Integration versus Outsourcing with Vertical Linkages*

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Abstract

In the model by Grossman and Helpman (2002) no industry has both vertically integrated and specialized producers in equilibrium. I generalize their model by assuming that final goods producers (irrespective of whether they are vertically integrated with the upstream stage or specialized in the downstream stage only) need a basket of differentiated commodities, in addition to labor, as a fixed requirement for production. I then show the existence of an equilibrium populated simultaneously by vertically integrated and disintegrated firms.

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1 Introduction

One feature of the model by Grossman and Helpman (2002) is that, generically, no industry has both vertically integrated and specialized producers in equilibrium.[1] The authors suggest that allowing productivity heterogeneity among entrants would be one way to alter this result and reconcile the model with the empirical evidence showing that vertically integrated and specialized firms do coexist in the same industry. Here I propose a simple generalization to Grossman and Helpman (2002) aimed at generating an equilibrium with both types of firms that takes another route.

What I do is to posit vertical linkages, in the sense that both integrated and specialized manufacturers need a basket of differentiated varieties, in addition to labor, to cover the fixed costs of production.[2] Since the fixed costs of production affect the fulfilment of the zero profits condition under both production modes, by making them to depend on the price index of the differentiated varieties through vertical linkages, I allow an additional degree of freedom which enables a positive amount of both types of firms to coexist at equilibrium. In other terms, while in the Grossman and Helpman (2002) standard framework entry by one type of firm in numbers that ensure zero profits guarantees losses for the other type, in my framework this is not necessarily true.

2 Setup of the model

2.1 Consumption

The preferences of a representative consumer are Cobb-Douglas:

$$U = C_\xi D^\xi Q^{1-\xi}, \tag{1}$$

where C_ξ is a positive constant such that $C_\xi^{-1} = (\xi)^\xi (1-\xi)^{1-\xi}$, with $\xi > 0$, D is a CES aggregate of a mass equal to N of differentiated varieties,

$$D \equiv \left[\int_0^N y_i^\alpha \mathrm{d}i \right]^{1/\alpha},$$

α is the substitutability parameter, y_i the consumption of each variety, Q is the amount of the agricultural commodity. Calling I consumer's income, the utility function is maximized subject to income not exceeding I. Maximization yields that the consumer's demand function for a particular variety is

$$y_i = A p_i^{-1/(1-\alpha)}$$

and

$$A \equiv \frac{\xi I}{\int_0^N p_i^{-\alpha/(1-\alpha)} di} = \frac{\xi I}{P},$$

[1] Grossman and Helpman (2002), Proposition 1.

[2] See Ottaviano and Thisse (2004) and Ottaviano and Robert-Nicoud (2006) for applications of the concept of vertical linkages to an economic geography setting.

where p_i is a variety's price and the index P is equal to

$$P \equiv \int_0^N p_i^{-\alpha/(1-\alpha)} di.$$

2.2 Production

The traditional good is produced under perfect competition and constant returns to scale using labor only, with γ units of labor needed for 1 unit of Q. The profit maximizing price of Q equals γ times the wage. By choosing labor as the numeraire, the wage is pinned down to 1, and the price of the agricultural good is γ.

The varieties of the modern good are produced under monopolistic competition and increasing returns to scale. There are three different organizational forms in this industry. The first is vertical integration: firms perform both the upstream and the downstream stage of the production process. The second form is specialization in the upstream stage of the production process: the firm produces an intermediate input that is customized specifically for a downstream producer, to whom the component is passed. Finally, the firm may be a specialized final good producer: it receives the intermediate input from the upstream firm and then processes it to deliver the final product. As for variable costs, I choose units of the differentiated commodity so that the production of a unit of the final good by a specialized downstream producer requires a unit of the customized component by a specialized upstream firm. If the component is produced within a vertically integrated firm λ units of labor are needed.

Vertically integrated firms bear a fixed cost in terms of an input that is a Cobb-Douglas composite of labor and differentiated varieties. The same kind of fixed cost is incurred by specialized final good producers. This assumption introduces *vertical linkages*, provided that the demand by final good firms for the fixed input generates additional demand for differentiated varieties. Specialized input suppliers bear a fixed cost in terms of labor only.

I assume that the fixed requirement for vertically integrated firms, f_v, differ from the fixed requirement for disintegrated downstream producers, f_s, so that $f_v \neq f_s$. In both cases, the production of f_z is Cobb-Douglas with constant returns to scale,

$$f_z = K^{\xi} l_f^{1-\xi} \tag{2}$$

where $z = \{v, s\}$ is the index related to the mode of organization, and K corresponds to the same CES aggregate that enters the utility function,

$$K = \left[\int_0^N b_i^{\alpha} \mathrm{d}i\right]^{1/\alpha}$$

with b_i the quantity of variety i needed for business purposes, while l_f is labor. The assumption that K is the same of D is standard. Another assumption that has been previously used in the literature is the equality of the Cobb-Douglas parameter in (1)and (2).[3] Each final good firm solves the following

[3]Ottaviano and Thisse (2004); Ottaviano and Robert-Nicoud (2006).

optimization problem to determine the optimal quantities of differentiated varieties and labor needed

$$\begin{cases} \min\limits_{b_i, l_f} & \int_0^N p_i b_i \mathrm{d}i + l_f \\ \text{s.t.} & K^{\xi} l_f^{1-\xi} = f_z \end{cases}$$

leading to a total cost equal to $C_{\xi} f_z P^{-\xi(1-\alpha)/\alpha}$.

The demand by a vertically integrated producer for a variety i is

$$b_{i,v} = \frac{\xi C_{\xi} f_v P^{-\xi(1-\alpha)/\alpha}}{\int_0^N p_i^{-\alpha/(1-\alpha)} di} p_i^{-1/(1-\alpha)},$$

while the demand from a specialized downstream producer is

$$b_{i,s} = \frac{\xi C_{\xi} f_s P^{-\xi(1-\alpha)/\alpha}}{\int_0^N p_i^{-\alpha/(1-\alpha)} di} p_i^{-1/(1-\alpha)}.$$

The total demand for variety i is the sum of the demands from consumers and final good producers:

$$y_i + s b_{i,s} + v b_{i,v} = (A + B) p_i^{-1/(1-\alpha)} \tag{3}$$

where

$$B \equiv \frac{s \xi C_{\xi} f_s P^{-\xi(1-\alpha)/\alpha} + v \xi C_{\xi} f_v P^{-\xi(1-\alpha)/\alpha}}{P}$$

so that

$$A + B = \frac{\xi I + s \xi C_{\xi} f_s P^{-\xi(1-\alpha)/\alpha} + v \xi C_{\xi} f_v P^{-\xi(1-\alpha)/\alpha}}{P}. \tag{4}$$

2.3 Business environment

Third parties cannot verify the quality of the component supplied by an upstream producer, hence enforceable contracts cannot be signed. Such contractual incompleteness precludes an efficient relationship-specific investment by upstream suppliers. The temporal structure of the model is: (1) firms enter as either vertically integrated producers, or specialized final goods' producers; (2) specialized downstream firms search for a specialized upstream partner, whose supply is perfectly elastic;[4] (3) all types of firms (vertically integrated or not) hire labor and buy differentiated varieties to meet the variable and fixed requirements for production, and the production of final goods simultaneously takes place. I assume that consumers own firms, so firms profits are distributed to them.

2.3.1 Vertical integration

In equilibrium, the price charged by a vertically integrated firm for the final good is

$$p_v = \frac{\lambda}{\alpha}$$

[4]Grossman and Helpman (2002) introduce search and matching frictions in this process. This is an unnecessary complication which does not alter the result of my paper.

and the resulting sales are

$$x_v = (A+B)\left(\frac{\alpha}{\lambda}\right)^{1/(1-\alpha)}.$$

The total cost function under vertical integration is

$$TC_v = C_\xi f_v P^{-\xi(1-\alpha)/\alpha} + \lambda x_v.$$

Total profits for vertically integrated firms are equal to total revenues minus total costs

$$\Pi_v = \frac{(1-\alpha)\lambda}{\alpha} x_v - C_\xi f_v P^{-\xi(1-\alpha)/\alpha}$$

which can be written also as

$$\Pi_v = (A+B)(1-\alpha)\left(\frac{\alpha}{\lambda}\right)^{\alpha/(1-\alpha)} - C_\xi f_v P^{-\xi(1-\alpha)/\alpha}. \qquad (5)$$

2.3.2 Outsourcing

The labor input requirement is set to one for the production of the intermediate component under outsourcing. The upstream firm has to determine how many units to supply to the downstream firm. As in Grossman and Helpman (2002), I assume Nash bargaining on the surplus of the relationship, that leaves the upstream firm with a share equal to ω of revenues. The upstream firm then maximizes variable profits equal to $(\omega p_s - 1)x_s$, where ω is the share of revenues going to the intermediates' producer, p_s is the market price of the final good, and x_s is the amount of components produced. In equilibrium, the price charged for the final good is

$$p_s = \frac{1}{\alpha\omega}$$

while sales are

$$x_s = (A+B)(\alpha\omega)^{1/(1-\alpha)}.$$

Total profits of a specialized upstream firm are

$$\Pi_m = (1-\alpha)\omega(A+B)(\alpha\omega)^{\alpha/(1-\alpha)} - f_m - T,$$

where f_m is the labor fixed cost and T is the upfront fee asked by the buyer to the supplier in order to participate to the relationship. Total profits of a downstream firm are

$$\Pi_s = (1-\omega)(A+B)(\alpha\omega)^{\alpha/(1-\alpha)} - C_\xi f_s P^{-\xi(1-\alpha)/\alpha} + T$$

and, due to the existence of a very large mass of potential suppliers, it turns out that

$$T = (1-\alpha)\omega(A+B)(\alpha\omega)^{\alpha/(1-\alpha)} - f_m.$$

As a consequence, the intermediate component supplier earns a payoff equal to zero from this relationship, because the final good producer secures for himself all profits, which become consequently

$$\Pi_s = (A+B)(\alpha\omega)^{\alpha/(1-\alpha)}(1-\alpha\omega) - C_\xi f_s P^{-\xi(1-\alpha)/\alpha} - f_m. \qquad (6)$$

3 Equilibrium analysis

3.1 Short-run equilibrium

In the short run the total number of each type of firm is fixed. Given that total income is equal to $I = L + s\Pi_s + v\Pi_v$, the expenditure on the manufacturing sector, $(A+B)P$, is equal to

$$(A+B)P = \xi L + \xi v \left[(A+B)(1-\alpha)\left(\frac{\alpha}{\lambda}\right)^{\alpha/(1-\alpha)} - C_\xi f_v P^{-\xi(1-\alpha)/\alpha}\right] +$$
$$\xi s \left[(A+B)(1-\alpha\omega)(\alpha\omega)^{\alpha/(1-\alpha)} - C_\xi f_s P^{-\xi(1-\alpha)/\alpha} - f_m\right] + \xi v C_\xi f_v P^{-\xi(1-\alpha)/\alpha} + \xi s C_\xi f_s P^{-\xi(1-\alpha)/\alpha} \tag{7}$$

where the index P is fixed to its short-run level,

$$P(s,v) = s(\alpha\omega)^{\alpha/(1-\alpha)} + v\left(\frac{\alpha}{\lambda}\right)^{\alpha/(1-\alpha)}.$$

After some manipulations I can write the total industry demand level $(A+B)$, which from now on I call $\mathcal{AB}$, as

$$\mathcal{AB}(s,v) = \frac{\xi(L - sf_m)}{v\,(\alpha/\lambda)^{\alpha/(1-\alpha)}\,[1-\xi(1-\alpha)] + s(\alpha\omega)^{\alpha/(1-\alpha)}[1-\xi(1-\alpha\omega)]}. \tag{8}$$

Given (8), if $s < L/f_m$ then $\mathcal{AB}(s,v) > 0$. This is a consistency condition that requires that the fixed cost incurred by upstream firms does not absorb the whole labor force. In the short run, the number of the different types of firms, s and v, is given and this also fixes $\mathcal{AB}$ to its short-run level.

3.2 Long-run equilibrium

I assume a simple law of motion for entry/exit of firms into the industry. To simplify the analysis, for vertically integrated firms it is $\dot{v} = \Pi_v(s,v)$, while for disintegrated downstream producers it is $\dot{s} = \Pi_s(s,v)$. So there is entry as far as firms make positive profits, and exit if profits are negative. I can write the dynamical system as follows:

$$\begin{cases} \dot{s} = \Pi_s(s,v) \\ \dot{v} = \Pi_v(s,v) \end{cases} \tag{9}$$

The long-run equilibrium (steady state) of the economy is characterized by the following equations:

$$\begin{cases} \dot{v} = 0 \\ \dot{s} = 0 \end{cases}$$

The solution to the dynamical system requires that the zero-profit condition under vertical integration, $\Pi_v = 0$, applies. From (5) this is the case if and only if

$$\Pi_v(s,v) = 0 \Leftrightarrow \mathcal{AB}(s,v) = \frac{(\lambda/\alpha)^{\alpha/(1-\alpha)}}{1-\alpha} C_\xi f_v P(s,v)^{-\xi(1-\alpha)/\alpha}. \tag{10}$$

The zero-profit condition under outsourcing, $\Pi_s(s,v)=0$, should also apply. From (6) this requires that

$$\Pi_s(s,v)=0 \Leftrightarrow \mathcal{AB}(s,v)=\frac{f_m+C_\xi f_s P(s,v)^{-\xi(1-\alpha)/\alpha}}{(1-\alpha\omega)(\alpha\omega)^{\alpha/(1-\alpha)}}. \tag{11}$$

Solving the system made by (10) and (11) I retrieve the value of $P(s,v)$ and $\mathcal{AB}(s,v)$ at steady state. For $P(s,v)$ I get

$$P(s,v)=\left\{\frac{(1-\alpha)f_m}{C_\xi[(1-\alpha\omega)(\lambda\omega)^{\alpha/(1-\alpha)}f_v-(1-\alpha)f_s]}\right\}^{\frac{-\alpha}{\xi(1-\alpha)}}\equiv P_E, \tag{12}$$

where P_E is a positive constant if and only if

$$\frac{f_s}{f_v}<\frac{1-\alpha\omega}{1-\alpha}(\lambda\omega)^{\alpha/(1-\alpha)}, \tag{13}$$

something that I assume to hold. There are several combinations of s and v such that $P(s,v)=P_E$. The locus of the points satisfying (12) is indeed a straight line in the (s,v) space:

$$s(\alpha\omega)^{\alpha/(1-\alpha)}+v\left(\frac{\alpha}{\lambda}\right)^{\alpha/(1-\alpha)}=P_E. \tag{14}$$

For $\mathcal{AB}(s,v)$ I get

$$\mathcal{AB}(s,v)=\frac{f_m f_v\,(\lambda/\alpha)^{\alpha/(1-\alpha)}}{(1-\alpha\omega)(\lambda\omega)^{\alpha/(1-\alpha)}f_v-(1-\alpha)f_s}\equiv\mathcal{AB}_E, \tag{15}$$

where $\mathcal{AB}_E$ is a positive constant under (13). Rearranging terms this implies another linear relationship in the (s,v) space:

$$v\mathcal{AB}_E\left(\frac{\alpha}{\lambda}\right)^{\alpha/(1-\alpha)}[1-\xi(1-\alpha)]+s\left\{\mathcal{AB}_E(\alpha\omega)^{\alpha/(1-\alpha)}[1-\xi(1-\alpha\omega)]+\xi f_m\right\}=\xi L. \tag{16}$$

The number of vertically integrated firms, v, and disintegrated firms, s, constituting the mixed general equilibrium is given by the linear system made of (14) and (16). The linearity of the system implies that there exists only one equilibrium. The solution is:

$$\begin{aligned} s &= \frac{\xi L-P_E\mathcal{AB}_E[1-\xi(1-\alpha)]}{\xi[f_m-(\alpha\omega)^{\alpha/(1-\alpha)}\mathcal{AB}_E\alpha(1-\omega)]}, \\ v &= \frac{\xi f_m P_E-(\alpha\omega)^{\alpha/(1-\alpha)}\left\{\xi L-P_E\mathcal{AB}_E[1-\xi(1-\alpha\omega)]\right\}}{\left(\frac{\alpha}{\lambda}\right)^{\alpha/(1-\alpha)}\xi[f_m-(\alpha\omega)^{\alpha/(1-\alpha)}\mathcal{AB}_E\alpha(1-\omega)]}. \end{aligned}$$

The slope of (14) is smaller in absolute value of that of (16), and this is true when

$$\frac{f_s}{f_v}<(\lambda\omega)^{\alpha/(1-\alpha)}. \tag{17}$$

Under (17) the condition that has to be satisfied for the existence of a mixed interior equilibrium ($s>0$ and $v>0$) is

$$\frac{P_E\mathcal{AB}_E[1-\xi(1-\alpha)]}{\xi}<L<\frac{P_E\mathcal{AB}_E[1-\xi(1-\alpha\omega)]}{\xi}+\frac{P_E f_m}{(\alpha\omega)^{\alpha/(1-\alpha)}}. \tag{18}$$

When

$$(\lambda\omega)^{\alpha/(1-\alpha)} < \frac{f_s}{f_v} < \frac{1-\alpha\omega}{1-\alpha}(\lambda\omega)^{\alpha/(1-\alpha)}, \tag{19}$$

(14) is steeper than (16) and the restriction that has to be satisfied for the existence of the mixed equilibrium is

$$\frac{P_E\mathcal{AB}_E[1-\xi(1-\alpha\omega)]}{\xi} + \frac{P_E f_m}{(\alpha\omega)^{\alpha/(1-\alpha)}} < L < \frac{P_E\mathcal{AB}_E[1-\xi(1-\alpha)]}{\xi}. \tag{20}$$

3.2.1 Stability of the long-run equilibrium

Given the dynamical system (9) it is possible to get a closed-form solution for the Jacobian matrix. However, it is hard to study the sign of eigenvalues. I then provide an assessment of the stability of the interior equilibrium by means of numerical simulations. First, I consider a case where $L = 7,190$, $\lambda = 1.15$, $\omega = .2$, $\alpha = .3$, $\xi = .35$, $f_m = 30$, $f_s = 20$, $f_v = 350$, so that (17) and (18) are verified. In this case, at equilibrium, $s \approx 23$, $v \approx 36$, with one eigenvalue of the Jacobian evaluated at the steady state being positive and the other being negative. The steady state is a saddle.

Then I consider a case where $L = 349,295$, $\lambda = 1.57$, $\omega = .4$, $\alpha = .28$, $\xi = .08$, $f_m = 70$, $f_s = 470$, $f_v = 560$. The restrictions that are verified are (19) and (20). The steady state is $s \approx 32$, $v \approx 31$ and, based on the sign of the eigenvalues of the Jacobian, it turns out to be a sink. The model exhibits rich stability properties of the mixed equilibrium.

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