Productivity Growth and Volatility: How Important are Wage and Price Rigidities?

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ABSTRACT We study the implications of having different sources of nominal rigidities on the relationship between productivity growth and shocks volatility in a model with procyclical R&D and imperfect competition in goods and labour markets. We show that the effects of uncertainty on long-term growth not only depends on the source of fluctuations, as recent literature shows, but also, and crucially, on whether prices and/or wages are rigid.
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1. INTRODUCTION

Traditionally macroeconomists considered growth and business cycles as two different fields of research. It wasn't until the Eighties and the seminal papers by Nelson and Plosser (1982) and Kydland and Prescott (1982) that scholars started questioning this view on the grounds that fluctuations are persistent and output does not show a strong tendency to return to its long-run trend after a shock. An explanation for this evidence is provided by endogenous growth models, where any temporary disturbance may affect growth-enhancing activities (i.e. savings, investments, R&D activities) thus producing everlasting effects on the level of output.[1]

The goal of this paper is to study how imperfect information in price and wage setting will affect the relationship between short-run fluctuations and productivity growth.

In the presence of market power by firms (or workers) business cycles are likely to involve inefficient fluctuations in the allocation of resources, in the sense that the surplus lost from unemployment above its natural rate during recessions is larger than the gain from a symmetric deviation (see Galí et al. 2007). In a standard New Keynesian model the key distortion producing this effect is monopolistic competition, which introduces a wedge between the marginal product of labor and the household's marginal rate of substitution between consumption and leisure due

[1] See Gaggl and Steindl (2007) and in Aizenman and Pinto (2005) for an overview on the literature.

to market power in the goods and labour market. Here we show that the inefficiency caused by fluctuations will compound over time if the rate of growth itself is negatively influenced by the business cycle. This happens in our model because R&D activity is stronger during expansions, even if its opportunity cost is higher as after the first period of production innovations are copied by other firms. Innovating is therefore more convenient when demand is higher.

As Galí et al. (2007) note, this inefficiency wedge can be decomposed into the product of a price markup (the ratio of the marginal product of labor to the real wage) and a wage markup (the ratio of the real wage to the household's marginal rate of substitution). Here, we show analytically how uncertainty will push up both of these markups on average, if there are price and wage rigidities. This is interesting as a number of authors have shown that price and wage rigidities substantially improve the capacity of DSGE models to match stylized facts.[2] While New Keyensian contributions are generally made in calibrated models, here we take a complementary line and consider a model with an explicit solution that makes more transparent the mechanisms at work. As in Cooper and John (1988), Ng (1980, 1992), Blackburn and Pelloni (2004), (2005), Wang and Wen (2011), Annicchiarico et al. (2011a), among others, firms and/or wage setters set prices without knowing the true costs they'll bear among many others.

Two sets of basic stylized facts motivate this paper: the first one is the existence of nominal rigidities, both on the goods and on the labour markets, the second refer to the negative relationship between volatility and growth and the strong procyclicality of both total factor productivity and R&D.

Recent direct evidence on price/wage rigidities for developed countries are provided by Bils and Klenow (2004), Dhyne et al. (2006), Nakamura and Steinsson (2008), Dickens et al. (2007) among others.

Since the seminal paper by Ramey and Ramey (1995), cross-country studies have found that volatility has a significant negative impact on long-run (trend) growth (see Andreou et al. 2008, Bredin and Fountas 2009, Hnatkovska and Loayza 2005, Kose et al. 2005, Martin and Rogers 2000, Badinger 2010 among many others). Moreover there seems to be a consensus in

[2]For a complete treatment of the relationship between nominal rigidities and fluctuations, see Bénassy (2002) and (2011), while for an introduction to the New Keynesian approach to monetary policy, see Galí (2008). For a recent explanation of how menu and information costs in price changing may produce nominal rigidities this effect see Alvarez et al. (2011).

the literature on the negative effects of inflation and/or money average growth and volatility on growth.[3] Barlevy (2004) and Comin and Gertler (2006) observe that, over the postwar period, many industrialized countries have experienced significant oscillations between periods of robust growth versus relative stagnation and suggest that these medium-frequency oscillations may, to a significant degree, be the product of business cycle disturbances at the high frequency. Fatàs (2000) shows that empirical evidence from a large sample of countries suggests that there is indeed a correlation between how persistent fluctuations are and the long-term growth rates of GDP. Complementary evidence is found by Benigno et al. (2010) who document the existence of a positive relationship between long-run unemployment and the variance of productivity growth.

There is strong empirical evidence in favour of the procyclicality of R&D expenses (see e.g. Barlevy 2007, Comin and Gertler 2006 and Walde and Woitek 2004). The positive comovement between R&D activity and output is a puzzle for theorists, since innovating has a cost in terms of current production, which is likely to be lower during recessions.[4] A possible explanation for the puzzle is that credit constraints are pervasive and, as a result, R&D has to be financed by current profits (see e.g. Aghion and Saint-Paul 1998, Aghion and Banerjee 2005, Aghion et al. 2010). The different– but by no means alternative– explanation for the puzzle we incorporate in our model is closer to Fatàs (2000), who shows that exogenous cyclical shocks may have persistent effects on growth through the impact they have on aggregate demand. The mechanism we focus on is the following: R&D activity is stronger during expansions, even if its opportunity cost is higher, because its rewards are higher too, as the new goods, over which after the first period of production the innovating firm will lose its monopoly power, will be sold on a bigger scale, as aggregate demand is higher.

Our paper contributes to an expanding theoretical literature that studies the link between the business cycle and growth.[5] However, in this literature, it has been proved difficult to reproduce

[3]Studies based on cross-section data (e.g. Barro 1997 and 2001, Turnovsky and Chattopadhyay 2003), panel data (e.g. Andrés and Hernando 1997, Judson and Orphanides 1999) and time series methods (see Grier and Perry 2000 and Elder 2004) find consistent results. In a multivariate GARCH approach, Grier et al. (2004), Fountas et al. (2006) and Andreou et al. (2008) find a generally negative relationship between volatility of money shocks and output growth for G7 countries, but a positive relationship between growth volatility and output growth.

[4]The so called Schumpeterian approach has particularly stressed that since the opportunity cost of R&D is lower during recessions, these may have a positive impact on growth.

[5]Studies in this literature include Acemoglu and Zilibotti (1997), Jones et al. (2000), Francois and Lloyd-Ellis (2003), Jones et al. (2005), Blackburn (1999), Pelloni (1997), de Heck (1999), Canton (2002), Martin and Rogers (1997), Blackburn and Galindev (2003), Blackburn and Varvarigos (2008), Evans and Kenc (2003), Dotsey and Sarte (2000), Varvarigos (2008), Annicchiarico et al. (2011a, 2011b).

theoretically the negative relationship between volatility and growth suggested by the empirical evidence, because a high volatility will induce more saving and thus more growth unless the degree of risk aversion is unplausibly low.

An ingredient for the generation of this negative relationship are nominal rigidities. In particular, in models with technology *à la* Romer (1986) and one-period preset wages, Blackburn and Pelloni (2004, 2005), Annicchiarico et al. (2011a), show the existence of an unambiguous negative relationship between nominal volatility and growth, while the relationship between real volatility and growth crucially depends on the source of fluctuations and on labour market organization. Wang and Wei (2011) find a negative relationship in the presence of extrinsic uncertainty using an AK model with variable capacity utilization and price setting by monopolistic firms.

A common limit of these papers is that no R&D activity is introduced. This activity is explicitly modelled here. Moreover we study, in turn, the role of wage and price rigidities on the relationship between long-term productivity growth and uncertainty due to different sources of stochastic fluctuations (i.e. technology and money supply). We also solve the model under the assumption that both wages and prices are set prior to the realizations of the shocks on the basis of one-period-optimal contracts. This is done in the context of a simple stochastic OLG endogenous growth model with imperfect competition in goods and labour markets, where money is the only store value and functions as a medium of exchange.

We find that, in general, the relationship between volatility and productivity growth is negative or null when technological uncertainty predominates, depending on whether prices are predetermined or not. When monetary volatility is concerned instead, we find that under nominal rigidities uncertainty tends to always have a negative effect on long-run productivity growth, but that the strength of this channel crucially depends on the source of nominal rigidities. From this point of view, we argue that different price and wage changes frequencies across countries may account for some of the ambiguous findings of the related empirical literature.

In our model the opportunity cost of doing research is higher during expansions, in the sense that the marginal productivity of labour used in research is lower during expansions, (while the wage may be higher), however this opportunity cost effect is more than offset by the demand effect seen above.

The paper is organized as follows. Section two describes the setup of an OLG stochastic monetary growth model with market power. In Section three we study a benchmark case where prices and wages are set when all shocks have already materialized. In Section four we derive the relationship between long-term productivity growth and volatility under the assumption of different sources of nominal rigidities: (i) preset wages, (ii) preset prices, (iii) preset prices and wages. Finally, some concluding comments are offered in Section five.

2. A SIMPLE ENDOGENOUS GROWTH MODEL WITH PROCYCLICAL R&D AND MARKET POWER

There is a continuum of firms indexed by $j \in [0,1]$ operating in a monopolistically competitive market. Productivity is endogenously determined and depends on the R&D activity of firms. The demand side is described by a two-period overlapping generations monetary model. The labour market is characterized by the presence of a monopolistic union setting nominal wages, whereby firms determine the level of employment. Time is discrete and indexed by $t \in \{0, 1, ...\infty\}$.

2.1. Firms

Each firm produces a differentiated good $Y_t(j)$ by combining $L_{Y,t}(j)$ units of labour according to the following technology:

$$Y_t(j) = A_t(j) L_{Y,t}(j)^{\alpha}, \ \alpha \in (0,1), \tag{1}$$

where $A_t(j)$ is firm's j total factor productivity (TFP) given by the stock of knowledge at its disposal. By doing R&D firms can increase their TFP in the current period:

$$A_t(j) = \Omega_t A_{t-1} L_{RD,t}^{\phi}(j), \tag{2}$$

with

$$\Omega_t = \Omega \exp\left(\varepsilon_{\Omega,t} - \frac{\sigma_\Omega^2}{2}\right), \ \varepsilon_{\Omega,t} \sim iid.N\left(0, \sigma_\Omega^2\right), \tag{3}$$

$$\Omega > 1, \ \phi \in (0,1), \ \alpha + \phi < 1,$$

where $L_{RD,t}(j)$ is the quantity of labour inputs devoted to the R&D activity, A_{t-1} is the previous period of publicly available stock of knowledge (that is to say that the productivity of individual firm's R&D activity increases with the stock of existing knowledge). Putting it in another way, equation (2) captures the so called "standing on shoulders" effect of knowledge accumulation, indicating that the productivity of researchers increases with the stock of ideas that have already been discovered. The specification of the exogenous variable Ω_t implies that $E(\Omega_t) = \Omega$ and therefore enables us to capture the pure effect of volatility.[6]

Parameter ϕ is less than one in order to capture the possibility of decreasing returns to labour in R&D.[7] Firm j faces a demand function of the form:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t, \tag{4}$$

where P_t is the aggregate price level, Y_t aggregate production, $P_t(j)$ the price of good j and $\theta > 1$ the elasticity of substitution between any pair of goods. Firm's j will set $L_{Y,t}(j)$ and $L_{RD,t}(j)$ so as to equate their marginal product. Moreover firms are assumed to be symmetric so we can drop the index j. At an optimum we have:

$$L_{RD,t} = \frac{\phi}{\alpha} L_{Y,t}, \tag{5}$$

$$Y_t = \Psi \Omega_t A_{t-1} L_t^{\alpha+\phi}, \tag{6}$$

where $L_t = L_{RD,t} + L_{Y,t}$, $\Psi \equiv [\phi/(\alpha+\phi)]^{\phi} [\alpha/(\alpha+\phi)]^{\alpha}$ and, given L_t, $L_{RD,t} = \phi L_t/(\alpha+\phi)$. The condition for the optimal price is contingent on the timing of its fixing, so we will consider it later. Notice that the marginal productivity of labour used in R&D, i.e. $\phi L_{RD}^{\phi-1} = \phi [\phi L_t/(\alpha+\phi)]^{\phi-1}$ is countercyclical, so the opportunity cost channel stressed by the 'creative destruction approach' is incorporated in the model.

[6] An increase in σ_Ω^2, in fact, implies an increase in the variance of Ω_t, which is found to be $\Omega^2 \left[\exp\left(\sigma_\Omega^2\right) - 1\right]$, leaving its mean unchanged, since $E\left[\exp\left(\varepsilon_{\Omega,t} - \frac{\sigma_\Omega^2}{2}\right)\right] = 1$.

[7] In reality, there could also be an externality due to the duplication of research activity taking place in other firms. This so called "stepping on toes" effect could induce an even stronger curvature of the aggregate knowledge production function with respect to labour.

2.2. Households

The typical household born at time t supplies labour N_t, consumes C_t in period t and $C_{t+1}^{'}$ in period $t+1$. The expected value of lifetime utility is:

$$U_t = E_t\left[\gamma \log C_t + \log C_{t+1}^{'} - (1+\gamma)\frac{1}{v}N_t^v\right], \quad \gamma > 0,\ v > 1, \tag{7}$$

where C_t is a consumption index given by $C_t \equiv \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$ with $C_t(j)$ being the quantity consumed of good j. At the optimum the demand for each good $j \in [0,1]$ is $C_t(j) = (P_t(j)/P_t)^{-\theta} C_t$, where $P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}$. It follows that $\int_0^1 P_t(j)C_t(j) = P_t C_t$.

Households save in the form of money (at time t, denoted by M_t) which is the unique asset of the economy. Their budget constraints in the two periods are given by:

$$P_t C_t + M_t = W_t N_t + \Pi_t, \tag{8}$$

$$P_{t+1} C_{t+1}^{'} = \mu_{t+1} M_t, \tag{9}$$

$$\mu_t = \mu \exp\left(\varepsilon_{\mu,t} - \frac{\sigma_\mu^2}{2}\right),\ \mu > 1,\ \varepsilon_{\mu,t} \sim iid.N\left(0, \sigma_\mu^2\right), \tag{10}$$

where Π_t denotes firms' nominal profits and μ_t is a stochastic multiplicative monetary shock and $\varepsilon_{\mu,t}$ is a monetary innovation.

The representative household will choose C_t and C_{t+1}' to maximize (7) subject to the budget constraints (8) and (9). By using the fact that in general equilibrium all markets must clear, so that $C_t + C_t' = Y_t$ and $M_t = H_t$, with H_t being money supply which evolves as $H_t = \mu_t H_{t-1}$, we have that (9) can be rewritten as $P_t C_t^{'} = M_t$, so that money is entirely held by the old who spend all of it. The consumption function of the young is

$$C_t = \frac{\gamma}{1+\gamma} Y_t, \tag{11}$$

and the condition for the money market equilibrium is

$$\frac{(1+\gamma) M_t}{P_t} = Y_t. \tag{12}$$

3. VOLATILITY AND GROWTH WITH FLEXIBLE PRICES AND WAGES

In this Section we present the remaining equilibrium conditions and model solution under the assumption that both wages and prices are flexible. By 'flexibility' we mean that the current realizations of the shocks are observed when prices and wages are set.

Firm's j will set $P_t(j)$ to maximise current profits $\Pi_t(j) = P_t(j) Y_t(j) - W_t [L_{Y,t}(j) + L_{RD,t}(j)]$ given the wage W_t, the demand for its own good (4) and the technology equations (1) and (2). We have already seen that (5) and (6) will hold. A further optimizing condition dictates that the optimal price will correspond to a markup over the marginal cost

$$P_t = \Theta \frac{W_t L_t}{(\alpha + \phi) Y_t}, \tag{13}$$

again omitting the j index to save on notation and with $\Theta \equiv \theta/(\theta - 1)$. Clearly, when prices are set after the realizations of the shocks the price markup, which we label $\mathcal{MU}^P$ for future reference, is always constant and equal to Θ. Finally, using (6) and (13) we can express labour demand as

$$L_t = \left(\frac{\Theta}{\alpha + \phi} \frac{W_t}{\Psi \Omega_t P_t A_{t-1}} \right)^{-\frac{1}{1-(\alpha+\phi)}}. \tag{14}$$

Coming to the labour market functioning, we adopt a "right to manage" assumption: a monopolistic trade union sets the nominal wage in order to maximise households' expected utility (7), given labour demand (14). As hinted above, we distinguish between the case in which this is done after the current shock is revealed (the "flexible wages" case), and the case in which this is done before ("preset wages" case), presented in the next section. Given (14), at the optimum we have:

$$\frac{W_t}{P_t} = \frac{1}{\alpha + \phi} \frac{(1 + \gamma) L_t^{\upsilon - 1} C_t}{\gamma}, \tag{15}$$

where we have used the fact that in equilibrium $L_t = N_{t.}$.

Notice that market power in the labour market introduces a wedge between the real wage rate, W_t/P_t, and the marginal rate of substitution between leisure and consumption, $[(1 + \gamma)/\gamma] L_t^{\upsilon-1} C_t$. Henceforth, we will refer to this wedge as the wage markup labeled as $\mathcal{MU}^W$. This is constant and equal to $(\alpha + \phi)^{-1}$ when wages are set after all shocks have materialized.

By combining (15) with (14) and using (11), the equilibrium level of employment is then found to be:

$$L_t = \overline{L} \equiv \left[\frac{(\alpha+\phi)^2}{\Theta} \right]^{1/v} . \tag{16}$$

From (16) the equilibrium level of employment is increasing in the elasticity of labour demand $\alpha+\phi$ and in the elasticity of substitution between intermediates θ, i.e. decreasing in the market power of producers. Finally, employment is lower the lower is the Frisch elasticity of labour supply $1/(v-1)$.

Let $g_A \equiv \Omega\left[\phi\overline{L}/(\alpha+\phi)\right]^{\phi}$ denote the deterministic growth rate of productivity,[8] i.e. the growth rate prevailing in the absence of shocks. Due to the fact that productivity growth is a linear function of the technological shocks and is not affected by money shocks, we can immediately infer the following:

PROPOSITION 1. *In the absence of nominal rigidities the equilibrium growth rate of productivity between two consecutive periods and its average are given by:*

$$\frac{A_t}{A_{t-1}} = \Omega_t \left(\frac{\phi}{\alpha+\phi}\overline{L} \right)^{\phi} , \tag{17}$$

$$E\left(\frac{A_t}{A_{t-1}}\right) = g_A \equiv \Omega\left[\phi\overline{L}/(\alpha+\phi)\right]^{\phi} , \tag{18}$$

implying that long-run growth is not affected by shocks volatility.

Proof. Using $L_{RD,t} = \phi L_t/(\alpha+\phi)$ in $A_t = \Omega_t A_{t-1} L_{RD,t}^{\phi}$, given (16), delivers the result (17). Taking expectations, given (3), yields (18). ∎

Summing up, without nominal rigidities, long-run growth is not affected by uncertainty. Producers and unions, in fact, set prices and wages after the shocks have materialized, being so able to keep price and wage markups at the desired optimal levels, i.e. Θ and $(\alpha+\phi)^{-1}$, as implied by equations (13) and (15).

[8] It can be easily shown that by removing producers and unions market power from the economy, the deterministic growth rate of productivity will be $g_A \equiv \Omega\left[\phi/(\alpha+\phi)\right]^{\phi}(\alpha+\phi)^{\phi/v}$, being $L_t = (\alpha+\phi)^{1/v}$ the equilibrium level of employment which differs from (16) by two constant factors of proportionality: $(\alpha+\phi)^{1/v}$ and $(1/\Theta)^{1/v}$, the first one reflecting the effect of monopolistic wage setting and the second one the inefficiency of the monopolistic competition in the final goods market. The presence of market power leads, in fact, to underemployment and low growth.

4. VOLATILITY AND GROWTH WITH NOMINAL RIGIDITIES

In this Section we study, in turn, the case of preset wages, preset prices and preset prices and wages.

4.1. Preset Wages

Consider the case in which in each period monopolistic unions set the nominal wage prior to the realizations of the shocks, so as to maximise the expected utility function of those working in the period (i.e. the young) given firms labour demand (14). The optimal nominal wage set at the beginning of period t is such that

$$W_t = \frac{1}{\alpha + \phi} \frac{(1+\gamma) E_{t-1} L_t^v}{\gamma E_{t-1} \left(\frac{L_t}{P_t C_t} \right)}. \tag{19}$$

In equilibrium the nominal wage, employment and its mean are then seen to be (see the Appendix for the full derivation):

$$W_t = \frac{(1+\gamma)(\alpha+\phi)}{\Theta} \bar{L}^{-1} M_{t-1} \mu \left[\exp(v-1) \frac{\sigma_\mu^2}{2} \right], \tag{20}$$

$$L_t = \overline{L} \frac{\mu_t}{\mu} \exp \left[-(v-1) \frac{\sigma_\mu^2}{2} \right], \tag{21}$$

so that:

$$E(L_t) = \overline{L} \exp \left[-(v-1) \frac{\sigma_\mu^2}{2} \right]. \tag{22}$$

This yields:

Lemma 1. *When wages are preset on the basis of one-period contract before the realizations of the shocks, the nominal wage is increasing in money volatility while equilibrium employment (and its average) is decreasing in it.*

Proof. From inspection of (20) and (21), recalling that $v > 1$. ∎

First notice that a technology shock has offsetting income and substitution effects on working hours, so the optimally chosen contract wage and, as a consequence, employment are not affected by technology shocks or their volatilities.[9] As a contrast, the nominal wage is increasing in money volatility, and, through labour demand, employment (and its expected value) are correspondingly decreasing in it. To understand these effects intuitively consider that, while under certainty the marginal rate of substitution between consumption and leisure is always equal to the real wage (bar the constant monopoly wedge), under uncertainty people will have to work more (so that the marginal rate of substitution goes up) when the real wage is lower and less when the real wage is higher. The way to reduce these deviations is to make them happen around a lower average level of employment (so the increases and decreases in the marginal rate of substitution will be less steep) and therefore set a higher nominal wage. In fact, it can be easily shown that on average the wage markup will be higher, the higher the level of monetary uncertainty (see the Appendix for details):

$$E(\mathcal{MU}_t^W) = \frac{1}{\alpha+\phi}\exp\left(v^2\sigma_\mu^2\right). \tag{23}$$

The presence of the wage friction is then a source of economic inefficiency made worse by volatility. The size of the effect depends on the preference parameter v. The higher is v, in fact, the lower is the Frisch elasticity of labour supply. In the limiting case where $v = 1$, a mean preserving spread increase in money uncertainty will leave expected employment unchanged. Conversely, higher α and ϕ will reduce the positive effect of money volatility on the expected markup. This is because the marginal productivity of labour and, therefore, the real wage will be less influenced by changes in employment, the higher these parameters are.

PROPOSITION 2. *When wages are preset on the basis of one-period contract before the realizations of the shocks, the equilibrium growth rate of productivity and its average are given by:*

$$\frac{A_t}{A_{t-1}} = \Omega_t\left(\frac{\phi}{\alpha+\phi}\overline{L}\right)^{\phi}\left(\frac{\mu_t}{\mu}\right)^{\phi}\exp\left[-\phi\left(v-1\right)\frac{\sigma_\mu^2}{2}\right], \tag{24}$$

$$E\left(\frac{A_t}{A_{t-1}}\right) = g_A\exp\left[-\phi\left(v-\phi\right)\frac{\sigma_\mu^2}{2}\right], \tag{25}$$

Proof. Using $L_{RD,t} = \phi L_t/(\phi+\alpha)$ and (21) into $A_t = \Omega_t A_{t-1} L_{RD,t}^{\phi}$ delivers the result (24). Taking expectations, given (10) and (3), delivers (25). ∎

[9]This result is consistent with the findings of Blackburn and Pelloni (2005).

From (24) we notice that the equilibrium growth rate of productivity is a linear function of Ω_t, as in the previous case with flexible wages. On the other hand, an increase in money supply in period t has now a positive effect on growth, through the effect that a monetary surprise has on aggregate demand and so on employment (21) and TFP growth (since in this setting R&D activity is procyclical). In this context, the effect of money volatility on long-run growth is negative, while the effect of technology volatility is null. Since the rate of technology growth is linear in technology shocks, its expected value is not influenced by their volatility. The negative effect of money volatility is mediated by the effect on employment and, in particular, by the degree of disutility deriving from non-leisure activity, measured by the parameter v, and by the parameter ϕ. However, the effect of ϕ is two-fold: on the one hand a higher value of ϕ has a moderating influence on the wage markup and, therefore, limits the negative effect of money volatility on average employment and so on average productivity growth, which is an increasing function of labour. On the other hand, a higher value of ϕ implies a stronger contribution of labour to innovation and so a more detrimental effect of lower employment to productivity growth.

4.2. Preset Prices

We now introduce nominal rigidities in the form of preset prices, while moving back to the assumption of flexible wages. Producers set prices for period t before observing the realizations of the shocks in order to maximise expected profits. At time $t-1$ firm j will set the price $P_t(j)$ in order to maximise the expected profit, $E_{t-1}\Pi_t(j)$, subject to the technological constraints represented by (1), (2) and (4) and given the wage W_t. First notice that (5) and therefore (6) are still valid, as necessary conditions for optimality. Under symmetry of firms, the optimal pricing decision taken at time $t-1$ is then:

$$P_t = \frac{\Theta E_{t-1} W_t L_t}{(\alpha + \phi) E_{t-1} Y_t}, \tag{26}$$

which now replaces (13), which holds only if prices are flexible. In equilibrium we then have (see the Appendix):

$$P_t = \frac{\overline{L}^{-(\alpha+\phi)}\left(1+\gamma\right)\mu M_{t-1}}{\Psi\Omega A_{t-1}}\exp\left[\frac{\alpha+\phi+v}{\alpha+\phi}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right)\right], \tag{27}$$

$$L_t = \overline{L}\left(\frac{\Omega}{\mu}\frac{\mu_t}{\Omega_t}\right)^{\frac{1}{\alpha+\phi}}\exp\left[-\frac{\alpha+\phi+v}{\left(\alpha+\phi\right)^2}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right)\right], \tag{28}$$

and, taking expectations:

$$E\left(L_t\right) = \overline{L}\exp\left[-\frac{v-1}{\left(\alpha+\phi\right)^2}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right)-\frac{1}{\alpha+\phi}\sigma_\mu^2\right]. \tag{29}$$

This yields:

Lemma 2. *When prices are preset on the basis of one-period contracts before the realizations of the shocks, the price level are increasing in money and technological volatilities, while equilibrium employment (and its mean) are decreasing in them.*

Lemma 3. *From inspection of (27)-(29).*

From (28) employment depends on both the realizations and the variances of the shocks. Since nominal wages are flexible, they will react to all changes in technology and money supply. Positive money shocks will translate into higher employment and positive technology shocks into lower employment. This is because an unexpected monetary expansion has an expansionary effect on demand and so on output and employment. On the other hand, a positive technological innovation increases total factor productivity in (2), but leads to an employment decline, because price rigidities prevent an increase in aggregate demand: the same production will then be obtained with less labour.

A higher level of volatility, whatever its source, is always harmful to employment. To understand why, let us consider the effect of volatility on price markups. While the wage markup will be constant and equal at $(\alpha+\phi)^{-1}$, price markups will now depend on the realizations of the shocks ad on their variances. From the convexity of both the cost function of firms and of the labour disutility function of workers, higher uncertainty in production and employment implies higher expected marginal costs and so a higher price level. On average firms' price markup is found to be (see the Appendix):

$$E\left(\mathcal{MU}_t^P\right) = \Theta \exp\left[\left(\frac{v}{\alpha+\phi}\right)^2 \sigma_\Omega^2 + v\frac{v+\alpha+\phi}{\left(\alpha+\phi\right)^2}\sigma_\mu^2\right], \tag{30}$$

which is clearly higher than its deterministic counterpart Θ.

The effect of volatility is weaker the higher the elasticity of the aggregate production function with respect to labour $(\alpha+\phi)$ and the higher the elasticity of labor supply. It is worth noting that for $v = 1$ a mean preserving spread increase in nominal volatility will still have negative effect on employment contrary to the case of preset wages analyzed in the previous section. The effect of money volatility is stronger than the effect of technology volatility: this is because with money volatility firms will have to produce more (less) just when the real wage is higher (lower), as nominal wages are free to move procyclically: this will increase the convexity of the firms' cost function (in general equilibrium that is when taking into account the reaction of the wage). With technological volatility real wages move countercyclically which flattens the cost curve with respect to the case of money volatility.

Given the above results we have:

Proposition 3. *When prices are preset on the basis of one-period contract before the realizations of the shocks, the equilibrium growth rate of productivity and its average are given by:*

$$\frac{A_t}{A_{t-1}} = \Omega_t\left(\frac{\phi}{\alpha+\phi}\overline{L}\right)^{\phi}\left(\frac{\Omega}{\mu}\frac{\mu_t}{\Omega_t}\right)^{\frac{\phi}{\alpha+\phi}} \exp\left[-\phi\frac{\alpha+\phi+v}{\left(\alpha+\phi\right)^2}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right)\right], \tag{31}$$

$$E\left(\frac{A_t}{A_{t-1}}\right) = g_A \exp\left[-\phi\frac{2\alpha+\phi+v}{\left(\alpha+\phi\right)^2}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right)\right]. \tag{32}$$

Proof. Using $L_{RD,t} = \phi L_t/(\phi+\alpha)$ and (28) into $A_t = \Omega_t A_{t-1} L_{RD,t}^{\phi}$ gives us (31). Taking expectations given (3) and (10), gives us (32). ∎

Recalling the results of Proposition 2, we notice that nominal volatility is more detrimental to productivity growth under preset prices than under preset wages, as $\phi\left(v-\phi\right) < \phi\frac{2\alpha+\phi+v}{(\alpha+\phi)^2}$. Moreover we have a negative effect of technology volatility, which was absent in the previous case. All these effects work through employment which is inefficiently low as a result of price rigidities.

4.3. Preset Prices and Wages

Finally, we solve the model of Section 2 under the assumption that both prices and wages are set on the basis of one-period contract before observing the shocks. In this case the optimal price and wage decided at time $t-1$ are

$$P_t = W_t \frac{\Theta}{\alpha+\phi} \frac{E_{t-1}L_t}{E_{t-1}Y_t}, \tag{33}$$

and

$$W_t = \frac{P_t}{\alpha+\phi} \frac{(1+\gamma) E_{t-1} L_t^v}{\gamma E_{t-1}\left(\frac{L_t}{C_t}\right)}. \tag{34}$$

The above equations now replace (13) and (15), implying that the real wage is now preset at time $t-1$. In equilibrium prices, wages, employment and its mean are (see the Appendix for the full derivation):

$$P_t = \frac{\overline{L}^{-(\alpha+\phi)} (1+\gamma) \mu M_{t-1}}{\Psi \Omega A_{t-1}} \exp\left[\frac{\alpha+\phi+v}{\alpha+\phi}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right) - B\frac{\sigma_\mu^2}{2}\right], \tag{35}$$

$$W_t = \frac{\alpha+\phi}{\Theta} (1+\gamma) \overline{L}^{-1} \mu M_{t-1} \exp\left[\frac{v-1}{(\alpha+\phi)^2}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right) + \frac{1-\alpha-\phi}{v(\alpha+\phi)}\sigma_\mu^2\right], \tag{36}$$

$$L_t = \overline{L}\left(\frac{\mu_t \Omega}{\mu \Omega_t}\right)^{\frac{1}{\alpha+\phi}} \exp\left[-\frac{\alpha+\phi+v}{(\alpha+\phi)^2}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right) + \frac{B}{\alpha+\phi}\frac{\sigma_\mu^2}{2}\right], \tag{37}$$

$$E(L_t) = \overline{L}\exp\left[-\frac{v-1}{(\alpha+\phi)^2}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right) - \frac{1-\alpha-\phi}{v(\alpha+\phi)}\sigma_\mu^2\right], \tag{38}$$

where $B \equiv 2\{1-[1-(\alpha+\phi)]/v\} > 0$.

The above expressions deliver the following result:

Lemma 4. *When both prices and wages are preset on the basis of one-period contract before the realizations of the shocks an increase in money and technological uncertainty has a positive* *effect on the equilibrium price and wage levels and a negative one on employment and its mean.*

Proof. From inspection of (35), (36) and (37) and by noting that $-[1+v/(\alpha+\phi)]+B<0$. ∎

Notice the effect on nominal wages is stronger for nominal uncertainty than for real uncertainty. To gain intuition as to why we observe the above results consider the markups. We have:

$$E\left(\mathcal{MU}^P\right) = \Theta \exp\left[\frac{1}{(\alpha+\phi)^2}\sigma_\Omega^2 + \frac{1-\alpha-\phi}{(\alpha+\phi)^2}\sigma_\mu^2\right], \tag{39}$$

and

$$E\left(\mathcal{MU}^W\right) = \frac{1}{\alpha+\phi}\exp\left[\frac{v(v-1)}{(\alpha+\phi)^2}\sigma_\Omega^2 + \frac{v(v+\alpha+\phi-1)}{(\alpha+\phi)^2}\sigma_\mu^2\right]. \tag{40}$$

Derivations are in the Appendix. It can be easily seen that, again, both price and wage markups are higher on average than without uncertainty. However the price markup is lower than when nominal wages are flexible, so to some extent, the two kinds of frictions offset each other. This is because with fixed nominal wages, the elasticity of labour supply is no longer relevant as regards the movements of the real wage over the business cycle (which is of course constant). Higher volatility employment implies higher expected marginal costs for firms and therefore a higher price level. However, the convexity of the cost function of firms is now reduced by the fact that nominal wages cannot move procyclically, so that firms' marginal costs tend to be more stable. Conversely, the wage markup is higher than when prices are flexible. First of all, since a technology shock does not influence the real wage but does push employment up or down, to minimize the deviations of the marginal rate of substitution from its (privately) optimal level, the average labour supply moves down in the face of increased technological uncertainty, an effect that was absent with predetermined wages and flexible prices. Moreover, the effect of money uncertainty on the wage markup is also higher than with flexible prices, (just by comparing (23) and (40) and noticing that $\frac{v(v+\alpha+\phi-1)}{(\alpha+\phi)^2} > v^2$). In fact, with preset prices an increase in aggregate demand will not be (partially) absorbed by inflation, so employment will fluctuate more. Unions will then find it optimal to set a higher nominal wage, so that the deviations between the marginal rate of substitution between consumption and leisure and the real wage are smaller.

From the above analysis, this result immediately follows:

Proposition 4. *When both prices and wages are preset on the basis of one-period contract* *before the realizations of the shocks, the equilibrium growth rate of productivity and its average are given by:*

$$\frac{A_t}{A_{t-1}} = \Omega_t \left(\frac{\phi}{\alpha + \phi} \overline{L} \right)^{\phi} \left(\frac{\mu_t \Omega}{\mu \Omega_t} \right)^{\frac{\phi}{\alpha+\phi}} \exp \left[-\phi \frac{\alpha + \phi + v}{(\alpha + \phi)^2} \left(\frac{\sigma_\mu^2}{2} + \frac{\sigma_\Omega^2}{2} \right) + \phi \frac{B}{\alpha + \phi} \frac{\sigma_\mu^2}{2} \right], \qquad (41)$$

and

$$E\left(\frac{A_t}{A_{t-1}} \right) = g_A \exp \left[-\phi \frac{2\alpha + \phi + v}{(\alpha + \phi)^2} \left(\frac{\sigma_\mu^2}{2} + \frac{\sigma_\Omega^2}{2} \right) + \phi \frac{B}{\alpha + \phi} \frac{\sigma_\mu^2}{2} \right]. \qquad (42)$$

Proof. Using $L_{RD,t} = \phi L_t / (\phi + \alpha)$ and (37) into $A_t = \Omega_t A_{t-1} L_{RD,t}^{\phi}$ delivers (41). the first result. Then take expectations, given (3) and (10) and note that $-\frac{2\alpha+\phi+v}{\alpha+\phi} + B < 0$. ∎

From (42) it should be noted that the coefficient on real volatility is as in (32), since it is only through price stickiness that technology uncertainty reflects on employment and hence on growth. As regards money volatility, we note that the the effect of nominal uncertainty is lower than when only prices are sticky by virtue of the more muted response of real marginal costs to money shocks

5. CONCLUSIONS

In this paper we construct an analytically solvable endogenous monetary model with growth driven by R&D to study the effect of wage and price rigidities on the relationship between long-term productivity growth and short-term volatility. We find that both nominal and real uncertainty may be harmful to secular growth, as suggested by the empirical evidence.

In the model, R&D is more profitable the higher the level of economic activity. This explains its observed procyclicality. On its turn, the level of economic activity is reduced by market power by firms and workers, as this inserts a wedge between the marginal product of labour and the marginal rate of substitution between leisure and consumption. We show that price and wage rigidities induce fluctuations in this wedge making it higher on average than its deterministic counterpart. We analyse how price and nominal wage distortions interact with each other in propagating real and nominal shocks. In particular we show that the effects on growth depend critically on the source of uncertainty and on the type of nominal rigidities.

We have derived our results in a very simple model in order to obtain analytically solvable solutions and to be able to better understand the economic mechanisms underlying the observed results. Future research should be oriented to investigate the role of different and more complex sources of wage and price rigidities on the relationship between uncertainty and long-run growth in more realistic models, allowing for capital accumulation and serially correlated shocks.

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APPENDIX

Derivation of (20)-(21)

To get the equilibrium nominal wage, after expressing C_t in terms of Y_t using (11), eliminate the nominal wage from (19) using (13). Simplifying, one obtains:

$$E_{t-1}L_t^v = \frac{1}{\Theta}\left(\alpha+\phi\right)^2. \tag{43}$$

Now, from (13) using (12) to eliminate Y_t and P_t, one gets:

$$L_t = \frac{\left(1+\gamma\right)\left(\alpha+\phi\right)}{\Theta}\frac{M_t}{W_t}. \tag{44}$$

This, combined with (43), gives us:

$$W_t = \frac{\left(1+\gamma\right)\left(\alpha+\phi\right)}{\Theta}\bar{L}^{-1}\left(E_{t-1}M_t^v\right)^{1/v}, \tag{45}$$

where we have used the fact that W_t is a control variable at time $t-1$. Substituting back in (44):

$$L_t = \bar{L}\frac{M_t}{\left(E_{t-1}M_t^v\right)^{1/v}}. \tag{46}$$

Recalling that $M_t = \mu_t M_{t-1}$ and (10) and exploiting the relationship between the mean of a lognormal variable and the moments of the underlying normal we get, from the last two expressions, (20) and (21) in the text.

Derivation of (23)

By definition, the wage markup $\mathcal{MU}_t^W$ is equal to

$$\mathcal{MU}_t^W = \frac{\frac{W_t}{P_t}}{\frac{1+\gamma}{\gamma}L_t^{v-1}C_t}. \tag{47}$$

Substituting (11), (12) in (47) to eliminate C_t and P_t, then using (20) and (21), and simplifying yield:

$$\mathcal{MU}_t^W = \frac{1}{\alpha+\phi} \frac{\exp v\left(v-1\right)\frac{\sigma_\mu^2}{2}}{\left[\exp\left(\varepsilon_{\mu,t} - \frac{\sigma_\mu^2}{2}\right)\right]^v}, \tag{48}$$

from which by taking expectations one gets (23).

Derivation of (27)-(28)

To find the reduced form expression for prices we proceed as follows. Consider equation (15) and express C_t and L_t in terms of Y_t, using respectively (11) and (6), to get:

$$W_t L_t = \frac{1}{\alpha+\phi}\left(\frac{\Psi^{-1}}{\Omega_t A_{t-1}}\right)^{\frac{v}{\alpha+\phi}} Y_t^{\frac{v+\alpha+\phi}{\alpha+\phi}} P_t. \tag{49}$$

We then plug the expression so found for $W_t L_t$ in (26). This gives us:

$$P_t = \frac{\Theta}{\left(\alpha+\phi\right)^2 \Psi A_{t-1}} \frac{E_{t-1}\left[\left(\frac{1}{\Omega_t}\right)^{\frac{v}{\alpha+\phi}} Y_t^{\frac{v+\alpha+\phi}{\alpha+\phi}} P_t\right]}{E_{t-1} Y_t}, \tag{50}$$

or since P_t is a control variable at time $t-1$,

$$E_{t-1} Y_t = \frac{\Theta}{\left(\alpha+\phi\right)^2 \Psi A_{t-1}} E_{t-1}\left(\frac{1}{\Omega_t}\right)^{\frac{v}{\alpha+\phi}} Y_t^{\frac{v+\alpha+\phi}{\alpha+\phi}}. \tag{51}$$

Again using (12) to express Y_t in terms of M_t and solving for P_t we obtain:

$$P_t = \frac{\bar{L}^{-(\alpha+\phi)}\left(1+\gamma\right)}{\Psi A_{t-1}} \left[\frac{E_{t-1}\left(\frac{1}{\Omega_t}\right)^{\frac{v}{\alpha+\phi}} \left(M_t\right)^{\frac{v+\alpha+\phi}{\alpha+\phi}}}{E_{t-1} M_t}\right]^{\frac{\alpha+\phi}{v}}. \tag{52}$$

Finally, exploiting the relationships between the moments of normal and lognormal variables we get the expression for P_t in the text (27).

To get (28), use (12) to express Y_t in terms of M_t, then substitute in (6) and finally plug (27).

Derivation of (30)

By definition, the price markup $\mathcal{MU}_t^P$ is equal to P_t over the nominal marginal cost, that is

$$\mathcal{MU}_t^P = \frac{P_t}{\frac{1}{\alpha+\phi}\frac{L_t}{Y_t}W_t}. \tag{53}$$

Substitute the wage equation (15) in (53), simplify, recalling (11), and then plug (28). This yields

$$\mathcal{MU}_t^P = \Theta \left\{ \frac{\exp\left[2\frac{\sigma_\mu^2}{2} + \frac{v}{\alpha+\phi}\left(\frac{\sigma_\mu^2}{2} + \frac{\sigma_\Omega^2}{2}\right)\right]}{\exp\left(\varepsilon_{\mu,t} - \varepsilon_{\Omega,t}\right)} \right\}^{\frac{v}{\alpha+\phi}}, \tag{54}$$

where we have used (3) and (10). From (54) by taking expectations one gets (30).

Derivation of (35)-(37)

First, combine (33) with (34) to eliminate W_t/P_t

$$E_{t-1}\left(\frac{L_t}{C_t}\right)(E_{t-1}Y_t) = \frac{\Theta}{(\alpha+\phi)^2}\frac{1+\gamma}{\gamma}\left(E_{t-1}L_t^v\right)\left(E_{t-1}L_t\right). \tag{55}$$

As usual, express C_t and L_t in terms of Y_t, using (11) and (6) and then use the LM (12). Simplifying one obtains the following expression for the price preset at time t:

$$P_t = \frac{\overline{L}^{-(\alpha+\phi)}(1+\gamma)}{\Psi A_{t-1}} \left\{ \frac{\left[E_{t-1}\left(\frac{M_t}{\Omega_t}\right)^{\frac{1}{\alpha+\phi}}\right]\left[E_{t-1}\left(\frac{M_t}{\Omega_t}\right)^{\frac{v}{\alpha+\phi}}\right]}{(E_{t-1}M_t)\left[E_{t-1}\left(\frac{1}{M_t}\left(\frac{M_t}{\Omega_t}\right)^{\frac{1}{\alpha+\phi}}\right)\right]} \right\}^{\frac{\alpha+\phi}{v}}. \tag{56}$$

Finally, using (3) and (10) and exploiting the relationships between the moments of normal and lognormal variables we get the expression for P_t in the text (35).

To find (37), substitute (35) into LM (12) to determine Y_t and substitute it in (6).

Finally, (36) can be obtained from (33), expressing Y_t in terms of L_t using (6), substituting (35), (37) and taking expectations.

Derivation of (39)-(40)

The expected price markup (39) can be derived as follows. Use the LM (12) to express Y_t in terms of M_t and substitute in (53). Given the result, use (37) and (36) to get:

$$\mathcal{MU}_t^P = \Theta \frac{\left(\frac{\mu_t}{\mu}\right)^{1-\frac{1}{\alpha+\phi}} \left(\frac{\Omega_t}{\Omega}\right)^{\frac{1}{\alpha+\phi}}}{\exp\left[-\frac{1}{(\alpha+\phi)^2}(\alpha+\phi+1)\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right)+\frac{1}{\alpha+\phi}\sigma_\mu^2\right]}, \tag{57}$$

and then take expectations.

Finally, substituting (11), (12) in (47) to eliminate C_t and P_t, then using (37) and (37), simplifying yields:

$$\mathcal{MU}_t^W = \frac{1}{\alpha+\phi}\frac{\mu}{\mu_t}\frac{\exp\left\{\frac{v-1}{(\alpha+\phi)^2}\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right)+\frac{1}{\alpha+\phi}\frac{1}{v}\left[1-(\alpha+\phi)\right]\sigma_\mu^2\right\}}{\left(\frac{\mu_t\Omega}{\mu\Omega_t}\right)^{\frac{v-1}{\alpha+\phi}}\exp\left[-\frac{v-1}{\alpha+\phi}\left(\frac{v+\alpha+\phi}{\alpha+\phi}\right)\left(\frac{\sigma_\mu^2}{2}+\frac{\sigma_\Omega^2}{2}\right)+\frac{v-1}{\alpha+\phi}B\frac{\sigma_\mu^2}{2}\right]}, \tag{58}$$

from which, taking expectations gives (40).