A Multi-layer Model of Order Book Dynamics

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Abstract Multi-layer networks give the chance to represent multiplicity of relations among financial operators. In particular, such a framework provides the natural environment to depict both the informative diffusion and the transactions phase in separate, though interacting, network levels. In this paper we present a two-layers order book model that implements the information spreading on the first layer, which exhibits Self-Organized Criticality (SOC) to describe herding behavior among traders, and the financial trading on the second one. The model is based on the relevant role played by the individual imitation in determining trading decisions. Despite its simplifying assumptions, results of numerical simulations show tailed Probability Density Functions (PDF) of financial returns and other interesting features typical of real financial markets.

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1 Introduction

Existing models of financial markets are often based on several interactions among interconnected heterogeneous agents. Trade decisions follow expectations and generally depend on different behavioral rules of investors. In true markets, several feedback mechanisms determine complexity of the aggregate behavior and unpredictability of prices. Further, extreme events typically characterize the dynamics.

The consciousness about the complex nature of aggregate markets is not a novelty among economists [1–4]. Nonetheless, during the Seventies and the Eighties, the rational expectations [5, 6] theory sustained the microeconomic approach to Macroeconomics that had, unfortunately, prevailed. Our view is that financial markets can be explained and understood only by means of a different, macroeconomic, approach, which put the core analysis on the relevance of informative signals and their distribution. We stress that not all the relevant information *can* be known by investors. Because of this incomplete information, agents try to look around in order to find suggestions for possible dynamic evolution of markets. Like in the famous Keynesian metaphor of the "beauty contest", agents soon discover the ugly truth: one should never try to predict what will happen, but what markets *think* that will happen, instead. Everyday history shows that market crises are not just matter of *perfect* information described in models of efficiency [7]: human interactions and individual psychology cannot be ignored, as financial markets dramatically showed in many situations [8].

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It is possible to implement methods and concepts coming from statistical and theoretical physics [9–11] in the economic analysis of financial markets, in order to describe several features, such as agents heterogeneity, psychological and imitative dynamics, emergent phenomena [12–15]. In this regard, agent-based models of individuals interacting in complex network structures, with different informative sets and simple behavioral rules have shown a quite strong attitude to describe complex economic dynamics of many socio-economic systems [16, 17], and in particular of financial markets [18–21]. Financial integration on a global scale is today so extreme that policy-makers must learn how to set innovative policy designs [22], in a context where the current "mainstream" economics approach has not yet shown the ability to prevent wild market fluctuations that frequently occur.

The model here proposed follows well-known agent-based approaches in financial markets analysis, see in refs. [23] and [24] for comprehensive surveys. In particular, the usual classification of agents has been adopted, by distinguishing them in two categories: fundamentalists and chartists. The former are traders with an eye on the fundamental value of assets; thus they decide whether to buy a share or not, by looking at its current price level and by comparing it with its fundamental value (which is calculated by considering its dividends, as it will be shown in what follows). The latter are technical analysts, who decide their strategies by following trends and graphic dynamics of past prices on charts. Such a differentiation is common in a very wide range of literature, among which [25–35].

Previous studies highlighted the specific role played by the information spreading in determining actual trading choices [18, 19, 36] and, more generally, by the awareness in consumer behavior [20]. In a large part of the literature related to financial markets, the imitative behavior of a trader has often been modeled by means of a switching oscillation from fundamentalists to chartists or vice versa. Recently, a more realistic kind of imitation has been proposed in [36], so that the imitation refers only to the trading decision, no matter which group the trader belongs to. On one hand, this approach may seem more realistic, since it is reasonable to presume that agents do not necessarily transform their "behavioral root" because of a single trading decision. On the other, it augments the descriptive ability of the model in exhibiting herding as the emergent phenomenon deriving from information cascades between agents [37].

Our model relies on the relevant advances in complex networks analysis driven by the recent developments in the multi-layer approach, see [38,39] for very detailed surveys. The main goal of the present paper is to provide a simple multi-layer network model for a truly operating financial order book model. More precisely, two layers will be adopted: (1) the first is the *informative* layer, which configures the global environment of social contacts among market participants: it is built as a small-world network of investors, since this kind of network capture the topological structure of social relationships; (2) the second is the *trading* layer, which represents the trading opportunities for purchases and sales: it is built as a fully connected network, since each investor, in principle, can trade with each other in the market. Differently from other models describing herding in financial markets [40, 41], the first layer shows the information dynamics as a composite pressure, of both global and individual nature, by recalling some features of a Self-Organized Criticality (SOC) model of earthquakes dynamics [42]; whereas the second layer adopts an order book mechanism that determines the asset price by the matching between supply and demand of heterogeneous traders, of any kind, who interact according to information, imitation and prospective utility. Their orders are placed in the order book and consequently executed. Different market mechanisms have been studied in the literature dealing with the market microstructure, such as in [44–49] among others.

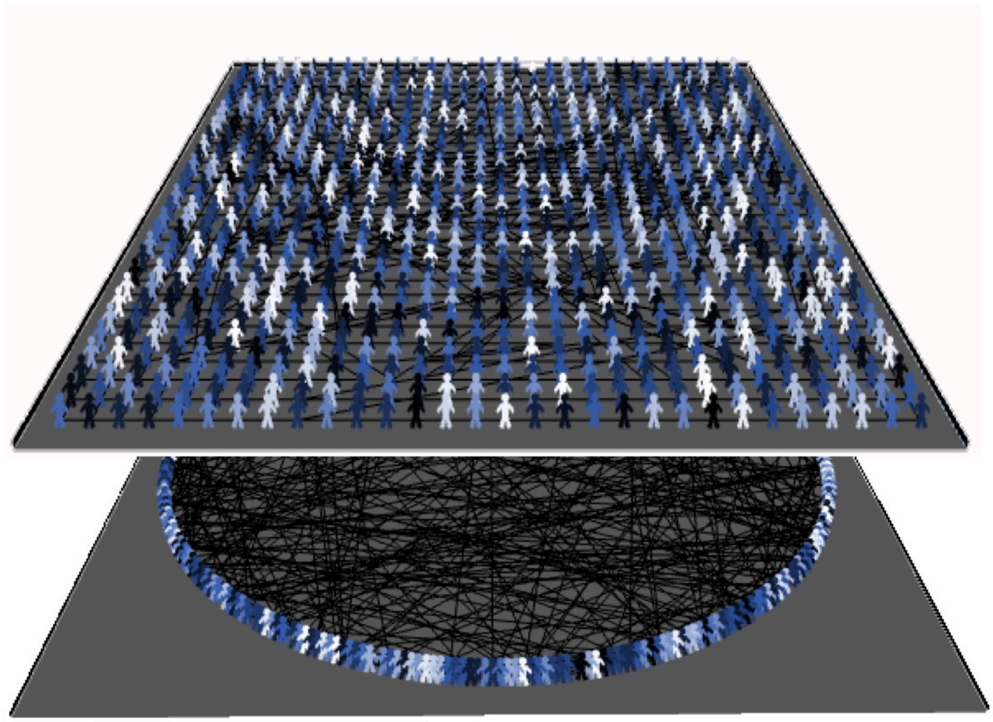

Figure 1: The two-layers configuration of the ML-CFP Model. The upper layer (or *informative* layer) is a 2D Small World square lattice with N traders, connected by means of short and long-range links. The lower layer (or *trading* layer) is a fully connected network where each trader is connected to all the others. In both layers, different colors represent different levels of information: the brighter a trader is, the more informed she is. Initial levels of information are distributed randomly.

The model here presented is based on the following simplified assumptions: first, in the market there exists just one asset; second, orders are just limit orders; third, orders are of quantity one. Dividends dynamics is introduced as the main determinant of the fundamental value of the asset (that, therefore, results to be variable). By considering information and trading ideally separated, this model combines together the influence of herding dynamics and the orders matching within a framework that counts on the expressive potential of multilayer networks. Summarizing, this paper aims to provide a realistic description of the following aspects of real financial markets: *a)* separation and interaction between the formation of the informative set of traders and the trading execution of their orders; *b)* an endogenously formed price series resulting from a truly operating order book mechanism for orders placement; *c)* individual imitation and herding among agents due to informational cascades. Such a descriptive framework is then used to test some stabilization policy.

The paper is organized as follows: section 2 contains the model description; section 3 reports simulation results and their discussion; section 4, presents some conclusions.

2 The ML-CFP Model

In order to simulate the operation of a financial market, the model here presented mainly extends the basic network framework contained in [36] by augmenting it with a second layer, devoted to the order book mechanism. Thus, the order-book-driven *Multi-Layer Contagion-Financial-Pricing* model (ML-CFP henceforth) is obtained (see Fig.1). Technically, this kind of multilayer network is called *multiplex*, since the nodes (traders) are the same in both the layers, changing only the meaning of the edges.

In the next subsections, the role of both the two layers will be described in detail. Here we summarize the info-trading dynamics, which proceeds at each simulation time-step t following this 2-phases evolution:

I- in the informative layer, according to the link configuration given by the network topology, agents collect and share information, therefore deciding their status (bidder, asker or holder) and the (ask or bid) price of their possible order, depending on the global price at time t and on the herding effect;

II- in the trading layer, investors put their orders in the order book, which provides a sort of *compensation room* to execute them, and the next global price

In phase I, all market participants are devoted to gain information about the market and the unique asset. As it will be described below, information enters the model as a twofold phenomenon, in order to replicate a realistic market setting: from a first (global) point of view, it is a generalized (exogenous) signal that reaches all agents, even if with different (random) weights; from a second (individual) point of view, it comes from known people in the social network, no matter if by means of professional competence or by simple word of mouth transmission. Such a configuration may, for example, represent real markets where all agents can receive general (and maybe low-quality) informative signals, while personal contacts, opinions, and persuasive behaviors of trusted people may suggest personal component of information. Under the influence of both the informative sources, and following the prescription of their group (fundamentalists or chartists), agents attempt to form their individual (heterogeneous) expectation for the future price. Then, according to it, they select their trading status (whether to set an order and to negotiate, or not) and, in case, the ask or bid price of the order. It is worth to notice that, when traders have to decide their status and price, the contagion mechanism comes into play, in order to take into account the chance that the euphoric or pessimistic expectations can generate information cascades of purchases and sales. In this respect, as explained in subsection 2.1, one can imagine that each trader is endowed with a sort of *confidence tank* that is filled by the accumulation of information coming from the general informative source. When, at a given time t, an agent - that we call *trigger agent* - overcomes her threshold, i.e. when she reaches a level of knowledge that she considers satisfying, she transmit the (individual) information about her status and price to her neighbors in the network. In turn, the neighbors can overcome their threshold too: in this case they imitate both the status and price of the first agent and transmit the same information to their neighbors, and so on. In such a way, the herding avalanche can develop. At the end of each avalanche, all the traders involved in the herding process operate in the same way, while the others act independently.

At this point, phase II can start and all the orders are organized in the order book, which operates the matching for the transactions to be actually done. Then, the new asset price will be determined as explained in subsection 2.2.

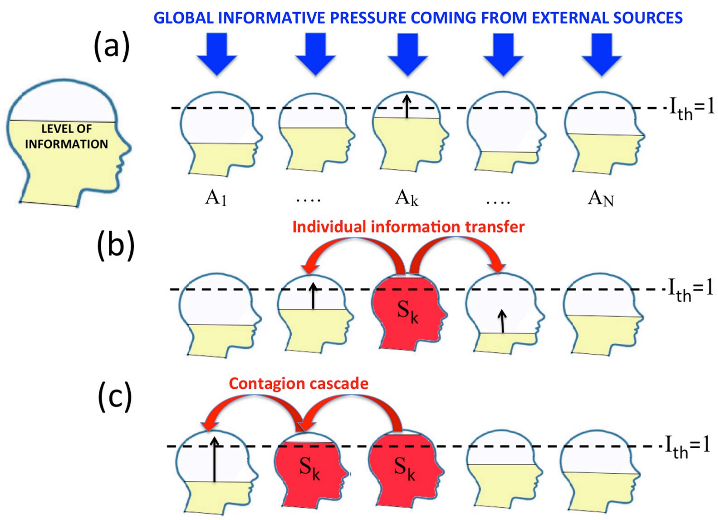

Figure 2: Informative pressure and contagion cascade.

2.1 The Informative Layer

Let a community of traders A_i (with $i = 1, ..., N$) be connected among themselves in a *Small World* (SW) network [19]. This topology, firstly introduced in [43], is usually adopted in order to describe realistic communities in social or economical contexts. In particular, we consider here a two-dimensional regular square lattice with open boundary conditions and an average degree $< k >= 4$. See ref. [19] for more details.

As above explained, each trader is exposed to two flows of information: a *global* one (a) and an *individual* one (b), [19, 21, 36].

(a) The global informative pressure reaches all investors uniformly at every time-step, from external sources. Each trader is endowed with a real variable $I_i(t)$ ($i = 1, 2, ..., N$) that represents her information at time t. Initially, at $t = 0$, the informative level of each trader is set randomly, in such a way that $I_i(t) \in [0, I_{th}]$, where $I_{th} = 1.0$ is a threshold assumed to be the same for all agents. Then, at any time-step $t > 0$, the information accumulated by each trader in her awareness tank is increased by a quantity δI_i, different for each agent and randomly extracted within the interval $[0, (I_{th} - I_{max}(t))]$. Such an accumulation process may lead a given trader A_k, before the other, to exceed her personal threshold value at a given time $t = t_{av}$. In this case, that trader becomes *active* and transmits her opinion, as an informative signal, to her neighbors;

(b) Such an opinion spreading represents the second (individual) flow of information, since every trader may receive signals from her neighbors (who have possibly passed their threshold). If it happens, it may cause, in turn, that also other agents exceed their thresholds because of this supplementary amount of information, which is additive with regards to the global one (a). Such a process explains how the informative cascades may generate herding in the market.

The information transfer happens in the information network according to the following simple mechanism [19], analogous to the energy transmission in earthquake dynamics modeled in networks of terrestrial crust pieces contained in [42]:

$$I_k > I_{th} \Rightarrow \begin{cases} I_k \to 0, \\ I_{nn} \to I_{nn} + \frac{\alpha}{N_{nn}} I_k, \end{cases} \tag{1}$$

where nn denotes the set of nearest-neighbors of the active agent A_k. N_{nn} is the number of direct neighbors, and the parameter α controls the level of dissipation of the information during the dynamics ($\alpha = 1$ if there is no dissipation): it is realistic to presume that part of the information content is lost in transmission, therefore in our simulations we always adopted $\alpha < 1$. As a consequence of the received amount of information, it has been said that other traders may be activated and they may, possibly, pass the threshold level as well. This process is shown in the three steps depicted in Fig.2: from step (a) to step (c), all the newly active traders will imitate the status S_k of the *first* agent and will transmit, in turn, their own signal to their neighbors following again Eq.1, and so on. In such a way, an informative avalanche will take place at time t_{av}, producing a contagion cascade of traders, who will share the same behavior and choice.

2.2 The Trading Layer

In our model we consider an ideal financial market where only one asset exists and where money has an ancillary function, just for transactions regulation. Let us imagine, first, that the trading layer is not influenced by the herding avalanches of the informative layer. In this case we could consider the process of status setting and price formation for an individual agent as independent from the behavior of the other agents. The traders A_i (with $i = 1, ..., N$) are endowed, at the beginning of each simulation, with an equally valued portfolio, composed by the same initial quantity of money $M_i = M$ $(\forall i)$ and the same initial quantity of the unique asset $Q_i = Q$ $(\forall i)$. The total wealth W_i of each trader is then defined as: $W_i = M_i + Q_i \cdot p_t$, where p_t is the global price of the asset at time t. As we know, two groups of traders exist: fundamentalists and chartists. At each time-step, traders will behave differently, according to their *character*:

F- Fundamentalists presume the existence of a *fundamental value*, FV_t, and believe that the market price of the asset will always tend to it. The fundamental value changes every t_f time-steps following the rule:

$$FV_{t+t_f} = FV_t + \mathbb{D}_t \tag{2}$$

where $FV_0 = 0$ and $\mathbb{D}_t$ is a random variable extracted from a normal distribution with zero mean and standard deviation σ_f. This corresponds to the assumption that dividends follow a random walk. The fundamental value is then used by each fundamentalist in order to build a personal opinion about the *correct* price for the asset, named *fundamental price*, p_t^F, being computed as

$$p_t^F = p_0 + FV_t + \Theta \tag{3}$$

where p_0 is the initial global asset price and Θ is randomly chosen in the interval $(-\theta, \theta)$, in order to account for the heterogeneity of investors. Thus, fundamentalists form their expected price for the asset according to

$$E[p_{t+1}] = p_t + \phi(p_t^F - p_t) + \epsilon \tag{4}$$

where the parameter ϕ is a sensitivity parameter that describes the expected speed of convergence to the fundamental price and ϵ is a stochastic noise term, randomly chosen in the interval $(-\sigma, \sigma)$. In order to limit the number of parameters, we let the value of ϕ be fixed even if, in principle, it could be different for each trader of this group.

C- Chartists decide their behavior according to their inspection of past prices. Therefore, before defining their expectation for the future, they will analyze the past dynamics of the asset price series. In particular, they consider the information coming from such an inspection as a *past reference value* PRV_t, computed at any t by averaging the previous prices over a time window of length T, different for each chartist and randomly chosen in the interval $(2, T_{max})$:

$$PRV_t = \frac{1}{T} \sum_{j=t-T}^{t} p_j. \tag{5}$$

Then, the expected price for the next time-step is determined by each chartist as

$$E[p_{t+1}] = p_t + \frac{\kappa}{T}(p_t - PRV_t) + \epsilon \tag{6}$$

where κ (a constant) is the sensitivity parameter and ϵ is, again, a stochastic noise term defined as in Eq.4).

In order to choose the status of the traders a sensitivity threshold τ has been introduced in the model, in such a way that, if the expectations are not *sufficiently* strong, the trader will decide to hold, without setting any order. In particular, only if $E[p_{t+1}] > p_t + \tau$ traders will expect a rise in the market price and decide to buy the asset, setting their status S_i on *bidder*. On the other hand, only if $E[p_{t+1}] < p_t - \tau$ traders will expect a fall in the market price and they will decide to sell the asset, setting their status on *asker*. Finally, if $p_t - \tau < E[p_{t+1}] < p_t + \tau$, traders will decide to hold on, without doing nothing. Of course, traders who decide to buy must have a positive amount of money ($M_i > 0$) and, similarly, those who decide to sell must have a positive amount of the asset ($Q_i > 0$).

Once the individual status has been decided, each trader sets her order in the book by choosing the preferred price for the transaction. Both in case of sales and purchases, the price chosen by each trader for the transcription in the order book (personal bid price for bidders and personal ask price for askers) is a function of the expectation that inspired the status of the same trader. Since orders are always of quantity 1, bid (ask) prices are decided by traders by means of simple price setting rules that describe their willingness to pay (to accept) according to their expectations, instead of being defined by means of function optimization procedures. The heterogeneity of traders is embedded in the model by defining feasible intervals from which each investor can extract her bid/ask price:

I- if the status is *bidder*, the chosen bid price will be extracted (with uniform probability) from a range whose minimum and maximum are defined as follows:

min: since it is not convenient for any buyer to set a bid price too low, because no seller would accept to sell, the lower bound for the bid price setting at time $t+1$ is equal to the best ask price (i.e. the lowest one) observed at time t;

max: since the reason why the investor is bidding is that her expected price is higher than the current one, the upper bound for the bid price setting is exactly that expected price (but, of course, in case the trader has not enough money, the maximum value that she can bid is limited to the owned money);

II- if the status is *asker*, the chosen ask price will be extracted (with uniform probability) from a range, whose minimum and maximum are defined as follows:

min: since the reason why the trader is selling is that her expected price is lower than the current one, the lower bound for the ask price setting is the worst scenario that she infers, i.e. the expected price;

max: since it is not convenient for any seller to set an ask price too high, because no buyer would accept to buy, the upper bound for the ask price setting in $t+1$ is the expected price plus twice the difference between the current price and the expected one.

After status and price setting activities, orders for a "+1" or "−1" quantity are posted in the book. This is exactly the point where the interaction between the trading layer and the informative one becomes crucial. Actually, as anticipated in subsection 2.1, in presence of herding avalanches all the traders involved in the avalanche, regardless of their character (fundamentalist or chartist), will imitate both the status and the price of the agent who started the avalanche itself. And this, of course, strongly influences the order book aspect.

Once posted all the orders in the book, both sides (buy and sell orders) are ranked accordingly with their associated prices. Bid prices are ranked in decreasing order of willingness to pay: in such a way, the trader who has set the highest bid price (namely the *best-bid*) will be the top of the list and will have the priority in transactions. Conversely, ask prices are ranked in increasing order of willingness to accept: the trader with the lowest willingness to accept (who sets the so-called *best-ask*) will be the top of the list and will have the priority in transaction execution. Then, the matching is done by comparing the best ask and the best bid. The number of transactions N_T that actually does occur between askers (whose total number is N_a) and bidders (whose total number is N_b) strictly depends on such a comparison. Actually, only if best-bid > best-ask we have $N_T > 0$, i.e. a given number of transactions do occur, depending on the matching among ask and bid prices present in the order book. After the first transaction, occurring among traders who posted their own order at the best price, both from the demand or the supply side, transactions continue following the order in the book (ascending for the ask list and descending for the bid list) until the bid price is greater than the ask price and all the transactions are regulated at the ask price. Finally, the new global asset price will be $p_{t+1} = p_L$, where p_L is the ask price of the last transaction occurred.

3 Simulations results

In this section we present the simulation results of a typical run of the ML-CFP model, analyzing both its macroscopic and microscopic details, and plotting the final distributions of its main quantities.

We consider a network of $N = 900$ traders, equally divided in 450 fundamentalists and 450 chartists. The (typical) initial setup for the values of the control parameters of the model is the

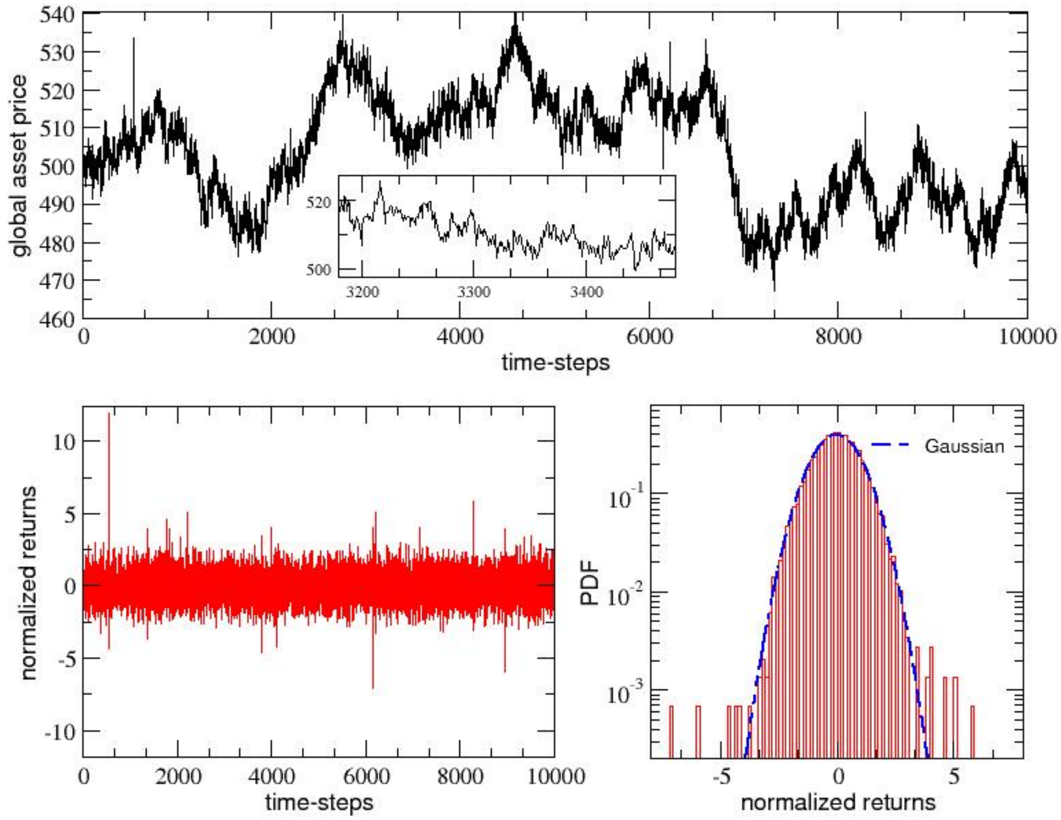

Figure 3: Top panel: typical time series for the global asset price; in the inset, a detail of the series. Bottom left panel: normalized returns of the price series. Bottom right panel: probability density distribution (PDF) of the normalized returns compared with a Gaussian distribution of unitary variance.

following: $p_0 = 500$ (initial price), $\alpha = 0.95$ (level of conservation of information), $\sigma_f = 2$ (standard deviation of the normal distribution for the fundamental value FV_t), $t_f = 10$ (time increment for FV_t), $\Theta = 30$ (range of variation for the fundamentalists' heterogeneity), $\phi = 0.5$ (sensitivity parameter for fundamentalists), T_{max} (maximum extension of the window for chartists), $\kappa = 2$ (sensitivity parameter for chartists), $\sigma = 30$ (maximum intensity of the stochastic noise for the expectation values), $\tau = 15$ (sensitivity threshold for the status setting), $M = 40000$ (initial quantity of money) and $Q = 200$ (initial quantity of the asset).

In the top panel of Fig.3 we show a typical time evolution of the global asset price. After a transient of 5000 time-steps without trading (not visible), need for the system to enter in the SOC regime (where power-law distributed avalanches can be observed in the informative layer), agents start to trade and the values of the asset price are plotted for the next 10000 time-steps, starting from the initial price $p_0 = 500$. In the inset, a detail of the same series is reported. Sometimes, very strong fluctuations are visible in the price series, due to the effect of herding avalanches. These fluctuations, of course, affect the volatility of the price, as emerges from the normalized returns time series, reported in the bottom left panel (normalized returns are defined as $r_t^{norm} = (r_t - r_{av})/r_{stdev}$, where $r_t = log(p_{t+1}) - log(p_t)$ are the logaritmic returns while r_{av} and r_{stdev} are, respectively, their mean and standard deviation calculated over the whole series). Consequently, the probability density functions (PDF) of normalized returns, plotted in the

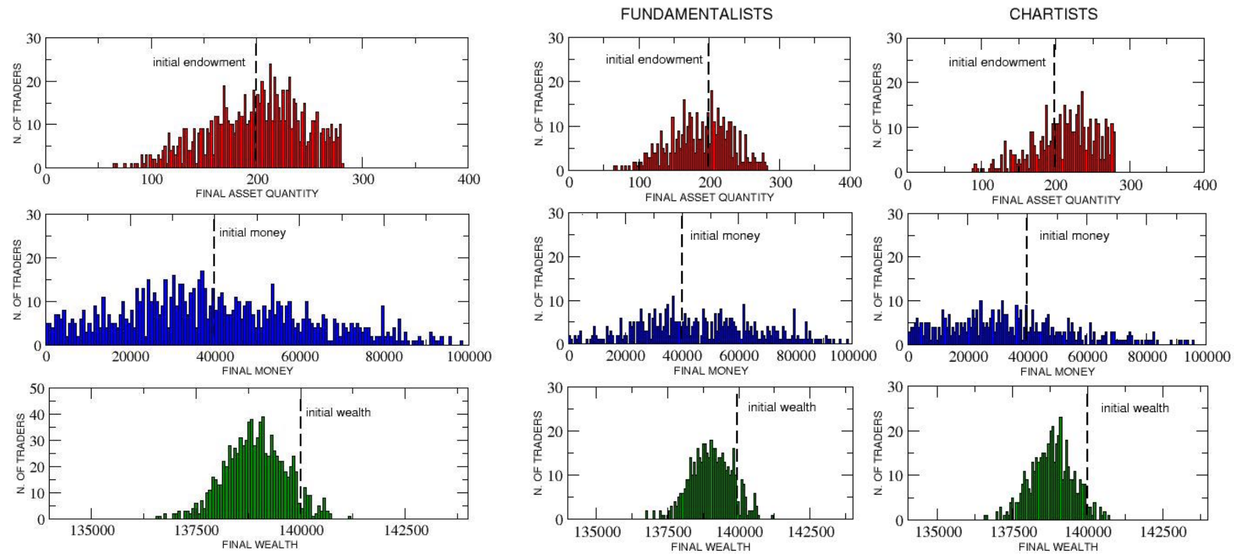

Figure 4: Left column: from top to bottom, final distributions of the asset quantity, money and wealth of all traders. Central and right columns: the same distributions are plotted separately for fundamentalists and chartists.

bottom right panel, shows the asymmetric fat tails characteristic of financial markets, symptom of the presence of extreme events.

On the other hand, the central part of the distribution is not peaked but follows a Normal shape, as visible from the comparison with a Gaussian curve with zero mean and unitary variance. This is probably a consequence of the strong approximation adopted in the ML-CFP model, where only one asset is considered and the orders are limited to one. Under these conditions, the system evidently is able to self-organize, maintaining a dynamical balance between purchase orders and sales. The latter feature clearly appears looking at the details of the transactions in the trading layer. Actually, the average numbers of askers and bidders calculated during the whole time period were, respectively, $N_a = 229.61$ and $N_b = 220.42$. Furthermore, over an average number of transactions equal to $N_T = 88.65$, the average numbers of buyers and sellers were, respectively, 52.02 and 52.49 for fundamentalists, and 51.2 and 50.75 for chartists, indicating a strong average equilibrium among the competing forces in the market.

Such an equilibrium can be also revealed by plotting, in Fig.4, the final distributions of the asset quantity, money and wealth for all the traders (left column) and, separately, for the fundamentalists (central column) and chartists (right column). The initial values of the three quantities, equal for all the traders, are also reported as dashed vertical lines. As one could expect, the final distributions appear to be widespread around their initial values but, for what concerns money and asset quantity, the trading dynamics seems to well balance between gains and losses. The only source of asymmetry is the small difference between the average number of buyers and sellers that changes its sign for fundamentalists and chartists, leading the former group to slightly favor purchases and the latter to slightly favor sales. This, in turn, induces fundamentalists to sell their assets, thus increasing their money, and chartists to buy new assets, thus decreasing money. But these two variables, together with the fluctuations in the asset price, evidently compensate in producing a similar final wealth distribution for the two groups.

The features observed for this typical run are quite robust and remain substantially unchanged if one varies the relative proportion of fundamentalists and chartists, the initial value of the asset price or the initial asset quantity and money. On the other hand, they are quite sensitive with respect to variations in some control parameters, like the sensitivity parameters for the expectation prices, the sensitivity threshold for the status setting or the maximum intensity of the stochastic noise: in this case, the observed equilibrium between bidders and askers becomes unstable and, typically, one of the two trading groups, fundamentalists or chartists, start to buy the asset much more than the other one, thus generating a spiral effect that leads fundamentalists or chartists to spend all its money thus taking, in fact, out of the market.

4 Conclusions

In this paper we have presented a simple multi-layer order-book model of financial market, namely the ML-CFP model, with heterogeneous agents. Its realistic framework is fruitfully coupled with interesting numerical results, which - despite the simplifying assumptions about assets and orders - adhere to some typical features of real financial markets. The obvious generalization of this model, achievable by introducing more than one asset or variable orders quantities, is in progress and will be the subject of further studies, together with a deeper analysis of stylized facts.

Acknowledgements

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