Internal vs. External Restructuring  
—Boundary of the Firm with an Endogenous Firm Structure

Qing Ma* and Susheng Wang**

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Abstract. We study the boundary of the firm with an endogenous firm structure. By investigating the firm’s restructuring options (internal vs. external), we determine the boundary of the firm depending on the optimal firm structure. We analyze the firm’s options based on market uncertainty, market size, market competition, synergy among divisions, and coordination cost. We find that when market uncertainty rises, a decentralized firm (D-firm) is more likely to carry out internal restructuring, while a centralized firm (C-firm) is more likely to carry out external restructuring. When market competition intensifies, a D-firm will stay put, while a C-firm is more likely to opt for either internal or external restructuring depending on whether there is positive synergy among divisions.

Keywords: internal restructuring, external restructuring, market uncertainty, market size, market competition, synergy among divisions

JEL classification: G34

*School of Economics, Southwestern University of Finance and Economics, Chengdu, Sichuan, China. E-mail: qingma@swufe.edu.cn

**Department of Economics, School of Business and Management, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. E-mail: s.wang@ust.hk
1. Introduction

In a changing market, a firm may decide to carry out internal restructuring or external restructuring for a product line. In a major restructuring program, a firm may simultaneously carry out internal restructuring for some product lines and external restructuring for other product lines. This paper focuses on a firm’s restructuring options (internal vs. external) when adapting to a changing business environment. We treat the boundary and structure of the firm as two interdependent factors in the firm’s options.

Research on the choice between internal and external restructuring is rare. Levine & Smith (2004) investigate the effect of organizational structure and horizontal disintegration on information sharing among divisions. They look specifically into the implications of division managers’ observing the outcomes of earlier investments by other division managers in a horizontally integrated firm. They find that a free rider problem may appear in which a division invests in an uninformative project and free ride off the information from an informative project that another division is investing in. The authors propose centralization or divestiture as possible remedies to the problem. Renucci (2008) considers conditions under which a firm is better off decentralizing its control structure. He assumes a capacity constraint for a centralized structure, while we assume a coordination cost; he assumes two independent projects, while we emphasize synergy among divisions. Since a centralized control structure is better for coordinating economic activities and leveraging synergy, synergy plays an important role in the firm’s choice not only between internal and external restructuring but also between decentralization and centralization. Alonso et al. (2015) focus on centralization vs. decentralization, while we focus on internal vs. external restructuring. They show that increased market competition (as reflected by increased price sensitivity of demand) implies a preference for centralization; in contrast, we show that it implies a preference for decentralization, which is consistent with prior literature. Decentralization offers better incentives for division managers so that the firm would become more competitive in the market. Besides, since our investigation includes both the internal and external options, our conclusion (Result 2) has a twist: the firm may also opt for divestiture if there is negative synergy among divisions.

Consistent with Williamson’s (1996, 2002) argument of coordinated adaptation, Harford (2012) states that “The traditional purpose of centralization is to make sure every business unit is coordinated and nobody is duplicating anyone else’s effort”. Centralization offers better coordination among divisions and better information to guide decision-making, but these two advantages are costly: a coordination cost is incurred and incentives for division managers are lower. Coordination plays two roles: coordinate divisions to better deal with uncertainty and allow them to better capture synergy. In contrast, decentralization offers better incentives and is less costly in decision-making, although there is no coordination among decision-makers.
and lack of overall market information available to guide their decisions. Besides internal restructuring, a firm can also carry out external restructuring by divesting a division. Divestiture offers the best incentives for managers and saves coordination cost, but it foregoes potential synergy among divisions. We analyze these options based on market uncertainty, market size, market competition, synergy, and coordination cost.

We consider a firm with two divisions, with positive or negative synergy between them. When coping with changes in the business environment, besides adjusting contractual relationships among managers, it may be better to locate some control rights at the division level in the hands of division managers and other rights at the top level in the hands of the chief executive officer (CEO). The firm can further consider divesting one of its divisions. Therefore, when facing a changing business environment, besides adjusting contractual relationships among managers, the firm has three structural options: stay put, internal restructuring (centralization or decentralization), and external restructuring (divestiture). When the chance that an option is the best one out of the three increases, we say that the firm is more likely to choose that option.

We find that when market uncertainty rises, a decentralized firm (D-firm) is more likely to carry out internal restructuring, while a centralized firm (C-firm) is more likely to carry out external restructuring. When market competition intensifies, a D-firm will stay put, while a C-firm is more likely to opt for either internal or external restructuring depending on whether there is positive synergy among divisions. When synergy increases, a D-firm will stay put, while a C-firm is more likely to carry out internal restructuring. When the market expands, a D-firm is more likely to stay put or carry out external restructuring, while a C-firm is more likely to carry out internal restructuring. When the coordination cost rises, a D-firm is more likely to carry out external restructuring, while a C-firm is more likely to carry out internal restructuring.

In sum, the general tendencies are: increasing market uncertainty induces firms to centralize (for the benefit of coordination); increasing market competition induces firms to decentralize (for better incentives and competitive pricing); lower synergy among divisions induces firms to divest (for better incentives); an expanding market induces firms to decentralize or divest (for better incentives); and a high coordination cost induces firms to divest (to avoid the coordination cost).

Our conclusions depend crucially on the fact that we allow both internal and external restructuring. If firm structure is not allowed to change, the effects on firms would be a lot different. For example, market competition may drive down firm profit, which has a negative effect on managerial incentives if firm structure is not adjusted; however, if the firm can decentralize its control structure or divest some divisions, managerial incentives may actually increase. In fact, we show that, when facing increasing market competition, a firm is likely to
decentralize its control structure or divest some divisions so as to improve managerial incentives (Result 2).

Our analysis is conditional on the boundary of the firm with an endogenous firm structure. By investigating the firm’s restructuring options (internal vs. external), we determine the boundary of the firm depending on the optimal firm structure. We contribute to the literature on the boundary of the firm by allowing an endogenous firm structure.

This paper proceeds as follows. Section 2 presents the model. Section 3 finds the solution. Section 4 analyzes the solution for the choice between internal and external restructuring. Section 5 applies our theory to several case studies. Section 6 offers conclusions. All proofs are given in the appendix.

2. The Model

The Firm

There is a firm with two divisions \( i = 1, 2 \). Each division produces a separate product. Division \( i \) produces output \( x_i \) and faces the following inverse demand:

\[
p_i = P_i(x_i, \tilde{a}_i),
\]

where \( p_i \) is the price and \( \tilde{a}_i \) is a random factor. Division \( i \)'s cost of production is

\[
c_i(x_i, x_j),
\]

where \( x_j \) is the other division’s output. This cost function allows division \( j \)'s output \( x_j \) to have a positive or negative effect on the production cost of division \( i \), implying an externality or synergy between the two divisions.

Control Structure

We take the incomplete contract approach. A control variable in the incomplete contract approach is similar to an effort or contractual variable in the complete contract approach. Instead of verifiability of an effort variable, we focus on the control rights or decision-making rights of a control variable.

Division \( i \)'s output \( x_i \) is a control variable, which may be decided by the manager of division \( i \) or by the CEO of the firm. When the control right of all outputs is situated at the division level, we call the firm a decentralized firm or \textit{D-firm}; when the control right of all outputs is situated at the firm level, we call the firm a centralized firm or \textit{C-firm}.

The relationships between the CEO and the division managers are defined by contracts. The CEO offers a contract to each division manager. In this contractual relationship, the CEO
is the principal and a division manager is the agent. The contract for division $i$’s manager is denoted by $S_i(x_i)$, specifying the payment $S_i(x_i)$ to the division manager when output is $x_i$. When the principal designs the contracts, she aims to maximize the firm’s overall profit; when a division manager decides on a division’s output, she aims to maximize her own income. Assume that a division’s output $x_i$ is verifiable, but the random factor $\tilde{a}_i$ and the division’s cost $C_i$ are not verifiable. The sequence of events is illustrated in the timeline of Figure 1.

![Figure 1. Timing of Events](image)

When the control right is situated at the firm level (centralized control), the CEO decides on both $x_1$ and $x_2$ ex post after the random factor $\tilde{a}_i$ is realized. There is a coordination cost $K(x_1, x_2)$ when decisions are made at the firm level. The coordination cost may arise from asymmetric information as in Alonso et al. (2015) or bureaucratic inefficiency as in Wang & Xiao (2009). When the control right is situated at the division level (decentralized control), the manager of division $i$ decides on $x_i$ ex post. The division managers cannot observe $\tilde{a}_i$. They decide on $x_1$ and $x_2$ simultaneously in Nash equilibrium in the ex post subgame. The solution in each case is a subgame perfect Nash equilibrium (SPNE).

In practice, while putting the CEO in control incurs a coordination cost, she does know the market better than anyone else. Various reports about markets go directly to the CEO. The chief information officer reports directly to the CEO. The chief operating officer, who looks after day-to-day activities including marketing and sales, provides feedback to the CEO. We hence assume that the division managers do not observe $\tilde{a}_i$; only the CEO does. In fact, the division managers in our model have no incentive to obtain market information $\tilde{a}_i$ since their income is based on their divisions’ own outputs $x_i$. The coordination cost at the firm level partially includes the cost of gathering such market information. Note that we do not consider the two divisions to be in two very different physical locations so that the division managers may have better local information than the CEO. When divisions are in the same location, a division manager is likely to be less knowledgeable of market information than the CEO, and synergy among divisions is an important factor for consideration. Also, a division manager

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1 We have used a simple contract form. The contract can also be of the form $S_i(P_i(x_i, a)x_i)\), $C_i(x_i, x_2)$ if the cost $C_i(x_i, x_2)$ is verifiable, and our main results would remain. That is, the form of contract itself does not influence our results. The reason is that in each case (a centralized structure, a decentralized structure, or with a spinoff), the contractual solution is efficient (maximizing the total surplus), implying that more complicated/sophisticated contracts will not improve firm profits.
typically knows her own market well, but not the markets of other divisions. In a firm where synergy exists among divisions, the CEO has the advantage of knowing all markets.

**Internal vs. External Restructuring**

When coping with a changing business environment, the CEO will adjust contractual relationships defined by \( S_1(\cdot) \) and \( S_2(\cdot) \). Besides, when the changes are substantial, the CEO may implement a structural change. The CEO can proceed in several ways. She may decentralize or centralize the divisions or divest one of the divisions. Divesting a division is one type of external restructuring, while decentralizing or centralizing the control structure within the firm is considered as internal restructuring. Depending on the business environment, internal restructuring may be better than external restructuring or vice versa. For example, when the market expands substantially, divestiture may prove to be the best coping strategy.

**Parametric Functions**

To analyze our solution, we use the following set of parametric functions:

\[
P_i(x_i, \bar{a}) = \bar{a} - \rho x_i, \quad C_i(x_i, x_j) = c x_i - \theta x_i x_j, \quad K(x_1, x_2) = 2k x_1 x_2,
\]

where \( \rho > 0, c > 0, k > 0 \) and \( \theta \in \mathbb{R} \). Here, \( k \) represents the coordination cost, \( c \) represents the marginal cost of production, and \( \theta \) represents synergy between the two divisions. If \( \theta > 0 \), there is positive synergy; if \( \theta < 0 \), there is negative synergy. We have a linear demand function for each product, where \( \rho \) is the slope of the demand curve and \( \bar{a} \) is the intercept. If \( \bar{a} \) increases (decreases), the demand curve shifts out (shifts in). For a larger (smaller) \( \rho \), demand is less (more) sensitive to the price. Since price sensitivity can be due to competition, \( 1/\rho \) is a measure of market competitiveness. Denote the density function of \( \bar{a} \) by \( f(\bar{a}) \), the mean value of \( \bar{a} \) by \( \bar{a} \equiv E(\bar{a}) \), and demand uncertainty by \( \sigma^2 \equiv Var(\bar{a}) \). If one of the divisions is divested, synergy disappears, and the cost and coordination functions for the two divisions become

\[
C_i(x_i) = c x_i, \quad K(x_i) = 2k x_i.
\]

For our purpose, we have assumed symmetric divisions (identical parametric functions but different products) in this parametric setting.

**3. The Solution**

**3.1. Decentralization**

With a decentralized control structure, the division managers have the right to decide on outputs. After accepting contracts \( S_1(x_1) \) and \( S_2(x_2) \), the two division managers choose outputs in an ex post subgame. Given division 2’s output, division 1’s problem is
\[
\max_{x_1} S_i(x_1) - C_i(x_1, x_2).
\]

Its first-order condition (FOC) implies an incentive compatibility (IC) condition \(IC_1\). Symmetrically, division 2’s FOC implies a second IC condition \(IC_2\). The two IC conditions imply a Nash equilibrium \((\hat{x}_1, \hat{x}_2)\) in the ex post subgame. Then, after taking into account the ex post individual rationality (IR) conditions \(IR_1\) and \(IR_2\), the principal’s ex ante problem is

\[
\Pi_d^* \equiv \max_{s_1(\cdot), s_2(\cdot), x_1, x_2} \int [P_1(x_1, a)x_1 + P_2(x_2, a)x_2 - S_1(x_1) - S_2(x_2)]f(a) \, da
\]

\[
s.t. \quad IC_1: S'_1(x_1) = C'_{1x_1}(x_1, x_2)
\]

\[
IC_2: S'_2(x_2) = C'_{2x_2}(x_1, x_2)
\]

\[
IR_1: S_1(x_1) \geq C_1(x_1, x_2)
\]

\[
IR_2: S_2(x_2) \geq C_2(x_1, x_2),
\]

where \(\Pi_d^*\) is the expected profit. Here, since the division managers cannot observe \(\bar{a}, x_1\) and \(x_2\) are independent of \(a\). With the parametric functions in (1), the solution is

\[
x^*_i = \frac{\bar{a} - c}{2(\rho - \theta)}, \quad S^*_i(x) = \frac{2c\rho - \theta\bar{a} - \theta c}{2(\rho - \theta)} x, \quad \Pi_d^* = \frac{(\bar{a} - c)^2}{2(\rho - \theta)}
\]

### 3.2. Centralization

With a centralized control structure, the CEO has the right to decide on outputs. After the two division managers have accepted contracts \(S_1(x_1)\) and \(S_2(x_2)\), the CEO decides on \(x_1\) and \(x_2\) by the following ex post problem:

\[
\max_{x_1, x_2} P_1(x_1, a)x_1 + P_2(x_2, a)x_2 - S_1(x_1) - S_2(x_2) - K(x_1, x_2).
\]

The two FOCs imply two IC conditions \(IC_1\) and \(IC_2\), which determine outputs \((\hat{x}_1(a), \hat{x}_2(a))\). Then, after taking into account the ex ante IR conditions \(IR_1\) and \(IR_2\), the CEO’s ex ante problem is

\[
\Pi^*_c \equiv \max_{s_1(\cdot), s_2(\cdot), x_1, x_2} \int [P_1(x_1, a)x_1 + P_2(x_2, a)x_2 - S_1(x_1) - S_2(x_2) - K(x_1, x_2)]f(a) \, da
\]

\[
s.t. \quad IC_1: P'_1x_1(x_1, a)x_1 + P_1(x_1, a) = S'_1(x_1) + K'_1(x_1, x_2)
\]

\[
IC_2: P'_2x_2(x_2, a)x_2 + P_2(x_2, a) = S'_2(x_2) + K'_2(x_1, x_2)
\]

\[
IR_1: \int [S_1(x_1) - C_1(x_1, x_2)]f(a) \, da \geq 0
\]

\[
IR_2: \int [S_2(x_2) - C_2(x_1, x_2)]f(a) \, da \geq 0,
\]

where \(x_1\) and \(x_2\) are functions of \(a\), and \(\Pi^*_c\) is the expected profit. With the parametric functions in (1), the solution is

\[
x^*_i(a) = \frac{a - c}{2(\rho - \theta + k)}, \quad S^*_i(x) = cx - \theta x^2, \quad \Pi^*_c = \frac{a^2 + (\bar{a} - c)^2}{2(\rho - \theta + k)}
\]

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3.3. Divestiture

The firm may do better by spinning off one of its divisions. Since the two divisions are symmetric, we assume that division 1 is divested. After spinning off, division 1 becomes an independent firm and decides on its own output $x_1$. Division 1’s problem is

$$\max_{x_1} P_1(x_1, a)x_1 - C_1(x_1),$$

which implies an optimal output $x_1^*(a)$. Then, the ex ante profit is

$$\Pi_1^* \equiv \int \{P_1[x_1^*(a), a]x_1^*(a) - C_1(x_1^*(a))\} f(a) da.$$

With the parametric functions in (1), the solution is

$$x_1^*(a) = \frac{a - c}{2\rho}, \quad \Pi_1^* = \frac{(\bar{a} - c)^2 + \sigma^2}{4\rho}.$$

A Decentralized Spinoff

After spinning off division 1, the firm still has division 2. If the firm has a decentralized control structure, the manager of division 2 has the right to decide on division 2’s output as follows:

$$\max_{x_2} S_2(x_2) - C_2(x_2).$$

Its FOC implies an IC condition $IC$. Then, after taking into account the ex post IR condition $IR$, the CEO’s ex ante problem is

$$\Pi_{2d}^* \equiv \max_{S_2(\cdot), x_2} \int [P_2(x_2, a)x_2 - S_2(x_2)] f(a) da$$

s.t. $IC: S_2'(x_2) = C_2'(x_2)$

$IR: S_2(x_2) \geq C_2(x_2).$

Since the division manager cannot observe $\bar{a}$, $x_2$ is independent of $a$. With the parametric functions in (1), the solution is

$$x_2^* = \frac{\bar{a} - c}{2\rho}, \quad S_2^*(x) = cx, \quad \Pi_{2d}^* = \frac{(\bar{a} - c)^2}{4\rho}.$$  

Then, the total expected profit is

$$\Pi_{ds}^* \equiv \Pi_1^* + \Pi_{2d}^* = \frac{2(\bar{a} - c)^2 + \sigma^2}{4\rho}.$$

A Centralized Spinoff

If the firm has a centralized control structure, the CEO has the right to decide on division 2’s output. With coordination cost $K(x_2)$, the CEO’s ex post problem is

$$\max_{x_2} P_2(x_2, a)x_2 - S_2(x_2) - K(x_2).$$
Its FOC implies an IC condition $I_C$. Then, after taking into account the ex ante IR condition $I_R$, the CEO's ex ante problem is

$$\Pi_{2c} \equiv \max_{s_2 \in S_2} \int \left[ P_2(x_2, a)x_2 - S_2(x_2) - K(x_2) \right] f(a) \, da$$

s.t.

$$I_C: P'_{2x_2}(x_2, a)x_2 + P_2(x_2, a) = S'_2(x_2) + K'(x_2)$$

$$I_R: \int \left[ S_2(x_2) - C_2(x_2) \right] f(a) \, da \geq 0,$$

where $x_2$ is a function of $a$. With the parametric functions in (1), the solution is

$$x^*_2(a) = \frac{a - c - 2k}{2\rho}, \quad S^*_2(x) = cx, \quad \Pi^*_2c = \frac{(\bar{a} - c - 2k)^2 + \sigma^2}{4\rho}.$$

Then, the total expected profit is

$$\Pi^*_c \equiv \Pi^*_1 + \Pi^*_2c = \frac{(\bar{a} - c)^2 + (\bar{a} - c - 2k)^2 + 2\sigma^2}{4\rho}.$$

4. Analysis

When coping with a changing business environment, besides adjusting contractual relationships among managers (adjusting implicitly in equilibrium), a firm may also carry out restructuring. The nature of restructuring can be either internal or external. An adjustment of the control structure within a firm is known as internal restructuring, while a divestiture is considered as external restructuring.

The purpose of this analysis is to offer an understanding of why a firm sometimes chooses external restructuring over internal restructuring and vice versa. We focus on a few influencing factors, including market uncertainty $\sigma^2$, market competition measured by price sensitivity $1/\rho$, synergy $\theta$ between two divisions, market size $\bar{a}$, and the coordination cost $k$. For convenience, assume

$$\rho > \theta, \quad \bar{a} > c + 2k.$$

**Proposition.** Each type of firm has two restructuring options: internal restructuring (decentralization or centralization), and external restructuring (divestiture).

(1) A D-firm will choose to centralize if

$$k(\bar{a} - c)^2 < (\rho - \theta)\sigma^2, \quad 2(\theta - k)(\bar{a} - c)^2 + (\rho + \theta - k)\sigma^2 > 0.$$  

(2) A D-firm will choose to divest if

$$2\theta(\bar{a} - c)^2 < (\rho - \theta)\sigma^2, \quad 2(\theta - k)(\bar{a} - c)^2 + (\rho + \theta - k)\sigma^2 < 0.$$  

(3) A C-firm will choose to decentralize if

$$k(\bar{a} - c)^2 > (\rho - \theta)\sigma^2, \quad \frac{\rho + \theta}{\rho - \theta} (\bar{a} - c)^2 > (\bar{a} - c - 2k)^2 + 2\sigma^2.$$
(4) A C-firm will choose to divest if
\[
\frac{\theta - k}{\rho - \theta + k}[(\bar{a} - c)^2 + \sigma^2] + 2k(\bar{a} - c - k) < 0, \\
\frac{\rho + \theta}{\rho - \theta}(\bar{a} - c)^2 < (\bar{a} - c - 2k)^2 + 2\sigma^2.
\] (13)

The firm has three restructuring options: stay put, internal restructuring (centralization or decentralization), and external restructuring (divestiture). When the chance that an option is the best one out of the three increases, i.e., the two corresponding conditions in the Proposition are more likely to hold, we say that the firm is more likely to choose that option. A “more likely” result is convenient for empirical testing. For example, if a D-firm finds that centralization is better than its current control structure and is also better than divestiture, i.e., the two conditions in (10) are satisfied, then the firm will carry out internal restructuring (centralization). If the firm’s current control structure is still the best, it will stay put; that is, the firm may carry out restructuring only if environmental changes are substantial enough. Table 1 lists the conditions under which the firm may choose to carry out internal or external restructuring based on the Proposition.

<table>
<thead>
<tr>
<th>Table 1. Internal vs. External Restructuring</th>
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<tbody>
<tr>
<td><strong>Internal restructuring</strong></td>
</tr>
<tr>
<td><strong>Conditions (10)</strong></td>
</tr>
<tr>
<td>( k(\bar{a} - c)^2 &lt; (\rho - \theta)\sigma^2, )</td>
</tr>
<tr>
<td>( 2(\theta - k)(\bar{a} - c)^2 + (\rho + \theta - k)\sigma^2 &gt; 0 )</td>
</tr>
<tr>
<td><strong>External restructuring</strong></td>
</tr>
<tr>
<td><strong>Conditions (11)</strong></td>
</tr>
<tr>
<td>( 2\theta(\bar{a} - c)^2 &lt; (\rho - \theta)\sigma^2, )</td>
</tr>
<tr>
<td>( 2(\theta - k)(\bar{a} - c)^2 + (\rho + \theta - k)\sigma^2 &lt; 0 )</td>
</tr>
<tr>
<td><strong>Conditions (12)</strong></td>
</tr>
<tr>
<td>( k(\bar{a} - c)^2 &gt; (\rho - \theta)\sigma^2, )</td>
</tr>
<tr>
<td>( \frac{\rho + \theta}{\rho - \theta}(\bar{a} - c)^2 &gt; (\bar{a} - c - 2k)^2 + 2\sigma^2 )</td>
</tr>
<tr>
<td><strong>Conditions (13)</strong></td>
</tr>
<tr>
<td>( \frac{\theta - k}{\rho - \theta + k}[(\bar{a} - c)^2 + \sigma^2] + 2k(\bar{a} - c - k) &lt; 0, )</td>
</tr>
<tr>
<td>( \frac{\rho + \theta}{\rho - \theta}(\bar{a} - c)^2 &lt; (\bar{a} - c - 2k)^2 + 2\sigma^2 )</td>
</tr>
</tbody>
</table>

**Result 1.**

(a) When market uncertainty \( \sigma^2 \) rises, if \( \rho + \theta > k \), a D-firm is more likely to carry out internal restructuring (centralization); if \( \rho + \theta < k \), a D-firm is more likely to carry out external restructuring (divestiture); and if \( \theta < k \), a C-firm is more likely to carry out external restructuring.

(b) When market uncertainty \( \sigma^2 \) drops, a D-firm will stay put, while a C-firm is more likely to carry out internal restructuring (decentralization).
A centralized control structure offers better coordination among divisions (in addition to a coordinating role of contracts) and can deal with uncertainty more effectively. Hence, a D-firm is more likely to centralize when market uncertainty increases. However, centralization incurs a coordination cost. Condition $\rho + \theta > k$ ensures that the coordination cost is sufficiently small so that centralization remains the best option. If the coordination cost is large, a D-firm may be better off carrying out external restructuring. Condition $\rho + \theta < k$ ensures that the coordination cost is sufficiently large or synergy among divisions is sufficiently small so that a D-firm will carry out external restructuring. However, a division with more demand uncertainty is less valuable to the firm. Hence, when demand uncertainty rises, coupled with a high coordination cost or low synergy among divisions such that $\theta < k$, a C-firm may be better off carrying out external restructuring.

When market uncertainty drops, the advantage of centralization in dealing with uncertainty diminishes. In addition, a decentralized control structure offers better incentives for division managers. Hence, when market uncertainty drops, a C-firm is more likely to carry out internal restructuring.

**Result 2.**

(a) When market competition intensifies ($\rho$ drops), a D-firm will stay put; if $\theta > 0$, a C-firm is more likely to carry out internal restructuring; and if $\theta < 0$, a C-firm is more likely to carry out external restructuring.

(b) When market competition subsides ($\rho$ rises), a D-firm is more likely to carry out internal restructuring, while a C-firm will stay put.

As expected, from the solutions in (4) and (6), a C-firm’s equilibrium output is on average lower than a D-firm’s equilibrium output, implying that a C-firm’s equilibrium price is on average higher than a D-firm’s equilibrium price. When market competition subsides or demand becomes less sensitive to the price, a higher price becomes a profitable option, inducing a D-firm to centralize its control structure. Symmetrically, when market competition intensifies or demand becomes more sensitive to the price, because of lower incentives and a higher price, centralization has a clear disadvantage. Hence, a C-firm may prefer to decentralize its control structure. But if there is negative synergy among divisions, the firm may be better off carrying out external restructuring. Our finding that market competition implies a preference for decentralization is consistent with prior literature but contradicts Alonso et al. (2015).

As expected, effects of market competition on firms differ substantially if firm structure is fixed. Schmidt (1997), Raith (2003), and Vives (2008) discuss effects of market competition on firms when firm structure is fixed. If firm structure is fixed, when market competition lowers a firm’s profit, it will have a negative effect on managerial incentives. However, in our model, the firm can decentralize its structure control or divest a division so as to alleviate the
incentive problem. In fact, a structural change can reverse the effect on incentives so that market competition would have a positive effect on managerial incentives.

**Result 3.**

(a) When synergy $\theta$ rises, a D-firm will stay put, while a C-firm is more likely to carry out internal restructuring.

(b) When synergy $\theta$ drops, both types of firms is more likely to carry out external restructuring.

When synergy $\theta$ rises, decentralization is the best control structure, since a divestiture foregoes synergy and a decentralized firm structure captures synergy without incurring a coordination cost. Hence, a C-firm is more likely to decentralize. When synergy $\theta$ drops, the benefit of keeping the two divisions within the firm diminishes, making divestiture more attractive.

**Result 4.**

(a) When the market expands ($\bar{a}$ rises), if $\theta > 0$, a D-firm will stay put; if $\theta < 0$, a D-firm is more likely to carry out external restructuring; and a C-firm is more likely to carry out internal restructuring.

(b) When the market shrinks ($\bar{a}$ drops), if $\theta > k$, both types of firms will stay put; if $\theta < k$, a D-firm is more likely to carry out internal restructuring.

When the market expands, the benefit of a decentralized structure in stimulating incentives increases, inducing a C-firm to carry out internal restructuring. When the market shrinks, centralization offers coordination to cope in this difficult time, inducing a D-firm to centralize.

**Result 5.**

(a) When the coordination cost $k$ rises, a D-firm is more likely to carry out external restructuring, while a C-firm is more likely to carry out internal restructuring.

(b) When the coordination cost $k$ drops, a D-firm is more likely to carry out internal restructuring.

The effect of the coordination cost is straightforward. An increase in the coordination cost makes centralization a less attractive option than decentralization, inducing a C-firm to carry out internal restructuring. At the same time, an increase in the coordination cost also makes external restructuring a more attractive option, inducing a D-firm to carry out external restructuring.
In sum, when market uncertainty rises, a D-firm is more likely to carry out internal restructuring, while a C-firm is more likely to carry out external restructuring. When market competition intensifies, a D-firm will stay put, while a C-firm is more likely to opt for either internal or external restructuring depending on whether synergy is positive. When synergy rises, a D-firm will stay put, while a C-firm is more likely to carry out internal restructuring. When the market expands, a D-firm is more likely to stay put or carry out external restructuring, while a C-firm is more likely to carry out internal restructuring. When the coordination cost rises, a D-firm is more likely to carry out external restructuring, while a C-firm is more likely to carry out internal restructuring.

5. Case Studies

In this section, we offer a few case studies in relation to our theory. In practice, divestiture has often been a strategy to spin off non-core or poorly-performing divisions; centralization has often been a strategy to streamline operations; and decentralization has often been a strategy to improve managerial incentives.

The US Federal Reserve System

The US Federal Reserve System (Fed) has implemented several major restructuring programs in its history. In 1919, due to expanding demand for US banking services during World War I, the Fed decentralized some of its control rights to the departmental level, which is consistent with our Result 4(a). In 1935, however, the Fed centralized many of its control rights (Wheelock, 1999) amid shrinking demand for banking services during the ten years of economic recession after the 1929 stock market crash, which is consistent with Result 4(b). In the last 20 to 30 years, US banks have been suffering from competition with nonbank financial intermediaries. Consequently, examples of external restructuring abound among US banks, which is consistent with Result 2(a).

Alfa Corporation

Alfa is a Mexican business group teetering on the edge of collapse in 1982 amid an economic crisis in Mexico. The company carried out restructuring in that same year and is today the fourth largest privately owned company in Mexico. The restructuring program centralized decision-making, strengthened the core business, regrouped management control of those subsidiaries with synergy, and divested non-productive or less important subsidiaries. It is consistent with Results 1(a), 2(a) and 4(b). Following the recovery of the Mexican economy in 1987, Alfa carried out a decentralization program, which is consistent with Results 1(b) and 4(a).
Hewlett-Packard (HP) Company

Up until 1996, HP had been performing well and repeatedly decentralized its organizational structure, which is consistent with Result 4(a). In early 2000, however, following stagnant revenues and a declining profit rate, HP centralized its 83 divisions into six centralized divisions. In March 2012, following declining profits, HP again centralized management of its product lines, which is consistent with Result 4(b). In 2014, as competition in the personal computer (PC) market intensified, HP tried to sell off its PC division (but failed), which is consistent with Result 2(a). In 2015, HP divested its Snapfish division to sharpen its focus; and in late 2015, HP further split into two entities: HP Inc. and HP Enterprise, which is consistent with Results 2(a) and 3(b).

Microsoft Corporation

Microsoft started facing a shrinking market in 2005, due to the growing popularity of tablets. In September that same year, the company centralized its decision-making; and in July 2013, it further centralized its technology decisions and combined eight divisions into four, which is consistent with Result 4(b).

Sony Corporation

In 2005, Sony started facing a shrinking market share with fierce competition from Samsung and Apple. In September that same year, the company centralized its decision-making, which is consistent with Result 4(b). In March 2012, it carried out another major restructuring program to centralize its decision-making and close down or sell off some divisions, which is consistent with Results 4(b) and 2(b).

Acer Inc.

In 1991, as a fast growing company, Acer decentralized many decision-making rights, which is consistent with Result 4(a). However, in 1998 and again in December 2000, with a shrinking PC market, Acer centralized its product management, manufacturing, customer services and brand management functions, which is consistent with Result 4(b). In 2001, with a fast growing market in mainland China, Acer spun off Wistron and BenQ, which is consistent with Result 4(a).
6. Conclusion

A centralized control structure offers better coordination among divisions and better information to aid decision-making, but it incurs a coordination cost and leads to lower incentives for division managers. A decentralized control structure offers better incentives and is less costly in decision-making, but there is no coordination among decision-makers and no overall market information available to guide their decisions. Divestiture offers the best incentives and saves coordination cost, but it foregoes synergy among divisions. Consequently, the general tendencies are: increasing market uncertainty induces firms to centralize (for the benefit of coordination); increasing market competition induces firms to decentralize (for better incentives and competitive pricing); lower synergy among divisions induces firms to divest (for better incentives); an expanding market induces firms to decentralize or divest (for better incentives); and a high coordination cost induces firms to divest (to avoid the coordination cost).

Globalization is a strong recent trend. Multinationals have been the driving force behind globalization. However, little is known about the boundary of these multinationals (see a survey by Antràs & Rossi-Hansberg (2009)). Why do multinationals often (although not always) choose external restructuring over internal restructuring? What factors limit a multinational’s size and determine its internal control structure? The breakup of DaimlerChrysler provides an example of external restructuring in a troubled time. Our study offers an understanding of how multinationals develop and evolve.

Internal restructuring is a reorganization of the firm, while external restructuring is a redefinition of the firm’s boundary. This paper is a study on the dependence of the firm’s boundary on its internal control structure. It presents a theory of the boundary of the firm with an endogenous firm structure.

Appendix

A.1. The Decentralization Solution

For problem (3), if the IR conditions are not binding in equilibrium, given optimal contracts $S_1(x_1)$ and $S_2(x_2)$, the CEO can always offer $S_1(x_1) - \varepsilon$ and $S_2(x_2) - \varepsilon$ for some $\varepsilon > 0$ to satisfy the IR conditions. Contracts $S_1(x_1) - \varepsilon$ and $S_2(x_2) - \varepsilon$ also satisfy the IC conditions. These contracts raise the firm’s profit, which contradicts the fact that contracts $S_1(x_1)$ and
$S_2(x_2)$ are optimal. Hence, the IR conditions must be binding in equilibrium. By the binding IR conditions, problem (3) becomes

$$
\Pi_d^* = \max_{s_1(.), s_2(.) \sim \mathcal{X}_1, x_2} \int [P_1(x_1, a)x_1 + P_2(x_2, a)x_2 - C_1(x_1, x_2) - C_2(x_1, x_2)] f(a) \, da \\
\text{s.t.} \\
I C_1: S_1'(x_1) = C_1'(x_1, x_2) \\
I C_2: S_2'(x_2) = C_2'(x_1, x_2) \\
I R_1: S_1(x_1) = C_1(x_1, x_2) \\
I R_2: S_2(x_2) = C_2(x_1, x_2). 
$$

(14)

Since $S_1(x_1)$ and $S_2(x_2)$ do not appear in the objective function of problem (14), the problem can be solved in two steps. First, we solve the following problem for optimal outputs $(x_1^*, x_2^*)$ without referring to contracts $S_1$ and $S_2$:

$$
\Pi_d^* = \max_{x_1, x_2} \int [P_1(x_1, a)x_1 + P_2(x_2, a)x_2 - C_1(x_1, x_2) - C_2(x_1, x_2)] f(a) \, da. 
$$

(15)

Second, given $(x_1^*, x_2^*)$, we find optimal contracts $S_1$ and $S_2$ that satisfy:

$$
\begin{align*}
S_1'(x_1) &= C_1'(x_1, x_2) \\
S_2'(x_2) &= C_2'(x_1, x_2) \\
S_1(x_1) &= C_1(x_1, x_2) \\
S_2(x_2) &= C_2(x_1, x_2). 
\end{align*}
$$

(16)

Since the division managers cannot observe $a$, if the CEO proposes $(x_1^*(a), x_2^*(a))$ which is dependent on $a$, the division managers may not accept $(x_1^*(a), x_2^*(a))$ when $\bar{a} = a$. Hence, the CEO has to propose a pair of fixed $(x_1^*, x_2^*)$ that is independent of $a$.

With functions in (1) and fixed $x_1$ and $x_2$, problem (15) becomes

$$
\Pi_d^* = \max_{x_1, x_2} (\bar{a} - \rho x_1)x_1 + (\bar{a} - \rho x_2)x_2 - (cx_1 - \theta x_1 x_2) - (cx_2 - \theta x_1 x_2). 
$$

The FOCs are

$$
\bar{a} - 2\rho x_1 - c + 2\theta x_2 = 0, \quad \bar{a} - 2\rho x_2 - c + 2\theta x_1 = 0,
$$

which imply

$$
x_1^* = x_2^* = \frac{(\theta + \rho)(\bar{a} - c)}{2(\rho^2 - \theta^2)} = \frac{\bar{a} - c}{2(\rho - \theta)}.
$$

Then, by (16), we need to find $S_1(x)$ such that

$$
S_1'(x_1) = c - \theta x_2^*, \quad S_1(x_1^*) = cx_1^* - \theta x_1^* x_2^*. 
$$

(17)

Consider a linear contract of the form $S_1 = \alpha x_1 + \beta$. Then, (17) implies

$$
\alpha = c - \theta x_2^* = c - \theta \frac{\bar{a} - c}{2(\rho - \theta)},
$$

and

$$
(c - \theta x_2^*) x_1^* + \beta = cx_1^* - \theta x_1^* x_2^*,
$$

implying $\beta = 0$. Hence,
\[ S_1^*(x) = \frac{2c\rho - \theta \tilde{a} - \theta c}{2(\rho - \theta)} x. \]

Then,
\[ p_1^* = \tilde{a} - \rho x_1^* = \frac{\rho(\tilde{a} - c)}{2(\rho - \theta)}, \]

and
\[ \Pi_\theta^* = 2(\tilde{a} - \rho x^*)x^* - 2[cx^* - \theta(x^*)^2] = \frac{(\tilde{a} - c)^2}{2(\rho - \theta)}. \]

**A.2. The Centralization Solution**

For problem (5), if the IR conditions are not binding in equilibrium, given optimal contracts \( S_1(x_1) \) and \( S_2(x_2) \), the CEO can always offer \( S_1(x_1) - \varepsilon \) and \( S_2(x_2) - \varepsilon \) for some \( \varepsilon > 0 \) to satisfy the IR conditions. Contracts \( S_1(x_1) - \varepsilon \) and \( S_2(x_2) - \varepsilon \) also satisfy the IC conditions. These contracts raise the firm’s profit, which contradicts the fact that contracts \( S_1(x_1) \) and \( S_2(x_2) \) are optimal. Hence, the IR conditions must be binding in equilibrium. By the binding IR conditions, problem (5) becomes

\[
\Pi_\varepsilon^* = \max_{s_1(.),s_2(.),x_1,x_2} \left\{ \int \left[ P_1(x_1,a)x_1 + P_2(x_2,a)x_2 - C_1(x_1,x_2) - C_2(x_1,x_2) - K(x_1,x_2) \right] f(a) da \right. \\
\left. \text{s.t.} \quad I_C_1: P_{1x_1}'(x_1,a)x_1 + P_1(x_1,a) = S_1'(x_1) + K_{x_1}'(x_1,x_2) \right. \\
\left. I_C_2: P_{2x_2}'(x_2,a)x_2 + P_2(x_2,a) = S_2'(x_2) + K_{x_2}'(x_1,x_2) \right. \\
\left. I_R_1: \int S_1(x_1)f(a) da = \int C_1(x_1,x_2)f(a) da \right. \\
\left. I_R_2: \int S_2(x_2)f(a) da = \int C_2(x_1,x_2)f(a) da. \right. 
\]

(18)

Since \( S_1(x_1) \) and \( S_2(x_2) \) do not appear in the objective function of problem (18), the problem can be solved in two steps. First, we solve the following problem for optimal outputs \((x_1^*, x_2^*)\) without referring to contracts \(S_1\) and \(S_2\):

\[
\Pi_\varepsilon^* = \max_{x_1,x_2} \left\{ \int \left[ P_1(x_1,a)x_1 + P_2(x_2,a)x_2 - C_1(x_1,x_2) - C_2(x_1,x_2) - K(x_1,x_2) \right] f(a) da \right. \\
\left. \text{s.t.} \quad I_C_1: P_{1x_1}'(x_1,a)x_1 + P_1(x_1,a) = S_1'(x_1) + K_{x_1}'(x_1,x_2) \right. \\
\left. I_C_2: P_{2x_2}'(x_2,a)x_2 + P_2(x_2,a) = S_2'(x_2) + K_{x_2}'(x_1,x_2) \right. \\
\left. I_R_1: \int S_1(x_1)f(a) da = \int C_1(x_1,x_2)f(a) da \right. \\
\left. I_R_2: \int S_2(x_2)f(a) da = \int C_2(x_1,x_2)f(a) da. \right. 
\]

(19)

Second, given \((x_1^*, x_2^*)\), we find optimal contracts \(S_1\) and an \(S_2\) that satisfy:

\[
P_{1x_1}'(x_1^*,a)x_1^* + P_1(x_1^*,a) = S_1'(x_1^*) + K_{x_1}'(x_1^*,x_2^*) \\
P_{2x_2}'(x_2^*,a)x_2^* + P_2(x_2^*,a) = S_2'(x_2^*) + K_{x_2}'(x_1^*,x_2^*) \\
\int S_1(x_1^*)f(a) da = \int C_1(x_1^*,x_2^*)f(a) da \\
\int S_2(x_2^*)f(a) da = \int C_2(x_1^*,x_2^*)f(a) da. 
\]

(20)

With functions in (1), problem (19) becomes

\[
\Pi_\varepsilon^* = \max_{x_1,x_2} \int \left[ (a - \rho x_1)x_1 + (a - \rho x_2)x_2 - (c x_1 - \theta x_1 x_2) - (c x_2 - \theta x_1 x_2) - 2 k x_1 x_2 \right] f(a) da 
\]

By the Hamilton method, the Euler equations are
\[ ax_1 - 2\rho x_1 - c + 2(\theta - k)x_2 = 0, \quad ax_2 - 2\rho x_2 - c + 2(\theta - k)x_1 = 0, \]

implying
\[ x_1^* = x_2^* = \frac{a - c}{2(\rho - \theta + k)}. \]

Then,
\[ p_i^* = a - \frac{\rho(a - c)}{2(\rho - \theta + k)}. \]

Assume \( S_1(x) = ax^2 + \beta x + \gamma \). (20) implies
\[ a - 2\rho x_1^* = 2ax_1^* + \beta + 2kx_1^*, \]
\[ aE([x_1^*(\bar{a})]^2) + \beta E(x_1^*(\bar{a})) + \gamma = cE(x_1^*(\bar{a})) - \theta E([x_1^*(\bar{a})]^2), \]

implying
\[
\left(1 - \frac{\alpha + k + \rho}{\rho - \theta + k}\right) a = \beta - \frac{\alpha + k + \rho}{\rho - \theta + k} c, \\
\frac{(\alpha + \theta)\sigma^2 + (\bar{a} - c)^2}{4(\rho - \theta + k)^2} + \frac{\beta - c(\bar{a} - c)}{2(\rho - \theta + k)} + \gamma = 0.
\]

Let
\[ \frac{\alpha + k + \rho}{\rho - \theta + k} = 1, \]

implying \( \alpha = -\theta \). Then, conditions in (21) imply \( \beta = c \) and \( \gamma = 0 \). Hence,
\[ S_1^*(x) = cx - \theta x^2. \]

Then,
\[ \Pi_i^* = E\left[2(a - \rho x_1^*)x_1^* - 2(cx_1^* - \theta(x_1^*)^2) - 2k(x_1^*)^2\right] = \frac{(\bar{a} - c)^2 + \sigma^2}{2(\rho - \theta + k)}. \]

A.3. The Divestiture Solution

With functions in (1), problem (7) becomes
\[ \max_{x_1} (a - \rho x_1 - cx_1), \]
which implies
\[ x_1^* = \frac{a - c}{2\rho}. \]

We then have
\[ \Pi_1^* = E\left[\frac{(\bar{a} - c)^2}{4\rho}\right] = \frac{(\bar{a} - c)^2 + \sigma^2}{4\rho}. \]
A Decentralized Spinoff

For problem (8), if the IR condition is not binding in equilibrium, given the optimal contract $S_2(x_2)$, the CEO can always offer $S_2(x_2) - \varepsilon$ for some $\varepsilon > 0$ to satisfy the IR condition. Contract $S_2(x_2) - \varepsilon$ also satisfies the IC condition. This contract raises the firm’s profit, which contradicts the fact that contract $S_2(x_2)$ is optimal. Hence, the IR condition must be binding in equilibrium. By the binding IR condition, problem (8) becomes

$$\Pi_{2d}^* = \max_{x_2} \int [P_2(x_2,a)x_2 - C_2(x_2)] f(a) da$$

s.t. IC: $S_2'(x_2) = C_2'(x_2)$
IR: $S_2(x_2) = C_2(x_2)$.

(22)

Since $S_2(x_2)$ does not appear in the objective function of problem (22), the problem can be solved in two steps. First, we solve the following problem for optimal output $x_2^*$ without referring to $S_2$:

$$\Pi_{2d}^* = \max_{x_2} \int [P_2(x_2,a)x_2 - C_2(x_2)] f(a) da.$$  

(23)

Second, given $x_2^*$, we find an $S_2$ that satisfies:

$$S_2'(x_2^*) = C_2'(x_2^*), \quad S_2(x_2^*) = C_2(x_2^*).$$

(24)

With functions in (1), problem (23) becomes

$$\Pi_{2d}^* = \max_{x_2} \int [(a - \rho x_2)x_2 - cx_2] f(a) da.$$  

Since $x_2$ does not depend on $a$ in this case, the problem becomes

$$\Pi_{2d}^* = \max_{x_2} (\bar{a} - c)x_2 - \rho x_2^2.$$  

which implies

$$x_2^* = \frac{\bar{a} - c}{2\rho}.$$  

Given $x_2^*$, we try to find an $S_2$ that satisfies (24). Consider a linear contract of the form $S_2(x_2) = ax_2 + \beta$. Then, (24) becomes

$$\alpha = c, \quad \alpha \frac{\bar{a} - c}{2\rho} + \beta = c \frac{\bar{a} - c}{2\rho},$$

implying $\alpha = c$ and $\beta = 0$. That is, there is indeed an optimal linear contract, which is

$$S_2^*(x) = cx.$$  

Then,

$$\Pi_{2d}^* = \frac{(\bar{a} - c)^2}{4\rho}.$$  

Thus,
\[ \Pi'_{ds} = \frac{2(\bar{a} - c)^2 + \sigma^2}{4\rho}. \]

**A Centralized Spinoff**

For problem (9), if the IR condition is not binding in equilibrium, given the optimal contract \( S_2(x_2) \), the CEO can always offer \( S_2(x_2) - \varepsilon \) for some \( \varepsilon > 0 \) to satisfy the IR condition. Contract \( S_2(x_2) - \varepsilon \) also satisfies the IC condition. This contract raises the firm’s profit, which contradicts the fact that contract \( S_2(x_2) \) is optimal. Hence, the IR condition must be binding in equilibrium. By the binding IR condition, problem (9) becomes

\[
\Pi^*_c = \max_{S_2(x_2)} \int \left[ P_2(x_2, a)x_2 - C_2(x_2) - K(x_2) \right] f(a) da \\
\text{s.t.} \quad \begin{align*}
\text{IC:} & \quad P_2'(x_2, a)x_2 + P_2(x_2, a) = S_2'(x_2) + K'(x_2) \\
\text{IR:} & \quad \int S_2(x_2)f(a) da = \int C_2(x_2)f(a) da.
\end{align*}
\] (25)

Since \( S_2(x_2) \) does not appear in the objective function of problem (25), the problem can be solved in two steps. First, we solve the following problem for optimal output \( x_2^* \) without referring to \( S_2 \):

\[
\Pi^*_c = \max_{x_2} \int \left[ P_2(x_2, a)x_2 - C_2(x_2) - K(x_2) \right] f(a) da .
\] (26)

Second, given \( x_2^* \), we find an \( S_2 \) that satisfies:

\[
\begin{align*}
P_2'(x_2^*, a)x_2^* + P_2(x_2^*, a) &= S_2'(x_2^*) + K'(x_2^*) \\
\int S_2(x_2^*)f(a) da &= \int C_2(x_2^*)f(a) da.
\end{align*}
\] (27)

With functions in (1), problem (26) becomes

\[
\Pi^*_c = \max_{x_2} \left[ (a - \rho x_2(a))x_2(a) - cx_2(a) - 2kx_2(a) \right] f(a) da.
\]

The Hamiltonian function can be defined as

\[ H(x_2, a) = (a - c - 2k)x_2 - \rho x_2^2. \]

The Euler equation implies

\[ x_2^* = \frac{a - c - 2k}{2\rho}. \]

Given \( x_2^* \), we try to find an \( S_2 \) that satisfies (27). Consider a linear contract of the form \( S_2(x_2) = ax_2 + \beta \). Then, (27) becomes

\[
-\rho \frac{a - c - 2k}{2\rho} + a - \rho \frac{a - c - 2k}{2\rho} = a + 2k \\
E \left( x_2 \frac{\bar{a} - c - 2k}{2\rho} + \beta \right) = E \left( \frac{\bar{a} - c - 2k}{2\rho} \right),
\]

implying \( a = c \) and \( \beta = 0 \). That is, there is indeed an optimal linear contract, which is
\[ S_2(x) = cx. \]

Then,
\[ \Pi_{2c}^* = \frac{(\bar{a} - c - 2k)^2 + \sigma^2}{4\rho}. \]

Thus,
\[ \Pi_{cs}^* = \Pi_1^* + \Pi_{2c}^* = \frac{(\bar{a} - c)^2 + (\bar{a} - c - 2k)^2 + 2\sigma^2}{4\rho}. \]

### A.4. Proof of Proposition

#### (1) A D-Firm Chooses to Centralize

A D-firm will switch to a centralized structure if and only if \( \Pi_d^* < \Pi_c^* \) and \( \Pi_{ds}^* < \Pi_c^* \), i.e.,
\[
\frac{(\bar{a} - c)^2}{2(\rho - \theta)} < \frac{(\bar{a} - c)^2 + \sigma^2}{2(\rho - \theta + k)}, \quad \frac{2(\bar{a} - c)^2 + \sigma^2}{4\rho} < \frac{(\bar{a} - c)^2 + \sigma^2}{2(\rho - \theta + k)}
\]

implying
\[
k(\bar{a} - c)^2 < (\rho - \theta)\sigma^2, \quad 2(\theta - k)(\bar{a} - c)^2 + (\rho + \theta - k)\sigma^2 > 0.
\]

With \( \rho > \theta \), the conditions become
\[
k(\bar{a} - c)^2 < (\rho - \theta)\sigma^2, \quad 2(\theta - k)(\bar{a} - c)^2 + (\rho + \theta - k)\sigma^2 > 0.
\]

#### (2) A D-Firm Chooses to Divest

A D-firm will spin off one division if and only if \( \Pi_d^* < \Pi_{ds}^* \) and \( \Pi_c^* < \Pi_{ds}^* \), i.e.,
\[
\frac{(\bar{a} - c)^2}{2(\rho - \theta)} < \frac{2(\bar{a} - c)^2 + \sigma^2}{4\rho}, \quad \frac{(\bar{a} - c)^2 + \sigma^2}{2(\rho - \theta + k)} < \frac{2(\bar{a} - c)^2 + \sigma^2}{4\rho},
\]

implying
\[
2\theta(\bar{a} - c)^2 < (\rho - \theta)\sigma^2, \quad 2(\theta - k)(\bar{a} - c)^2 + (\rho + \theta - k)\sigma^2 < 0.
\]

#### (3) A C-Firm Chooses to Decentralize

A C-firm will switch to a decentralized structure if and only if \( \Pi_c^* < \Pi_d^* \) and \( \Pi_{cs}^* < \Pi_{d}^* \), i.e.,
\[
\frac{(\bar{a} - c)^2 + \sigma^2}{2(\rho - \theta + k)} < \frac{(\bar{a} - c)^2}{2(\rho - \theta)}, \quad \frac{(\bar{a} - c)^2 + (\bar{a} - c - 2k)^2 + 2\sigma^2}{4\rho} < \frac{(\bar{a} - c)^2}{2(\rho - \theta)},
\]

implying
\[
k(\bar{a} - c)^2 > (\rho - \theta)\sigma^2, \quad (\rho + \theta)(\bar{a} - c)^2 > (\rho - \theta)(\bar{a} - c - 2k)^2 + 2(\rho - \theta)\sigma^2.
\]
(4) A C-Firm Chooses to Divest

A C-firm will spin off one division if and only if $\Pi^*_C < \Pi^*_C$ and $\Pi^*_d < \Pi^*_C$, i.e.,

$$\frac{(a - c)^2 + \sigma^2}{2(\rho - \theta + k)} < \frac{(a - c)^2 + (a - c - 2k)^2 + 2\sigma^2}{4\rho},$$

$$\frac{(a - c)^2}{2(\rho - \theta)} < \frac{(a - c)^2 + (a - c - 2k)^2 + 2\sigma^2}{4\rho},$$

implying

$$\frac{(\rho + \theta - k)(a - c)^2 + 2(\theta - k)\sigma^2}{(\rho - \theta + k)(a - c - 2k)^2},$$

$$\frac{(\rho + \theta)(a - c)^2}{(\rho - \theta)(a - c - 2k)^2 + 2(\rho - \theta)\sigma^2}. $$

A.5. Proof of the Results

If $\theta \geq k$, the first condition of (13) and the second condition of (11) cannot hold. This makes sense: if synergy among the divisions is strong enough, neither type of firm would consider external restructuring. Hence, when we discuss conditions in (11) and (13), we need to assume $\theta < k$.

Proof of

**Result 1:** When $\sigma^2$ rises, the first conditions in (10) and (11) are more likely to hold, but the second conditions in (10) and (11) depend on the sign of $\rho + \theta - k$. If $\rho + \theta > k$, conditions in (10) are more likely to hold, implying that a D-firm is more likely to carry out internal restructuring; if $\rho + \theta < k$, conditions in (11) are more likely to hold, implying that a D-firm is more likely to carry out external restructuring. Also, when $\sigma^2$ rises, conditions in (12) are less likely to hold; and if $\theta < k$, conditions in (13) are more likely to hold.

When $\sigma^2$ drops, conditions in (10) and (11) are less likely to hold; but conditions in (12) are more likely to hold.

Proof of Result 2: If $\rho$ drops, conditions in (10) and (11) are less likely to hold; and if $\theta > 0$, conditions in (12) are more likely to hold; if $\theta < 0$, conditions in (13) are more likely to hold.

For a D-firm, if $\rho$ rises, conditions in (10) are more likely to hold; but for a C-firm, conditions in (12) and (13) are less likely to hold, whereas for conditions in (13) we need to assume $\theta < k$.

Proof of Result 3: When $\theta$ rises, conditions in (10) and (11) are less likely to hold, implying that a D-firm will stay put. However, conditions in (12) are more likely to hold.

When $\theta$ drops, conditions in (11) and (13) are more likely to hold.
Proof of Result 4: When $\bar{a}$ rises, conditions in (10) are less likely to hold. If $\theta > 0$, conditions in (11) are less likely to hold, but if $\theta < 0$, conditions in (11) are more likely to hold. Further, we have

$$\frac{\partial}{\partial \rho} \left[ \frac{\rho + \theta (\bar{a} - c)^2 - (\bar{a} - c - 2k)^2 - 2\sigma^2}{\theta (\bar{a} - c)} \right] = 2 \frac{2\theta}{\rho - \theta} (\bar{a} - c) + 4k > 0.$$ 

Hence, conditions in (12) are more likely to hold.

When $\bar{a}$ drops, if $\theta > k$, the second condition in (11) and the first condition in (13) simply cannot hold, and conditions in (10) and (12) are less likely to hold. If $\theta < k$, conditions in (10) are more likely to hold.

Proof of Result 5: When $k$ rises, conditions in (11) and (12) are more likely to hold. When $k$ drops, conditions in (10) are more likely to hold.

References


