Imperfect Substitutes for Perfect Complements: Solving the Anticommons Problem*

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Abstract

An integrated monopoly, where two complements forming a composite good are offered by a single firm, is typically welfare superior to a complementary monopoly. This is "the tragedy of the anticommons". We analyze the robustness of such result when competition is introduced for one or both complements. Particularly, competition in only one of the two markets may be welfare superior to an integrated monopoly if and only if the substitutes differ in their quality so that, as their number increases, average quality and/or quality variance increases. Then, absent an adequate level of product differentiation, favoring competition in some sectors while leaving monopolies in others may be detrimental for consumers and producers alike. Instead, competition in both markets may be welfare superior if goods are close substitutes and their number in each market is sufficiently high, no matter the degree of product differentiation.

JEL Codes: D43, K21, L13, L41.

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1 Introduction

A complementary monopoly is characterized by the presence of multiple sellers, each producing a complementary good. It has been known for quite some time in the literature that such market structure is worse than an integrated monopoly, in which a single firm offers all complements (Cournot, 1838). In fact, a firm producing a single good takes into account only the impact of a price raise on its own profits, without considering the negative externality imposed on the sellers of other complementary goods[1]. As a consequence, prices will be higher with separate producers than with an integrated monopolist, generating a lower consumer surplus.[2]

The complementary monopoly problem is also known as "the tragedy of the anticommons", in analogy with its mirror case, the more famous "tragedy of the commons" and has been applied in the legal literature to issues related to the fragmentation of physical and intellectual property rights.[3] Strictly speaking, such literature is applicable only to situations in which the markets for all complementary goods are monopolies. However, pure monopolies are quite rare in the real world. More often, each complement is produced in an oligopolistic setting. Consider, for instance, software markets, where each component of a system is produced by many competing firms, such as Microsoft, Apple, Unix and Linux for operating systems; Microsoft, Google, Apple, Mozilla for Internet browsers, and so on. Similarly, consider the market for photographic equipment, in which both camera bodies and lenses are produced by many competing companies (Nikon, Canon, Olympus, Pentax, etc.), some of which are active only in the market for lenses (Tamron, Sigma, Vivitar). In such cases, an integrated market structure may reduce the extent of the tragedy on the one hand, while lowering welfare because of reduced competition on the other.

The case of software markets is particularly relevant in this respect. In the last ten years, some important antitrust cases, both in the United States and in Europe, have brought to the attention of the economics profession the potential tradeoff between competition and the tragedy of the anticommons. For instance, in the Microsoft case, the American Court of Appeals ordered the firm to divest branches of its business other than operating systems, creating a new company dedicated to application development. The break-up (later abandoned) would have created two firms producing complementary goods, with the likely result of increasing prices in the market. However, far from being unaware of the potential tragedy of the anticommons, Judge Jackson motivated his decision with the need to reduce the possibility for Microsoft to engage in limit pricing, thus deterring entry. Separation would have facilitated entry and favoured competition, possibly driving prices below pre-separation levels.[4] A similar economic argument motivated

[1] The quantity demanded would be reduced for everyone, but each seller benefits fully of an increase in its own price.

[2] Complementary monopoly is similar to the problem of double marginalization in bilateral monopoly, with the important difference that here each monopolist competes "side by side", possibly without direct contacts with the others. In bilateral monopoly, the "upstream" monopolist produces an input that will be used by the "downstream" firm, who is then a monopsonist for that specific input (see Machlup and Taber, 1960).

[3] For an application to property rights, see Heller (1998), Buchanan and Yoon (2000) and Parisi (2002).

[4] United States v. Microsoft Corp., 97 F. Supp. 2d. 59 (D.D.C. 2000). See Gilbert and Katz (2001) for a thorough analysis of the Microsoft case.

the European Commission's Decision over the merger between General Electric and Honeywell.[5] In such case, the EC indicated that the post-merger prices would be so low as to injure new entrants, so that a merger would reduce the number of potential and actual competitors in both markets.[6]

Both these decisions indicate that separation may not be an issue (and may even be welfare improving) if the post-separation market configuration is not a complementary monopoly in the Cournot's sense, i.e., the market for each complement is characterized by competition. The initially higher prices due to the tragedy may in fact encourage entry in the market and, if competition increases sufficiently, the resulting market structure may yield lower prices and higher welfare than in the initial integrated monopoly. The question then is how much competition is needed in the supply of each complement in order to obtain at least the same welfare as in the original monopoly.

Investigating the impact of competition on welfare when complementary goods are involved, Dari-Mattiacci and Parisi (2007) note that, when n perfect complements are bought together by consumers and firms compete *à la* Bertrand, two perfect substitutes for $n-1$ complements are sufficient to guarantee the same social welfare experienced when an integrated monopolist sells all n complements. In fact, all competitors in the $n-1$ markets price at marginal cost, thus allowing the monopolist in the n-th market to extract the whole surplus, fixing its price equal to the one that would be set by an integrated monopolist for the composite good. Therefore, the negative externality characterizing a complementary monopoly disappears and the tragedy of the anticommons is solved by competition.

Our analysis maintains this framework when it considers perfect complements but then extends it in several directions. First, differently from previous literature, the competing goods are both imperfect substitutes[7] and vertically differentiated. Second, we consider the presence of substitutes in all components' markets.

Particularly, we consider two perfect complements, proving first that, if one complementary good is still produced in a monopolistic setting and if competition for the other complement does not alter the average quality in the market, an integrated monopoly remains welfare superior to more competitive market settings. In fact, with imperfect substitutability the competing firms retain enough market power as to price above their marginal cost, so that the monopolist in the first market is not able to fully extract the surplus enjoyed by consumers. As a result, the equilibrium prices of the composite goods remain higher than in an integrated monopoly, implying that favoring competition in some sectors while leaving monopolies in others may actually be detrimental for consumers. A competitive setting may still be welfare superior, but only if the substitutes of the complementary good differ in their quality, so that average quality and/or quality variance increase as their number increases.

[5] See European Commission Decision of 03/07/2001, declaring a concentration to be incompatible with the common market and the EEA Agreement Case, No. COMP/M.2220 - General Electric/Honeywell.

[6] On the possibility that an integrated monopolist engages in limit pricing to deter entry, see Fudenberg and Tirole (2000).

[7] Imperfect substitutability in this case means that the cross-price elasticity is lower than own-price elasticity.

Results change when competition is introduced in the supply of both components. In this case we find that the tragedy may be solved for a relatively small number of competing firms in each sector provided that goods are sufficiently close substitutes. Not surprisingly, the higher the degree of substitutability and the number of competitors in one sector, the more concentrated the remaining sector can be and still yield a higher consumer surplus.

The welfare loss attached to a complementary monopoly has been analyzed, among others, by Economides and Salop (1992), who present a generalized version of the Cournot complementary monopoly in a duopolistic setting. Differently from our contribution, however, their model does not consider quality differentiation, so that the tragedy always prevails whenever goods are not close substitutes. Moreover, they don't study if and how the tragedy can be solved when the number of substitutes for each complement increases.[8]

McHardy (2006) demonstrates that breaking up producing complementary goods may lead to substantial welfare losses. However, if the break-up stops limit-pricing practices by the previously merged firm, even a relatively modest degree of post-separation entry may lead to higher welfare than an integrated monopoly. He assumes a setting in which firms producing the same component compete *à la* Cournot, whereas competition is *à la* Bertrand among complements (i.e., among sectors). Differently from McHardy (2006), we analyze the impact of complementarities and entry in a model in which all firms compete both intra and inter layer and in such framework we also study the impact of product differentiation and imperfect substitutability.[9]

The paper is organized as follows. Section 2 introduces the model when one sector is a monopoly and presents the benchmark cases of complementary and integrated monopoly. Section 3 analyzes the impact of competition on welfare when one complement is produced by a monopolist while Section 4 extends the model considering competition in the markets for all complements. Section 5 concludes. Appendix A contains some technical material while Appendix B contains the proofs of the Lemmas and Propositions.

2 The Model

Consider a composite good (a system) consisting of two components, A and B. The two components are perfect complements and are purchased in a fixed proportion (one to one for simplicity). Initially, we assume that complement A is produced by a monopolist, whereas complement B

[8] Gaudet and Salant (1992) study price competition in an industry producing perfect complements and prove that welfare-improving mergers may fail to occur endogenously. Tan and Yuan (2003) consider a market in which two firms sell imperfectly substitutable composite goods consisting of several complementors. They show that firms have the incentive to divest along complementary lines, because the price raise due to competition among producers of complements counters the downward pressure on prices due to Bertrand competition in the market for imperfect substitutes. Alvisi et al. (2011) analyze break-ups of integrated firms in oligopolistic complementary markets when products are vertically differentiated, focusing in particular on the role of quality leadership. When an integrated firm is the quality leader for all complements, divestitures lower prices, enhancing consumer surplus. When quality leadership is shared among different integrated firms, "disintegrating" them may lead to higher prices, as only in this second case there exists a tragedy of the anticommons.

[9] Previous literature on the relationship between complementary goods and market structure is scanty and deals mostly with bundling practices. See Matutes and Regibeau (1988), Anderson and Leruth (1993), Denicolò (2000), Nalebuff (2004).

is produced by n oligopolistic firms.[10] Marginal costs are the same for all firms and are normalized to zero.[11] Firms compete by setting prices. We also assume full compatibility among components, meaning that the complement produced by the monopolist in sector A can be purchased by consumers in combination with any of the n versions of complement B. We make this assumption because we are interested in the effect of competition on the pricing strategies of the firms operating in the various complementary markets. If firms could restrict compatibility, competition may be limited endogenously (for instance, the monopolist could allow combination only with a subset of producers in sector B) and the purpose of our analysis would be thwarted.[12] Finally, we assume that the n systems of complementary goods have different qualities and that consumers perceive them as imperfect substitutes.[13]

More specifically, the representative consumer has preferences represented by the following utility function, quadratic in the consumption of the n available systems and linear in the consumption of all the other goods (as in Dixit, 1979, Beggs, 1994):

$$U(q,I)=\sum_{j=1}^{n}\alpha_{1j}q_{1j}-\frac{1}{2}\left[\beta\sum_{j=1}^{n}q_{1j}^{2}+\gamma\sum_{j=1}^{n}q_{1j}\left(\sum_{s\neq j}q_{1s}\right)\right]+I \tag{1}$$

where I is the total expenditure on other goods different from the n systems, $q=[q_{11},q_{12},..,q_{1n}]$ is the vector of the quantities consumed of each system and q_{1j} represents the quantity of system $1j$, $j=1,....,n$, obtained by combining q_{1j} units of component A purchased from the monopolist, indexed by the number 1 (component $A1$), and $q_{Bj}=q_{1j}$ units of component B purchased from the j-th firm in sector B (component Bj).[14]Also, $\alpha=(\alpha_{11},\alpha_{12},..,\alpha_{1n})$ is the vector of the qualities of each system (with α_{1j} representing the quality of system $1j$, $(j=1,....,n)$, γ measures the degree of substitutability between any couple of systems, $\gamma\in[0,1]$, and β is a positive parameter. The representative consumer maximizes the utility function (1) subject to a linear budget constraint of the form $\sum_{j=1}^{n}p_{1j}q_{1j}+I\leq M$, where

$$p_{1j}=p_{A1}+p_{Bj},\quad j=1,....,n \tag{2}$$

is the price of system $1j$ (expressed as the sum of the prices of the single components set by firm 1 in sector A and firm j in sector B, respectively) and M is income.

[10]We will remove this assumption later and consider a market configuration in which n_1 firms produce complement A, whereas n_2 firms produce complement B.

[11]This assumption is with no loss of generality, because results would not change for positive, constant marginal costs (see Economides and Salop, 1992).

[12]The assumption of perfect compatibility is common to many contributions in the literature on complementary markets, see Economides and Salop (1992), McHardy (2006), Dari-Mattiacci and Parisi (2007).

[13]This implies that the consumption possibility set consists of n imperfectly substitutable systems. Later on, when we consider n_1 components in sector A, consumers will have the opportunity to combine each of these components with any of the n_2 complements produced in Sector B. We would then have $n_1\times n_2$ imperfectly substitutable systems in the market.

[14]Note that when referring to a particular system, we use a couple of numbers indicating the two firms in sector A and B, respectively, selling each component of such system. When referring instead to separate components, we use a couple of one letter and one number, the first indicating the sector (the component) and the second the particular firm selling it. This might appear redundant for $A1$ when component A is sold by a monopolist, but it will become useful when we introduce competition in sector A.

The first order condition determining the optimal consumption of system $1k$ is

$$\frac{\partial U}{\partial q_{1k}} = \alpha_{1k} - \beta q_{1k} - \gamma \sum_{j \neq k} q_{1j} - p_{1k} = 0 \tag{3}$$

Summing (3) over all firms in sector B, we obtain the demand for system $1k$

$$q_{1k} = \frac{(\beta + \gamma(n-2))(\alpha_{1k} - p_{A1} - p_{Bk}) - \gamma \left(\sum_{j \neq k} \alpha_{1j} - (n-1)p_{A1} - \sum_{j \neq k} p_{Bj} \right)}{(\beta - \gamma)\,(\beta + \gamma(n-1))} \tag{4}$$

Using (4), we sum the demands of all firms in sector B to obtain the total market size

$$Q = \sum_{j=1}^{n} q_{1j} = \frac{\sum_{j=1}^{n} (\alpha_{1j} - p_{Bj}) - np_{A1}}{\beta + \gamma\,(n-1)} \tag{5}$$

Following Shubik and Levitan (1980), we set

$$\beta = n - \gamma(n-1) > 0 \tag{6}$$

to prevent changes in γ and n to affect Q, so that, substituting such expression into (5), the normalized market size becomes

$$Q = \bar{\alpha} - \bar{p}_B - p_{A1} \tag{7}$$

where $\bar{\alpha} = \frac{\sum_{j=1}^{n} \alpha_{1j}}{n}$ is the average quality of the n available systems and $\bar{p}_B = \frac{\sum_{j=1}^{n} p_{Bj}}{n}$ is the average price in the market for the second component.[15]

Note that component $A1$ is part of all the n systems, so that (7) also represents the demand function for the monopolist in sector A. Its profit can then be written as $\Pi_{A1} = p_{A1}Q = (\bar{\alpha} - \bar{p}_B)\,p_{A1} - p_{A1}^2$, whereas the profit of a single producer of component B is $\Pi_{Bk} = p_{Bk} \cdot q_{Bk}$, where $q_{Bk} = q_{1k}$ is given in (4). Bertrand equilibrium prices for the monopolist $A1$ and for the k-th oligopolist are, respectively

$$p_{A1}^M = \frac{\bar{\alpha}(n-\gamma)}{n(3-\gamma) - 2\gamma} \tag{8}$$

$$p_{Bk}^M = \frac{\bar{\alpha} n(1-\gamma)}{n(3-\gamma) - 2\gamma} + \frac{n(\alpha_{1k} - \bar{\alpha})}{2n - \gamma} \tag{9}$$

where the superscript M stands for "monopoly in sector A". Note first that p_{A1}^M is increasing in $\bar{\alpha}$. In fact, $A1$ is part of all systems, so that an increase in their average quality increases the representative consumer's willingness to pay for them and allows the monopolist to set a higher price p_{A1}^M, which also depends positively on the number n of systems, and on the

[15] The second order conditions for the maximization of $U(q, I)$ require $\gamma \leq \beta$, i.e., $\gamma < 1$.

degree of substitutability γ.[16] As we will show below, the increase in competition in the market of the second component (due either to a greater number of firms or to a higher degree of substitutability) reduces oligopolistic prices. Then, as γ or n increases, *ceteris paribus*, the monopolist in sector A is able to extract a bigger share of consumer surplus setting a higher p_{A1}^{M}.[17] Not surprisingly, from (9), producers of below-average quality charge lower than average prices (since $(\alpha_{1k} - \bar{\alpha}) < 0$), whereas the opposite is true for producers of above-average quality. However, quality "premiums and discounts" cancel out on average. In fact, the average price in the market for the second component is

$$\bar{p}_B = \frac{\sum_{k=1}^{n} p_{Bk}^{M}}{n} = \frac{\bar{\alpha}n(1-\gamma)}{n(3-\gamma)-2\gamma} \tag{10}$$

Combining (8) and (9), the equilibrium price of system $1k$ is

$$p_{1k}^{M} = p_{A1}^{M} + p_{Bk}^{M} = \frac{(n(2-\gamma)-\gamma)\bar{\alpha}}{n(3-\gamma)-2\gamma} + \frac{n\left(\alpha_{1k}-\bar{\alpha}\right)}{2n-\gamma} \tag{11}$$

so that, the average system price becomes

$$\bar{p}_{1k}^{M} = p_{A1}^{M} + \bar{p}_B = \frac{(n(2-\gamma)-\gamma)\bar{\alpha}}{n(3-\gamma)-2\gamma} \tag{12}$$

Finally, using (4), (8) and (9), we derive the equilibrium quantities

$$q_{1k}^{M} = \frac{\bar{\alpha}(n-\gamma)}{n(n(3-\gamma)-2\gamma))} + \frac{(\alpha_{1k}-\bar{\alpha})(n-\gamma)}{n(2n-\gamma)(1-\gamma)} \tag{13}$$

We are now ready to compute profits and consumer welfare. Given (8) and (4), the monopolist's profits in sector A are equal to

$$\Pi_{A1}^{M} = p_{A1}^{M}\sum_{j=1}^{n} q_{1j}^{M} = \frac{\bar{\alpha}^2(n-\gamma)^2}{(n(\gamma-3)+2\gamma)^2} \tag{14}$$

whereas, after some algebraic manipulation (reported in Appendix A), the $k-$th oligopolist's profit is

$$\Pi_{Bk}^{M} = t\left(q_{1k}^{M}\right)^2 = \frac{n^2(1-\gamma)}{(n-\gamma)}\left(\frac{\bar{\alpha}(n-\gamma)}{n(n(3-\gamma)-2\gamma)} + \frac{(\alpha_{1k}-\bar{\alpha})(n-\gamma)}{n(2n-\gamma)(1-\gamma)}\right)^2, \tag{15}$$

[16] This result is not surprising in our setting because an increase in average quality comes at no cost, particularly in sector B. If we assume instead that a system's quality can be increased only through a costly investment by the firm producing component B for that system, then conclusions might be less obvious. In fact, even in the symmetric case, where α is the same for all systems, increasing average quality implies higher costs for the whole set of n firms in sector B, so that prices will need to be higher. In other terms, quality investment might be considered a way to relax price competition in sector B and then to limit the ability the monopolist's ability to extract consumer surplus. In conclusion, p_{A1}^{M}could in principle decrease with $\bar{\alpha}$ if this effect counterbalances the increased willingness-to-pay of the representative consumer when systems' qualities are higher.

[17] It should be noted that the impact of an increase in n on p_{A1}^{M} is analyzed assuming a constant $\bar{\alpha}$, which implies that we are concentrating on mean-preserving distributions of quality across firms.

so that aggregate profits in sector B are equal to

$$\Pi_B^M = \sum_{j=1}^{n} \Pi_{Bj}^M = t \sum_{j=1}^{n} \left(q_{1j}^M\right)^2 = n(1-\gamma)(n-\gamma)\left(\frac{\bar{\alpha}^2}{n(n(3-\gamma)-2\gamma)^2} + \frac{\sigma_\alpha^2}{n(2n-\gamma)^2(1-\gamma)^2}\right) \tag{16}$$

where $\sigma_\alpha^2 = \frac{\sum_{j=1}^{n}(\alpha_{1j}-\bar{\alpha})^2}{n}$ represents the variance of the qualities of the n available systems.

Similarly, after some complex algebraic steps illustrated in Appendix A and following Hsu and Wang (2005), consumer surplus can be defined as

$$CS = \frac{n^2(1-\gamma)}{2}\tilde{B}^2\sigma_\alpha^2 + \frac{n^2}{2}\tilde{A}^2\bar{\alpha}^2 \tag{17}$$

where $\widetilde{A} = \frac{(n-\gamma)}{n(n(3-\gamma)-2\gamma)}$ and $\widetilde{B} = \frac{(n-\gamma)}{n(1-\gamma)(2n-\gamma)}$.

In the next Section we will compare the equilibrium prices, profits and welfare of our model with those obtained under both an integrated and a complementary monopoly, *ceteris paribus*. In this respect, we report here the main tfindings in these two alternative regimes.

Particularly, a profit-maximizing integrated monopoly producing both complements would set its system price at $p_{IM} = \frac{\alpha_{IM}}{2}$, selling $Q_{IM} = \frac{\alpha_{IM}}{2}$ systems, so that profits and consumer surplus would amount to

$$\Pi_{IM} = \frac{\alpha_{IM}^2}{4}; \qquad CS_{IM} = \frac{\alpha_{IM}^2}{8}. \tag{18}$$

In a complementary monopoly, two independent firms $A1$ and $B1$ produce one component each of the composite good (i.e., $n = 1$) and, in equilibrium, they set their prices at $p_{CM}^i = \frac{\alpha_{CM}}{3}$, $i = A, B$ (where CM stands for "complementary monopoly"). Hence, consumers pay a system price $p_{CM} = \frac{2\alpha_{CM}}{3}$ and purchase $Q_{CM} = \frac{\alpha_{CM}}{3}$ units of the system. Profits and consumer surplus are:

$$\Pi_{CM}^i = \frac{\alpha_{CM}^2}{9}, \quad i = A, B; \;\; CS_{CM} = \frac{\alpha_{CM}^2}{18}, \tag{19}$$

where $CS_{CM} < CS_{IM}$, obviously.

3 Competition and Welfare When Sector A is a Monopoly

In this section we verify the impact of changes in the number of firms in Sector B, n, in the degree of substitutability among systems,γ, and in the distribution of the quality parameters (the α_{1k}'s) on equilibrium prices and welfare.

Along the way, we will verify how the assumption of imperfect substitutability changes the impact of n on the extent of the tragedy of the anticommons when compared to the case studied by Dari-Mattiacci and Parisi (2007).[18]

First of all, comparing prices and quantities when sector A is a monopolist with those obtained in an integrated monopoly, it can be noticed immediately that, when $\sigma_\alpha^2 = 0$, and $\alpha_{1k} = \bar{\alpha} = \alpha_{IM} = \alpha^*$, $k = 1, ..., n$, individual component prices in sector B are lower than p_{IM}, while system

[18] One should recall that, in their simple model, two firms competing in the market for the second component would be enough to guarantee a surplus equal to that attained in the presence of a single, integrated firm.

prices are higher. In fact,

$$p_{Bk}^{M} - p_{IM} = -\frac{\alpha^*(n-\gamma)}{(3n-\gamma(2+n))} < 0 \tag{20}$$

$$p_{1k}^{M} - p_{IM} = \frac{\alpha^*(n-\gamma(4-n))}{2(3n-\gamma(2+n))} > 0 \tag{21}$$

for all $\gamma \in [0,1]$. Thus, while competition certainly lowers prices in the oligopolistic sector, the monopolist in sector A optimally reacts by extracting more surplus and setting higher prices, so that overall $p_{1k}^{M} > p_{IM}$. This has a negative impact on the number of systems sold in the market. In fact, it is immediate to check that $Q_M = nq_{1k}^{M} < Q_{IM}$.

Similarly, when $\sigma_\alpha^2 = 0$ and the common quality level among all systems coincides with that of a complementary monopoly (again, $\alpha_{1k} = \bar{\alpha} = \alpha_{CM}$, $k = 1, ..., n$), component and system prices are lower with competition than with a complementary monopoly (i.e. $p_{Bk}^{M} < p_{CM}^{B}$and $p_{1k}^{M} < p_{CM}$, respectively). This implies that $Q_M > Q_{CM}$, even if each oligopolist sells less than a complementary monopolist ($q_{1k}^{M} < Q_{CM}$).

The following Lemma illustrates the relationship between oligopolistic prices p_{Bk}^{M}, substitutability γ, and competition in sector B, given by n.

Lemma 1. *Oligopolistic prices decrease with n and γ.*

Proof. See Appendix B. □

The negative relationship between p_{Bk}^{M}, γ and n is intuitive. The higher the number of firms in sector B and the degree of substitutability among systems, the fiercer the competition for the second component and the lower the Bertrand equilibrium prices. Similarly, it is immediate to verify from (10) that the impact of a change in n and γ on $\bar{p}_B$ is the usual and negative one.[19]

When checking instead the relationship between n, γ and system prices, we notice from (11) that p_{1k}^{M} is influenced by opposite forces. On the one hand, p_{A1}^{M} increases as either n or γ increase, whereas p_{Bk}^{M} decreases. The following Proposition indicates however that the first effect is always dominated by the second, so that, overall, p_{1k}^{M} decreases with n and γ.

Proposition 1. *The equilibrium system prices decrease with n and γ. Then, consumer surplus increases with n and γ.*

Proof. See Appendix B. □

As stated in Lemma 1, when the number of firms in sector B increases, then p_{Bk}^{M} decreases. The monopolist's best response would be to increase p_{A1}, given the complementarity between goods $A1$ and Bk. However, such an increase would negatively affect the demand of all the n systems. Then, the monopolist internalizes such negative externality and limits the increase in

[19] p_{Bk}^{M} is also decreasing with $\bar{\alpha}$. In fact, it is defined for a given α_{1k}, so that if $\bar{\alpha}$ increases it is because the quality of some systems other than $1k$ has increased. In such circumstance, the ratio $\frac{\alpha_{1k}}{\bar{\alpha}}$ actually decreases, reducing the price that firm k can charge. On the other hand, the average price $\bar{p}_B$ is positively affected by $\bar{\alpha}$: as the average quality of the available systems increases, their average price also increases.

p_{A1}. As a result, the equilibrium system prices decrease with n and the same applies to the degree of substitutability γ.

As for welfare comparisons, we notice first from (21) that, with a common quality value, no matter the extent of competition in sector B (i.e., no matter n), "separating" the two components of the system produced by an integrated monopolist and having them sold by two independent firms, always leads to higher prices. This clearly indicates that, when goods are not perfect substitutes, the tragedy of the anticommons is never solved by introducing competition in sector B, contrarily to what happens with perfect substitutes (Dari-Mattiacci and Parisi, 2007).[20] In order to confirm such prediction, we now compare consumer surplus when sector B is an oligopoly with the one enjoyed under an integrated monopoly, establishing the following result

Proposition 2. *When sector A is a monopoly and n firms compete in sector B,*

1) if $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$ $(k = 1, ..., n)$, consumer surplus with an oligopoly in sector B is always lower than with an integrated monopoly but higher than with a complementary monopoly ($CS_{CM} < CS^M < CS_{IM}$).

2) if systems differ in quality, consumer surplus is higher with an oligopoly in sector B than with an integrated monopoly if and only if

$$\sigma_{\bar{\alpha}}^2 > \sigma_{CS}^2 = \frac{1}{(1-\gamma)\widetilde{B}^2}\left[\frac{\alpha_{IM}^2}{4n^2} - \widetilde{A}^2\bar{\alpha}^2\right] \tag{22}$$

where σ_{CS}^2 is decreasing in γ and n. If quality variance is sufficiently high, competition may be preferred even if $\bar{\alpha} < \alpha_{IM}$.

Proof. See Appendix B. □

When goods are imperfect substitutes and quality is the same across systems and market structures, then, competition in one sector can certainly improve consumer welfare with respect to a complementary monopoly, but it is never enough to solve the anticommons problem ($CS_M < CS_{IM}$).[21]

Competition can effectively increase consumer surplus above CS_{IM} only if both average quality and variance play a role. Particularly, while it is not surprising that competition increases consumer welfare when it also increases average quality, from (17) it can be verified that quality variance has a positive effect, as well. In other words, our representative consumer benefits from variety (*varietas delectat*). Moreover, both parameters n and γ have a negative effect on σ_{CS}^2.

[20] Our conclusion seem to contradict also the results obtained by McHardy (2006). In his paper, a very low number of competitors selling imperfect substitutes is sufficient to attain the level of social welfare of a complementary monopoly, even if the other sector remains monopolistic.

[21] Interestingly this result still holds even if the number of competitors in sector B is endogenized. In the absence of barriers to entry in sector B and for a common quality level ($\sigma_{\bar{\alpha}}^2 = 0$), the equilibrium number of firms will tend to be infinitely large. In fact, by Lemma 1, p_{Bk} decreases with n, but it stays above marginal cost. Particularly, using l'Hôpital's Rule, $lim_{n\to\infty} p_{Bk}^M = \frac{\bar{\alpha}(1-\gamma)}{3-\gamma} > 0$ for $\gamma < 1$. Moreover, $lim_{n\to\infty} p_{A1}^M = \frac{\bar{\alpha}}{3-\gamma}$, so that $lim_{n\to\infty}(p_{A1}^M + p_{Bk}^M) = \frac{\bar{\alpha}(2-\gamma)}{3-\gamma} > \frac{\bar{\alpha}}{2} = p_{IM}$ and $lim_{n\to\infty} CS_M = \frac{\bar{\alpha}^2}{2(3-\gamma)^2} < CS_{IM}$. Then, contrarily to Dari-Mattiacci and Parisi (2007), under imperfect substitutability, the tragedy is never solved as long as sector A is a monopoly.

This is because an increase in n and γ decreases equilibrium prices under competition, thus raising consumer surplus, *ceteris paribus*.[22]

The results in Proposition 2 are shown graphically in Figure 1, presenting simulations for different parameter values. Panel a) illustrates a case in which $n = 2$, $\bar{\alpha} = \alpha_{IM} = \alpha_{CM} = 1$ and $\sigma_\alpha^2 = 0$. Panel b) represents the same case, this time letting the number of firms n vary and setting $\gamma = \frac{1}{3}$. Both panels clearly show that $CS^M < CS_{IM}$. Panel c) considers a case in which $\sigma_\alpha^2 = 0.25$. and again $\bar{\alpha} = \alpha_{IM} = \alpha_{CM} = 1$. It is possible to verify that now $CS_M > CS_{IM}$ for a sufficiently high value of γ. Finally, panel d) depicts the case in which average quality under competition is slightly lower than the quality of an integrated monopoly ($\bar{\alpha} = 0.95$ and $\alpha_{IM} = 1$). Here, $\gamma = \frac{1}{3}$ and variance is set sufficiently high ($\sigma_\alpha^2 = 0.37$), so that, for $n > 4$, the representative consumer prefers an oligopoly in sector B to an integrated monopoly. [23]

When turning to equilibrium profits and producer surplus in the various market configurations, we establish first the following results regarding equilibrium quantities.

Lemma 2. *(a) Q_M is increasing in n and γ; (b) q_{1k}^M is decreasing in n, $k = 1, ..., n$; (c) There exists $\hat{\alpha}_{1k} < \bar{\alpha}$, such that q_{1k}^M is increasing in γ for $\alpha_{1k} > \hat{\alpha}_{1k}$ and is decreasing in γ for $\alpha_{1k} < \hat{\alpha}_{1k}$.*

Proof. See Appendix B. □

As for part (a), note that when n increases, both oligopolistic prices and total system prices in (9) and (11) decrease due to enhanced competition. Moreover, as assumed, such increase in the number of competing firms takes place leaving average quality $\bar{\alpha}$ unchanged, so that the difference $\alpha_{1k} - \bar{\alpha}$ is not affected by the entry of a new available system. Thus, overall, total demand for all systems raise proportionately. However, given that the total market size does not change with n, as implied by the normalization in (6), the increase in the number of available systems will result in a reduction in the demand for each single system.

The case in which γ changes is more complex. As γ increases, systems become closer substitutes and their prices decrease (see Lemma 1). However, this does not necessarily translate into a greater demand for each of them. In fact, as implied by the utility function (1), consumers have a taste for quality so that, *ceteris paribus*, they prefer systems characterized by a higher α_{1k}. Then, as systems become closer substitutes, consumers will demand more high-quality systems at the detriment of low-quality ones. Consequently, the demand for some of the latter ones (those with $\alpha_{1k} < \hat{\alpha}_{1k}$) decreases as γ increases. This has immediate repercussions on profits, as we will see below.

The following Corollary and Proposition use Lemmas 1 and 2 to discuss and compare equilibrium profits.

Corollary 1. *Π_{A1}^M is increasing in n and γ. Both Π_{Bk}^M and Π_B^M are decreasing in n.*

[22] Obviously, when $\alpha_{IM} > \bar{\alpha}$, the greater the gap between α_{IM} and $\bar{\alpha}$, the greater the valued of σ_{CS}^2 needed to compensate for lower quality.

[23] In the simulations presented here, consumer surplus in complementary monopoly, (CS_{CM}), is always lower than CS_M whenever $\alpha_{IM} = \alpha_{CM}$. This is due to the assumption that $\bar{\alpha}$ is only slightly smaller than or equal to α_{IM}. If $\bar{\alpha}$ were smaller enough, we might have $CS_M < CS_{CM}$, at least for low values of γ and n.

Corollary 1 states that the monopolist in sector A always benefits from an increase in competition in sector B. This is because both the monopolist's equilibrium price p_{A1}^M and total demand Q_M increase in n and γ (from Lemma 2). The Corollary also establishes a clear negative relationship between the number of firms in sector B and their profits: as n increases, competition gets fiercer and each firm sets a lower price, sells a lower quantity and obtains lower profits (see Lemmas 1 and 2). This implies that also aggregate profits in sector B decrease with n, "counterbalancing" the growth in the monopolist's profit level in sector A. Regarding the relationship between γ and Π_{Bk}^M, we know from Lemmas 1 and 2 that both p_{Bk}^M and q_{1k}^M decrease with γ for low-quality systems, but that q_{1k}^M increases with γ when the quality of system $1k$ is sufficiently high, i.e. $\alpha_{1k} > \hat{\alpha}_{1k} > \bar{\alpha}$. Then for high-quality systems such positive impact of γ on quantities might prevail and Π_{Bk}^M can be increasing with γ. Such possibility also influences the relationship between γ and Π_B^M, as the following Proposition shows. Particularly, this is more likely to happen when quality variance is high and then the chance of having firms in sector B with $\alpha_{1k} > \hat{\alpha}_{1k}$ is greater, *ceteris paribus*.

Proposition 3. *If $\bar{\alpha} = \alpha_{IM} = \alpha_{CM}$,*

(a)$\Pi_{IM} > \Pi_{A1}^M > \Pi_{CM}^A$ for any $n \geq 2$ and $\gamma \in [0,1]$. When systems are perfect substitutes ($\gamma = 1$), $\Pi_{A1}^M = \Pi_{IM} > \Pi_{CM}^A$;

(b)Π_B^M is increasing in γ if and only if σ_α^2 is sufficiently high;

(c) If $\sigma_\alpha^2 = 0$ then Π_B^M is lower than Π_{CM}^B and Π_{IM}. Also, Producer Surplus ($PS \equiv \Pi_B^M + \Pi_{A1}^M$) is such that $\Pi_{IM} > PS > \Pi_{CM}^A + \Pi_{CM}^B$.

(d) If $\sigma_\alpha^2 > 0$, $n = 2$, then $\Pi_B^M < \Pi_{IM}$. If $\sigma_\alpha^2 > 0$, $n \geq 3$, then$\Pi_B^M \geq \Pi_{IM}$ for sufficiently high σ_α^2. Also, for $n \geq 2$, $PS \geq \Pi_{IM}$ if and only σ_α^2 is sufficiently high.

Proof. See Appendix B. □

The positive relationship between Π_{A1}^M and n illustrated in Corollary 1 explains why, as indicated in part (a) of Proposition 3, the monopolist's profits are higher when sector B is an oligopoly than when the market is a complementary monopoly. However, whenever $\gamma < 1$ the monopolist's profits are always lower than Π_{IM} so that, even an infinite number of competitors would not allow the monopolist to obtain the same profits of an integrated monopolist. This is because systems are not perfect substitutes, so that prices in the oligopolistic sector remain, on average, above marginal cost and the negative externality of the tragedy is not fully overcome.[24] As for part (b) of Proposition 3, it confirms the previous intuition that industry profits in sector B are increasing with γ when quality variance in sufficiently high. In such case, the increase in profits of high-quality producers more than compensates the decrease in the profits of low-quality ones.

In the remaining two parts, Proposition 3 compares both industry profits in sector B and producer surplus with their respective values under a complementary and an integrated monopoly.

[24]Only in the limit case in which $\gamma = 1$, the monopolist in sector A is able to extract the whole surplus from sector B, thus behaving like an integrated monopolist. One should notice the analogy between this case and the results in Dari-Mattiacci and Parisi (2007).

In the simple case of a common quality level (part (c)), industry profits in sector B (and then *a fortiori* individual firm profits) are smaller than both the profits of a complementary and of an integrated monopolist producing the same quality level. The relationship between Π_B^M and Π_{IM} is not surprising and is a direct implication of the results at the beginning of this setion, according to which $p_{Bk}^M < p_{IM}$ and $Q_M < Q_{IM}$, no matter the number of competing firms. Note that in the same section we also established that both q_{Bk}^M and p_{Bk}^M are lower than q_{CM} and p_{CM}^B, respectively, so that $\Pi_{Bk}^M < \Pi_{CM}^B$. Part (c) now states that this result holds in aggregate, as well, and that $\Pi_B^M < \Pi_{CM}^B$: introducing competition in sector B unambiguously lowers industry profits, no matter the degree of substitutability.

As for producer surplus, results are ambivalent. On the one hand, the idea that post-separation entry of new firms in sector B never solves the tragedy is supported also in terms of the sum of all firms' profits in the economy (so that $\Pi_B^M + \Pi_{A1}^M < \Pi_{IM}$). On the other, we verify that competition in sector B increases the monopolist's profits in sector A in a way that more than compensates the losses in industry profits in sector B, so that overall producer surplus under competition is greater than under a complementary monopoly ($\Pi_B^M + \Pi_{A1}^M > \Pi_{CM}^A + \Pi_{CM}^B$).

Finally, in part (d) we establish that industry profits in sector B can actually be larger than those of an integrated monopolist (and then *a fortiori*, of a complementary monopolist) when variance is positive. As indicated by equation (16), the higher the quality variance, the larger the value of aggregate profits in sector B, so that it may indeed happen that $\Pi_B^M \geq \Pi_{IM}$. Then, for a sufficiently large value for σ_α^2, producer surplus under competition might also be greater than in an integrated monopoly.[25] In conclusion, quality variance is an indicator of product differentiation and *varietas delectat* not only for consumers, but also for sector B as a whole. Then, joining the results in Propositions 2 and 3, the following Corollary holds when sector B is an oligopoly.

Corollary 2. *(a) Total Surplus increases with quality variance. (b) When $\sigma_\alpha^2 = 0$ and $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$ $(k = 1, ..., n)$, total surplus is greater than with a complementary monopoly but lower than in an integrated monopoly. (c) When $\sigma_\alpha^2 > 0$, and $\bar{\alpha} = \alpha_{IM}$, there exists a value for σ_α^2 such that total surplus is greater than with an integrated monopoly.*

Summing up, consumers are always worse off in a complementary monopoly. Moreover, they might sometimes prefer competition in sector B to an integrated monopoly if quality variance is high. In fact, in such case they would fully enjoy the benefits of product differentation. Similarly, when variance is large enough, producers in sector B might earn greater industry profits than those obtained by an integrated monopolist. In such circumstance, as indicated in proposition 3, some very high-quality firms are able to earn sufficiently high profits to offset both the low profits of their low-quality competitors and the loss in market power due to competition vis à

[25]Note that, for a given average quality, variance is obviously weakly increasing in the number of firms in sector B. In other terms, the higher n, the higher the maximum value that quality variance can take while still satisfying the constraints of the model (that is non-negative prices). This is the reason why this result holds only if $n \geq 3$. Two firms only in sector B are not enough to generate a sufficiently high quality variance (or equivalently a sufficiently high value of the parameter α_{1k}^{max}, introduced in the proof of Lemma 1) such that $\Pi_B^M \geq \Pi_{IM}$.

vis an integrated monopoly. Moerover, when quality variance is high, such possibility is actually *favoured* by an high degree of substitutability, given that in such instance Π_B^M increases with γ.

Total surplus follows a similar trend. As long as quality is uniform across systems, the tragedy prevails and competition in sector B is never able to raise social welfare above the corrisponding integrated monopoly level. However, this does not necessary hold with a sufficiently high product differentiation, with important implications for antitrust regulation of complementary-good markets. In fact, according to such results the break-up of an integrated firm into independent units producing one component each can be welfare improving if this generates competition for at least one component and if the competing systems in the market exhibit enough quality differentiation. Note that in Proposition 3 we assumed that $\bar{\alpha} = \alpha_{IM} = \alpha_{CM}$, but our result would be qualitatively the same for $\bar{\alpha} \neq \alpha_{IM}$. Particularly, competition in one sector can still be welfare enhancing even if post-separation entry in such sector reduces average quality, provided a sufficiently high value for quality variance.[26]

In the next Section, we extend the model to consider competition in Sector A, too.

4 Oligopolies in the markets for both complements

In this Section we assume that both complements A and B are produced in oligopolistic markets. Particularly, component A is produced by n_1 different firms, whereas component B is produced by n_2 firms. Again, firms compete by setting prices.

Since consumers can "mix and match" components at their own convenience, there are $n_1 \times n_2$ systems in the market and the utility function in (1) becomes

$$U(q,I) = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\alpha_{ij}q_{ij} - \frac{1}{2}\left[\beta\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}q_{ij}^2 + \gamma\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left(q_{ij}\sum_{z=1}^{n_1}\sum_{s=1}^{n_2}q_{zs} - q_{ij}^2\right)\right] + I \tag{23}$$

where q_{ij} represents the quantity of system ij, $(i = 1, ..., n_1;\ j = 1, ..., n_2)$, obtained by combining q_{ij} units of component A purchased from the ith firm in sector A (component Ai), and q_{ij} units of component B purchased from the jth firm in sector B (component Bj). Also in this case, $\alpha_{ij} > 0$ $(i = 1, ..., n_1;\ j = 1, ..., n_2)$, $\gamma \in [0, 1]$. The budget constraint now takes the form $\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} p_{ij}q_{ij} + I \leq M$, where $p_{ij} = p_{Ai} + p_{Bj}$ $(i = 1, ..., n_1;\ j = 1, ..., n_2)$ is the price of system ij.

The first order condition determining the optimal consumption of system tk is

$$\frac{\partial U}{\partial q_{tk}} = \alpha_{tk} - (\beta - \gamma)\, q_{tk} - \gamma\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} q_{ij} - p_{tk} = 0 \tag{24}$$

[26] In this respect, our paper integrates the main conclusion in Economides (1999), according to which separation of the monopolized production of complementary goods may damage quality.

After some tedious algebra, we obtain the demand function for system tk

$$q_{tk} = \frac{b\left(\alpha_{tk} - p_{At} - p_{Bk}\right) - \gamma\left[\sum_{j\neq k}\left(\alpha_{tj} - p_{Bj}\right) - p_{At}\left(n_2 - 1\right)\right] - \gamma\sum_{i\neq t}\sum_{j=1}^{n_2}\left(\alpha_{ij} - p_{ij}\right)}{\left(\beta - \gamma\right)\left[\beta + \gamma\left(n_1 n_2 - 1\right)\right]} \tag{25}$$

where $b = \beta + \gamma\left(n_1 n_2 - 2\right)$.

As before, to prevent total market size to change with γ, n_1 and n_2 we normalize β as follows[27]

$$\beta = n_1 n_2 - \gamma(n_1 n_2 - 1) \tag{26}$$

Given that component At is possibly bought in combination with all n_2 components produced in sector B, total demand and then profits for firm t in sector A are obtained summing q_{tk} in (25) over all possible values of k, i.e., $\Pi_{At} = p_{At}D_{At} = p_{At}\sum_{j=1}^{n_2} q_{tj}$. Similarly, profits for firm k in sector B are $\Pi = p_{Bk}D_{Bk} = p_{Bk}\sum_{i=1}^{n_1} q_{ik}$. Then, equilibrium prices p_{At}^O and p_{Bk}^O (the superscript "O" stands for "oligopoly in both sectors") are, respectively

$$p_{At}^O = A\bar{\alpha} + B\left(\bar{\alpha}_t - \bar{\alpha}\right) \tag{27}$$

$$p_{Bk}^O = C\bar{\alpha} + D\left(\bar{\alpha}_k - \bar{\alpha}\right) \tag{28}$$

where $\bar{\alpha} = \frac{\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\alpha_{ij}}{n_1 n_2}$ is the average quality of all systems available in the market, $\bar{\alpha}_t = \frac{\sum_{j=1}^{n_2}\alpha_{tj}}{n_2}$ is the average quality of the systems containing component t, and $\bar{\alpha}_k = \frac{\sum_{i=1}^{n_1}\alpha_{ik}}{n_1}$ is the average quality of systems containing component k. Parameters A, B, C and D are defined as follows: $A = \frac{n_1(1-\gamma)(n_2-\gamma)}{n_1 n_2(3-2\gamma)+\gamma^2(1+n_1+n_2)-2\gamma(n_1+n_2)}$, $B = \frac{n_1}{2n_1-\gamma}$, $C = \frac{n_2(1-\gamma)(n_1-\gamma)}{n_1 n_2(3-2\gamma)+\gamma^2(1+n_1+n_2)-2\gamma(n_1+n_2)}$ and $D = \frac{n_2}{2n_2-\gamma}$. The equilibrium price of system tk, $p_{tk}^O = p_{At}^O + p_{Bk}^O$, is therefore

$$p_{tk}^O = (A + C)\,\bar{\alpha} + B\left(\bar{\alpha}_t - \bar{\alpha}\right) + D\left(\bar{\alpha}_k - \bar{\alpha}\right) \tag{29}$$

Equilibrium quantities are

$$q_{tk}^O = z\bar{\alpha} + \frac{\alpha_{tk} - \bar{\alpha}}{n_1 n_2(1-\gamma)} + \frac{\bar{\alpha}_t - \bar{\alpha}}{n_2(2n_1-\gamma)(1-\gamma)} + \frac{\bar{\alpha}_k - \bar{\alpha}}{n_1(2n_2-\gamma)(1-\gamma)} \tag{30}$$

where

$$z = \frac{(n_1-\gamma)(n_2-\gamma)}{n_1 n_2(n_1 n_2(3-2\gamma) + \gamma^2(1+n_1+n_2) - 2\gamma(n_1+n_2))} \tag{31}$$

Note that component At ($t = 1, ..., n_1$) is sold in combination with all its n_2 complements, so that each firm's profits in sector A are equal to $\Pi_{At}^O = p_{At}^O \cdot q_{At}^O = p_{At}^O \sum_{k=1}^{n_2} q_{tk}^O$. Similarly, total profits from the sale of complement Bk ($k = 1, ..., n_2$) amount to $\Pi_{Bk}^O = p_{Bk}^O \cdot q_{Bk}^O = p_{Bk}^O \sum_{t=1}^{n_1} q_{tk}^O$.

As for consumer surplus in this $n_1 \times n_2$ model, we adopt the same procedure followed in section 2.2 to obtain

$$CS = \frac{n_1^2 n_2^2}{2}\left(z^2\bar{\alpha}^2 + (1-\gamma)Var(q)\right) \tag{32}$$

[27] Again, the second-order condition requires $\gamma \leq 1$.

where

$$Var(q) = \frac{\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}\left(q_{ij}^{O} - \bar{q}\right)^2}{n_1 n_2} \tag{33}$$

is the variance of the systems' quantities sold in equilibrium in the whole market.[28]

In the remainder of this section we want to investigate the impact that the introduction of competition in sector A has on consumer surplus and on profits, compared to less competitive options, particularly, complementary or integrated monopoly or a situation in which sector A is a monopoly ($n_1 = 1$). The comparison is rather straightforward when all systems produced in oligopoly have the same quality (so that, by symmetry, $Var(q) = 0$). For more general cases, however, the complexity of the expressions for prices, quantities and profits renders the algebraic analysis rather difficult. We will therefore perform numerical simulations.

First, we assume that $Var(q) = 0$, with $\alpha_{tk} = \alpha_{IM} = \alpha_{CM} = \alpha^*$, $(t = 1, ..., n_1;\ k = 1, ..., n_2)$ and we establish the following results.

Proposition 4. *When both sectors are oligopolies and* $Var(q) = 0$*,* $\alpha_{tk} = \alpha_{IM} = \alpha_{CM} = \alpha^*$ $(t = 1, ..., n_1;\ k = 1, ..., n_2)$*,*

(a) $CS_O > CS_{CM}$*;*

(b) $CS_O > CS_{IM}$ *if and only if*

$$n_1 > n_1^* = \frac{(n_2 - 1)\,\gamma^2}{n_2(2\gamma - 1) - \gamma^2} \tag{34}$$

where n_1^* *decreases both with* n_2 *and* γ*.*

(c) Oligopolistic profits Π_{At} *and* Π_{Bk} *are always smaller than* Π_{CM}^{i}*, hence than* Π_{IM}*.*

Proof. See Appendix A. □

The threshold n_1^* is decreasing in n_2, indicating quite intuitively that when the number of firms in one of the two sectors is high (and then competition there is particularly aggressive, benefiting consumers), the tragedy can be solved also for a relatively low number of firms in the other sector.[29] Moreover, a closer look to the expressions for CS_O and CS_{IM} makes us conclude that a competitive industry may be preferred to an integrated monopoly also even when both n_1 and n_2 are relatively low. Particularly, two firms in both sectors may be already enough to solve the tragedy when γ is sufficiently high, as shown in Figure 2.

Figure 2 is obtained assuming $Var(q) = 0$, $\alpha^* = 1$, $n_1 = 2$ and $\gamma = 0.62$. As it can be readily verified, consumer surplus is always higher under competition than in a complementary monopoly. Moreover, it increases with n_1, lying below CS_{IM} for low n_1 and becoming larger than CS_{IM} for $n_1 > 4$ ($n_1^* = 4.021$). Part (b) of the proposition also suggests that the degree of competition required in one sector (say, sector A) to increase consumer surplus above CS_{IM}

[28] See Appendix

[29] Of course, it would be possible to establish a symmetric threshold for n_2, which would then be decreasing in n_1.

decreases as either the number of firms in the other sector or the degree of substitutability increase (in fact, n_1^* is decreasing in both n_2 and γ). This happens because an increase in n_2 and/or in γ not only reduces the prices of each single component sold in sector B but also the prices of all systems, thus increasing consumer welfare.[30] Finally, part (c) confirms the relationships among profits found in the $n \times 1$ case, with oligopolists always earning the lowest profits and an integrated monopolist the highest.

If firms produce different qualities and $Var(q) > 0$, the number of competing firms required to make consumer surplus under competition preferred to that obtained in an integrated monopoly decreases. In fact, a positive $Var(q)$ increases CS_O in (32), thus increasing the range of the parameters for which $CS_O > CS_{IM}$.[31] The exact changes in prices, quantities, profits and welfare as the number of firms and the degree of substitutability between systems vary are analyzed in the following two simulations.

In both, we assume that the two sectors A and B are characterized by different quality distributions which get reflected on systems' qualities. Specifically, in the first simulation the entry of new firms in one sector allows the composition of ever better systems, so that competition increases average quality in the market. We set α_{tk} $(t = 1, ..., n_1;\ k = 1, ..., n_2)$ as follows

$\alpha_{11} = 8$	$\alpha_{12} = 8.5$	$\alpha_{13} = 9$	$\alpha_{14} = 9.5$	$\alpha_{15} = 10$
$\alpha_{21} = 7.5$	$\alpha_{22} = 8$	$\alpha_{23} = 8.5$	$\alpha_{24} = 9$	$\alpha_{25} = 9.5$

Due to our chosen values, the set of systems $\{1k\}$ $(k = 1, ..., 5)$ has high average quality than the set $\{2k\}$ and systems denoted by higher k are better in quality. Table 1 reports equilibrium prices, quantities and welfare when competition increases in sector B. It can be verified that quantity q_{11} decreases with n_2. Moreover, prices in sector A increase with n_2, whereas prices in sector B decrease. System prices however decrease in n_2. Unsurprisingly, prices are higher with $\gamma = 0.2$ than with $\gamma = 0.62$, since competition is fiercer in the second case. When $\gamma = 0.2$, consumer and producer surplus are higher under integrated monopoly. Things change when $\gamma = 0.62$; now fiercer competition among closer substitutes leads to substantially lower system prices, thus benefiting consumers (for $n_2 \geq 3$). This more than compensates for the lower producer surplus, so that total surplus in oligopoly is the highest. Complementary monopoly yields the lowest surplus, both for consumers and producers. Individual profits decrease in sector B as n_2 increases, whereas sector A takes advantage of this by increasing its own prices and profits.[32]

In the second simulation, we assume that competition worsens average quality in the market, so that, the larger the number of active firms, the lower $\bar{\alpha}$, $\bar{\alpha}_t$ and $\bar{\alpha}_k$. Again, with no loss of

[30] As we will also see in the simulations below, oligopolists in sector A react to a decrease in the prices in the complementary sector B by increasing their own price. Such increase is however limited, and total system prices overall decrease.

[31] Clearly, a fortiori, $CS_O > CS_{CM}$ always when quality variance is positive.

[32] In Table 1 both consumer surplus and profits under monopolistic configurations increase in n_2. This happens because each oligopoly structure (for each n_2) is compared with both types of monopoly at the same average quality and here, by assumption, $\bar{\alpha}$ increases with n_2.

generality, we assume that competition increases in sector B, whereas $n_1 = 2$ throughout the simulation. To obtain the effect of a decreasing quality level as competition gets fiercer, we set α_{tk} $(t = 1, ..., n_1;\ k = 1, ..., n_2)$ as follows[33]

$\alpha_{11} = 10$	$\alpha_{12} = 9.5$	$\alpha_{13} = 9$	$\alpha_{14} = 8.5$	$\alpha_{15} = 8$
$\alpha_{21} = 9.5$	$\alpha_{22} = 9$	$\alpha_{23} = 8.5$	$\alpha_{24} = 8$	$\alpha_{25} = 7.5$

When $\gamma = 0.2$, Table 2 shows that individual firms' and system prices decrease with competition. Interestingly, prices are declining and lower in sector A. This reverts the trend observed in the previous simulation, in which the sector not affected by competition was able to limit the impact or even to take advantage of the increased competition in the complementary sector. Such change is indeed driven by the decline in quality. Moreover, demand decreases with competition. (in Table 2 we report q_{11}).[34] Even at declining prices and quantities, firms in sector A enjoy however higher profits than firms in sector B and are able to extract a higher surplus than their complementors operating in the more competitive sector. Overall, producer surplus is lower than in an integrated monopoly but higher than in a complementary monopoly. As for consumer surplus, it decreases with competition: lower prices and increased variance are in fact not enough to compensate for the decline in quality. Simmetrically to producer surplus, consumer surplus is highest in integrated monopoly and lowest in complementary monopoly.[35]

When $\gamma = 0.62$, a fifth firm in sector 2 obtains no demand because of a too low quality level. This is why the most competitive feasible market structure is at $n_2 = 4$. System prices and quantities decrease as n_2 increases (and prices are lower than in the $\gamma = 0.2$ case, whereas quantities are higher). Interestingly, comparing consumer surplus across market configurations, it can be noticed that $CS_O < CS_{IM}$ for $n_2 = 2$ but $CS_O > CS_{IM}$ for $n_2 \geq 3$. This happens because the comparison is performed for the same quality level (α_{IM} is set equal to $\bar{\alpha}$ for each value of n_2), but quality variance is increasing. Similarly to the $n \times 1$ case, then, as variance increases, consumer welfare might be greater in competition than with an integrated monopoly. Finally, although p_{B1} has the usual pattern (as competition increases in sector B, p_{B1} decreases), p_{A1} has a non-monotonic behavior. First, it increases when n_2 increases from $n_2 = 2$ to $n_2 = 3$. When $n_2 = 4$, however, p_{A1} gets significantly lower than before: average quality is getting so low that firms in sector A are forced to reduce their prices. The initial positive relationship with n_2 was caused by the high degree of substitutability γ, that rendered competition especially fierce in sector B. When a further increase of n_2 takes quality to very low levels, however, this is not possible anymore. Profits follow the same pattern: they increase in sector A when n_2 goes from 2 to 3 but then decrease. In other terms, the fiercer competition due to high substitutability does not allow firms in sector A to counteract the decline in demand due to lower average quality

[33]It should be noticed that the coefficients α_{tk} are the same as in Simulation 2, but in reversed order.

[34]At $n_2 = 6$ the quantity of the lowest quality system becomes negative, implying that increased competition is not sustainable in such market configuration. That's why simulation 2 considers n_2 only up to 5.

[35]Here consumer surplus and profits under monopolistic configurations decrease in n_2 since $\bar{\alpha}$ decreases with higher n_2.

with a profit-enhancing price reduction, as it happened when $\gamma = 0.2$. As for profits in sector B, they always decrease and so do total profits. However, $\Pi_O > \Pi_{IM} > \Pi_{CM}$ because of the high quality variance exogenously produced in the simulation, and this result, combined with the trend observed for consumer surplus, produces an increasing trend for social welfare. In fact, as n_2 increases, total surplus increases as well, surpassing the corresponding integrated monopoly value for $n_2 \geq 3$.

Finally, from Table 1 and 2 it is also immediate to check the positive effect that the increase in competition in sector A has on consumer surplus. In fact, no matter the degree of substitutability γ, $CS_O > CS_M$. Then, even when either γ or n_2 are low (so that they yield lower consumer surplus than an integrated monopoly) and an integrated monopoly is not a viable solution, introducing some competition in sector A is desirable.

5 Conclusions

Complementary monopoly is tipically dominated in welfare terms by an integrated monopoly, in which all such complementary goods are offered by a single firm. This is "the tragedy of the anticommons". We have considered the possibility of competition in the market for each complement, presenting a model in which n imperfect substitutes for each perfect complement are produced. We have proved that, if at least one complementary good is produced in a monopoly, an integrated monopoly is always welfare superior to a more competitive market setting. Consequently, favoring competition in some sectors, leaving monopolies in others may be detrimental for consumers. Competition may be welfare enhancing if and only if the goods produced by competitors differ in quality, so that also average quality and variance become important factors to consider. We have also proved that, when competition is introduced in each sector, the tragedy may be solved for relatively small numbers of competing firms in each sector if systems are close substitutes, and this even in the limit case of a common quality level across systems. Unsurprisingly, the higher the degree of substitutability and the level of competition in one sector, the more concentrated the other sector can be, while still producing higher consumer surplus than an integrated monopoly.

Throughout the paper we have assumed that quality is costless and exogenously distributed across systems. It would be interesting to extend our model and explicitly consider quality as a costly investment in complementary-good markets. Particularly, a study of the incentives for the monopolist A to discourage innovation and quality improvements in sector B seems a very promising line of research. Heller and Eisenberg (1998) have already argued that patents may produce an anticommons problem in that holders of a specific patent may hold up potential innovators in complementary sectors. Particularly, they focus on the case of biomedical research, showing how a patent holder on a segment of a gene can block the development of derivative innovations based on the entire gene. Emblematic, in this respect, the case of Myriad Genetics Inc., which held patents on specific applications of the BRCA1 and BRCA2 genes, and blocked the development of cheaper breast-cancer tests.[36]

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A Computing oligopoly profits and consumer surplus

A.1 Sector A is a monopoly

From (9), note first that

$$p_{Bk}^{M} = t \cdot q_{1k}^{M} \tag{35}$$

where $t = \frac{n^2(1-\gamma)}{(n-\gamma)}$. Hence

$$\Pi_{Bk}^{M} = t\left(q_{1k}^{M}\right)^2 = \frac{n^2(1-\gamma)}{(n-\gamma)}\left(\frac{\bar{\alpha}(n-\gamma)}{n(n(3-\gamma)-2\gamma)} + \frac{(\alpha_{1k}-\bar{\alpha})(n-\gamma)}{n(2n-\gamma)(1-\gamma)}\right)^2, \tag{36}$$

Following Hsu and Wang (2005), consumer surplus can be written as

$$CS = \frac{n(1-\gamma)}{2}\sum_{j=1}^{n} q_{1j}^2 + \frac{\gamma}{2}\left(\sum_{j=1}^{n} q_{1j}\right)^2 = \frac{n(1-\gamma)}{2}\sum_{j=1}^{n}\left(q_{1j}-\bar{q}\right)^2 + \frac{n^2}{2}\left(\bar{q}\right)^2 \tag{37}$$

where $\bar{q} = \frac{\sum_{j=1}^{n} q_{1j}}{n} = \frac{Q}{n}$ is average quantity. Using (13), we can write

$$\bar{q} = \widetilde{A}\bar{\alpha} \tag{38}$$

and

$$q_{1k} - \bar{q} = \widetilde{B}\left(\alpha_{1k} - \bar{\alpha}\right) \tag{39}$$

where $\widetilde{A} = \frac{(n-\gamma)}{n(n(3-\gamma)-2\gamma)}$ and $\widetilde{B} = \frac{(n-\gamma)}{n(1-\gamma)(2n-\gamma)}$. Also, using (39),

$$\sum_{j=1}^{n} (q_{1j} - \bar{q})^2 = \widetilde{B}^2 n \sigma_\alpha^2 \tag{40}$$

Finally, substituting (38) and (40) into (37), we obtain

$$CS^M = \frac{n^2(1-\gamma)}{2}\tilde{B}^2\sigma_\alpha^2 + \frac{n^2}{2}\tilde{A}^2\bar{\alpha}^2 \tag{41}$$

A.2 Oligopolistic markets for both complements

It is immediate to obtain the total amount of component At ($t = 1, ..., n_1$) sold in equilibrium if we sum q_{tk}^O over the n_2 complements which At is sold with, that is

$$q_{At}^O = \sum_{k=1}^{n_2} q_{tk}^O = \frac{(n_1-\gamma)(\gamma-n_2)\overline{\alpha}}{n_1(\gamma(n_2(2-\gamma)-\gamma)+n_1((2-\gamma)\gamma+n_2(2\gamma-3)))} + \frac{(n_1-\gamma)(\bar{\alpha}_t-\bar{\alpha})}{n_1(2n_1-\gamma)(1-\gamma)} \tag{42}$$

Similarly,

$$q_{Bk}^O = \sum_{t=1}^{n_1} q_{tk}^O = \frac{(n_2-\gamma)(\gamma-n_1)\overline{\alpha}}{n_2(\gamma(n_1(2-\gamma)-\gamma)+n_2((2-\gamma)\gamma+n_1(2\gamma-3)))} + \frac{(n_2-\gamma)(\bar{\alpha}_k-\bar{\alpha})}{n_2(2n_2-\gamma)(1-\gamma)} \tag{43}$$

As for consumer surplus, we generalize Hsu and Wang (2005) and rewrite it as

$$CS = \frac{n_1 n_2(1-\gamma)}{2}\sum_{i=1}^{n_1}\sum_{j=1}^{n_2}(q_{ij}-\bar{q})^2 + \frac{n_1^2 n_2^2}{2}\bar{q}^2 \tag{44}$$

Using (30), we find that

$$\bar{q} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} q_{ij}^O = z\bar{\alpha} \tag{45}$$

so that we can define $Var(q)$in equation (33). Finally, substituting (45) and (33) into (44), we obtain equation (32).

B Proofs

B.1 Proof of Lemma 1

In order to prove that $\frac{\partial p_{Bk}^M}{\partial \gamma} < 0$ we note first that this is always true if $\alpha_{1k} < \bar{\alpha}$. In fact, $\frac{\partial}{\partial\gamma}\frac{n(1-\gamma)}{n(\gamma-3)-2\gamma} = -\frac{2(n-1)n}{(n(3-\gamma)-2\gamma)^2} < 0$ and $\frac{\partial}{\partial\gamma}\frac{n}{2n-\gamma} = \frac{n}{(2n-\gamma)^2} > 0$. If $\alpha_{1k} > \bar{\alpha}$, it may be that $\frac{\partial p_{Bk}^M}{\partial\gamma} > 0$ for a sufficiently high value of α_{1k}, and in particular for $\alpha_{1k} > \tilde{\alpha}_{1k}$, where $\tilde{\alpha_{1k}}$ is obtained solving $\frac{\partial p_{Bk}^M}{\partial\gamma} = 0$ with respect to α_{1k}. We then check whether $\tilde{\alpha}_{1k}$ is a feasible value for an above-average quality. To do that, we compute first the highest α_{1k} compatible with a given average $\bar{\alpha}$, α_{1k}^{max}, which is obtained when the remaining $n-1$ firms produce such low-quality systems $\alpha_{1s}^{min} < \bar{\alpha}$, $s \neq k$ as to optimally set their price equal to marginal cost (so that they remain active in sector B), that is $p_{Bs}^M = 0$. From (9), we obtain:

$$\alpha_{1s}^{min} = \frac{\bar{\alpha}(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma} \tag{46}$$

Setting $\alpha_{1s} = \alpha_{1s}^{min}$ for all firms $s \neq k$, we obtain α_{1k}^{max} solving

$$\frac{(n-1)\,\alpha_{1s}^{min} + \alpha_{1k}^{max}}{n} = \bar{\alpha} \tag{47}$$

i.e., $\alpha_{1k}^{max} = n\bar{\alpha} - (n-1)\,\alpha_{1s}^{min}$. Substituting such value into $\frac{\partial p_{Bk}^M}{\partial \gamma}$, we have

$$\left.\frac{\partial p_{Bk}^M}{\partial \gamma}\right|_{\alpha_{1k}=\alpha_{1k}^{max}} = \frac{(n-1)n\bar{\alpha}\left[2\gamma^2 - n\left(1+4\gamma-\gamma^2\right)\right]}{(2n-\gamma)\left[n(\gamma-3)+2\right]^2} < 0.$$

Hence, $\alpha_{1k}^{max} < \tilde{\alpha}_{1k}$ always and $\frac{\partial p_{Bk}^M}{\partial \gamma} < 0$ for all $\gamma \in [0,\,1]$.

Similarly, in order to prove that $\frac{\partial p_{Bk}^M}{\partial n} < 0$ for all $n \geq 2$, we note from (9) that $\frac{\partial p_{Bk}^M}{\partial n} < 0$ always if $\alpha_{1k} > \bar{\alpha}$, since $\frac{\partial}{\partial n}\frac{n(1-\gamma)}{n(3-\gamma)-2\gamma} = -\frac{2(1-\gamma)}{(n(3-\gamma)-2\gamma)^2} < 0$ and $\frac{\partial}{\partial n}\frac{n}{2n-\gamma} = -\frac{\gamma}{(2n-\gamma)^2} < 0$. If $\alpha_{1k} < \bar{\alpha}$, it may be that $\frac{\partial p_{Bk}^M}{\partial n} > 0$ for a sufficiently low value of α_{1k}, but substituting to α_{1k} in (46) its minimun value, α_{1k}^{min}, we obtain $\left.\frac{\partial p_{Bk}^M}{\partial n}\right|_{\alpha_{1k}=\alpha_{1k}^{min}} = \frac{n\bar{\alpha}\gamma(\gamma^2-1)}{(2n-\gamma)(n(3-\gamma)-2\gamma)^2}$, which is negative for the whole parameters' range.

B.2 Proof of Proposition 1

Differentiating $\frac{\partial p_{1k}^M}{\partial \bar{\alpha}} = -\frac{(2\gamma-1)[1+(n-2)\gamma]}{[3+\gamma(2n-5)][2+\gamma(2n-3)]} > 0$ for all $\gamma < 1$.

We now prove that $\bar{p}_{1k}^M$ decreases with n. From (8) it can be readily verified that $\frac{\partial p_{A1}^M}{\partial n} > 0$, whereas Lemma 1 demonstrates that $\frac{\partial p_{Bk}^M}{\partial n} < 0$. It is then sufficient to prove that $\frac{\partial p_{A1}^M}{\partial n} < \left|\frac{\partial p_{Bk}^M}{\partial n}\right|$ when $\left|\frac{\partial p_{Bk}^M}{\partial n}\right|$ takes its minimum value with respect to α_{1k}, *ceteris paribus*. Note first that $\frac{\partial p_{Bk}^M}{\partial n} = -\frac{\gamma(\alpha_{1k}-\bar{\alpha})}{[2+(2n-3)\gamma]^2} - \frac{2(1-\gamma)\bar{\alpha}}{[3+(2n-5)\gamma]^2}$, which reaches its minimum value when $\alpha_{1k} = \alpha_{1k}^{min}$ (α_{1k}^{min} is defined in the proof of Lemma 1), since $-\frac{\gamma(\alpha_{1k}-\bar{\alpha})}{[2+(2n-3)\gamma]^2}$ is positive and maximum at α_{1k}^{min}. It is then easy to verify that $\frac{\partial p_{A1}^M}{\partial n} - \left|\frac{\partial p_{Bk}^M}{\partial n}\right|_{\alpha_{ik}=\alpha_{ik}^{min}} = -\frac{\alpha(1-\gamma)(2+4\gamma(n-\bar{2})+(8-7n+2n^2)\gamma^2)}{(1+\gamma(n-1))(3+\gamma(2n-5))^2(2+\gamma(2n-3))} < 0$ for all γ and n.

A similar proof works for $\frac{\partial p_{1k}^M}{\partial \gamma}$.

The effect on CS_M is a direct consequence of the influence of γ and n on system prices.

B.3 Proof of Proposition 2

Part 1). In this case $\alpha_{1k} = \bar{\alpha}$, $(k = 1, ..., n)$ and $\sigma_\alpha^2 = 0$. From (17), $CS_M = \frac{n^2}{2}\tilde{A}^2\bar{\alpha}^2$. Comparing such expression with consumer surplus under integrated and complementary monopoly (given by (18) and (19), respectively), we note immediately that the difference $CS^M - CS_{IM} = \frac{\bar{\alpha}^2 n(1-\gamma)(n(\gamma-5)+4\gamma)}{8(n(3-\gamma)-2\gamma)^2}$ is negative, while the difference $CS^M - CS_{CM} = \frac{\bar{\alpha}^2\left(n(6\gamma(n-1)+\gamma)+5\gamma^2\right)}{18(n(3-\gamma)-2\gamma)^2}$ is positive, for all $n \geq 2$ and $\gamma \in [0,1]$.

Part 2). When $\sigma_\alpha^2 > 0$, subtracting CS_{IM} from CS_M and solving for σ_α^2, we obtain σ_{CS}^2 in expression (22). Note that $\sigma_{CS}^2 > 0$ iff $\bar{\alpha} < \frac{\alpha_{IM}}{2\tilde{A}n}$. It can be verified that $\alpha_{IM} < \frac{\alpha_{IM}}{2\tilde{A}n}$, so that it is possible to have a case in which $\bar{\alpha} < \alpha_{IM}$ and $CS_M > CS_{IM}$.

Finally, given that CS_M is increasing in n and γ, the minimum value of σ_α^2 required to have $CS_M \geq CS_{IM}$, σ_{CS}^2, must be decreasing in n and γ.

B.4 Proof of Lemma 2

Differentiating q_{ik}^M in (13) with respect to γ we get

$$\frac{\partial q_{ik}^M}{\partial \gamma} = \frac{(n-1)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} + \frac{(2n^2+\gamma^2-n(1+2\gamma))(\alpha_{1k}-\bar{\alpha})}{n(1-\gamma)^2(2n-\gamma)^2} \tag{48}$$

When $n \geq 2$ and $\gamma \in [0,1]$, the first term on the right-hand side of (48) is positive. The second term is positive if $\alpha_{1k} > \bar{\alpha}$ and negative otherwise. Thus, $\frac{\partial q_{ik}^M}{\partial \gamma} > 0$ always if $\alpha_{1k} > \bar{\alpha}$. If $\alpha_{1k} < \bar{\alpha}$, the maximum negative value of the second term in (48) is obtained when α_{1k} reaches its minimum feasible value, α_{1s}^{min} (see equation (46) in the proof of Lemma 1). Evaluating $\frac{\partial q_{ik}^M}{\partial \gamma}$ at $\alpha_{1k} = \alpha_{1s}^{min}$ we obtain $\left.\frac{\partial q_{ik}^M}{\partial \gamma}\right|_{\alpha_{1k}=\alpha_{1s}^{min}} = -\frac{(n(4n-6\gamma-1)+\gamma^2(2+n))(n-\gamma)\bar{\alpha}}{n(2n-\gamma)(1-\gamma)(n(3-\gamma)+2\gamma^2} < 0$. Thus, given that $\frac{\partial q_{ik}^M}{\partial \gamma}$ is continuous in α_{1k}, there exists $\hat{\alpha}_{1k} < \bar{\alpha}$ such that $\frac{\partial q_{ik}^M}{\partial \gamma} \geq 0$ for $\alpha_{1k} \geq \hat{\alpha}_{1k}$ and negative otherwise.

Differentiating q_{ik}^M in (13) with respect to n we get

$$\frac{\partial q_{ik}^M}{\partial n} = \frac{((3-\gamma)n(2-n)-2\gamma^2)\bar{\alpha}}{n^2(n(3-\gamma)-2\gamma)^2} + \frac{(2n(n-2\gamma)+\gamma^2)(\alpha_{1k}-\bar{\alpha})}{n^2(\gamma-1)(2n-\gamma)^2} \tag{49}$$

When $n \geq 2$ and $\gamma \in [0,1]$, the first term on the right-hand side of (49) is negative. The second term is negative if $\alpha_{1k} > \bar{\alpha}$ and positive otherwise. Thus, $\frac{\partial q_{ik}^M}{\partial n} < 0$ always if $\alpha_{1k} > \bar{\alpha}$. If $\alpha_{1k} < \bar{\alpha}$, the maximum positive value for the second term of (49) occurs when $\alpha_{1k} = \alpha_{1s}^{min}$. Evaluating $\frac{\partial q_{ik}^M}{\partial n}$ at at this value we obtain $\left.\frac{\partial q_{ik}^M}{\partial n}\right|_{\alpha_{1k}=\alpha_{1s}^{min}} = -\frac{(n-\gamma)\gamma(1-\gamma)\bar{\alpha}}{n(2n-\gamma)(n(3-\gamma)-2\gamma)^2} < 0$. Thus, $\frac{\partial q_{ik}^M}{\partial n} < 0$.

Finally, define total quantity as

$$Q^M \equiv \sum_{k=1}^{n} q_{ik}^M = \frac{\bar{\alpha}(n-\gamma)}{n(3-\gamma)-2\gamma} \tag{50}$$

Differentiating (50) with respect to γ and n we obtain $\frac{\partial Q^M}{\partial \gamma} = \frac{n(n-1)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} > 0$ and $\frac{\partial Q^M}{\partial n} = \frac{\gamma(1-\gamma)\bar{\alpha}}{(n(3-\gamma)-2\gamma)^2} > 0$ in the admissible range of the parameters.

B.5 Proof of Proposition 3

Part (a). Comparing Π_{A1}^M in (14) and Π_{CM}^A in (19), we obtain $\Pi_{A1}^M - \Pi_{CM}^A = \frac{(n-1)\bar{\alpha}^2\gamma(n(6-\gamma)-5\gamma)}{9(n(3-\gamma)-2\gamma)^2} > 0$ in the relevant parameters' range. Similarly, $\Pi_{A1}^M - \Pi_{IM} = -\frac{n\bar{\alpha}(1-\gamma)(n(5-\gamma)-4\gamma)}{4(n(3-\gamma)-2\gamma)^2} < 0$. Note that $lim_{n\to\infty}\Pi_{A1}^M = \frac{\bar{\alpha}^2}{(3-\gamma)^2}$, which is in any case smaller than Π_{IM} when $\gamma \in [0,1]$. Only at $\gamma = 1$ we would have $\Pi_{A1}^M = \Pi_{IM}$.

Part (b). From Lemmas 1 and 2, both p_{Bk}^M and q_{1k}^M decrease with n. Then both Π_{Bk}^M and Π_B^Malso decrease with n.

To prove the impact of γ on Π_B^M, let us differentiate expression (16) with respect to γ. We find:

$$\frac{\partial \Pi_B^M}{\partial \gamma} = \frac{n(n(2n-3\gamma)+\gamma(2-\gamma))}{(2n-\gamma)^3(1-\gamma)^2}\sigma_\alpha^2 - \frac{n(n-1)(n+\gamma(n-2))}{(n(3-\gamma)-2\gamma)^3}\bar{\alpha} \tag{51}$$

It might then happen that $\frac{\partial \Pi_B^M}{\partial \gamma} > 0$ if σ_α^2 is high enough for given $\bar{\alpha}$. It is a well-known result in statistics that the maximum variance $\sigma_{\alpha max}^2$ in a discrete distribution is attained when $\frac{n}{2}$ firms have quality equal to the minimum value in the range and $\frac{n}{2}$ firms have quality equal to the maximum value in the range (see Plackett, 1947). In our specific case, the minimum value in the range is given by α_{1s}^{min}, whereas the maximum value can be computed given the average $\bar{\alpha}$ and the fact than $\frac{n}{2}$ firms produce α_{1s}^{min}. Define such maximum $\breve{\alpha} = \bar{\alpha}\left(n - \frac{(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma}\right)$. Then maximum variance would be $\sigma_{max}^2 = \frac{1}{2}\bar{\alpha}^2\left(\frac{(1-\gamma)^2(2n-\gamma)^2}{(n(3-\gamma)-2\gamma)^2}\right) + \left(n-1+\frac{(n-\gamma)(1+\gamma)}{n(3-\gamma)-2\gamma}\right)^2$. By differentiating Π_B^M with respect to γ and solving the derivative with respect to σ_α^2, it is possible to verify that $\frac{\partial \Pi_B^M}{\partial \gamma} \geq 0$ iff $\sigma_\alpha^2 \geq \sigma_0^2 = \frac{(n-1)\bar{\alpha}^2(2n-\gamma)^3(1-\gamma)^2(n-2\gamma+n\gamma)}{(n(3-\gamma)-2\gamma)^3(2n^2-3n\gamma-\gamma(1-2\gamma))}$. To compare σ_0^2 with σ_{max}^2, we evaluate the expression $\sigma_{max}^2 - \sigma_0^2$ numerically for all admissible values of γ and we find that $\sigma_{max}^2 > \sigma_0^2$ for all $n \geq 2$, implying that $\frac{\partial \Pi_B^M}{\partial \gamma} > 0$ for σ_α^2 sufficiently high.

Part (c). When $\sigma_\alpha^2 = 0$, then all systems have the same quality level α_{1k}, $k = 1, ..., n$. Moreover, if $\alpha_{1k} = \alpha_{IM} = \alpha_{CM}$, then the difference $\Pi_B^M - \Pi_{CM}^B = -\frac{(n-1)\bar{\alpha}^2\gamma(3n+\gamma(n-4))}{9(n(3-\gamma)-2\gamma)^2}$ is always negative in the admissible parameters' range. (We know already that $\Pi_{CM}^B < \Pi_{IM}$. Hence, *a fortiori*, $\Pi_B^M - \Pi_{IM} < 0$). As for Producer Surplus, $PS \equiv \Pi_{A1}^M + \Pi_B^M = \frac{\bar{\alpha}^2\left[n^2(2-\gamma)-n(3-\gamma)\gamma+\gamma^2\right]}{[n(3-\gamma)-2\gamma]^2}$. It is easy to verify that $PS - \Pi_{IM} = -\frac{n\bar{\alpha}^2(1-\gamma)^2}{4(n(3-\gamma)-2\gamma)^2} < 0$. Also, $\Pi_{A1}^M + \Pi_B^M - \Pi_{CM}^A - \Pi_{CM}^B = PS - 2\Pi_{CM}^i = \frac{(n-1)\bar{\alpha}^2\gamma(n(3-2\gamma)-\gamma)}{9(n(3-\gamma)-2\gamma)^2}$ which is always positive in the relevant parameters' range.

Part (d). The final result is immediate and is obtained solving $\Pi_{Bk}^M = \Pi_{IM}$ with respect to σ_α^2. Then $\Pi_{Bk}^M \geq \Pi_{IM}$ iff $\sigma_\alpha^2 \geq \sigma_{\Pi_B}^2 = \frac{(n-1)\bar{\alpha}^2(1-\gamma)\gamma(2n-\gamma)^2(n(3+\gamma)-4\gamma)}{9n(n-\gamma)(n(3-\gamma)-2\gamma)^2}$, where $\sigma_{\Pi_B}^2 < \sigma_{max}^2$ for all $n \geq 3$ (numerical evaluation for all admissible values of γ). For $n = 2$, $\sigma_{\Pi_B}^2 > \sigma_{max}^2$, implying that $\Pi_{Bk}^M < \Pi_{IM}$. As for Producer Surplus, the result is obtained solving $\Pi_{A1}^M + \Pi_B^M = \Pi_{IM}$ with respect to σ_α^2. Then $\Pi_{A1}^M + \Pi_B^M \geq \Pi_{IM}$ iff $\sigma_\alpha^2 \geq \sigma_{PS}^2 = \frac{n\bar{\alpha}^2(1-\gamma)^2(2n-\gamma)^2}{4(n-\gamma)(n(3-\gamma)-2\gamma)^2}$. Also, it is possible to establish (through numerical evaluation) that $\sigma^2_{PS} < \sigma^2_{max}$ for all $n \geq 2$.

B.6 Proof of Proposition 4

Part (a). The proof is immediate, setting $Var(q) = 0$ in (32) and comparing the resulting expression with CS_{CM}.

Part (b). Solving $CS_O - CS_{IM} = 0$ with respect to n_1, i.e. $\frac{n_1^2 n_2^2}{2} z^2 q^2 - \frac{\alpha^{*2}}{8} = 0$, yields two solutions, $n_{11} = \frac{(n_2-1)\gamma^2}{n_2(2\gamma-1)-\gamma^2}$ and $n_{12} = \frac{\gamma(n_2(4-\gamma)-3\gamma)}{n_2(5-2\gamma)-(4-\gamma)\gamma}$, so that $CS_O > CS_{IM}$ iff either $n_1 < n_{12}$ or $n_1 > n_{11}$. It is possible to verify, however, that $n_{12} < 1$ for all γ and n_2 in the admissible range of the parameters. Therefore, $CS^O \geq CS_{IM}$ iff $n_1 \geq n_{11}$ and $n_{11} = n_1^*$ in (34).

Finally, Differentiating (34) with respect to γ yields $\frac{\partial n_1^*}{\partial \gamma} = -\frac{2(n_2-1)n_2(1-\gamma)\gamma}{(n_2(1-2\gamma)+\gamma^2)^2} < 0$, whereas differentiating it with respect to n_2 yields $\frac{\partial n_1^*}{\partial n_2} = -\frac{(1-\gamma)^2\gamma^2}{(n_2(1-2\gamma)+\gamma^2)^2} < 0$.

Part (c). For this part, it suffices to prove that either Π_{At} or Π_{Bk} is smaller than Π^i_{CM}. The remaining inequality would be implied by the clear symmetry. Moreover, being $\Pi^i_{CM} < \Pi_{IM}$, in such case Π_{At} and Π_{Bk} would also be smaller than Π_{IM}. By comparing Π_{At} with Π^A_{CM}, we find that

$$\Pi_{At} - \Pi^A_{CM} = \frac{1}{9}\alpha^{*2}\left(\frac{9(n_1-\gamma)(n_2-\gamma)^2(1-\gamma)}{n_2(\gamma(n_2(\gamma-2)+\gamma)+n_1(n_2(3-2\gamma)+(\gamma-2)\gamma))^2} - 1\right) \tag{52}$$

Numerically solving (52) with respect to n_1 for given values of n_2 and considering all the admissible values for γ, it is possible to check that (52) admits two solutions $\tilde{n}_a$ and $\tilde{n}_b$ and that both are always lower than 1 when not imaginary. Simulations show that $\Pi_{At} - \Pi^A_{CM} \geq 0$ for $\tilde{n}_a \leq n_1 \leq \tilde{n}_b$ (when $\tilde{n}_a$ and $\tilde{n}_b$ are real) and $\Pi_{At} - \Pi^A_{CM} < 0$ when $\tilde{n}_a$ and $\tilde{n}_b$ are imaginary. This implies that $\Pi_{At} - \Pi^A_{CM} < 0$ in the relevant range of the parameters. The same proof can be applied to Π_{Bk}.

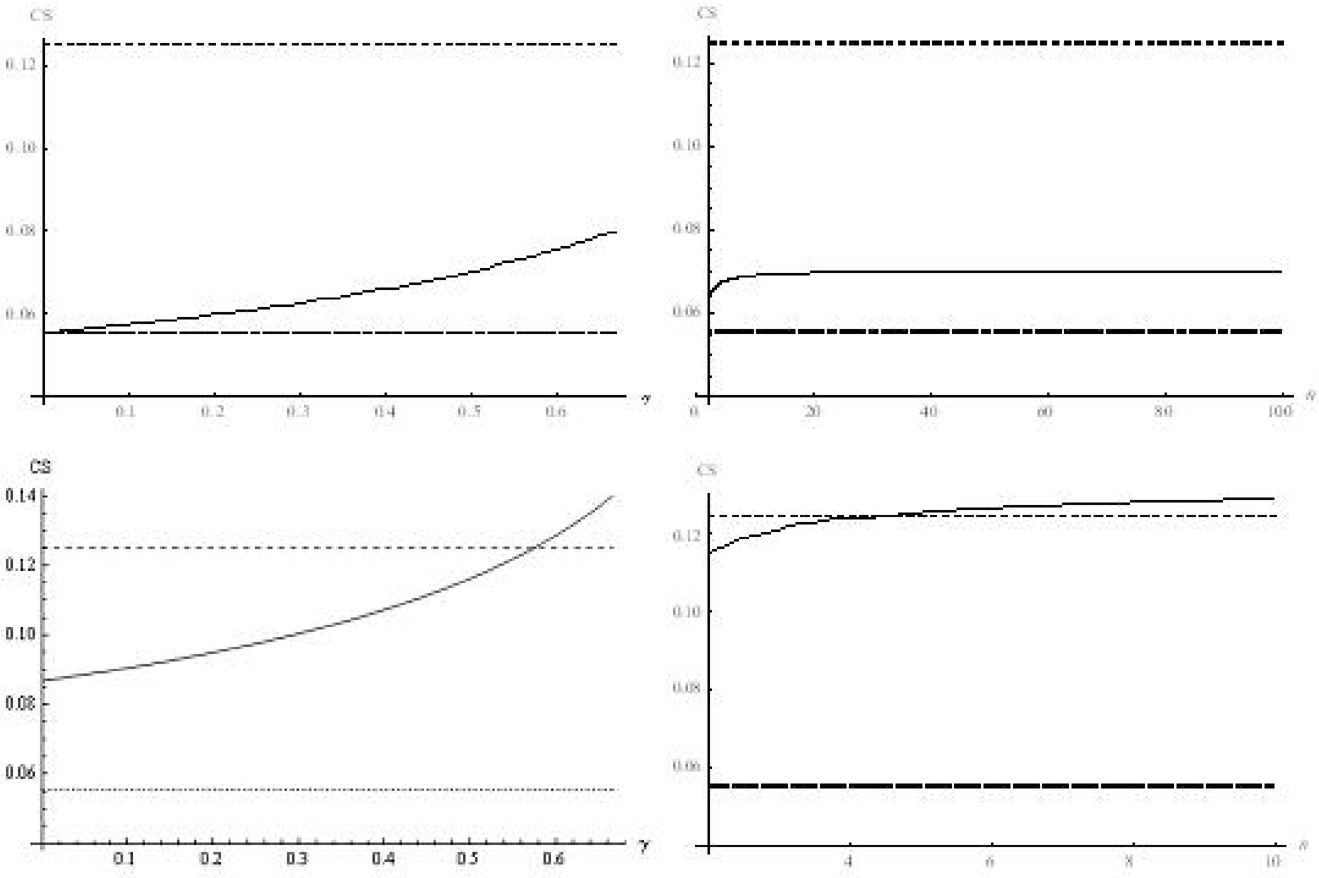

Figure 1: Comparing consumer surplus under three regimes – competition, integrated and complementary monopoly (___ CS^M, --- CS_{IM}, ... CS_{CM}).

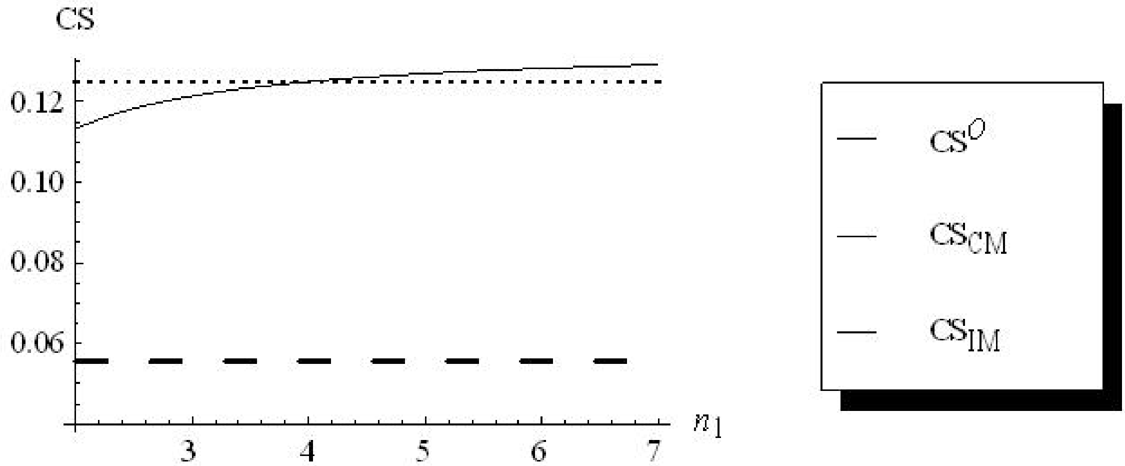

Figure 2: Consumer surplus under three different regimes when competition is present in both sectors.

	$\gamma = 0.2$				$\gamma = 0.62$			
	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$
p_{A1}	2.69	2.80	2.90	2.99	2.24	2.39	2.49	2.58
p_{B1}	2.69	2.32	2.24	2.17	2.24	1.67	1.50	1.38
p_{11}	5.38	5.12	5.14	5.16	4.49	4.06	3.99	3.96
q_{11}	0.72	0.42	0.27	0.18	0.95	0.51	0.28	0.14
CS_O	4.33	4.84	5.38	5.97	7.66	9	10.21	11.47
CS_{IM}	8	8.51	9.03	9.57	8	8.51	9.03	9.57
CS_{CM}	3.55	3.78	4.01	4.25	3.55	3.79	4.01	4.25
CS_M	4.09	4.45	4.78	5.12	5.24	5.92	6.46	6.98
Π_{A1}	4.07	4.42	4.72	5.02	4.57	5.17	5.62	6.02
Π_{B1}	4.07	2.09	1.48	1.13	4.57	1.94	1.26	0.88
Π_O	14.78	15.83	16.88	17.96	16.04	17.16	18.31	19.51
Π_{IM}	16	17.01	18.06	19.14	16	17	18.06	19.14
Π_{CM}	14.22	15.125	16.05	17.01	14.22	15.12	16.05	17.01
TS^O	19.11	20.67	22.27	23.94	23.70	26.16	28.52	30.98
TS_{IM}	24	25.52	27.09	28.72	24	25.52	27.09	28.71
TS_{CM}	17.78	18.91	20.07	21.27	17.78	18.91	20.07	21.27

Table 1: Impact of competition when firms are heterogeneous and competition increases quality.

	$\gamma = 0.2$				$\gamma = 0.62$			
	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$	$n_2 = 2$	$n_2 = 3$	$n_2 = 4$	$n_2 = 5$
p_{A1}	3.17	3.12	3.06	2.99	2.64	2.66	2.62	-
p_{B1}	3.17	3.14	3.16	3.19	2.64	2.46	2.43	-
p_{11}	6.34	6.26	6.22	6.18	5.28	5.12	5.05	-
q_{11}	1.1	0.8	0.65	0.56	1.65	1.27	1.07	-
CS_O	6.03	6	5.97	5.96	10.63	11.12	11.3	-
CS_{IM}	11.3	10.7	10.12	9.57	11.28	10.7	10.12	-
CS_{CM}	5	4.75	4.5	4.25	5	4.75	4.5	-
CS_M	5.70	5.55	5.34	5.12	6.32	6.41	6.25	-
Π_{A1}	5.65	5.5	5.27	5.02	7.32	7.39	7.21	-
Π_{B1}	5.65	3.84	2.96	2.44	6.32	4.22	3.28	-
Π_O	20.83	19.88	18.91	17.97	22.6	21.5	20.5	-
Π_{IM}	22.56	21.39	20.25	19.14	22.6	21.4	20.25	-
Π_{CM}	20.05	19.01	18	17	20	19	18	-
TS_O	26.86	27.13	24.88	23.94	33.22	32.66	31.8	-
TS_{IM}	33.86	32.08	30.37	28.71	33.84	32.08	30.04	-
TS_{CM}	25.07	23.77	22.5	21.27	27.07	23.77	22.5	-

Table 2: Impact of competition when firms are heterogeneous and competition decreases quality.