\lambda}_C^A q_j^A \\
p_j^R &= \widehat{R} + \widehat{\gamma}^R q_j^R + \widehat{\gamma}_C^R q_i^R + \widehat{\lambda}^A q_j^A + \widehat{\lambda}_C^A q_i^A \\
p_i^A &= \widehat{A} + \widehat{\gamma}^A q_i^A + \widehat{\gamma}_C^A q_j^A + \widehat{\lambda}^R q_i^R + \widehat{\lambda}_C^R q_j^R \\
p_j^A &= \widehat{A} + \widehat{\gamma}^A q_j^A + \widehat{\gamma}_C^A q_i^A + \widehat{\lambda}^R q_j^R + \widehat{\lambda}_C^R q_i^R
\end{aligned}$$

The monopoly profit to be maximized is:

$$\max_{q_i^R, q_i^A, q_j^R, q_j^A} \left(\widehat{R} + \widehat{\gamma}^R q_i^R + \widehat{\gamma}_C^R q_j^R + \widehat{\lambda}^A q_i^A + \widehat{\lambda}_C^A q_j^A\right) q_i^R + \left(\widehat{R} + \widehat{\gamma}^R q_j^R + \widehat{\gamma}_C^R q_i^R + \widehat{\lambda}^A q_j^A + \widehat{\lambda}_C^A q_i^A\right) q_j^R +$$
$$+ \left(\widehat{A} + \widehat{\gamma}^A q_i^A + \widehat{\gamma}_C^A q_j^A + \widehat{\lambda}^R q_i^R + \widehat{\lambda}_C^R q_j^R\right) q_i^A + \left(\widehat{A} + \widehat{\gamma}^A q_j^A + \widehat{\gamma}_C^A q_i^A + \widehat{\lambda}^R q_j^R + \widehat{\lambda}_C^R q_i^R\right) q_j^A$$

At the optimum,

$$\begin{aligned}
\frac{\partial \pi}{\partial q_i^R} &= \left(\widehat{R} + 2\widehat{\gamma}^R q_i^R + \widehat{\gamma}_C^R q_j^R + \widehat{\lambda}^A q_i^A + \widehat{\lambda}_C^A q_j^A\right) + \widehat{\gamma}_C^R q_j^R + \widehat{\lambda}^R q_i^A + \widehat{\lambda}_C^R q_j^A = 0 \\
\frac{\partial \pi}{\partial q_j^R} &= \left(\widehat{R} + 2\widehat{\gamma}^R q_j^R + \widehat{\gamma}_C^R q_i^R + \widehat{\lambda}^A q_j^A + \widehat{\lambda}_C^A q_i^A\right) + \widehat{\gamma}_C^R q_i^R + \widehat{\lambda}^R q_j^A + \widehat{\lambda}_C^R q_i^A = 0 \\
\frac{\partial \pi}{\partial q_i^A} &= \left(\widehat{A} + 2\widehat{\gamma}^A q_i^A + \widehat{\gamma}_C^A q_j^A + \widehat{\lambda}^R q_i^R + \widehat{\lambda}_C^R q_j^R\right) + \widehat{\gamma}_C^A q_j^A + \widehat{\lambda}^A q_i^R + \widehat{\lambda}_C^A q_j^R = 0 \\
\frac{\partial \pi}{\partial q_j^A} &= \left(\widehat{A} + 2\widehat{\gamma}^A q_j^A + \widehat{\gamma}_C^A q_i^A + \widehat{\lambda}^R q_j^R + \widehat{\lambda}_C^R q_i^R\right) + \widehat{\gamma}_C^A q_i^A + \widehat{\lambda}^A q_j^R + \widehat{\lambda}_C^A q_i^R = 0
\end{aligned}$$

Given symmetry (implying $q_i^R = q_j^R$ and $q_i^A = q_j^A$), we can replace:

$$\begin{aligned}
\frac{\partial \pi}{\partial q_i^R} &= \frac{\partial \pi}{\partial q_j^R} = \widehat{R} + 2\widehat{\gamma}^R q^R + \widehat{\gamma}_C^R q^R + \widehat{\lambda}^A q^A + \widehat{\lambda}_C^A q^A + \widehat{\gamma}_C^R q^R + \widehat{\lambda}^R q^A + \widehat{\lambda}_C^R q^A = 0 \\
\frac{\partial \pi}{\partial q_i^A} &= \frac{\partial \pi}{\partial q_j^A} = \widehat{A} + 2\widehat{\gamma}^A q^A + \widehat{\gamma}_C^A q^A + \widehat{\lambda}^R q^R + \widehat{\lambda}_C^R q^R + \widehat{\gamma}_C^A q^A + \widehat{\lambda}^A q^R + \widehat{\lambda}_C^A q^R = 0
\end{aligned}$$

After simplification, one obtains:

$$q^R = \frac{-\widehat{R} - q^A\left(\widehat{\lambda}^A + \widehat{\lambda}_C^A + \widehat{\lambda}^R + \widehat{\lambda}_C^R\right)}{2\widehat{\gamma}^R + \widehat{\gamma}_C^R + \widehat{\gamma}_C^R}$$

$$q^A = \frac{-\widehat{A} - q^R\left(\widehat{\lambda}^A + \widehat{\lambda}_C^A + \widehat{\lambda}^R + \widehat{\lambda}_C^R\right)}{\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}_C^A\right)}$$

$$q^R = \frac{-\widehat{R} - \frac{-\widehat{A} - q^R\left(\widehat{\lambda}^A + \widehat{\lambda}_C^A + \widehat{\lambda}^R + \widehat{\lambda}_C^R\right)}{\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}_C^A\right)}\left(\widehat{\lambda} \ldots\right.}{2\widehat{\gamma}^R + \widehat{\gamma}_C^R + \widehat{\gamma} \ldots}$$

$$q^R = -\widehat{R} + \frac{\widehat{A}\left(\widehat{\lambda}^A + \widehat{\lambda}_C^A + \widehat{\lambda}^R + \ldots\right.}{\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}_C^A\right)\left(2\widehat{\gamma}^R + \ldots\right.}$$

$$q^R\left(\frac{\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}_C^A\right)\left(2\widehat{\gamma}^R + \widehat{\gamma}_C^R + \widehat{\gamma}_C^R\right) - \left(\widehat{\lambda}^A + \widehat{\lambda}_C^A + \widehat{\lambda}^R + \widehat{\lambda}_C^R\right)^2}{\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}_C^A\right)\left(2\widehat{\gamma}^R + \widehat{\gamma}_C^R + \widehat{\gamma}_C^R\right)}\right). = \frac{-\widehat{R}\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}_C^A\right)\left(2\widehat{\gamma}^R + \ldots\right.}{\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}^A \ldots\right.}$$

$$q^R. = \frac{-\widehat{R}\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}_C^A\right)\left(2\widehat{\gamma}^R + \widehat{\gamma} \ldots\right.}{\left(2\widehat{\gamma}^A + \widehat{\gamma}_C^A + \widehat{\gamma}_C^A\right)\left(2\widehat{\gamma}^R + \widehat{\gamma}_C^R \ldots\right.}$$

6 Conclusion

This paper shows that, depending on the parameter values, we may observe lower-than-monopoly output prevailing on one of the two markets in a two-sided market. This happens because low quantities may reduce the incentives to deviate, thereby stabilizing the cartel. This occurs because, while lower quantities increase the deviation profit (and therefore make collusion less stable) in the same market, it may decrease the deviation profit in the other market.

In particular, the cartel is stabilized by a decrease in quantity below monopoly when the externality between the two market is positive and strong (in the case of a change in readers' quantity, this requires $\frac{-\left(\lambda_C^R\gamma^A - \lambda^R\gamma_C^A\right)}{(\gamma^A)^2 - \left(\gamma_C^A\right)^2} > 0$, that is $\frac{\gamma^A}{\gamma_C^A} < \frac{\lambda^R}{\lambda_C^R}$, related to stronger complementarities between the two sectors). Basically, the positive network externality adds a complementarity aspect to the two sides.