Bank Regulation and Market Structure *

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September 20, 2016

*Funding from the following research projects is gratefully acknowledged: "The Challenges of the Crisis. Rethinking Micro and Macro Policies", UCSC research project D3.2, "Local interactions and global dynamics in economics and finance: models and tools", PRIN-2009, MIUR, and "Teorie e modelli matematici per le scienze economiche", D.1-2013, Catholic University of Milan.

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Abstract Following the recent and on-going tightenings of capital requirements in response to the financial crisis many experts have predicted a decline in the importance of the banking sector as banks struggle to compete with other providers of financial intermediation. The purpose of our research is to investigate this hypothesis in a model where the need for bank regulation is explained from first principles.
Our analysis suggests that, in a monopolistic competition setting, with higher capital requirements the bank sector may see bank competition reduced as the sector becomes dominated by fewer banks. Parallel with this development depositors may stop using banks in favor of an outside option that can be interpreted as banks which are either unregulated or subject to other, possibly less stringent, types of regulation. It is noteworthy that the first effect goes against an objective often stated by regulators, namely to reduce the number of large banks in the market, banks which may be "too big to fail".
We explain tightened capital requirements as the regulator's response to financial innovation. In our model such innovation increases the riskiness of certain asset classes without improving on their expected return. The only way to prevent banks from using these new assets is by increasing the capital requirements for all asset classes. The costs of these requirements are ultimately born by depositors and may lead them to look for better returns elsewehere.
Our model builds on the Salop (1979) model and in doing so also sheds some light on its properties. In particular we show that so-called monopoly equilibria only exist when the number of banks is constrained to be an integer.

Keywords: Bank regulation, capital requirements, market leakage, financial repression, moral hazard, market structure, bank size, Salop model.

JEL: D43, G21, G28.

1 Introduction

Recently there has been a surge in interest in the side effects of capital requirements used as an instrument for regulating banks. Most of the related research is macroeconomic oriented and often comes under the headline "procyclicality", the idea being that capital requirements may amplify shocks to the economy, as they become more stringent and costly when market conditions are tight.[3]

Our paper focuses on side effects of bank regulation of a more microeconomic nature and in particular asks how tightening capital requirements may affect the competitive structure and size of the bank sector. We show how such tightenings may reduce the number of banks in the market, while increasing the size of each individual bank. If we take into account that there can only be an integer number of banks in the market, tightened capital requirements may lead to some depositors leaving the market, i.e. market leakage, an effect which is particular important when the market is small to begin with. These effects of capital requirements constitute an extra cost for society that should be added to the direct costs (here the cost of capital) when assessing the desirability of this type of regulation.

Crucial for our results is the (natural) assumption that depositors have an alternative to bank deposits for their savings, here in the form of an exogenously defined return. Formally, the exact nature of this outside option is not important, however it may be interpreted as banks outside, geographically or legally, the jurisdiction of the regulator. In the former case, the interpretation is that depositors have the possibility of placing their savings in shadow banks, a phenomena which has recently been much discussed by regulators and industry experts.[4] In the latter case, we may have a situation akin to regulatory competition where more lax rules outside the jurisdiction of regulators may attract some potential depositors.

Our paper, following Repullo (2004) and extensions in Nielsen and Weinrich (2015a), assumes a Salop style banking sector where banks are price setters and new banks enter as long as this is profitable.[5] Like in Hellman et al. (2000), Nielsen and Weinrich (2015a) and Repullo (2004), banks may invest their deposits in either a socially suboptimal risky asset (called the gambling asset) or in a safe asset, called prudent. Since their choice is not observable and since there is limited liability, a moral hazard problem appears, which is the reason why bank regulation is needed. Specifically, in our model capital requirements

[3]For contributions to this strand of research, see Covas and Fujita (2010) and Gordy and Howells (2006).

[4]See for instance Errico et al. (2010).

[5]Casual observations suggest that the banking sector is not perfectly competitive, something which has for instance been empirically confirmed by De Bandt and Davis (2000).

are used to prevent banks from using the gambling asset.

We also follow the main literature in assuming that capital requirements are expensive to banks (else there would be no need to impose them to begin with) and it is also theoretically well known that these costs are likely to be ultimately born by depositors in terms of lower deposit rates.

Main Results

When we do not impose integer constraints there are two possible regimes of competition, *Monopolistic Competition* and *Constrained Oligopoly.* The first occurs for low returns of the outside option, or equivalently for low levels of capital requirements. This regime is characterized by banks engaged in competition with each other, in particular, if one bank lowers its deposit rates other banks will capture some of its depositors. In the constrained oligopoly regime this is not so, since after a decrease in the deposit rate some depositors will leave the bank in favor of the outside option. Nonetheless, in this equilibrium all potential depositors are served by some bank. Finally, for sufficiently high k, the bank sector cannot sustain itself, i.e. there is no equilibrium with a positive number of banks. We show by means of a numeric example how the competitive regime changes with k.

When we ignore the integer constraints, the unit circle will always be filled out by banks. In contrast, if we acknowledge the integer constraint, for large k, we end up in a regime, which we call a *monopolistic equilibrium*, where not all depositors are served by banks. Ignoring the integer constraint may be reasonable when there is a large number of banks. In the other case the possibility that, even when banks are symmetric in terms of cost structure, in equilibrium some depositors use the outside option, must be taken seriously. In such an equilibrium banks do not compete with each other, but with the outside option available to agents.

We show that, as the capital requirement k increases, the deposit rate offered by banks decreases but initially the number of banks stays constant. At a certain critical point for k, the nature of the equilibrium changes from monopolistically competitive to constrained oligopoly. From then on, as k increases further, the number of banks decrease. The decrease in deposit rates followed by a reduction in the number of banks serving the market hurts depositors who now have a larger distance to the bank (or a product further away from their ideal one), while also receiving a lower return on their savings.[6] It also means that each bank remaining in the market becomes larger, since it is serving more depositors. When the number of banks is small, as k increases further the number of

[6]In a model without free entry, changes in the market structure would translate into higher profits which would hurt depositors. In our model, the effect of changing market structures is through the number of banks.

banks may become so small that they effectively are local monopolies implying that a significant number of depositors are denied access to the banking system.

We propose to model financial innovation, possibly in response to tightened capital requirements, as an increase in the riskiness of the gambling asset that leaves its expected return unchanged. In response to this, regulators need to increase capital requirements further - else banks will use the gambling asset. These increases may then have the effects on market structure and size we just outlined.

We have derived these effects in a very simple model, however with the important property that the need to regulate banks is derived from first principles. We think there are good reasons to think that the effects we identified will also be present in more realistic and complex settings. For example, to compete effectively with off-shore financial systems in a situation where regulation becomes more costly, banks need to become bigger so as to explore better their economies of scale. But bigger banks also means less competition and more costs for consumers either in the form of smaller deposit rates or in the form of lower levels of service - in our model captured by longer distances to banks.

It has been an important objective of recent regulatory overhauls to limit the number of large banks since these may constitute a risk to the entire financial system. Our results suggest that regulation using capital requirements may work in the exact opposite direction. Another insight offered by our model is that the existence of attractive outside options, for instance in the form of shadow banks or more lax regulatory regimes elsewhere, may make financial regulation more costly.

Organization

In the following section we present the model and, for the case where the integer constraint is ignored, the definitions of the various types of equilibria. Section 3 characterizes them and shows how the level of capital requirements determines which type is realized. This section also contains a numerical example showing how, as the riskiness of assets increases, and consequently capital requirements are tightened, the nature of the market equilibrium changes. In Section 4 we define equilibria and study their properties as function of the capital requirements for the case when the integer constraint is imposed. We also provide a numerical example parallel to the one presented in Section 3, but now showing how monopoly equilibria may arise due to higher capital requirements. Section 5 concludes.

2 Model and Equilibrium

2.1 The Model

Our model is, except for one important assumption, which we return to below, identical to the one of Nielsen and Weinrich (2015a) (which is in turn based on Repullo, 2004, and Hellman et al., 2000). For this reason we shall be brief in presenting it and refer to some results already reported in Nielsen and Weinrich (2015a).

In this monopolistic competition economy banks are free to enter at a cost C and, in equilibrium, the number n of banks (endogenously determined) place themselves uniformly on the unit circle which is inhabited by potential depositors each endowed with D units of money. The deposits received by any bank can in each period be placed either in a prudent asset, yielding a net return $\alpha > 0$ with probability 1, or a gambling asset which has net return $\gamma > \alpha$ with probability $1 - \pi$ and $\beta > -1$ with probability π. By assumption,

$$\alpha > (1 - \pi)\gamma + \pi\beta \tag{1}$$

so the regulator would like to induce banks to use the prudent asset. However, since $\gamma > \alpha$ and the investment decisions of banks are assumed unobservable a well known moral hazard problem arises, that is, banks may prefer to invest in the gambling asset. Since regulated banks are covered by deposit insurance, depositors do not worry about the riskiness of the dispositions of banks, but only about the deposit rate they are offered.

The cost of "walking to the bank" (which may be interpreted as the cost of getting a product which is not the optimal product from the depositor's point of view) is μ per unit of distance traveled. Potential depositors have an alternative to using the banks giving them a (net) rate of return $\sigma \geq -1$. When deciding whether to deposit with a bank or use this alternative, they take into account the interest rates offered by the two closest banks as well as the distance to these banks.

The existence of an alternative return is central to our conclusion and therefore deserves some comments. One interpretation is that σ is a reduced form return from using shadow banks, i.e. banks which are not subject to regulation and which are more risky, also because they are not covered by deposit insurance. Another, closely related, interpretation is that depositors may choose to use banks outside the area of jurisdiction of the regulator, so not necessarily shadow banks, but banks subject to other types of regulation. In both cases, since we assume that the rate σ is independent of the location of the depositor, this means that this outside option is of the same attractiveness to all depositors. For instance, if the alternative involves placing funds in banks with no local network of branches, the "distance" to foreign branches may, for all practical purposes, be identical for all depositors. Under both interpretations, if, in equilibrium, some depositors use this alternative, we shall speak of *deposit market leakage*. An alternative, more straightforward interpretation of σ is that it is the return from using the funds in the current period, rather than in the future, or the return of a storage technology.[7] In this case, if not all depositors use the bank, the appropriate term is *financial repression* as f.i. considered in the development economics literature (see e.g. Shaw, 1973).

We assume that depositors live for two periods, consuming the proceeds from their deposits in the second period of their lives, while once a bank has entered the market, it will only exit when bankrupt (i.e. when it cannot fully repay its depositors). Besides the deposits received, banks may also invest their own capital in any of the two assets, however since we assume an opportunity cost of such capital equal to

$$\rho > \alpha \tag{2}$$

they will only do so when there are capital requirements in place. In each period a bank in the market chooses what asset class to invest in (it will only invest in one, as proved in Nielsen and Weinrich, 2015a) and what deposit rate to offer. This is with the purpose of maximizing the present value of profits using ρ as discount factor.

Note that (1), (2) and the fact that $\beta \geq -1$ together imply that

$$1 + \rho > (1 + \gamma)(1 - \pi) + (1 + \beta)\pi \geq (1 + \gamma)(1 - \pi)$$

that is

$$\gamma < \overline{\overline{\gamma}} \equiv \frac{\rho + \pi}{1 - \pi} \tag{3}$$

In each period, first potential banks decide to enter or not, then, with the number n of banks being given, a subgame perfect Nash equilibrium in terms of investment choice and deposit rate is reached.

We study symmetric equilibria where all banks offer the same interest rate and invest in the same asset class. Regulators choose a level of capital requirements $k \geq 0$ to prevent gambling equilibria, i.e. equilibria where all banks use the gambling asset, from coming into existence. This means that, for every unit of deposits received and invested, banks are required to supply the capital k.

2.2 Equilibria without integer constraints

In our model the number of banks is determined by the free entry condition: as long as a bank can enter without suffering losses it does so. In dealing with this, we shall have

[7] If the utility function is $u(c_1, c_2) = c_1 + \frac{1}{1+\delta} c_2$ then $\sigma = 1 + \delta$.

to take into account that n must be an integer. It turns out to be convenient to split up our analysis in two parts, first allowing n to be a continuous number and then extend the results to the integer case. Ignoring the integer constraints makes for a simpler analysis and provides the basis for an understanding of the other case. It may anyway be a good approximation when the number of banks is large. We return to the integer constraint in Section 4.

2.2.1 Equilibrium definition

When there is an outside option, a depositor at distance z from bank j has three possibilities: deposit his money with bank j, getting the deposit rate r_j, deposit his money with the other bank available to him at a distance $1/n - z$, getting the deposit rate r and using an alternative investment opportunity giving him the rate σ. Thus his return is $\max\{r_j - \frac{\mu}{D}z, r - \frac{\mu}{D}(\frac{1}{n} - z), \sigma\}D$. Consequently, the marginal depositor (on one side of the bank) for bank j is defined by the distance z which satisfies

$$r_j - \frac{\mu}{D}z = \max\left\{r - \frac{\mu}{D}(\frac{1}{n} - z), \sigma\right\} \tag{4}$$

We concentrate on symmetric situations, implying that neighbor banks of bank j charge the same interest rate r. From (4) we obtain

$$z = \min\left\{(r_j - r)\frac{D}{2\mu} + \frac{1}{2n}, (r_j - \sigma)\frac{D}{\mu}\right\}.$$

Multiplying by $2D$ gives the demand functions

$$\mathcal{D}^c(r_j, r, n) = \left(\frac{r_j - r}{\mu}D + \frac{1}{n}\right)D \tag{5}$$

for the case where bank j is constrained by the interest rates of neighboring banks (the superscript 'c' refers to monopolistic competition) and

$$\mathcal{D}^m(r_j, \sigma) = 2\frac{r_j - \sigma}{\mu}D^2 \tag{6}$$

for the case where bank j is constrained by the outside option of its depositors (the superscript 'm' refers to (local) monopoly).

Figure 1 presents a segment of length $\frac{1}{n}$ of the circle, which is bordered by two banks. The downward sloping lines show the net return per unit of deposit as a function of the distance z, that is $r - \frac{\mu}{D}z$. In a symmetric equilibrium banks charge the same interest rate, hence the lines meet at $\frac{1}{2n}$. The outside option is not binding since $r - \frac{\mu}{D}\frac{1}{2n} > \sigma$. If bank j (to the right) lowers the interest rate, two things may happen: if the reduction is small, it will loose depositors to the neighboring bank, while if the reduction is large, some of its depositors will use the outside option instead, i.e. it will loose more depositors than if there were no outside option. If, on the other hand, it chooses a higher interest rate, some depositors of the neighboring bank will become its costumers.

[Figure 1]

Finally we let

$$\mathcal{D}(r_j, r, n, \sigma) = \min \{\mathcal{D}^c(r_j, r, n), \mathcal{D}^m(r_j, \sigma)\} \tag{7}$$

which, ignoring non-negativity constraints, is the demand function that bank j faces as it chooses its interest rate. We have $\partial\mathcal{D}^m/\partial r_j = 2D^2/\mu > D^2/\mu = \partial\mathcal{D}^c/\partial r_j$ and there is a kink of $\mathcal{D}$ at r^o defined by $\mathcal{D}^c(r^o, r, n) = \mathcal{D}^m(r^o, \sigma)$, i.e.

$$r^o(r, n; \sigma) = 2\sigma - r + \frac{\mu}{Dn}. \tag{8}$$

Beyond the interest rate, bank j can also choose which of the two asset classes to invest in. Thus let

$$\Pi^p(r_j, D; k) = -kD + \frac{1}{1+\rho}\left[\alpha - r_j + (1+\alpha)k\right] D$$

be the short run profit when (i) bank j sets the interest rate r_j and uses the prudent asset, (ii) it receives D in deposits and (iii) capital requirements are k. Likewise let

$$\Pi^g(r_j, D; k) = -kD + \frac{1-\pi}{1+\rho}[\gamma - r_j + (1+\gamma)k]D$$

be the short run expected profit when instead bank j uses the gambling asset. Here we take into account that, when the gambling asset is being used, with probability $1-\pi$ the bank will not receive any return on its investment.

We then define the present value of bank j (already in the market) using the prudent asset as

$$V_p(r, n; k, \sigma) = \max_{r_j} \Pi^p(r_j, \mathcal{D}(r_j, r, n, \sigma); k)) + \frac{1}{1+\rho}V_p(r, n; k, \sigma) \tag{9}$$

and we denote the solution to the maximization problem in (9) by $r^p(r,n;k,\sigma)$, and in a parallel fashion for the gambling asset

$$V_g(r,n;k,\sigma) = \max_{r_j} \Pi^g(r_j, \mathcal{D}(r_j,r,n,\sigma);k) + \frac{1-\pi}{1+\rho} V_g(r,n;k,\sigma) \tag{10}$$

where we take into account that with probability $1-\pi$ the bank is bankrupt and thus its future value is zero. The solution to the maximization problem is denoted $r^g(r,n;k,\sigma)$.

Replacing $\mathcal{D}(r_j,r,n,\sigma)$ with $\mathcal{D}^c(r_j,r,n)$ in (9) we get $V_p^c(r;k,n) = max_{r_j}\Pi^p(r_j,\mathcal{D}^c(r_j,r,n);k)) + \frac{1}{1+\rho}V_p^c(r;k,n)$. The maximization problem on the right hand side, which is equivalent to

$$\max_{r_j} -k\mathcal{D}^c(r_j,r,n) + \frac{1}{1+\rho}\left[\alpha - r_j + (1+\alpha)k\right]\mathcal{D}^c(r_j,r,n) \tag{11}$$

has as solution

$$r^{pc}(r,n;k) = \frac{\alpha + r - \frac{\mu}{Dn}}{2} - \frac{\rho-\alpha}{2}k \tag{12}$$

while if we instead substitute with $\mathcal{D}^m(r_j,\sigma)$ we get the (parallel) maximization problem

$$\max_{r_j} -k\mathcal{D}^m(r_j,\sigma) + \frac{1}{1+\rho}\left[\alpha - r_j + (1+\alpha)k\right]\mathcal{D}^m(r_j,\sigma) \tag{13}$$

which has as solution

$$r^{pm}(k,\sigma) = \frac{\alpha+\sigma}{2} - \frac{\rho-\alpha}{2}k. \tag{14}$$

Similarly, for the gambling strategy, when replacing $\mathcal{D}(r_j,r,n,\sigma)$ with respectively $\mathcal{D}^c(r_j,r,n)$ and $\mathcal{D}^m(r_j,\sigma)$ we get solutions

$$r^{gc}(r,n;k) = \frac{\gamma + r - \frac{\mu}{Dn}}{2} - \frac{\frac{1+\rho}{1-\pi} - (1+\gamma)}{2}k \tag{15}$$

and

$$r^{gm}(k,\sigma) = \frac{\gamma+\sigma}{2} - \frac{\frac{1+\rho}{1-\pi} - (1+\gamma)}{2}k. \tag{16}$$

We then have the following

Lemma 1 *(i) Suppose there is a point of non-negative profits on the demand curve $\mathcal{D}$. If $r^{pm}(k,\sigma) < r^o(r,n;\sigma)$, $r^{pm}(k,\sigma)$ is the solution to the maximization problem in (9), and if $r^{pc}(r,n;k) > r^o(r,n;\sigma)$, $r^{pc}(r,n;k)$ is the solution. If neither of these conditions hold, $r^o(r,n;\sigma)$ is the solution to (9). (ii) Similarly, assuming a point of non-negative profit on the demand curve, if $r^{gm}(k,\sigma) < r^o(r,n;\sigma)$, $r^{gm}(k,\sigma)$ is the solution to (10), and if $r^{gc}(r,n;k) > r^o(r,n;\sigma)$, $r^{gc}(r,n;k)$ is the solution. If neither of these conditions hold, $r^o(r,n;\sigma)$ is the solution to (10).*

Proof. The bank will never offer an interest higher than $\overline{r} = \alpha + (1+\alpha)k$, at which point profits are negative, so there is a solution to the profit maximization problem (9) where (because positive profit is possible) the quantity q of deposits received is strictly positive. Iso-profit curves in $r - q$ plane, described by

$$[\alpha - r - (\rho - \alpha)k]\,\frac{q}{\rho} = \overline{v}$$

are increasing and convex (when positive, profit is decreasing in r, increasing in q). A profit maximum on the $\mathcal{D}^m$ curve thus gives no smaller profit than a profit maximum on the lower envelope of $\mathcal{D}^m$ and $\mathcal{D}^c$. It follows that a maximum on $\mathcal{D}^m$ that is also on the lower envelope is a maximum for the lower envelope. Since $\partial\mathcal{D}^m/\partial r_j > \partial\mathcal{D}^c/\partial r_j$, this can be the case only if $r^m(k,\sigma) \leq r^o(r;\sigma,n)$. The analogous argument applies to $\mathcal{D}^c$ in which case necessarily $r^c(r;k,n) \geq r^o(r;\sigma,n)$. In case there is no tangency between an iso-profit curve and the lower envelope, $r^o(r;\sigma,n)$ is the maximizing interest rate for the bank. The proof for the gambling asset case is completely parallel ■

Note that the solution $r^o(r,n;\sigma)$ in the above lemma can also be obtained when we add in (11) the constraint $\mathcal{D}^c(r_j,r,n) \leq \mathcal{D}^m(r_j,\sigma)$ and/or in (13) the constraint $\mathcal{D}^m(r_j,\sigma)$ $\leq \mathcal{D}^c(r_j,r,n)$. As long as neither of them is binding, we have either $r^{pc}(r,n;k)$ or $r^{pm}(k,\sigma)$ as solution, but when one of them is binding, so is the other, and $r^o(r,n;\sigma)$ is the solution.

Our equilibrium concept is that of a subgame perfect Nash equilibrium, which exists if it is immune to one-shot deviations. If, in a symmetric equilibrium where the $n^p - 1$ other banks use the prudent asset offering r^p in deposit rate, a bank deviates, in the current period only, to using the gambling asset, it will solve the problem:

$$\max_{r_j} \Pi^g(r_j, \mathcal{D}(r_j, r^p, n^p, \sigma); k) + \frac{1-\pi}{1+\rho} V_p(r^p, n^p; k, \sigma) \tag{17}$$

Likewise, a one-period deviator in a gambling equilibrium (with n^g banks offering r^g) will solve

$$\max_{r_j} \Pi^p(r_j, \mathcal{D}(r_j, r^g, n^g, \sigma); k) + \frac{1-\pi}{1+\rho} V_g(r^g, n^g; k, \sigma) \tag{18}$$

We can now define respectively a prudent and a gambling equilibrium for our model:

Definition 1 *Symmetric Prudent Equilibrium*

A pair $(r^p, n^p) \in \mathbb{R}^2$ s.t.
(i) $r^p = r^p(r^p, n^p; k, \sigma)$;
(ii) $V_p(r^p, n^p; k, \sigma) = C$;

(iii) $C \geq \max_{r_j} \Pi^g(r_j, \mathcal{D}(r_j, r^p, n^p, \sigma); k) + \frac{1-\pi}{1+\rho} C$.

Condition (i) states for any bank that, given it uses the prudent asset, its choice of deposit rate maximizes its discounted profit. Condition (iii) says that, for any bank, it is optimal to use the prudent asset rather than the gambling one, while condition (ii) states that no bank wants to enter the market (or regrets having entered it).

Definition 2 *Symmetric Gambling Equilibrium*

A pair $(r^g, n^g) \in \mathbb{R}^2$ s.t.
(i) $r^g = r^g(r^g, n^g; k, \sigma)$;
(ii) $V_g(r^g, n^g; k, \sigma) = C$;
(iii) $C \geq \max_{r_j} \Pi^p(r_j, \mathcal{D}(r_j, r^g, n^g, \sigma); k) + \frac{1}{1+\rho} C$.

Similarly to Salop (1979) and using Lemma 1 we can now classify the equilibria as follows:

Definition 3 We say that a Symmetric Prudent Equilibrium (r^p, n^p) is *monopolistically competitive* if $r^p > r^o(r^p, n^p; \sigma)$ (in which case $r^p = r^{pc}(r^p, n^p; k)$); *constrained oligopolistic* if $r^p = r^o(r^p, n^p; \sigma)$; *monopolistic* if $r^p < r^o(r^p, n^p; \sigma)$ (in which case $r^p = r^{pm}(r^p, n^p; k)$). Likewise we classify a Symmetric Gambling Equilibrium (r^g, n^g) as *monopolistically competitive* if $r^g > r^o(r^g, n^g; k)$ (in which case $r^g = r^{gc}(r^g, n^g; k)$), *constrained oligopolistic* if $r^g = r^o(r^g, n^g; \sigma)$ and *monopolistic* if $r^g < r^o(r^g, n^g; \sigma)$ (in which case $r^g = r^{gm}(r^g, n^g; k)$).

The Constrained Oligopoly Equilibrium corresponds to what Salop (1979) calls a *kink equilibrium.* We prefer our terminology since this equilibrium represents an intermediate case between monopolistic competition and monopoly. In fact, as we shall see, in such an equilibrium the number of banks will be smaller than in a monopolistic competition equilibrium, however banks are still constrained by the presence of other banks.

2.2.2 Equilibria without the outside option

Suppose $\sigma = -\infty$, so that depositors always choose some bank as recipient of their wealth. This means that only monopolistically competitive equilibria exist. Suppose we are in a symmetric equilibrium with n^{pc} banks all using the prudent asset and with the deposit rate r^{pc}. Then from (12) we have, using $r = r^{pc}$ and, redefining by a slight abuse of notation the equilibrium value of r^{pc},

$$r^{pc} = r^{pc}(n^{pc}; k) := \alpha - \frac{\mu}{Dn^{pc}} - (\rho - \alpha)k \tag{19}$$

and the present value of the firm becomes

$$V_p^c = \frac{\mu}{\rho n^2}.$$

Setting this equal to the entry cost C we get the equilibrium value for the number of banks:

$$n^{pc} = \sqrt{\frac{\mu}{\rho C}} \tag{20}$$

In a parallel fashion we find the equilibrium interest rate in a symmetric gambling equilibrium. Define

$$r^{gc}(n;k) := \gamma - \frac{\mu}{Dn} - \left[\frac{1+\rho}{1-\pi} - (1+\gamma)\right] k \tag{21}$$

and then this equilibrium is described by the pair (r^{gc}, n^{gc}) where

$$n^{gc} = \sqrt{\frac{\mu}{C}}\sqrt{\frac{1-\pi}{\rho+\pi}} \tag{22}$$

and

$$r^{gc} = r^{gc}(n^{gc}; k) \tag{23}$$

Coming back to problem (17), its solution is

$$r^{dp} = \frac{r^{pc}(n^{pc}; k) + r^{gc}(n^{pc}; k)}{2} \tag{24}$$

One can show (see Nielsen and Weinrich, 2015a) that $r^{dp} > r^{pc}$.

Likewise, the solution to (18) is

$$r^{dg} = \frac{r^{pc}(n^{gc}; k) + r^{gc}(n^{gc}; k)}{2}$$

Regarding the occurrence of prudent versus gambling equilibrium, in Nielsen and Weinrich (2015a) we show that there is a value of capital $\tilde{k}^c$ defined as follows:

$$\tilde{k}^c(\gamma) = \frac{2\frac{\sqrt{\mu\rho C}}{D}\left(\sqrt{\frac{\rho+\pi}{(1-\pi)\rho}} - 1\right) - (\gamma - \alpha)}{\rho - \alpha - \frac{1+\rho}{1-\pi} + (1+\gamma)} \tag{25}$$

such that when $k > \tilde{k}^c(\gamma)$ only a prudent equilibrium exists and when $k < \tilde{k}^c(\gamma)$ only a gambling equilibrium exists while, when we have equality, both types of equilibria exist.

Lemma 2 *$\tilde{k}^c$ is increasing in γ and tends to ∞ as γ tends to $\alpha + \frac{1+\rho}{1-\pi} - (1+\rho)$*

Proof. Because of Assumptions (1) and (2), both the nominator and denominator are negative (see Nielsen and Weinrich, 2015a). When γ increases towards $\alpha + \frac{1+\rho}{1-\pi} - (1+\rho)$, the nominator decreases and the denominator increases towards 0, so $\tilde{k}^c$ increases towards ∞. ■

3 Conditions for existence of a prudent equilibrium without integer constraints

We now turn to the central question of this paper, namely what is the smallest capital requirement k needed for the existence of a prudent equilibrium and, for this given k, what type of prudent equilibrium we have. In the next subsection, we focus on the last issue.

3.1 Characterization of Prudent Equilibria

In this subsection we establish how the type of prudent equilibrium depends on σ or, equivalently, k. To do this, we ignore the possibility that banks may use the gambling asset, i.e. we provide necessary conditions for the existence of the various regimes of prudent equilibria.

Monopolistic Competition Equilibrium

For (r^{pc}, n^{pc}) to constitute an equilibrium, we require $r^{pc} > r^o(r^{pc}, n^{pc}; \sigma)$, i.e.

$$r^{pc} > 2\sigma - r^{pc} + \frac{\mu}{n^{pc}D} \Leftrightarrow r^{pc} > \sigma + \frac{\mu}{2n^{pc}D}.$$

$\Leftrightarrow$

$$\sigma < r^{pc} - \frac{\mu}{2n^{pc}D} =: \underline{\sigma} \tag{26}$$

where $\underline{\sigma}$ is the rate of return of the bank's marginal depositor. This is the condition for the existence of a monopolistic competition equilibrium. As we shall see, for $\sigma \geq \underline{\sigma}$ a constrained oligopoly equilibrium will occur.

Monopoly Equilibrium

We next turn to studying a monopoly equilibrium. Inserting r^{pm} in $\mathcal{D}^m$ we arrive at the amount of deposits received and measure of depositors $\mathcal{M}$ served by each bank:

$$\mathcal{D}^m(\frac{\alpha+\sigma}{2} - \frac{\rho-\alpha}{2}k, \sigma) = \frac{\alpha-\sigma-(\rho-\alpha)k}{\mu}D^2 \text{ and } \mathcal{M} = \frac{\alpha-\sigma-(\rho-\alpha)k}{\mu}D.$$

This means that the number of banks in this equilibrium is

$$n^{pm}(k,\sigma) = \frac{\mu}{[\alpha - \sigma - (\rho - \alpha)k]\,D}. \tag{27}$$

From Lemma 1 we require that $r^{pm} < 2\sigma - r^{pm} + \frac{\mu}{n^{pm}D}$, i.e.

$$2r^{pm} < 2\sigma + \frac{\mu}{n^{pm}D} \Leftrightarrow \alpha + \sigma - (\rho - \alpha)k < 2\sigma + \frac{\mu}{n^{pm}D} \Leftrightarrow \alpha < \frac{\sigma + \rho k + \frac{\mu}{n^{pm}D}}{1+k} \Leftrightarrow \alpha < \alpha$$

where in the last step we have used (27). We conclude that a monopoly equilibrium without integer constraint does not exist.[8]

Constrained Oligopoly Equilibrium

Whatever is the number of banks n in equilibrium, the equilibrium deposit rate r^o by (8) must satisfy $r^o = 2\sigma - r^o + \frac{\mu}{Dn}$. Denote it, by a slight abuse of notation,

$$r^o(n,\sigma) := \sigma + \frac{\mu}{2nD}. \tag{28}$$

Since there is free entry and exit, the number of banks present in equilibrium has to be such that $V = C$ which, using $\mathcal{D}^m(r^o(\sigma,n),\sigma) = \mathcal{D}^c(r^o(\sigma,n), r^o(\sigma,n), n) = D/n$, becomes

$$V_p^c = [\alpha - r^o(\sigma,n) - (\rho-\alpha)k]\frac{D}{\rho n} = C.$$

Inserting (28) we obtain

$$[\alpha - \sigma - \frac{\mu}{2nD} - (\rho-\alpha)k]\frac{D}{\rho n} = C$$

and, multiplying through by $2n^2\rho/D$ and rearranging,

$$\frac{2\rho C}{D}n^2 - 2[\alpha - \sigma - (\rho-\alpha)k]n + \frac{\mu}{D} = 0.$$

This yields

$$n_{\pm} = \frac{\alpha - \sigma - (\rho-\alpha)k] \pm \sqrt{[\alpha - \sigma - (\rho-\alpha)k]^2 - 2\rho C\mu/D^2}}{2\rho C/D}. \tag{29}$$

[8] Salop (1979) does not treat the monopoly case as it "requires the very restrictive condition that the exogenously given average curve be tangent to the exogenously given demand curve ... We ignore this limiting case for the remainder of the analysis" (p. 148). However, in his Figure 6, p. 146, "MONOPOLY EQUILIBRIUM", he depicts a situation which suggests that such a configuration be possible. As in our model, without integer constraint this is not the case. At most there can be a borderline monopoly/kinked equilibrium as indicated by the dashed line in Salop's figure.

Real solutions exist only for σ such that $\delta := [\alpha - \sigma - (\rho - \alpha)k]^2 - 2\rho C\mu/D^2 \geq 0$. For $\delta > 0$ there exist two solutions of which only the larger one is relevant as, with free entry, at the smaller value of n banks could enter and make positive profits, thus n_- would not persist. This follows from the following lemma.

Lemma 3 *At $n = n_-$ the function $V(n) := [\alpha - r^o(\sigma, n) - (\rho - \alpha)k]\frac{D}{\rho n}$ is increasing.*

Proof. We calculate the derivative of V and examine when it is positive.

$$\begin{aligned} V'(n) &= -\frac{\partial r^o}{\partial n} \cdot \frac{D}{\rho n} - [\alpha - r^o(\sigma, n) - (\rho - \alpha)k]\frac{D}{\rho n^2} \\ &= \frac{D}{\rho n^2}\left\{\frac{\mu}{2nD} - \left[\alpha - \sigma - \frac{\mu}{2nD} - (\rho - \alpha)k\right]\right\} \\ &= \frac{D}{\rho n^2}\left\{\frac{\mu}{nD} - [\alpha - \sigma - (\rho - \alpha)k]\right\} > 0 \end{aligned}$$

$\Leftrightarrow$

$$\frac{\mu}{nD} - [\alpha - \sigma - (\rho - \alpha)k] > 0 \Leftrightarrow n < \frac{\mu}{D} \cdot \frac{1}{\alpha - \sigma - (\rho - \alpha)k} =: \overline{n}.$$

Setting now $A := \alpha - \sigma - (\rho - \alpha)k$ we get from (29)

$$n_- = \frac{A - \sqrt{A^2 - 2\rho C\mu/D^2}}{2\rho C/D}$$

which is smaller than $\overline{n}$ if

$$\frac{A - \sqrt{A^2 - 2\rho C\mu/D^2}}{2\rho C/D} < \frac{\mu}{D} \cdot \frac{1}{A}.$$

Setting $B := 2\rho C\mu/D^2$, this is equivalent to

$$A - \sqrt{A^2 - B} < B/A \Leftrightarrow A - B/A < \sqrt{A^2 - B} \Leftrightarrow A^2 - 2B + (B/A)^2 < A^2 - B$$

$$\Leftrightarrow (B/A)^2 < B \Leftrightarrow B < A^2$$

which is true whenever there are two solutions for n. ■

The benchmark value of σ for which $\delta = 0$ is determined by

$$\alpha - \sigma - (\rho - \alpha)k = \sqrt{2\rho C\mu}/D$$

$\Leftrightarrow$

$$\sigma = \alpha - (\rho - \alpha)k - \sqrt{2\rho C\mu}/D = \alpha - (\rho - \alpha)k - \sqrt{2}\sqrt{\frac{\rho C}{\mu}\frac{\mu}{D}}$$

$\Leftrightarrow$

$$\sigma = \alpha - (\rho - \alpha)k - \sqrt{2}\frac{\mu}{n^c D} = r^c - \left(\sqrt{2} - 1\right)\frac{\mu}{n^c D} =: \overline{\sigma}. \tag{30}$$

Next, consider how δ varies as σ varies. We have

$$\frac{\partial \delta}{\partial \sigma}(\sigma) = -2[\alpha - \sigma - (\rho - \alpha)k]$$

which at $\sigma = \overline{\sigma}$ is smaller than zero iff

$$-2\left[\alpha - \left(\alpha - \frac{\mu}{n^c D} - (\rho - \alpha)k\right) + \left(\sqrt{2} - 1\right)\frac{\mu}{n^c D} - (\rho - \alpha)k\right] < 0$$

iff $-2\sqrt{2}\frac{\mu}{n^c D} < 0$ which is true. Thus for $\sigma > \overline{\sigma}$ there exists no n such that $V = C$, and $\overline{\sigma}$ is the largest value of σ such that $V = C$ can be achieved. Comparing $\overline{\sigma}$ with $\underline{\sigma}$ we obtain

$$\underline{\sigma} < \overline{\sigma} \Leftrightarrow -\frac{\mu}{2n^c D} - \frac{\mu}{n^c D} < -\sqrt{2}\frac{\mu}{n^c D} \Leftrightarrow \frac{3}{2} > \sqrt{2}$$

which is true. In virtue of this discussion we can now set

$$n^o(k,\sigma) = \frac{\alpha - \sigma - (\rho - \alpha)k + \sqrt{[\alpha - \sigma - (\rho - \alpha)k]^2 - 2\rho C\mu/D^2}}{2\rho C/D}, \ \underline{\sigma} \leq \sigma \leq \overline{\sigma}. \tag{31}$$

Using (26) and (30), respectively, it can be verified that

$$n^o(k, \underline{\sigma}) = n^{pc} \tag{32}$$

and

$$n^o(k, \overline{\sigma}) = \left(1/\sqrt{2}\right) n^{pc} = \left(1/\sqrt{2}\right)\sqrt{\frac{\mu}{\rho C}} \tag{33}$$

Moreover, $n^o(k,\sigma)$ is decreasing in both k and σ. We summarize in the following lemma.

Lemma 4 *Without integer constraints for the number of banks the alternative investment with rate of return σ is relevant only if $\sigma > \underline{\sigma}$. For any $\sigma < \underline{\sigma}$ there is a unique equilibrium which is of type monopolistic competition. For any $\underline{\sigma} \leq \sigma \leq \overline{\sigma}$ we have a unique constrained oligopoly equilibrium. There the number of banks shrinks from n^{pc} for $\sigma \leq \underline{\sigma}$ to $\left(1/\sqrt{2}\right) n^{pc}$ for $\sigma = \overline{\sigma} > \underline{\sigma}$. For $\sigma > \overline{\sigma}$ no equilibrium with non-negative net profits exists. Moreover, monopoly equilibria never exist.*

Now observe that both $\underline{\sigma}$ and $\overline{\sigma}$ are linearly decreasing functions (through r^{pc}) of k, $\underline{\sigma} = \underline{\sigma}(k)$ and $\overline{\sigma} = \overline{\sigma}(k)$. Thus, since $\underline{\sigma}(k) < \overline{\sigma}(k)$ for all k, for given $\sigma > 0$ there are $\underline{k} < \overline{k}$ such that $\sigma = \underline{\sigma}(\underline{k})$ and $\sigma = \overline{\sigma}(\overline{k})$. This implies the following

Corollary 1 *In the model without integer constraints, if $k < \underline{k}$ we have monopolistic competition equilibrium with $n = n^{pc}$ banks whereas if $\underline{k} \leq k \leq \overline{k}$ we have constrained oligopoly equilibrium with $\left(1/\sqrt{2}\right) n^{pc} \leq n < n^{pc}$ banks, where $\left(1/\sqrt{2}\right) n^{pc}$ relates to $\overline{k}$ and n^{pc} to $\underline{k}$. If $k > \overline{k}$ the banking sectors collapses.*

Now consider the function $\underline{\sigma}(\cdot) : k \mapsto r^{pc} \mapsto \underline{\sigma}$ as defined by the composition of (19) and (26). Its explicit form is

$$\underline{\sigma}(k) = \alpha - \frac{3\mu}{2n^{pc}D} - (\rho - \alpha)k$$

which can be inverted to become

$$\underline{k}(\sigma) = \frac{\alpha - \frac{3\mu}{2n^{pc}D} - \sigma}{\rho - \alpha} = \frac{\alpha - \frac{3}{2}\frac{\sqrt{\mu\rho C}}{D} - \sigma}{\rho - \alpha} \tag{34}$$

For given value of σ, $\underline{k}(\sigma)$ is the threshold value of k at which the outside option becomes relevant. It is the value of capital requirement that separates monopolistic competition from constrained oligopoly equilibrium.

Likewise, the function $\overline{\sigma}(\cdot) : k \mapsto r^{pc} \mapsto \overline{\sigma}$ given by (19) and (30), or

$$\overline{\sigma}(k) = \alpha - \sqrt{2}\frac{\mu}{n^{pc}D} - (\rho - \alpha)k,$$

gives rise to

$$\overline{k}(\sigma) = \frac{\alpha - \sqrt{2}\frac{\mu}{n^{pc}D} - \sigma}{\rho - \alpha} = \frac{\alpha - \sqrt{2}\frac{\sqrt{\mu\rho C}}{D} - \sigma}{\rho - \alpha}$$

which is the value of k beyond which there is no prudent equilibrium where banks break even.

Example 1. To illustrate the above results we choose the following parameter values: $\alpha = 0.06$, $\gamma = 0.1$, $\beta = -0.1$, $\pi = 0.20504$, $\rho = 0.1$, $\mu = 1$, $C = 0.025$ and $D = 10$. Then (25) yields $\widetilde{k}^c = 0.12478$ as the value of k needed and sufficient to avoid the gambling equilibrium. Moreover, $n^{pc} = 20$, $r^{pc} = 0.050009$, $\underline{\sigma} = 0.047509$, $\overline{\sigma} = 0.047938$ and $r^{pm}(\widetilde{k}, \overline{\sigma}) = 0.051473$. This is illustrated in Figure 2. It shows the locus $V = C$ and the graphs of $\mathcal{D}(\cdot, r^{pc}, n^{pc}, \underline{\sigma}) = \min\{\mathcal{D}^c(\cdot, r^{pc}, n^{pc}), \mathcal{D}^m(r_j, \underline{\sigma})\}$ and $\mathcal{D}(\cdot, r^{pm}, n^{pm}, \overline{\sigma}) =$

$\min\{\mathcal{D}^c(\cdot, r^{pm}, n^{pm}), \mathcal{D}^m(r_j, \overline{\sigma})\}$. The bold part on the $V = C$ locus coincides with all possible equilibrium allocations when varying σ, depicting at point E^c a monopolistic competition equilibrium, i.e. when $\sigma \leq \underline{\sigma}$, and at point E^m a borderline monopoly equilibrium, i.e. when $\sigma = \overline{\sigma}$. All points E^o between E^c and E^m are constrained oligopoly equilibrium allocations. For example, for $\sigma = 0.047509$, $\underline{k}(\sigma) = 0.12478$ and $\overline{k}(\sigma) = 0.1355$.

[Figure 2]

Now assume that due to financial innovation the gambling asset's return in the good state increases to $\gamma' = 0.1023$ but β decreases such that the expected return of the gambling asset does not increase. Then $\widetilde{k}^c$ has to increase to 0.1355 and the number of banks decreases to $n^o(0.1355, 0.047509) = 14.142 = \left(1/\sqrt{2}\right) n^{pc}$. If there is a further financial innovation such that $\widetilde{k}^c > \overline{k}(\sigma)$ the banks disappear. Figure 3 illustrates this scenario. Until $\underline{k}$ equilibrium is of type monopolistic competition, for $\underline{k} < k \leq \overline{k}$ it is constrained oligopolistic, and for $k > \overline{k}$ no equilibrium with a positive number of banks exists anymore.

[Figure 3]

3.2 Implementation of the Prudent Equilibrium

We now turn to establishing the capital requirements needed to sustain the two types of prudent equilibria. We have already shown that, when $\sigma = -\infty$, for a monopolistically competitive equilibrium to exist we require $k \geq \tilde{k}^c(\gamma)$. However, we need to take into account that deviations that are profitable when $\sigma = -\infty$ may no longer be so when $\sigma \in \mathbb{R}$. Lemma 5, which is relatively straightforward, clarifies this issue. In Lemma 6 we turn to establishing when prudent constrained oligopolistic equilibria exist.

Lemma 5 *(i) There is a monopolistically competitive prudent equilibrium only if $k \geq \tilde{k}^c(\gamma)$; when this inequality holds, a sufficient condition for the pair (r^{pc}, n^{pc}) as defined in (19) and (20) to be an equilibrium is that $r^{pc} > r^o(r^{pc}, n^{pc}; \sigma)$ or equivalently that $k \leq \underline{k}$. (ii) Suppose $k \leq \tilde{k}^c(\gamma)$ and that for the pair (r^{gc}, n^{gc}) as defined in (23) and (22) we have $r^{gc} > r^o(r^{gc}, n^{gc}; \sigma)$. Then (r^{gc}, n^{gc}) constitutes a monopolistically competitive gambling equilibrium.*

Proof. (i) Suppose $k < \tilde{k}^c(\gamma)$ and that there were a monopolistically competitive prudent equilibrium pair (r^{pc}, n^{pc}). We know that, when $\sigma = -\infty$, if a bank wanted to deviate, it would set its deposit rate to r^{dp}. However, since $r^{dp} > r^{pc} \geq r^o(r^{pc}, n^{pc}; \sigma)$, r^{dp} is possible also when $\sigma \in \mathbb{R}$. Then a deviator can still use r^{dp} which is profitable. Hence the pair (r^{pc}, n^{pc}) is not resistant to a deviation. Suppose next that $k \geq \tilde{k}^c(\gamma)$. When $\sigma = -\infty$, (iii) of Definition 1 holds for (r^{pc}, n^{pc}). But then it also holds when $\sigma \in \mathbb{R}$ (adding a constraint only makes it more difficult for a potential deviator to a gambling equilibrium). (ii) This follows using the same logic as for the first part of (i). If (r^{gc}, n^{gc}) constitutes a monopolistically competitive gambling equilibrium when $\sigma = -\infty$, this is also so when $\sigma \in \mathbb{R}$. ■

In the following we use the function $\tilde{k}^c(\gamma)$ which, by Lemma 2, is strictly increasing. We showed in Corollary 1 that there is a $\underline{k}$ s.t. $k > \underline{k}$ implies that no prudent monopolistic competition equilibrium exists. It follows that there is a $\underline{\gamma}$ s.t. $\gamma > \underline{\gamma}$ implies that there is no prudent monopolistic competition equilibrium, where $\underline{\gamma}'$ is defined by $\tilde{k}^c(\underline{\gamma}') = \underline{k}$.

We now investigate if, when $\gamma > \underline{\gamma}'$, there is a $\tilde{k}^m(\gamma)$ s.t. a prudent constrained oligopolistic equilibrium exists when the capital requirements are $\tilde{k}^m(\gamma)$.

We consider a particular potential Prudent Constrained Oligopoly Equilibrium (r^o, n^o) and note, using (15) that a deviator would like to choose

$$r' = \frac{\gamma + r^o - \frac{\mu}{Dn^o}}{2} - \frac{\frac{1+\rho}{1-\pi} - (1+\gamma)}{2} k = \frac{r^{gc}(n^o; k) + r^o}{2} \tag{35}$$

where the last equality follows from (21). If $r' \geq r^o$ i.e. $r^{gc}(n^o; k) \geq r^o$ (which will be the case) and the deviation is not profitable, then (r^o, n^o) is an equilibrium.[9]

To find the value of an optimal deviation, we insert the expression for r' (from (35)) into the profit function from (2.2.1): $\Pi^g(r'^o, \mathcal{D}^c(r'^o, n^o); k) + \frac{1-\pi}{1+\rho} C$ to get

[9] If it were the case that $r' < r^o$, we would be on the segment $\mathcal{D}^m$ of the demand curve, and the profit of the deviation derived in the following, which uses the segment $\mathcal{D}^c$, would be larger than what the deviator would in fact achieve.

$$
\begin{aligned}
& -k\left[\frac{r^{gc}(n^o;k)-r^o}{2\mu}D^2+\frac{D}{n^o}\right]+\frac{1-\pi}{1+\rho}\left[\gamma-\frac{r^o+r^{gc}(n^o;k)}{2}+(1+\gamma)k\right] \\
& \times\left[\frac{r^{gc}(n^o;k)-r^o}{2\mu}D^2+\frac{D}{n^o}\right]+\frac{1-\pi}{1+\rho}C \\
= \; & \frac{1-\pi}{1+\rho}\left[\gamma-\frac{r^{gc}(n^o;k)+r^o}{2}+(1+\gamma)k-\frac{1+\rho}{1-\pi}k\right]\left[\frac{r^{gc}(n^o;k)-r^o}{2\mu}D^2+\frac{D}{n^o}\right] \\
& +\frac{1-\pi}{1+\rho}C \\
= \; & \frac{1-\pi}{1+\rho}\left[r^{gc}(n^o;k)+\frac{\mu}{n^oD}-\frac{r^{gc}(n^o;k)+r^o}{2}\right]\left[\frac{r^{gc}(n^o;k)-r^o}{2\mu}D^2+\frac{D}{n^o}\right] \\
& +\frac{1-\pi}{1+\rho}C \\
= \; & \frac{1-\pi}{1+\rho}\left[\frac{r^{gc}(n^o;k)-r^o}{2}+\frac{\mu}{n^oD}\right]\left[\frac{r^{gc}(n^o;k)-r^o}{2\mu}D^2+\frac{D}{n^o}\right]+\frac{1-\pi}{1+\rho}C \\
= \; & \frac{1-\pi}{1+\rho}\left[\frac{[(r^{gc}(n^o;k)-r^o)D]^2}{4\mu}+\frac{(r^{gc}(n^o;k)-r^o)D}{n^o}+\frac{\mu}{(n^o)^2}\right]+\frac{1-\pi}{1+\rho}C
\end{aligned}
$$

Thus for the original equilibrium to exist it is sufficient that

$$
\frac{1-\pi}{1+\rho}\left[\frac{[(r^{gc}(n^o;k)-r^o)D]^2}{4\mu}+\frac{(r^{gc}(n^o;k)-r^o)D}{n^o}+\frac{\mu}{(n^o)^2}\right]+\frac{1-\pi}{1+\rho}C\leq C \tag{36}
$$

or equivalently

$$
\left[\frac{[(r^{gc}(n^o;k)-r^o)D]^2}{4\mu}+\frac{(r^{gc}(n^o;k)-r^o)D}{n^o}+\frac{\mu}{(n^o)^2}\right]\leq\frac{\rho+\pi}{1-\pi}C \tag{37}
$$

From this relation we can derive the following lemma, showing one necessary condition for a prudent constrained oligopolistic equilibrium to exist:

Lemma 6 *Suppose*

$$
\frac{\rho}{1+2\rho}\leq\pi \tag{38}
$$

and $\underline{\gamma}' < \overline{\overline{\gamma}}$ *hold, and let* $\gamma > \underline{\gamma}'$ *be given. Then there is a* $\tilde{k}^m(\gamma)$ *s.t. if a prudent constrained oligopolistic equilibrium exist, then* $k \geq \tilde{k}^m(\gamma)$*.*

Proof. The condition (38) is equivalent to $\frac{1-\pi}{1+\rho}(2\rho+1)C \leq C$ which, using (33) (which implies that $\frac{\mu}{[n^o(\overline{k},\sigma)]^2} = 2\rho C$) in turn is equivalent to

$$\frac{1-\pi}{1+\rho}\left[\frac{\mu}{[n^o(\overline{k},\sigma)]^2} + C\right] \leq C$$

This means that, if there is a $\hat{\overline{k}}$ s.t. $r^{gc}(n^o(\hat{\overline{k}},\sigma),\hat{\overline{k}}) = r^o(n^o(\hat{\overline{k}},\sigma),\sigma)$, (37) holds with strict inequality. We have, using (21) and (28):

$$r^{gc}(n^o(k,\sigma),k) - r^o(n^o(k,\sigma),\sigma) = \gamma - \left[\frac{1+\rho}{1-\pi} - (1+\gamma)\right]k - \sigma - \frac{3}{2}\frac{\mu}{n^o(k,\sigma)D} =: H(\gamma,k) \tag{39}$$

The function H is increasing both in γ and k (since $n^o(k,\sigma)$ is decreasing in k) and if $H(\gamma,k) \geq 0$, the optimal deviation value, as described in (35) is larger than $r^o(n^o(k,\sigma),\sigma)$ so that the deviation is to the $\mathcal{D}^c$ segment of the demand curve.

We now show that $H(\gamma,\underline{k}) > 0$. Recall (32) that $n^o(\underline{k},\sigma) = n^{pc}$ and that $r^o(n^{pc},\sigma) = r^{pc}(n^{pc},\underline{k})$.[10] Finally, as we mentioned earlier, Nielsen and Weinrich (2015a) show that $r^{gc}(n^{pc},\underline{k}) > r^{pc}(n^{pc},\underline{k})$ for $\gamma \leq \underline{\gamma}''$. Thus $H(\underline{\gamma}',\underline{k}) > 0$ and this inequality then also holds for $\gamma > \underline{\gamma}'$.

Since $H(\gamma,\underline{k}) > 0$, as is evident from the the format of (39), $\hat{\overline{k}}$ then exists and we shall write it as a function of γ. To summarize, for $k = \hat{\overline{k}}(\gamma)$, (37) holds with strict inequality.

We already noted that, if $k = \underline{k}$, a deviation to $r' = \frac{r^{gc}(n^o(\underline{k},\sigma);\underline{k}) + r^o(n^o(\underline{k},\sigma),\sigma)}{2}$ is to the segment $\mathcal{D}^c$ of the demand curve. Furthermore, since $\gamma > \underline{\gamma}'$ and $\tilde{k}^c$ is strictly increasing, $\underline{k} < \tilde{k}^c(\gamma)$ which means that (37) does not hold for $k = \underline{k}$.

We have, for the given $\gamma > \underline{\gamma}'$ found $\hat{\overline{k}}$ s.t. (37) holds with strict inequality and also noted that (37) does not hold for $k = \underline{k}$. By continuity, there is then $k' \in (\underline{k}, \hat{\overline{k}}(\gamma))$ s.t. for $k = k'$, (37) holds with equality. We let $\tilde{k}^m(\gamma)$ be the smallest $k' \in (\underline{k}, \hat{\overline{k}}(\gamma))$, s.t. this is so (see Figure 2 below, for an illustration). When $k = \tilde{k}^m(\gamma)$ a deviating bank is choosing a maximum r' on $\mathcal{D}^c$ such that $r' > r^o$. This means that r' also is the maximum on the minimum of $\mathcal{D}^m$ and $\mathcal{D}^c$ (this is the same logic as in Lemma 1). ■

We conclude with the following

Theorem 1 *Suppose (38) holds. There are $\underline{\gamma}$ and $\overline{\gamma}$ s.t. for $\gamma \in (0,\underline{\gamma}]$ a prudent monopolistically competitive equilibrium can be implemented by setting $k = \tilde{k}^c(\gamma)$ while for*

[10] Combining (20) and (34), we get $r^{pc}(n^{pc},\underline{k}) = \sigma + \frac{\sqrt{\mu\rho C}}{2D}$, which by (28) is $r^o(n^{pc},\sigma)$.

$\gamma \in (\underline{\gamma}, \overline{\gamma})$ a prudent constrained oligopolistic equilibrium can be implemented by setting $k = \tilde{k}^m(\gamma)$.

Proof. If $\overline{\overline{\gamma}} \leq \underline{\gamma}'$, set $\underline{\gamma} = \overline{\gamma} = \overline{\overline{\gamma}}$.

If $\overline{\overline{\gamma}} > \underline{\gamma}'$, set $\underline{\gamma} = \underline{\gamma}'$. If $\gamma > \underline{\gamma}$ and if $\tilde{k}^m(\gamma) \leq \overline{k}$, then, with $k = \tilde{k}^m(\gamma)$, a prudent constrained oligopolistic equilibrium exists. This follows since by construction $\underline{k} < \tilde{k}^m(\gamma)$ so (Corollary 1) that without the gambling asset a prudent constrained oligopolistic equilibrium exists, and since with $k = \tilde{k}^m(\gamma)$, a deviation to the gambling asset is not profitable. Since, at $\underline{\gamma}$, $k'(\gamma) < \overline{k}$, there is by continuity an $\gamma' > \underline{\gamma}$ s.t $\hat{k}(\gamma) < \overline{k}$ on $(\underline{\gamma}, \gamma')$.

We finally show that there is a $\overline{\gamma} > \underline{\gamma}$ s.t. for $\gamma \in (\underline{\gamma}, \overline{\gamma})$, $k'(\gamma) < \overline{k}$. Thus suppose that for some $\gamma' \in (\underline{\gamma}, \overline{\overline{\gamma}})$, $k' = \tilde{k}^m(\gamma') \leq \overline{k}$, i.e. a prudent constrained oligopolistic equilibrium exists when $k = k'$. Consider $\gamma'' \in (\underline{\gamma}, \gamma')$ close to γ'. At (γ', k') (37) holds with equality and $H(\gamma', k') > 0$. Since H is increasing in γ we have $0 < H(\gamma'', k') < H(\gamma', k')$ (where the first inequality follows since γ'' is close to γ'). But this means that (37) now holds with strict inequality while it does not hold at $(\gamma'', \underline{k})$. There must then be some $k'' \in (\underline{k}, k')$ such that (37) holds with equality, which is what we wanted to prove.

There are now two possibilities. One is that $\tilde{k}^m(\gamma) \leq \overline{k} \forall \gamma \in (\underline{\gamma}, \overline{\overline{\gamma}})$ in which case we set $\overline{\gamma} = \overline{\overline{\gamma}}$. The other is that $\tilde{k}^m(\overline{\overline{\gamma}}) > \overline{k}$ in which case we let $\overline{\gamma}$ be defined by $\tilde{k}^m(\overline{\gamma}) = \overline{k}$. ■

[Figure 4]

Figure 4 illustrates a case where $\overline{\overline{\gamma}} > \underline{\gamma}'$ and hence $\underline{\gamma} = \underline{\gamma}'$. The two parts of $\tilde{k}(\cdot)$, namely $\tilde{k}^c(\cdot)$ for $\gamma \leq \underline{\gamma}$ and $\tilde{k}^m(\cdot)$ for $\gamma > \underline{\gamma}$ are also illustrated. In the figure, at a point before $\overline{\overline{\gamma}}$, $\tilde{k}^m$ meets $\overline{k}$ and thus this meeting point defines $\overline{\gamma}$. We have also illustrated $\hat{\overline{k}}(\gamma)$ which we used in Lemma 6 to show that $\tilde{k}^m(\cdot)$ exists.

4 The Model with integer constraints

In the following we confine our attention to prudent equilibria as we assume that k is sufficiently large so as to exclude gambling equilibria. In the model so far there is no leakage. As we shall see, this is due to the simplifying assumption that the number of banks can be any non-negative real number. This is not adequate if the number of banks is relatively small which is the case we have in mind now. To remedy for this we first have to adjust our definitions of equilibrium. As indicated in Salop (1979) we apply the concept of "deterrence equilibrium", that is, banks enter until their profit after entry is non-negative. More precisely:

Definition 4 A *Monopolistic Competition Equilibrium with integer constraint* is, for given k and σ, a pair $(\widehat{r}^c, \widehat{n}^c) \in \mathbb{R} \times \mathbb{N}$ s.t.

(i) $\widehat{r}^c = r^{pc}(\widehat{n}^c, k)$;
(ii) $V_p^c(\widehat{r}^c, \widehat{n}^c; k) \geq C$ and $V_p^c(r; k, \widehat{n}^c + 1) < C$ for all r;
(iii) $\widehat{r}^c > r^o(\widehat{n}^c; \sigma)$.

For any real number x we now define $[x]$ to be the largest integer smaller than or equal to x.

Definition 5 A *Monopoly Equilibrium with integer constraint* is, for given k and σ, a pair $(\widehat{r}^m, \widehat{n}^m) \in \mathbb{R} \times \mathbb{N}$ s.t.

(i) $\widehat{r}^m = r^{pm}(k, \sigma)$;
(ii) $\widehat{n}^m = [D/\mathcal{D}^m(\widehat{r}^m, \sigma)]$;
(iii) $\widehat{r}^m < r^o(\widehat{n}^m, \sigma)$.

Definition 6 A *Constrained Oligopoly Equilibrium with integer constraint* is, for given k and σ, a pair $(\widehat{r}^o, \widehat{n}^o) \in \mathbb{R} \times \mathbb{N}$ s.t.

(i) $\widehat{r}^o = r^o(\widehat{n}^o, \sigma)$;
(ii) $V_p^c(\widehat{r}^o, \widehat{n}^o; k) \geq C$ and $V_p^c(r, \widehat{n}^o + 1; k) < C$ for all r.

The number of banks in a monopolistic competition equilibrium is $\widehat{n}^c = [n^{pc}]$ where n^{pc} is given by (20). Thus banks can possibly realize a positive profit, but if one more bank entered, all banks' profits would become negative. The expressions corresponding to (19) and (26) now become

$$\widehat{r}^c := \alpha - \frac{\mu}{\widehat{n}^c D} - (\rho - \alpha) k$$

and

$$\underline{\widehat{\sigma}} := \widehat{r}^c - \frac{\mu}{2\widehat{n}^c D}.$$

Since $\widehat{n}^c \leq n^{pc}$, $\widehat{r}^c \leq r^{pc}$ and $\underline{\widehat{\sigma}} \leq \underline{\sigma}$. When $\sigma < \underline{\widehat{\sigma}}$ we have a monopolistic competition equilibrium as defined above.

In case $\sigma \geq \underline{\widehat{\sigma}}$ equilibrium is determined as follows. First set $\widehat{n}^o(k, \sigma) = [n^o(k, \sigma)]$. Then determine $r^o(\widehat{n}^o(k, \sigma), \sigma)$ according to (28) and compare it with $r^{pm}(k, \sigma)$ as defined in (14). This yields the ultimate equilibrium values as $\widehat{n}^o(k, \sigma)$ for the number of banks and $\min\{r^o(\widehat{n}^o(k, \sigma), \sigma), r^m(k, \sigma)\}$ for the deposit rate.

More precisely, as long as $r^o(\widehat{n}^o(k,\sigma),\sigma) \leq r^{pm}(k,\sigma)$ we have a constrained oligopoly equilibrium and there is no leakage since in this case banks would even like to enlarge the set of depositors they serve which, however, they cannot do as they are constrained by the other banks' activities: $\mathcal{D}^m(r_j,\sigma)$ cannot exceed $\mathcal{D}^c(r_j, r^o(\widehat{n}^o(k,\sigma),\sigma), \widehat{n}^o(k,\sigma))$. Instead, when $r^{pm}(k,\sigma) < r^o(\widehat{n}^o(k,\sigma),\sigma)$, a monopoly equilibrium obtains. This occurs when σ is close to $\overline{\sigma}$ and $n^o(k,\overline{\sigma})$ does not happen to be an integer, in which case banks do not cover the whole set of depositors. Then the quota $1 - \widehat{n}^o(k,\sigma)/n^o(k,\sigma)$ remains unserved and engages in outside investment. For example, since $n^o(k,\overline{\sigma}) = \left(1/\sqrt{2}\right) n^{pc}$, in an economy with, say, initially $\widehat{n}^c = n^{pc} = 7$ banks, as $\left(1/\sqrt{2}\right) 7 \approx 4.95 < 5$ this gives rise to up to $1 - 4/4.95 \approx 20\%$ of leakage.

To be more precise, we assert the following:

Lemma 7 *In case $\widehat{n}^o(k,\overline{\sigma}) < n^o(k,\overline{\sigma})$ there is a real number $\widehat{\sigma} < \overline{\sigma}$ such that $D/\widehat{n}^o(k,\sigma) > \mathcal{D}^m(r^{pm}(k,\sigma),\sigma)$ if and only if $\widehat{\sigma} < \sigma \leq \overline{\sigma}$. The $\widehat{n}^o(k,\sigma)$ banks in the market choose the deposit rate $r^{pm}(k,\sigma)$ and contract the amount of deposits $\mathcal{D}^m(r^{pm}(k,\sigma),\sigma)$. Thus there is a leakage of $D - \widehat{n}^o(k,\sigma)\mathcal{D}^m(r^{pm}(k,\sigma),\sigma) > 0$ of funds which are not deposited in the banking system.*

Proof. Observe that by assumption $n^o(k,\overline{\sigma})$ is not an integer. By continuity of $n^o(k,\cdot)$ as given by (31) there thus exists $\widehat{\sigma}_1 := \inf\{\sigma | \widehat{n}^o(k,\sigma) = \widehat{n}^o(k,\overline{\sigma})\} = \inf\{\sigma | n^o(k,\sigma) < \widehat{n}^o(k,\overline{\sigma}) + 1\} < \overline{\sigma}$. Then $\widehat{n}^o(k,\sigma) = \widehat{n}^o(k,\overline{\sigma})$ iff $\sigma \in (\widehat{\sigma}_1, \overline{\sigma}]$. Next consider $n^{pm}(k,\sigma)$. From (27) it follows that $n^{pm}(k,\cdot)$ is continuous. We shall show later that

$$n^o(k,\overline{\sigma}) = n^{pm}(k,\overline{\sigma}). \tag{40}$$

Then $\widehat{n}^o(k,\overline{\sigma}) < n^{pm}(k,\overline{\sigma})$ and there exists $\widehat{\sigma} := \inf\{\sigma > \widehat{\sigma}_1 | \widehat{n}^o(k,\overline{\sigma}) < n^{pm}(k,\sigma)\} < \overline{\sigma}$. Then $\widehat{n}^o(k,\sigma) < n^{pm}(k,\sigma)$ iff $\sigma \in (\widehat{\sigma}, \overline{\sigma}]$. But this is equivalent to

$$\widehat{n}^o(k,\sigma) < D/\mathcal{D}^m(r^{pm}(k,\sigma),\sigma) \Leftrightarrow D - \widehat{n}^o(k,\sigma)\mathcal{D}^m(r^{pm}(k,\sigma),\sigma) > 0.$$

Since for $\sigma \in (\widehat{\sigma}, \overline{\sigma}]$ banks can realize their monopoly deposit rate and quantity combination $r^{pm}(k,\sigma)$ and $\mathcal{D}^m(r^{pm}(k,\sigma),\sigma)$, they choose this rather than the combination $r^o(\widehat{n}^o(k,\sigma),\sigma)$ and $\mathcal{D}^c(r^o(\widehat{n}^o(k,\sigma),\sigma), r^o(\widehat{n}^o(k,\sigma),\sigma), \widehat{n}^o(k,\sigma)) = D/\widehat{n}^o(k,\sigma)$.

There remains to be shown (40). $n^o(k,\overline{\sigma})$ is known from (33). Regarding $n^{pm}(k,\overline{\sigma})$, from (27), (30) and (19) we obtain

$$\begin{aligned}
n^{pm}(k,\overline{\sigma}) &= \frac{\mu}{\left[\alpha-\overline{\sigma}-k(\rho-\alpha)\right]D} = \frac{\mu}{\left[\alpha-\left(r^{pc}-\left(\sqrt{2}-1\right)\frac{\mu}{n^{pc}D}\right)-k(\rho-\alpha)\right]D} \\
&= \frac{\mu}{\left[\alpha-\left(\alpha-\frac{\mu}{n^{pc}D}-(\rho-\alpha)k-\left(\sqrt{2}-1\right)\frac{\mu}{n^{pc}D}\right)-k(\rho-\alpha)\right]D} \\
&= \frac{\mu}{\left[\sqrt{2}\frac{\mu}{n^{pc}D}\right]D} = \frac{1}{\sqrt{2}}n^{pc} = n^o(k,\overline{\sigma}).
\end{aligned}$$

■

Finally, as in the case of the number of banks a continuous number, the values of σ and k are linked to each other as follows.

Proposition 1 *Requiring the number of banks n to be an integer there are values $\underline{k}$, $\widehat{k}$ and $\overline{k}$ such that we have for capital requirement $k < \underline{k}$ monopolistic competition equilibrium with $n = n^{pc}$, for $\underline{k} \leq k \leq \widehat{k}$ constrained oligopoly equilibrium with $n \leq n^{pc}$ but no leakage, and for $\widehat{k} < k \leq \overline{k}$ monopoly equilibrium with $n < n^{pc}$ and leakage of funds which are not deposited in the banking system. For $k > \overline{k}$ none of the equilibria defined exists and the banking sector disappears.*

Example 2. We build on the Example 1 ($\alpha = 0.06$, $\gamma = 0.1$, $\beta = -0.1$, $\rho = 0.1$, $C = 0.025$ and $D = 10$) but change μ which now is taken to be $\mu = 0.1225$. This means that it is less costly for depositors to travel to banks who thus have less market power, and hence a smaller number of banks will be able to stay in the market. In fact, the number of banks in a monopolistic competition equilibrium is now $n^{pc} = \sqrt{0.1225/(0.1 * 0.025)} = 7$. Also we change π to $\pi = 0.231$ so as to keep $\widetilde{k}^c$ at its previous value $\widetilde{k}^c = 0.12478$. Then $\underline{\sigma} = 0.052384 < \overline{\sigma} = 0.052534 < r^{pc} = 0.053259 < r^{pm}(k,\overline{\sigma}) = 0.053771$. Consider now how the situation evolves as σ is supposed to increase. The industry remains in a monopolistic competition equilibrium as long as $\sigma \leq \underline{\sigma}$. Since profit in that equilibrium is zero, as soon as σ is larger than $\underline{\sigma}$ profit becomes negative and one bank has to exit. In the subsequent constrained oligopoly equilibrium the remaining 6 banks realize a positive profit as long as σ is not too high, or more precisely as long as $V(r,q) = [\alpha - r - (\rho-\alpha)k]\frac{q}{\rho} \geq C$ for $q = D/n = 10/6$. Solving for r yields $r \leq \alpha - (\rho-\alpha)k - \rho C n/D =: r_1 = 0.053509$. By (28) this corresponds to a value of $\sigma \leq r_1 - \mu/(2nD) =: \sigma_1 = 0.052634$. When σ increases beyond σ_1, the number of banks decreases further, first to 5 and subsequently, for ever larger values of σ, to possibly ever smaller values. Where does this process stop?

To determine this we calculate the (non-integer) number of banks n^{pm} at $\sigma = \overline{\sigma}$. From (27) it is given by

$$n^{pm}(k, \overline{\sigma}) = \frac{0.1225}{(0.06 - 0.052534 - 0.12478(0.1 - 0.06))\,10} = 4.95$$

which is of course also equal to $7/\sqrt{2} = \left(1/\sqrt{2}\right) n^{pc} = n^{o}(k, \overline{\sigma})$. With the integer requirement and σ close to $\overline{\sigma}$ there is space for 4 banks only. As they are unconstrained by other banks' activities, they form a monopoly equilibrium. In the limit $\sigma = \overline{\sigma}$ only the quota $4/4.95 = 0.808$ of all depositors is served. Said equivalently, there is a leakage of almost 20%. Figure 5 illustrates the scenario.

[Figure 5]

5 Conclusions

In a companion paper, Nielsen and Weinrich (2015b), we suggest that bank regulation in the form of risk weighted capital requirements may have the side effect of reducing the lending to firms by banks, in other words, may restrict the role of banks as intermediaries between depositors and firms. In the present paper we seek to highlight another way this may happen: tighter capital requirements are costly and these costs are ultimately born by depositors in the form of lower deposit rates. This may lead to the exodus of some depositors seeking alternative ways of placing their funds.

Tighter capital requirements may also lead to a reduction in the number of banks and, concurrently, a reduction in the degree of competitiveness in the market.[11] In this situation fewer banks serve the same number of depositors and accept the same amount of deposits. Thus capital requirements may work against one of the often states goals of regulators, namely to reduce the number of large banks, which individually may constitute a risk to the economic system.

[11]It is noteworthy that such a reduction in the number of banks will not happen if the regulatory instrument is deposit rate ceilings, while an exodus of depositors is still possible.

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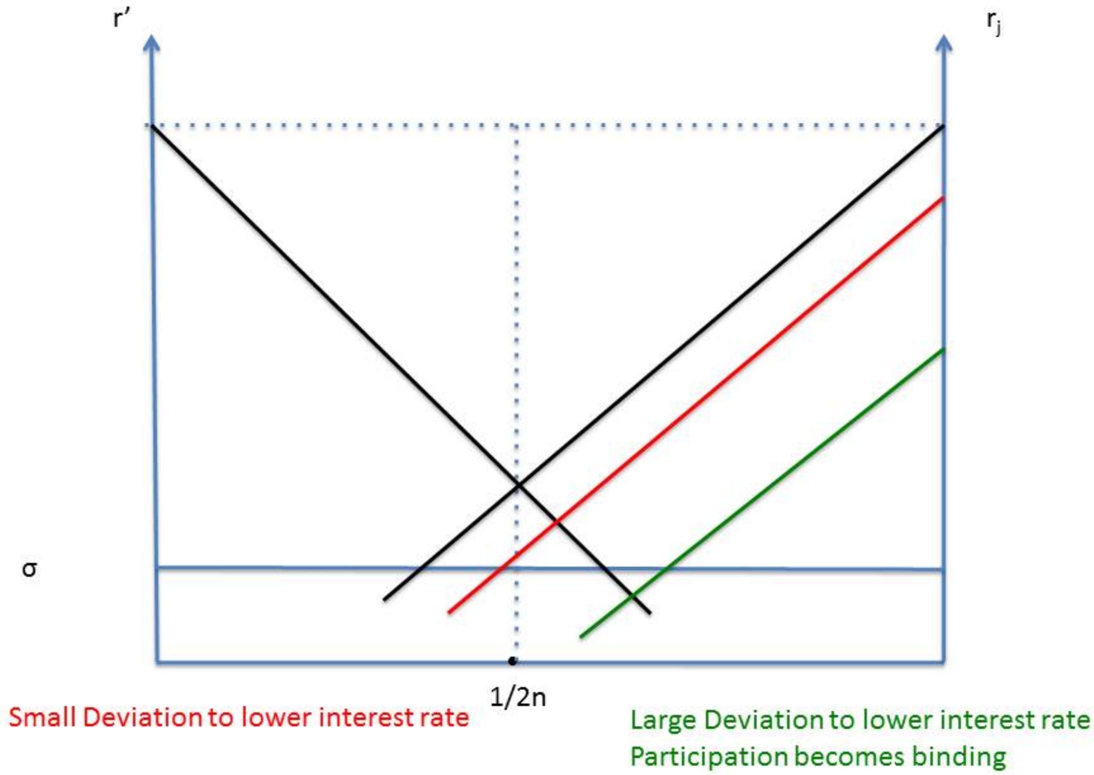
Figure 1.jpg


r′
rj
σ
1/2n
Small Deviation to lower interest rate
Large Deviation to lower interest rate
Participation becomes binding

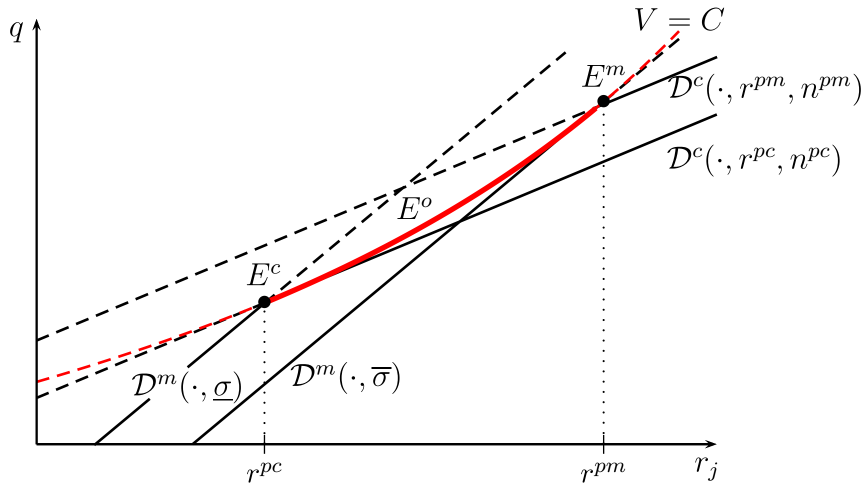


Fig. 2: Locus $E^c - E^o - E^m$ shows all possible equilibrium allocations for varying values of σ.

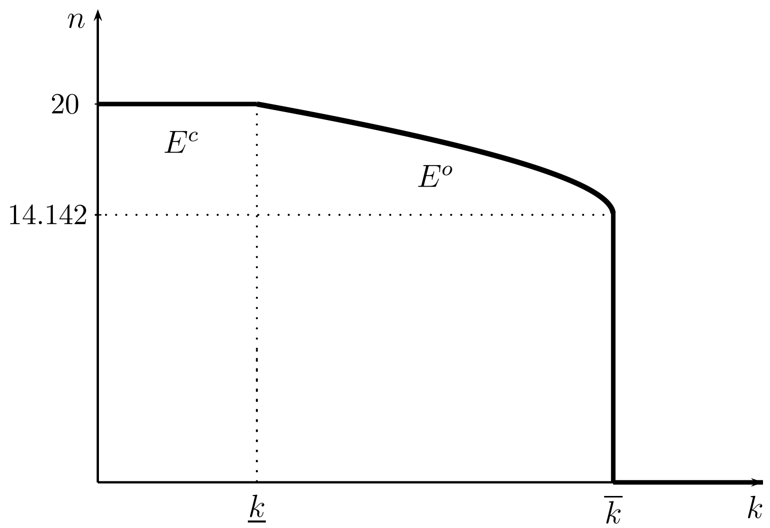
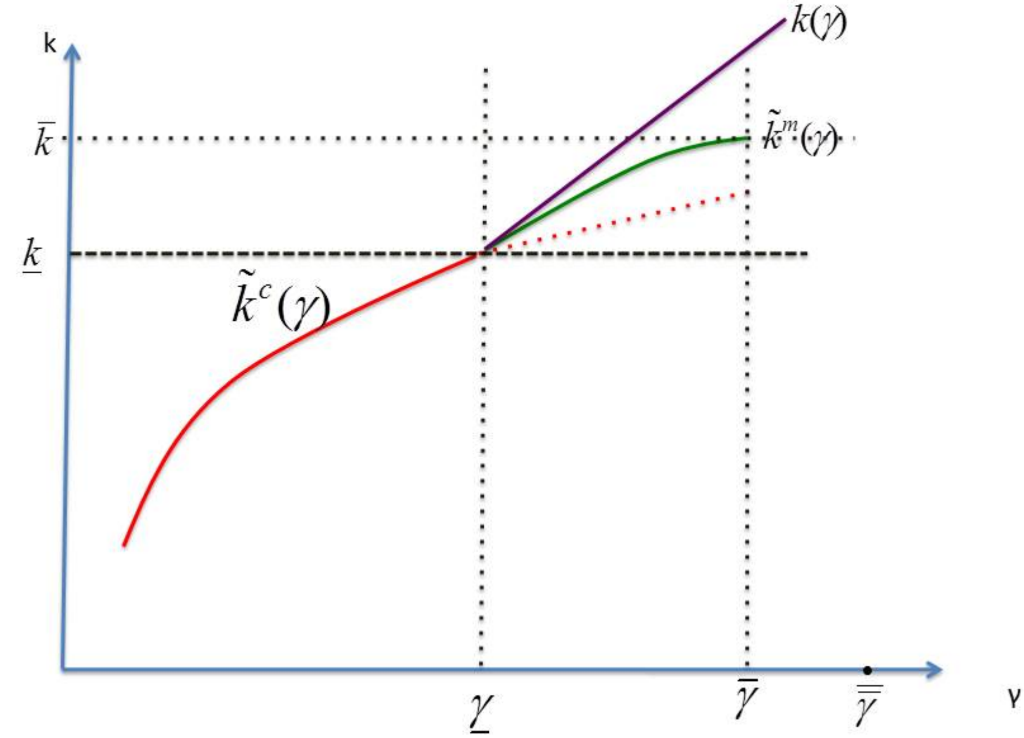


Fig. 3: Number of banks without the integer constraint ($\mu = 1$)

Figure 4.jpg



k
$\overline{k}$
$\underline{k}$
$\hat{\bar{k}}(\gamma)$
$\tilde{k}^m(\gamma)$
$\tilde{k}^c(\gamma)$
$\underline{\gamma}$
$\overline{\gamma}$
$\overline{\overline{\gamma}}$
γ

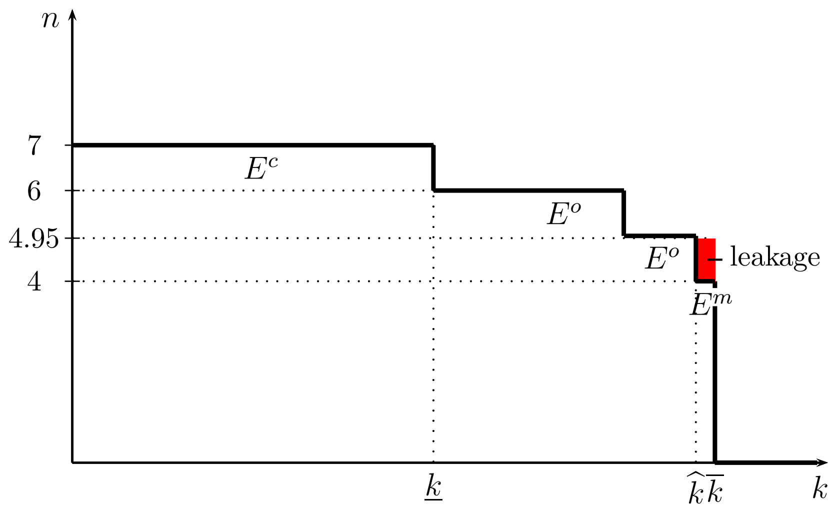


Fig. 5: Number of banks with the integer constraint ($\mu = 0.1225$)