An Alternative Position Auction Mechanism

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Abstract: We examine position auctions where the search engines such as Google, Microsoft, and Yahoo! sell advertising slots on internet to their sponsors. These three giant search engines use a generalized per-click second-price auction procedure where the least valuable spot is allocated earlier and the payment of a slot is determined by the per-click price for the slot multiplied by the realized number of clicks. Literature shows that their procedure raises the same expected revenue as the VCG mechanism although truth telling is not an equilibrium strategy. We notice that a Japanese advertisement company uses a different procedure for position auctions. Its procedure is a sequential first-price sealed-bid auction where the order of the sales is opposite to the big three's auction procedure and the most valuable slot is auctioned off earlier. In addition, it does not adopt per-click price mechanism and auctions off each slot as a whole. We characterize the equilibrium bidding function and calculate the expected revenue of the Japanese company's auction procedure. Further, we compare them with the big three's auction procedure and conclude that it achieves an efficient allocation and raises the same expected revenue as the big three's auction procedure and the VCG mechanism.

Keywords: Position auctions, Internet advertisement.

JEL classification numbers: D44, D82, L86, M37.

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1. Introduction

The giant internet search engine, Google, sells slots on online advertisement linked to keyword search. It is almost the only revenue source for Google whose market value is more than $150 billion. When one types some keywords, several sponsored links appear at the top and on the right side of the search results. Usually, the slot on the top is more valuable to those on the right side. Among slots on the right side, k^{th} ranked slot is more valuable than $(k+1)^{th}$ ranked slot. Google sells these slots via per-click price auctions which is called position auctions. In its dynamic auction, the k^{th} highest bidder wins the k^{th} slot and pays the $(k+1)^{th}$ highest bid. Edelman, Ostrovsky, and Schwarz (2007) (hereafter EOS) analyze this auction and show that it is not equivalent to the VCG mechanism in the sense that truth telling is not an equilibrium strategy, but it raises the same expected revenue as the VCG mechanism. If we use a dynamic ascending price auction to implement the EOS mechanism, the less valuable slot is allocated earlier than the more valuable slots. Chen and He (2006) and Athey and Ellison (2008) keep the basic features of the EOS mechanism and endogenize the values of the slots. Chen and He (2006) introduce product differentiation where a consumer learns about her valuation and the desirability of the product only when she pays search cost and clicks the website of a sponsor. Athey and Ellison (2008) introduce costly consumer search to the EOS mechanism to determine the values of the slots.

Although the giant search engine, Google, adopts a generalized second price auction procedure to position auctions, D2 communication in Japan uses a different auction procedure from Google. In D2 communication's auction, k^{th} highest bidder wins k^{th} slot as in the case of the EOS auction, but pays the k^{th} highest bid, that is, pays his own bid. In addition, the more valuable slot is auctioned off earlier than the less valuable slot in D2 communication's auction procedure. Therefore, the order of sales is completely opposite to the EOS auction.

We characterize the equilibrium bidding function of D2 communication's auction and the expected revenue. Then, we compare them to those of the EOS auction. We conclude that D2 communication's auction achieve an efficient allocation and raise the same expected revenue as the EOM auction and the VCG mechanism although the payment system and the order of sales are quite different from the EOS auction.

Our paper is also related to literature identifying the source of decreasing prices observed in practical auctions. Ashenfelter (1989) reported evidences from wine auctions that prices decrease in sequential auctions. Kahn and Wiggans (1997) observed the same phenomenon in livestock auctions and Vanderporten (1992) did in the condominium auctions. McAfee and Vincent (1993) attributed it to buyer's being risk averse. Black and Meza (1993) introduced "buyer's option" to explain decreasing prices. "Buyer's option" means that the winner of the first auction has an opportunity to buy additional objects at the same price that he paid for the first one. Since this option increases the value of the first object, they obtain decreasing prices. Lambson and Thurston (2006) analyzed a sequential ascending auction with two identical items and positive minimum bid increments are imposed. Each bidder demands more than one item and the winner of the first auction has the privilege of beginning the starting price of the second auction. They showed that the price in the second auction can be lower than that in the first auction, but only by minimum bid increment. Bernhardt and Scoones (1994) and Engelbrecht-Wiggans (1994) considered stochastically equivalent objects in two objects model on the unit demand assumption and obtain decreasing prices. Our model appeals to an alternative reason for decreasing prices: Price decreases when the more valuable objects are sold earlier and it happens when the seller can endogenously choose the order of selling the heterogeneous items.

The rest of the paper is organized as follows: Section 2 explains the model. Section 3 analyzes the sequential sealed-bid first-price auction. Section 4 characterizes the expected revenue maximizing auction mechanism and addresses how to implement it. Section 5 summarizes the result, mentions about the author's contribution to the advancement of business, shows several directions to extend the basic model, and discuss future research prospects. All the proofs are collected in the appendices.

2. The Model

There are $n+1>m$ ($m\geq 1$) potential buyers for m objects. N is the set of potential buyers and M is the set of items. Each buyer i draws his private signal x_i independently from some common probability distribution function $F(\bullet)$ whose support is $[0,\bar{x}\,]$[1] which has a continuous density function $f(\bullet)$There are m common public signals, $t_1 \geq t_2 \geq ... \geq t_m$. When buyer i with private signal x_i obtains the kth item at price p_k , he receives net utility of $U(t_k, x_i) - p_k$ where $U(t_k, x_k)$ is a supermodular. We also assume that each potential buyer i demands at most one object, or equivalently, that the seller restricts each buyer to obtain at

[1] We address a bidder "he" for simplicity and there is no intention of sexual differentiation.

most one object. The structure of a game is common knowledge among the players except each bidder only knows his realized private signal but not others.

First, we analyze sequential sealed-bid first-price auctions with price announcements. There, the seller auctions one object at each period following a predetermined order. At the beginning of each period, the seller announces the price paid in the previous period. This allows all losing bidders to form the same belief on their opponents' private signals at the beginning of each period. Further, the bidder who submits the highest bid obtains the object and pays the amount of his own bid. It is possible to consider other information releasing procedures such as no announcement at all about the price of the previous period's auction. In this case, losing bidders only know that the fact that they "lost." However, it is worth noting that we can show that bidders' strategies remain the same on the equilibrium path as the case with price announcement.

We adopt the Bayesian perfect equilibrium as an equilibrium concept and analyze symmetric equilibrium strategies. A strategy for a bidder, $b_k(x;\ \bullet)$ $(\forall\ k \in M)$, is a combination of functions whose domain is $R \times R^{k-1}$ and whose range is R. The first argument indicates his realized private signal and the second represents any information available up to period k such as the realized prices in the previous auctions. In other words, $b_k(x;\ \bullet)$ indicates how much a bidder is willing to bid in the period k auction when he loses all the auctions prior to period k. Later, we need to consider each bidder's belief on their opponents' private signals at the beginning of each period to characterize equilibrium bidding strategies. For this purpose, we need additional notation. X_i is a random variable of bidder i's private signal ($\forall\ i \in N$) and x_i is its realization. Further, Y_r (r =1,2,...,n) is the rth higher order statistics of $X_2, X_3, ..., X_{n+1}$ and y_r is a realization of Y_r.

3. Sequential Sealed-Bid First-Price Auctions

First, we analyze the model with price announcement, that is, the seller announces the price of the previous auction at the beginning of each period before bidders submit their bids. This allows all the remaining bidders to have the same belief on the opponents' private signals in each period. The following proposition characterizes the symmetric Bayesian perfect equilibrium strategies in this case.

Proposition 1 (with price announcement):

Monotonically increasing equilibrium bidding functions, $b_k(x; y_1, y_2, ..., y_{k-1}) = b_k(x)$ ($\forall\, k \in M$), exist if the objects are sold by the predetermined order of $t_1 \geq t_2 \geq ... \geq t_m$ and characterized as follows:

If $y_{k-1} \geq x$,

$$\begin{cases} b_k(x) = \int_0^x (n-k+1)\left(\frac{F(s)}{F(x)}\right)^{n-k+1}\left(\frac{f(s)}{F(x)}\right)\left((U(t_k, x) - U(t_{k+1}, x))s + b_{k+1}(s)\right)ds \quad for 1 \leq k \leq m-1 \\ b_m(x) = \int_0^x U(t_m, s)(n-m+1)\left(\frac{F(s)}{F(x)}\right)^{n-m+1}\left(\frac{f(s)}{F(x)}\right)ds \end{cases}$$

Otherwise, $b_k(x) = b_k(b_{k-1}^{w\,-1}(p_{k-1})) \quad for 1 \leq k \leq m$.

Proof:

See Appendix.

We can immediately obtain the following corollary from proposition1.

Corollary1 (without price announcement)

Suppose that the seller does not announce the price of the auctions at all. Then, there exists equilibrium bidding functions that are the same as those in Proposition 1 on the equilibrium path.

Next, we define unadjusted and adjusted prices to examine the price path

Definition (unadjusted price):

The unadjusted price in period k $(\forall k \in M)$, p_k, is the winning bid of the period k auction for the sequential sealed-bid first-price auction and the highest loser's bid for the sequential sealed-bid second-price auction.

Definition (adjusted price):

The adjusted price at period k $(\forall k \in M)$ is defined as the unadjusted price divided by t_k, (i.e., $\frac{p_k}{t_k}$) when $U(t_k, x) = t_k x$.

The following proposition and corollary examine the pattern of the price paths.

Proposition 2 (declining adjusted prices):

In the sequential sealed-bid first-price auction with predetermined selling order of $t_1 \geq t_2 \geq ... \geq t_m$, the unconditional expected adjusted prices decline on the equilibrium path.

Proof:

See Appendix.

We can immediately obtain the following corollary from proposition2.

Corollary2 (declining unadjusted prices):

In the sequential sealed-bid first-price auction with predetermined selling order of $t_1 \geq t_2 \geq ... \geq t_m$, the unconditional expected adjusted prices decline on the equilibrium path.

4. The Expected Revenue Maximizing Auction Mechanism

This section characterizes the expected revenue maximizing (optimal) auction mechanism in our setting and shows that both the sequential sealed-bid first-price and second-price auctions implement the optimal auction mechanism. First, we characterize the optimal auction mechanism. To simplify the analysis, we impose the following regularity condition.

Regularity condition:

The regularity condition is satisfied if $U(t_k, x_i) - \left(\frac{\partial U(t_k, x_i)}{\partial x_i}\right)\left(\frac{1 - F_i(x_i)}{f_i(x_i)}\right)$ increases w.r.t. x_i, $\forall\, i \in N$ and $k \in M$.

Proposition 3 (the optimal mechanism):

Assume that the regularity condition holds and bidders are symmetric ex-ante. Then, the optimal auction mechanism is characterized by the following three conditions:

(1) The payment of player i $(\forall i \in N)$ who reports his private signal as x_i is

$$c(x_i)= E\left[\left\{\sum_{k=1}^{m} U(t_k, x_i) p^*_{ik}(\vec{X}) - \int_0^{x_i} \frac{\partial U(t_k, s)}{\partial t_k} p^*_{ik}(s, \vec{X}_{-i}) ds\right\} \middle| X_i = x_i\right],$$

where,

$$\vec{X} = (X_1, X_2, ..., X_{n+1}), \quad \vec{x} = (x_1, x_2, ..., x_{n+1}), \quad \vec{X}_{-i} = (X_1, X_2, ..., X_{i-1}, X_{i+1}, ..., X_{n+1})$$

(2) $$p^*_{ik}(x) = \begin{cases} 1 & \text{if } x_i \text{ is the kth highest among } \vec{x} \\ 0 & \text{otherwise} \end{cases}$$

(3) The required minimum reported private signal is x_k^r $(\forall k \in M)$ s.t.

$$U(t_k, x_k^r) - \left(\frac{\partial U(t_k, x)}{\partial x}\bigg|_{x=x_k^r}\right)\left(\frac{1-F(x_k^r)}{f(x_k^r)}\right)=0.$$

Proof:

Proof is omitted because we can obtain the result by slight modification of the argument in Myerson (1981). Q.E.D.

Proposition 4 (implementation):

The sequential sealed-bid first-price auctions with or without price announcement with the minimum required reported private signal x_k^r $(\forall k \in M)$ implement the optimal selling mechanism.

Proof:

Obvious from propositions1 and3. Q.E.D.

5. Discussion

This paper analyzes the position auction procedure adapted by D2 communication in Japan. Their auction procedure to sell advertising slots on internet keyword search is quite different from the one designed by Google. Google uses a generalized VCG mechanism where (1) k^{th} highest bidder wins the k^{th} slot and pays the $(k+1)^{th}$ highest bid, (2) the payment is "per click," and (3) the less valuable slot is auctioned off earlier than the more valuable slot. D2 communication in Japan uses a very different auction procedure. First, k^{th} highest bidder wins the k^{th} slot as in the case of Google's auction, but pays his own bid, namely it is a first-price sealed-bid auction. Secondly, the payment is not "per click," but k^{th} highest bidder pays for being awarded the k^{th} slot regardless of the actual number of clicks realized later. Thirdly, the order of sales is reversed and the more valuable slot is auctioned off earlier in D2 communication's auction procedure. Nevertheless, we show that D2 communication's auction procedure achieves an efficient allocation and raises the same expected revenue as the EOS auction and the VCG mechanism.

Further, decreasing prices anomaly is observed in the sealed-bid first-price auctions. Although researches on decreasing prices are originated from the Ashenfelter's wine auctions' example and, accordingly, they assume some kind of homogeneity across items (either in deterministic or stochastic sense), we believe that showing that auctions of heterogeneous items can also experience decreasing prices because of the seller's strategic behavior sheds a new light on this stream of the literature. Note that “prices decreasing anomaly” implies unequal treatment of the winners and it causes serious problem when the government designs a selling procedure to privatize state-owned properties[2]. For example, in the Japanese land auctions, it means that different winners pay different per square meter price for the adjacent pieces of land are sometimes not approval.

There are a few important points for future research. First, though we assume that bidders’ private signals are i.i.d., this should be only the first step to analyze more practical cases such as affiliation. Next, there are examples in the practical world where each bidder is demands more than one objects. The FCC radio spectrum license auctions are one of the most important examples among others. There, we also need to consider the interdependency across different objects to characterize bidders’ strategies. This will complicate the analysis, but be worth challenging.

Appendix

Proof of Proposition 1:

We use mathematical induction to prove Proposition 1. There are four steps in the argument. The first two steps characterize bidding functions on the equilibrium path. The third step examines the off-the-equilibrium -path strategies. The fourth step shows that the strategies in Proposition 1 attain global maximum.

Step 1 (period m).

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[2] Possibility of auctions satisfying efficiency, revenue maximization, and equal treatment of

Without loss of generality, we can focus on the problem for bidder 1. Assume that bidder 1 loses all the (*m*-1) auctions held prior to period *m* and consider bidder 1's problem for the last period (period m) auction. Since we assume that the seller announces the price of the previous period auction at the beginning of the next before bidders submit their bids, all the remaining bidders have the same belief on the opponents' private signals at the beginning of the last period's auction. Namely, all the bidders know the value of the winning bids up to period *m*-1, which we denote by $b_1^w, b_2^w, ..., b_{m-1}^w$.

On the equilibrium path, this means that the remaining bidders actually know the value of $y_1, y_2, ..., y_{m-1}$ from $b_1^w, b_2^w, ..., b_{m-1}^w$ by inverting bidding functions, since we assume that there exists monotonically increasing equilibrium bidding strategy w.r.t. private signal. This means that all the remaining bidders know that all the remaining opponents' private signals are i.i.d. from the probability distribution function $\frac{F(.)}{F(y_{m-1})}$. So, the bidder 1's problem in period *m* auction is the same as one in the situation where there are $(n+1)-(m-1)$ potential buyers whose private signals are i.i.d. of the truncated probability distribution function $\frac{F(.)}{F(y_{m-1})}$.

Therefore, we can write bidder 1's problem as follows:

$$\underset{x_m}{Max} \quad \left(\frac{F(x_m)}{F(y_{m-1})}\right)^{n-m+1} (U(t_m, x) - b_m(x_m))$$

The F.O.C. w.r.t. x_m evaluating (1) at $x_m = x$ yields

$$(n-m+1)\left(\frac{F(x)}{F(y_{m-1})}\right)^{n-m}\left(\frac{f(x)}{F(y_{m-1})}\right)(U(t_m, x) - b_m(x)) - \left(\frac{F(x)}{F(y_{m-1})}\right)^{n-m} b_m'(x) = 0 \qquad ...(2)$$

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the winners is analyzed in Baba (1999).

Solving (2) with boundary condition of $b_m(0) = 0$ leads to

$$b_m(x) = \int_0^x U(t_m, s)(n-m+1)\left(\frac{F(s)}{F(x)}\right)^{n-m}\left(\frac{f(s)}{F(x)}\right)ds \qquad ...(3)$$

<u>Step 2 (period *k*=1,2,...,*m*-1).</u>

Suppose that there exist monotonically increasing bidding functions from period k+1 to period m, $b_{k+1}(x), b_{k+2}(x), ..., b_m(x)$, which do not depend on the realization of the price path. Then, we would like to show that there exists a monotonically increasing bidding function in period k, $b_k(x)$, which does not depend on the realization of the price path, either.

Bidder 1's problem is expressed as follows:

$$\underset{x_k}{Max} \qquad U_k\left(x_k, x; \quad Y_1 = y_1, Y_2 = y_2, ..., Y_{k-1} = y_{k-1}\right)$$

where, $U_k\left(x_k, x; \quad Y_1 = y_1, Y_2 = y_2, ..., Y_{k-1} = y_{k-1}\right)$ is the expected utility of bidder 1 when his true private signal is x and his reported private signal is x_k, given that $Y_1 = y_1, Y_2 = y_2, ..., Y_{k-1} = y_{k-1}$. The first order condition w.r.t. x_k for the case of $x_k > x$ and for the case of $x_k < x$ evaluated at $x_k = x$ coincide and is expressed as follows.

$$b_k'(x) + (n-k+1)\left(\frac{f(x)}{F(x)}\right)b_k(x) = (n-k+1)\left(\frac{f(x)}{F(x)}\right)(U(t_k, x) - U(t_{k+1}, x) + b_{k+1}(x)) \qquad ...(4)$$

Solving (4) for $b_k(x)$ with the boundary condition $b_k(0) = 0$ yields

$$b_k(x) = \int_0^x (n-k+1)\left(\frac{F(s)}{F(x)}\right)^{n-k}\left(\frac{f(s)}{F(x)}\right)\left(U(t_k, s) - U(t_{k+1}, s) + b_{k+1}(s)\right)ds \qquad ...(5)$$

Together with (3), the bidding functions $b_k(x)$ $(\forall\, k \in M)$ are recursively determined as follows:

$$b_m(x) = \int_0^x U(t_m, x)\left(\frac{F(s)}{F(x)}\right)^{n-m+1}\left(\frac{f(s)}{F(x)}\right)ds$$.

or $b_k(x) = \int_0^x (n-k+1)\left(\frac{F(s)}{F(x)}\right)^{n-k+1}\left(\frac{f(s)}{F(s)}\right)\left(U(t_k, s) - U(t_{k+1}, s) + b_{k+1}(s)\right)ds$, for $1 \le k \le$ m-1

Step 3 (off the equilibrium path).

If the sequence of the realized values of $y_1, y_2, ..., y_{k-1}$ obtained from inverting $b_1^w, b_2^w, ..., b_{k-1}^w$ is not decreasing, let bidders re-order $y_1, y_2, ..., y_{k-1}$ to yield a decreasing sequence of $z_1, z_2, ..., z_{k-1}$ and apply the argument in step 1 and 2 to the sequence of $z_1, z_2, ..., z_{k-1}$. Since we characterize the Bayesian perfect equilibrium, this procedure works.

Step 4 (Global maximum).

The global optimality follows from the usual argument and it is straightforward.

Q.E.D.

The following two lemmata are useful to prove proposition2.

Lemma 1 (Weber 1983):

Suppose that there are (*n*+1) bidders and that *m* homogeneous objects are up for sequential sealed-bid first-price or second-price auction. Then unconditional expected values of the prices remain the same for all the periods for both types of auction.

Lemma 1':

Suppose $\exists k \in \{1,2,...,m-1\}, t_k = t_{k+1} \geq t_{k+2} \geq ... \geq t_m$. Then, the unconditional expected values of both the adjusted and the unadjusted prices are the same for period k and period (k+1) for both sequential sealed-bid first-price and second-price auctions.

Proof of lemma1':

We can obtain the results from plugging $t_k = t_{k+1}$ into (15) in the proof of proposition 1.

Q.E.D.

Proof of Proposition2:

We use Lemma1' later. First, recall that proposition 1 tells us the equilibrium bidding functions in our case are:

$$b_k(x) = \int_0^x (n-k+1)\left(\frac{F(s)}{F(x)}\right)^{n-k+1}\left(\frac{f(s)}{F(s)}\right)\left((t_k - t_{k+1})s + b_{k+1}(s)\right)ds \text{ , for } k = 1,2,..., m-1$$

...(1)

Similarly,

$$b_{k+1}(x) = \int_0^x (n-k+1)\left(\frac{F(s)}{F(x)}\right)^{n-k}\left(\frac{f(s)}{F(s)}\right)\left((t_{k+1} - t_{k+2})s + b_{k+2}(s)\right)ds \quad ,$$

for $k = 1,2,..., m-2$ ...(2)

Differentiate (2) w.r.t. t_{k+1},

$$\frac{\partial b_{k+1}(x)}{\partial t_{k+1}} = \int_0^x (n-k)\left(\frac{F(s)}{F(x)}\right)^{n-k}\left(\frac{f(s)}{F(s)}\right)s \; ds \qquad ...(3)$$

Using (3),

$$\frac{\partial\left(b_{k+1}(x)\big/t_{k+1}\right)}{\partial t_{k+1}} = \frac{1}{t_{k+1}^{\ 2}}\left(\frac{\partial b_{k+1}(x)}{\partial t_{k+1}}t_{k+1} - b_{k+1}(x)\right)$$

$$\frac{\partial\left(b_{k+1}(x)\big/t_{k+1}\right)}{\partial t_{k+1}} = \frac{1}{t_{k+1}^{\ 2}}\left(\frac{\partial b_{k+1}(x)}{\partial t_{k+1}}t_{k+1} - b_{k+1}(x)\right)$$

$$= \frac{1}{t_{k+1}^{\ 2}}\left\{\int_0^x \left((n-k)\left(\frac{F(s)}{F(x)}\right)^{n-k}\left(\frac{f(s)}{F(s)}\right)\right)\left(\left(t_{k+1} - t_{k+1}\right)s + t_{k+2}s - b_{k+2}(s)\right)ds\right\} > 0 \qquad \text{...(4)}$$

On the other hand,

$$\frac{\partial\left(b_k(x)\big/t_k\right)}{\partial t_{k+1}} = \frac{1}{t_k}\left(\frac{\partial b_k(x)}{\partial t_{k+1}}\right) = \int_0^x (n-k+1)\left(\frac{F(s)}{F(x)}\right)^{n-k+1}\left(\frac{f(s)}{F(s)}\right)\left(-s + \frac{\partial b_{k+1}(s)}{\partial t_{k+1}}\right)ds \qquad \text{...(5)}$$

Further,

$$\left(-s + \frac{\partial b_{k+1}(s)}{\partial t_{k+1}}\right) = -s + \int_0^s (n-k)\left(\frac{F(t)}{F(s)}\right)^{n-k}\left(\frac{f(t)}{F(t)}\right)t\ \ dt$$

$$= -s + \left(\frac{1}{F(s)}\right)^{n-k}\int_0^s (n-k)F(t)^{n-k+1} f(t)t\ \ dt$$

$$= -s + \left(\frac{1}{F(s)}\right)^{n-k}\left\{\left[F(t)^{n-k}t\right]_0^s - \int_0^s F(t)^{n-k}dt\right\}$$

$$= -s + s - \int_0^s F(t)^{n-k}dt \quad < 0 \qquad \text{...(6)}$$

From (5) and (6),

$$\frac{\partial\left(b_k(x)\big/t_k\right)}{\partial t_{k+1}} < 0 \qquad \text{...(7)}$$

From, (4), (7) and Lemma 1’, we can obtain the desired result as follows. First, Lemma 1’ tells us that the difference between unconditional expected adjusted prices of the period k and

(k+1) auctions equals to 0 when $t_k = t_{k+1}$, or equivalently, $\frac{t_{k+1}}{t_k}$=1. Further, (4) and (7) imply that the difference between unconditional expected adjusted prices of the period k and (k+1) auctions is a decreasing function w.r.t. $\frac{t_{k+1}}{t_k}$. Therefore, we know that the difference between unconditional expected adjusted prices of the period k and (k+1) auctions are strictly positive as long as $t_k > t_{k+1}$ and this is the desired conclusion. Q.E.D.

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