The Carbon Bubble: Climate Targets in a Fire-Sale Model of Deleveraging*

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Comments/Corrections Welcome

Abstract

Since the market valuation of fossil fuels companies is contingent upon the value of their reserves, and the market valuations of many other firms are contingent on their being able to purchase cheap energy as an intermediate input, the credible implementation of climate change targets could cause large asset price falls. If investors are leveraged, this may precipitate a fire-sale as investors rush for the exits, and generate a large and persistent fall in output and investment. Maintaining productive capacity by massively expanding low carbon infrastructure whilst carbon intensive infrastructure is retired is vital if society is to successfully implement policies which will address climate change. There may thus be a trade-off between the quantity of low carbon investment that the market can deliver and the rate at which climate policy mandates that carbon intensive infrastructure is retired. We model this trade-off by introducing a second investment good in a Kiyotaki and Moore (1997) economy. We find that it is welfare enhancing to gradually liquidate carbon intensive capital rather than to take strong and immediate action, and for the government to take an active role in recapitalising investors who have been impacted by the implementation of climate policy.

Keywords: Carbon Bubble, fire-sale, Kiyotaki and Moore's (1997) model, deleveraging, carbon tax, resource substitution, 2°C target.

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1 Introduction

The financial crisis of 2007-09, and the very slow recovery since, has its roots in the mispricing of subprime mortgages. The financial crisis started when it was realised that these assets were over-valued. However, according to DeLong (2011), this realisation possibly represents a perceived loss of wealth of perhaps $500 billion dollars. In the context of a world economy with $80 trillion dollars of financial wealth, this is a loss of perhaps 0.6%.[1] This small loss to investors precipitated the deepest recession since World War II.

The severity of the financial crisis has proven that a financial market disruption of a very small portion of the economy can cause a deep recession: a globally insignificant problem in the US subprime mortgage market snowballed into a global financial disaster. The deleveraging of the financial sector results in declining property prices and consequent decreases in the debt capacity of the non financial sector. As economic activity worsens, the property price drop fuels further debt capacity reductions in a downward spiral. This is the so called "financial accelerator" mechanism of feedback between the financial and non financial sectors.

In 1996, EU Governments set a global temperature target of two degree Celsius (°C) above pre-industrial level which was made international policy at the 2009 United Nations Climate Change Conference in Copenhagen.[2] A global mean temperature increase of 2°C is considered as a threshold separating safety from extreme events: significant extinctions of species, reductions in water availability and food production (EU Climate Change Expert Group, 2008), and catastrophic ice sheet disintegration and sea level rise. The Potsdam Climate Institute has calculated that if we want to reduce the probability of exceeding 2°C warming to 20%, then only one-fifth of the Earth's proven fossil fuel reserves can be burned unabated[3] (Carbon Tracker Initiative, 2011).[4] As a consequence, the global "carbon budget" is only 20% of worldwide reserves, whilst the rest is "unburnable carbon". The Carbon Tracker Initiative (2011) report warns that, analogously to the subprime mortgage problem, the global economy is once again mis-pricing risks as markets are

[1]Other estimates vary. The total value of subprime mortgages outstanding accounted for $1.2 trillion in the second quarter of 2008. Even if this number is very large in absolute term, it accounts for less than 5% of the value of residential real estate in the United States and probably for less than 2.5% of the value of total private wealth in the United States (Hellwig, 2009). Moreover, global wealth was estimated by the Boston Consulting Group to be $109.5 trillion at the end of 2007. In any case, the loss to a well diversified investor is small.

[2]See Jaeger and Jaeger (2011) for a summary of how the target emerged and evolved.

[3]By "unabated" we mean without the use of e.g. Carbon Capture and Storage (CCS) technology in which the fossil fuels are burned, but the carbon dioxide does not reach the atmosphere.

[4]The latest estimate is from McGlade and Ekins (2015) who suggest that "globally, a third of oil reserves, half of gas reserves and over 80 per cent of current coal reserves should remain unused ... in order to meet the target of 2°C".

overlooking this "unburnable carbon" problem.[5]

This issue is termed the "Carbon Bubble" because the imposition of climate policy consistent with the Potsdam Climate Institute's calculations would mean that the fundamental value of many fossil fuel assets must be zero as they cannot be used. Their current market value must therefore be made up of a zero fundamental value, and a "bubble" component: the Carbon Bubble. Note that a bubble of this form is not consistent with the rational bubbles that are described in the economics literature that follows from Tirole (1985). What has been termed the Carbon Bubble is a real asset which has positive fundamental value in one state of the world (no regulation) but not in another (with regulation) - it is not a bubble at all in the economic sense. However, this is the terminology that has been adopted.[6]

Despite climate science based claims that not even all existing fossil fuel assets can be used, capital markets place a positive value on fossil fuel reserves. Investors use the reserves that companies claim to own as an indicator of future revenues: the share price of these companies is heavily influenced by the reserves on their books. This means that companies have the incentive to invest to find new reserves, and to invest in new technology that will allow the exploitation of currently unprofitable resources, even though the exploitation of these deposits is inconsistent with the climate change targets that the world's governments have signed up to.

If politicians enforce policy responses in compliance with the 2°C target, markets will begin to recognise that the values of the reserves on these companies' books are untenable, and the value of the companies will fall considerably as a consequence of these *stranded assets*. Also, the values of companies who have cheap fossil fuels as an input are also likely to fall. It is not only the equity of companies that is exposed to this, the quality of the debt they have issued is also exposed and there will be defaults and ratings downgrades.

We believe that the issue of the Carbon Bubble cannot be considered without reference to the financial accelerator mechanism. If fossil fuel companies are using their balance sheets as collateral, or if investors are using their holdings of exposed financial assets as collateral, then the write-offs associated with a credible climate policy implementation will likely lead to a breakdown of credit relationships and a general decline in the amount of total credit supplied to the economy. If a limitation in the total carbon budget was imposed suddenly, this could cause a "sudden stop" (Mendoza, 2010) akin to, or worse than, the 2008 Financial Crisis: the Carbon Bubble could burst. Perhaps such a crisis could be avoided if any agreement was phased gradually?

[5]Though significant players in the market are starting to take an interest in this topic, in particular the Bank of England has signalled that it is investigating this issue - see Carney (2014).

[6]According to Google Trends, web search on the term "Carbon Bubble" reached a high around April 2013. One of the most cited news item on this topic is "Global Warming's Terrifying New Math" by Bill McKibben, published in Rolling Stone in June 2012.

This paper models the consequences of a major write-down of assets that would follow some climate policy response. Given a binding cumulative emissions allowance (a carbon budget), the social welfare objective will be closely related to the stock of zero carbon productive capacity that is built by the time the carbon budget is exhausted. In a frictionless model, simply writing-off carbon emitting capital will raise the return on investments in zero carbon capital, incentivising high investment rates. However, since a major channel for asset values to impact the real economy is through their use as collateral, we must incorporate the financial accelerator framework, and allow for the possibility that writing-off carbon emitting capital will lower the economy's ability to generate investment in zero carbon capital. We incorporate a financial accelerator mechanism by using the credit amplification mechanism of Kiyotaki and Moore (1997), where entrepreneurs borrow from savers using their current asset holdings as collateral.

The contribution of this article is to link, for the first time, the issue of the Carbon Bubble with the financial accelerator mechanism. This reveals that the issue of deciding on appropriate climate change policy cannot be separated from an analysis of how the investment in zero carbon infrastructure will be funded. If the aim of climate policy is to replace our current carbon emitting productive capacity with zero carbon productive capacity whilst remaining within our carbon budget, the impact of announcing certain assets to be worthless upon collateral and investment is of central importance. A sudden "bursting of the carbon bubble" could throw the economy into a deep recession, depriving green technology of investment funds when they are most needed. Even if the fossil fuels assets really should be written-off if disastrous global warming is to be avoided, the implementation of such a policy must pay cognisance to the impact that it will have upon investors' balance sheets. These balance sheets will be used to fund investment in zero carbon infrastructure, a substantial stock of which is needed for prosperity at the point at which the carbon budget is exhausted.

The remainder of the paper is organized as follows: Section 2 reviews some of the literature on the financial accelerator and on a cumulative carbon emissions constraint that are relevant to the issue outlined in this paper; Section 3 outlines the technical details of the model. Section 4 describes the steady state equilibria of the model, and the comparative static that we look at when considering steady states is the level of low carbon investment. We show that it can be better, in terms of the investment flow directed towards zero carbon infrastructure, to allow some investment in high carbon infrastructure in steady state. Of course, a cumulative emissions constraint implies zero steady state emissions of carbon, and in Section 5 we look at dynamic simulations of an economy in which the planner bans investment in new high carbon infrastructure, and implements restrictions on the usage of existing carbon assets. We consider two possible actions for the planner: a tax funded write-off of investors' debts alongside the restrictions on the use of assets; and a dishonest policy whereby the planner can vary the amount by which it tells the market that current high carbon infrastructure stocks are usable. We see that the planner can achieve a better outcome, in discounted utility terms, in the first case by implementing debt write-offs, and in the second case by lying to the market and implementing two smaller "burstings of the carbon bubble", rather than honestly announcing the carbon budget and causing a single large burst. Section 6 concludes.

2 Relevant Literature

This paper uses the financial accelerator model of Kiyotaki and Moore (1997). In their model, relatively patient savers lend to relatively impatient entrepreneurs. There is a financial friction because it is possible for the entrepreneurs to repudiate their debt by walking away from their assets. An entrepreneur cannot pre-commit to work[7] and thus all the savers can recover on default is the value of the entrepreneur's assets, not the asset value plus expected output from production. The savers therefore require that the credit that they advance is fully collateralised by the value of the assets. The risk neutral entrepreneurs produce using a more productive technology than the savers, and so borrow until they are credit constrained to fund their activities.

Kiyotaki and Moore (1997) show how the financial accelerator mechanism can exaggerate the fluctuations of output and investment following a relatively small temporary shock to the economy. When the economy experiences a negative productivity shock, there is a dynamic feedback process between the fall in asset values and the level of borrowing. The asset plays two roles: as an input to production and also as collateral for borrowing. The fall in asset values means that the entrepreneurs have to sell assets to repay debt, which means they have lower debt carrying capacity, since the assets were used as collateral for debt. The high productivity sector has reduced its asset demand, and the low productivity sector has to meet this demand in equilibrium. For this sector to increase its demand, asset values will need to fall. Therefore a fall in asset values precipitates forced sales to ensure borrowing and collateral requirements are aligned, but this forced sale causes prices to fall again which causes further forced sales, and further price falls, and so on. These are the dynamics of a "fire-sale". The process stops when the assets become so unproductive in the hands of the savers that the entrepreneurs can once again afford the lowered price.

The model presented in Kiyotaki and Moore (1997) does not return to steady state following a very large negative shock. In order to allow for large shocks, we follow Cordoba and Ripoll (2004) and develop a model with a more complex timing of production decisions, which mitigates the price response to large negative shocks, and ensures the model returns to steady state. Therefore in this paper we use the model of Kiyotaki and

[7] Hart and Moore (1994) refer to this option as "inalienability of human capital".

Moore (1997) with the renegotiation process introduced by Cordoba and Ripoll (2004) as a specific financial accelerator model that is suitable for our purposes. However, as shown by Gerke et al. (2013), most models of the financial accelerator share qualitatively similar features.

In Kiyotaki and Moore (1997) the assumption is that any shock is realised once the entrepreneurs have already taken their labour input decisions. For large negative shocks this means that the net worth of entrepreneurs is negative. With no debt renegotiation process specified, there will be no assets allocated to the productive sector in the period following the shock.

In Cordoba and Ripoll (2004) it is assumed that markets are open during the day, shocks occur at dusk, and then there is a window of opportunity for debt renegotiation to take place, before production occurs overnight. If entrepreneurs want to, they can default on the debt, crucially, before production takes place: the lender gets the ownership of the assets back but loses the outstanding value of the debt. They may be able to do better by renegotiating the outstanding value of the debt down to the new value of the collateral and incentivising the entrepreneurs to engage in production. This shares the burden of the fall in asset values with the lenders and ultimately limits the decrease in asset prices and output with respect to Kiyotaki and Moore (1997). Following a positive shock, entrepreneurs do not have the incentive to default and so no renegotiation of the debt occurs. The possibility of default and renegotiation modifies the response of the economy following a negative shock and thus introduces an asymmetry in the dynamics.

We further extend the (full) Kiyotaki and Moore (1997, Chapter III) model by introducing two flavours of capital[8] that entrepreneurs can develop: a high carbon variety and a zero carbon variety. Carbon based production is assumed to cause a global externality that the infinitesimal entrepreneurs will take as given: in the absence of policy they will choose to produce using the high carbon variety. Policy can however induce the entrepreneurs to use zero carbon production. This framework allows us to model the Carbon Bubble, which has hitherto not been considered as part of the extensive investigations conducted into the economics of climate change.

The standard approach to the economics of climate change, Nordhaus's (2008) Integrated Assessment Model (IAM), considers climate change in an optimal economic growth framework which includes damages from climate change. Typically IAMs balance the economic benefits of fossil fuel emissions for production against the economic damages from climate change, to produce some optimal timepath for emissions reduction which is implemented with a timepath of carbon taxes. The scientific literature, on the other hand, suggests that the first order impact of emissions in any given period is related to their contribution to the overall cumulative emissions, which is the main driver behind

[8] Or "tree" in the notation of Kiyotaki and Moore (1997).

climate change (Allen et al., 2009).[9] In this paper, we will take the contribution from Allen et al. (2009) as definitive. This is consistent with the headlines from the Carbon Tracker Initiative (2011) report. But it is also for simplicity since it makes the modelling exercise easier: we model a cumulative emissions limit separating non catastrophic damages, which are broadly undetectable in the social welfare function, from catastrophic damages which cause infinitely negative social welfare[10] and so must be avoided at all costs.

One way to think about imposing such a cumulative emissions constraint that embeds it within the standard approach is to say that we are arguing probabilistically, and invoke Weitzman (2009). Perhaps the damages associated with climate change have an uncertainty that grows with their median size. With low emissions, within our allowed carbon budget, we have low median damages and further, the uncertainty on these damages has a thin-tailed distribution: the product of the infinitely negative impact of catastrophic damages with the zero chance of them occurring is zero. The expected impact of such emissions is close to the medium impact and it is almost undetectable in terms of overall social welfare. Our carbon budget represents some threshold between a thin-tailed and a fat-tailed distribution for damages from emissions. With a fat-tailed distribution of damages, the product of the infinitely negative impact of catastrophic damages with the zero chance of them occurring is infinitely negative. Therefore, for emissions greater than the carbon budget, although the median impact is smoothly increasing in emission levels, the expected value tends to infinity across this threshold. Therefore, treating climate damages as approximately zero within the carbon budget and infinite beyond the carbon budget can be rationalised, and it simplifies the modelling substantially.

3 The Model

We develop a two-agent closed economy model which extends the "full version" of Kiyotaki and Moore (1997, Chapter III) by allowing entrepreneurs to choose between two types of investment good with different productivity.[11] We also introduce a very trivial government.

Time is discrete and indexed by $t = 0, 1, 2, ..., \infty$. There are two types of infinitely lived agents: a continuum of entrepreneurs of mass m_e, and a continuum of savers of mass m_s.[12] For simplicity, m_e is normalised to unity, and m_s is referred to as m. Entrepreneurs

[9] The IPCC (2014) report agrees: "Cumulative emissions of CO2 largely determine global mean surface warming by the late 21st century and beyond".

[10] E.g. human extinction.

[11] In the terminology of the original paper, "farmers" can choose between two types of "trees".

[12] Variables regarding the savers are identified by the prime. Aggregate variables will be capitalized. Steady state variables will be starred. For a list of variables and their definitions, see Appendix A.1.

and savers have the following preferences

$$\max_{\{x_s\}} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} x_s \right] \qquad \text{and} \qquad \max_{\{x'_s\}} E_t \left[\sum_{s=t}^{\infty} (\beta')^{s-t} x'_s \right] \tag{1}$$

i.e. they both maximize the expected discounted utilities from consumption: x_t and x'_t represent consumption at date t of the entrepreneur and the saver respectively; $0 < \beta < 1$ and $0 < \beta' < 1$ indicate the discount factors; and E_t indicates expectations formed at date t. Both types of agents are risk neutral but they differ in their rates of time preference: entrepreneurs are more impatient i.e. they have a lower discount factor than savers.

Assumption A $\qquad \beta < \beta'$

Exogenous ex-ante heterogeneity on the subjective discount factors ensures the model simultaneously has borrowers and lenders.

There are three types of goods: a durable asset, an investment good and a non durable commodity. The durable asset (K) may be thought of as land, as in Kiyotaki and Moore (1997), or capital, as in Cordoba and Ripoll (2004). The investment good (Z) has two flavours: one may be thought of as fossil fuel based infrastructure, while the other is zero carbon infrastructure. The non durable commodity cannot be stored but can be consumed or invested. The durable asset does not depreciate and is available in a fixed aggregate amount, given by $\bar{K}$, while both types of investment good depreciate at rate $1 - \lambda$ per period.

The government can levy a tax on the output of an entrepreneur who uses high carbon investment goods i.e. a carbon tax. The tax is then used to make lump-sum transfers on a per capita basis. The government runs a balanced budget, and its only activity, at this stage, is to decide on the level of the carbon tax.[13]

At the beginning of each time t, there is a competitive asset market and a competitive one-period credit market. In the former, one unit of the durable asset is exchanged for q_t units of the commodity; in the second, one unit of the commodity at date t is exchanged for R_t units of the commodity at date $t + 1$. The commodity is assumed to be the numeraire, so that its price is normalized to unity. q_t then represents the durable asset price, and R_t is the gross interest rate.

3.1 Representative Entrepreneur

A typical entrepreneur produces a quantity of the commodity, y, with a one-period Leontief production function: collateralizable assets, k, are combined with investment goods, z, in $1 : 1$ proportion.

[13] Later we will introduce the additional roles of limiting the stocks of high carbon investment goods, and considering mitigating policies that maximise social welfare.

This period's production decisions affect next period's production. The entrepreneur can choose between two technologies. Choosing the first, k_{t-1} units of assets are combined with z^L_{t-1} units of the low carbon investment good, producing y_t units of the commodity for the entrepreneur, according to

$$y_t = F_L(k_{t-1}, z^L_{t-1}) = (a^L + c) \times \min(k_{t-1}, z^L_{t-1}). \tag{2}$$

Choosing the second, k_{t-1} units of the asset are combined with z^H_{t-1} units of the high carbon investment good, producing y_t units of the commodity. However, the use of the high carbon technology means that the after tax output available to the entrepreneur will be reduced by any proportional carbon tax implemented, $\tilde{\tau}_t$:[14]

$$\begin{aligned} y_t = F_H(k_{t-1}, z^H_{t-1}) &= (a^H + c) \times \min(k_{t-1}, z^H_{t-1}) \\ (1 - \tilde{\tau}_t) y_t &= (a^H - \tau_t + c) \times \min(k_{t-1}, z^H_{t-1}). \end{aligned} \tag{3}$$

No matter the technology used, ck_{t-1} units of the y_t units of output produced at date t are not tradable and must be consumed by the entrepreneurs (who therefore must pay any carbon tax levied out of tradable output).[15]

The dichotomous variable $a^i + c$ represents the productivity of durable assets in the hands of entrepreneurs, and is given by the constant $a^H + c$ if the high carbon investment good is used in the production, and by $a^L + c$ otherwise. We assume that low carbon investment goods are less productive than high carbon investment goods:

Assumption B $\quad a^H > a^L$

The commodity can be consumed or invested. For that portion of their output which is invested, the entrepreneur converts ϕ units of the commodity into one unit of the investment good: ϕ is the output cost of investing in one unit of investment good.[16]

Two critical assumptions in Kiyotaki and Moore (1997) are imposed here. Firstly, the entrepreneur cannot pre-commit to work and can freely decide to withdraw their labour: Hart and Moore (1994) refer to this option as "inalienability of human capital". Secondly,

[14] In the remainder of the paper we discuss only $\tau_t = \tilde{\tau}_t(a^H + c)$, a positive bijective transformation of the tax rate into units that can be compared to the productivities of the two alternative technologies.

[15] The ratio $a^i/(a^i + c)$ represents an upper bound on the entrepreneur's savings rate and is introduced in Kiyotaki and Moore (1997) to avoid the possibility that the entrepreneur keeps postponing consumption. Indeed, since preferences are linear, entrepreneurs would like to not consume and increase investment. While this assumption and the presence of linear preferences but different discount factors can be considered as unorthodox modelling choices, Kiyotaki and Moore (1997, Appendix) show that the same qualitative results can be obtained using an overlapping generations model with standard concave preferences and conventional saving/consumption decisions.

[16] Note that, instead of writing the model in terms of differing productivities of high and low carbon technologies, $\{a^H, a^L\}$, we reach qualitatively the same results by writing the model in terms of differing output costs of investing in investment goods, $\{\phi^H, \phi^L\}$. Since results are qualitatively similar, we do not present this alternative model here.

the entrepreneur's technology and investment good are idiosyncratic. Thus, if they decide to withdraw their labour between dates t and $t+1$, there would be only the asset k_t and no output at $t+1$. Given these assumptions, a constraint arises limiting the debt of an entrepreneur. An entrepreneur may want to repudiate their contract when their debt becomes too onerous. The lender knows this possibility and asks the entrepreneur to back the loan with collateral. Rather than the amount of collateral depending upon the relative bargaining power of the agents, Hart and Moore (1994) suggest that the lender will require the full value of their counterpart's assets as collateral. Thus, for an amount of debt b_t and current asset holdings k_t, the entrepreneur must repay $R_{t+1}b_t$ next period, at which time their asset holdings will be worth $q_{t+1}k_t$. Entrepreneurs are therefore subject to the following borrowing constraint:

$$b_t \leq \frac{q_{t+1}k_t}{R_{t+1}}. \tag{4}$$

Consider an entrepreneur who holds k_{t-1} units of assets and has gross debt b_{t-1} at the end of period $t-1$. At date t they produce $a^i k_{t-1}$ units of tradable output (depending on the technology used), they incur a new loan b_t and acquire more assets, $k_t - k_{t-1}$. Having experienced depreciation and having increased their asset holdings, the entrepreneur will have to convert part of the tradable output to investment goods. In general, they will have to invest $\phi(k_t - \lambda k_{t-1})$ in order to have enough investment goods to cover depreciation and new asset acquisition. They then repay the accumulated debt, $R_t b_{t-1}$, pay the carbon tax if appropriate, $\tau_t k_{t-1}$, and choose how much to consume in excess of the amount of non tradable output, $(x_t - ck_{t-1})$. Moreover, they receive a per capita transfer from the government, g_t. Thus, the entrepreneur's flow-of-funds constraint is given by

$$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + R_t b_{t-1} + (x_t - ck_{t-1}) + \tau_t k_{t-1} = a^H k_{t-1} + b_t + g_t \tag{5a}$$

$$q_t(k_t - k_{t-1}) + \phi(k_t - \lambda k_{t-1}) + R_t b_{t-1} + (x_t - ck_{t-1}) = a^L k_{t-1} + b_t + g_t. \tag{5b}$$

The first line refers to an entrepreneur who uses the high carbon investment good, while the second relates to the use of the low carbon investment good.

Each period only a fraction, $0 \leq \pi \leq 1$, of entrepreneurs have an investment opportunity.[17] Thus, with probability $1-\pi$, the entrepreneur cannot invest and must downsize its scale of operation, since the depreciation of their investment goods implies $z_t^i = \lambda z_{t-1}^i$. This probabilistic investment assumption,[18] when combined with Leontief production,

[17]The arrival rate of the investment opportunity is independent through time and across agents.

[18]This assumption is introduced by Kiyotaki and Moore (1997, page 229 - 230) to capture "the idea that ... investment in fixed assets is typically occasional and lumpy".

means that with probability $1 - \pi$ the entrepreneur also faces the constraint:

$$k_t \leq \lambda k_{t-1}. \tag{6}$$

3.2 Representative Saver

Savers are willing to lend assets to entrepreneurs in return for debt contracts, and they also produce commodities by means of a decreasing return to scale technology which uses the asset as input and takes one period, according to

$$y'_t = \Psi(k'_{t-1}) \qquad \text{with } \Psi' > 0, \quad \Psi'' < 0. \tag{7}$$

Savers are never credit constrained because they can trade all their output and there is nothing idiosyncratic in their production process. Savers solve the maximization problem in (1), subject to their budget constraint

$$q_t(k'_t - k'_{t-1}) + R_t b'_{t-1} + x'_t = \Psi(k'_{t-1}) + b'_t + g_t. \tag{8}$$

Equation (8) should be read as follows: a saver who produces $\Psi(k'_{t-1})$ units of the commodity, incurs in new debt, b'_t, and receives the per capita government expenditure, g_t, (right-hand side) can cover the cost of buying new assets, $q_t(k'_t - k'_{t-1})$, repaying the previous debt (interests included), $R_t b'_{t-1}$, and consuming, x'_t, (left-hand side). Note that b'_{t-1} and b'_t can (and will in equilibrium) be negative.

3.3 Competitive Equilibrium

In general, an equilibrium consists of a sequence of prices $\{(q_t, R_t, \tau_t)\}$, allocations for the entrepreneur $\{(x_t, k_t, z_t, b_t)\}$ and the saver $\{(x'_t, k'_t, b'_t)\}$ such that, taking the prices as given, each entrepreneur solves the maximization problem in (1) subject to the technological constraints in either (2) or (3) and, if appropriate, (6), the borrowing constraint in (4) and the flow-of-funds constraint in (5a) or (5b); each saver maximizes (1) subject to the technological constraint in (7) and the budget constraint in (8); the government always runs a balanced budget; and the markets for goods, assets and credit clear.

Using γ_t to indicate the share of aggregate entrepreneurs' asset holdings dedicated to high carbon production at time t, let Z_t^H, Z_t^L, B_t, $mb'_t \equiv B'_t$, K_t, $mk'_t \equiv K'_t$, X_t, $mx'_t \equiv X'_t$, Y_t, $my'_t \equiv Y'_t$, $(1+m)g_t \equiv G_t$, $\tau_t \gamma_t K_t \equiv T_t$ be aggregate investment, aggregate borrowing, aggregate asset holdings, aggregate consumption, aggregate output, aggregate government expenditures and taxes. Then the government budget constraint and the market clearing conditions for assets, credit and goods are, respectively,

$$T_t = G_t \tag{9a}$$

$$K_t + K'_t = \bar{K} \tag{9b}$$

$$B_t + B'_t = 0 \tag{9c}$$

$$X_t + Z_t^H + Z_t^L + X'_t + G_t - T_t = Y_t + Y'_t. \tag{9d}$$

Note that, given assumption A, the impatient entrepreneurs borrow from the patient savers in equilibrium. Moreover, given that savers are risk neutral and there is no uncertainty, the rate of interest, R_t, is constant and determined by the patient saver's rate of time preference[19] i.e. $R_t = 1/\beta' \equiv R$.

To characterize equilibrium, we start with the less productive sector as the saver's maximization problem is not affected by the carbon tax. Since the savers are not credit constrained, their asset purchases are such that they are indifferent between lending and buying assets. This is the case if the rate of return from buying assets is equal to the rate of return of lending[20]

$$\frac{\Psi'(k'_t)}{u_t} = R \tag{10}$$

where

$$u_t \equiv q_t - \frac{q_{t+1}}{R} \tag{11}$$

has a dual role. This "user cost of capital" is defined from the point of view of the entrepreneur as the down payment required to purchase one unit of the assets,[21] but it is also the opportunity cost of holding assets for the saver.

Using (9b) together with (10), the following asset market equilibrium condition is obtained:

$$u_t = \frac{1}{R}\Psi'\left(\frac{\bar{K} - K_t}{m}\right) \equiv u(K_t). \tag{12}$$

The ratio $(\bar{K} - K_t)/m$ is the representative saver's asset holdings. An increase in the saver' demand for assets causes the middle term of equation (12) to decrease, given the assumption of decreasing marginal productivity in (7). Equivalently, an increase in entrepreneurs' demand for assets needs a decrease in savers' demand for the market to clear: this is achieved by a rise in the user cost, u_t. Thus, $u' > 0$.

[19]The rate of time preference is given by the inverse of the discount factor.

[20]Equivalently, savers' asset purchases are such that they equate the marginal product of assets, $(1/R)\Psi'(k'_t)$, obtained by using assets to produce, and the opportunity cost of not selling the assets this period at price q_t and waiting until the next when, from the point of view of today, they will be worth $(1/R)q_{t+1}$.

[21]It represents the amount an agent has to prepare when buying new assets and it is given by the difference between the price of one unit of the assets and the amount the entrepreneur can borrow using that unit as collateral.

Now consider a constant carbon tax rate, τ, such that the after-tax productivity of the high carbon technology is equal to the productivity of the low carbon technology i.e. $a^L = a^H - \tau$. In this scenario, the entrepreneur is indifferent between the two technologies. To characterize the equilibrium, we indicate with $0 \leq \gamma \leq 1$ the share of aggregate entrepreneurs' asset holdings dedicated to high carbon resources in equilibrium. Entrepreneurs who can invest at date t will prefer borrowing up to the limit and investing, rather than saving or consuming, hence limiting their consumption to the current non tradable output ($x_t = ck_{t-1}$). Thus, the credit constraint in (4) is binding and the flow-of-funds constraint in (5) can be rearranged as[22]

$$k_t = \frac{1}{q_t + \phi - \frac{q_{t+1}}{R}} \left[(q_t + \lambda\phi + a^L)k_{t-1} - Rb_{t-1} + g_t \right]. \tag{13}$$

At the beginning of period t, the net worth of an entrepreneur, were their assets to be liquidated, is given by the expression in the square brackets and consists of the value of the tradable output, assets and remaining investment goods, net of debt repayment, Rb_{t-1}, plus the transfer from the government. The net worth is used by the entrepreneur to cover that part of total investment, $k_t(q_t + \phi)$, exceeding the amount they can borrow using their assets as collateral, $k_t q_{t+1}/R$.

An entrepreneur who cannot invest at t will consume only their non tradable output and pay off their debt.[23] Since they will not want to waste their remaining stock of investment goods, equation (6) will hold with equality i.e.

$$k_t = \lambda k_{t-1}. \tag{14}$$

Since the previous equations are all linear in k_{t-1} and b_{t-1}, we can derive the equations of motion for the entrepreneurs' aggregate asset holdings[24]

$$K_t = (1 - \pi)\lambda K_{t-1} + \frac{\pi}{q_t + \phi - \frac{q_{t+1}}{R}} \left[\left(q_t + \phi\lambda + a^L \right) K_{t-1} - RB_{t-1} + \frac{\gamma\tau}{1+m} K_{t-1} \right] \tag{15}$$

[22] The following relationship is derived by noticing that (5a) applies to a share γ of entrepreneur's asset holdings, while (5b) to the remaining $1 - \gamma$, and by using $b_t = q_{t+1}k_t/R$ from (4) and $\tau = a^H - a^L$.

[23] We refer the interested reader to Kiyotaki and Moore (1997, footnote 22) for the full proof of the claims on the behaviour of investing and non-investing entrepreneurs. They show that by Assumption A investment strictly dominates saving while Assumption G in Appendix A.2 ensures that an entrepreneur prefers to invest (if they can) or save (if they cannot invest) rather than consuming the marginal unit of tradable output.

[24] This is obtained by noticing that equation (13) refers to a fraction π of investors, while equation (14) applies to the remaining $1 - \pi$. Moreover, we express the total transfers from the government to the entrepreneurs as the fraction $1/(1 + m)$ of the total tax revenue, $\gamma\tau K_{t-1}$.

and borrowing[25]

$$B_t = q_t(K_t - K_{t-1}) + \phi(K_t - \lambda K_{t-1}) + RB_{t-1} - a^L K_{t-1} - \frac{\gamma\tau}{1+m} K_{t-1}. \qquad (16)$$

One interesting implication of equation (15) is that demand for assets by the entrepreneurial sector increases given an increase, in equal proportion, of both today's and tomorrow's asset prices. A rise in the current asset price increases entrepreneur's net worth and a rise in future asset prices strengthens the value of the collateral (thus allowing the entrepreneurs to borrow more) and this more than compensates for the price increase induced reduction in demand.

We are now able to characterize, for given K_{t-1} and B_{t-1}, the perfect foresight competitive equilibrium from date t onward as the paths of aggregate entrepreneurs' asset holdings and debts, as well as asset prices, $\left\{K_{t+s}, B_{t+s}, q_{t+s}\right\}_{s=0}^{\infty}$, such that equations (12), (15), and (16) are satisfied for all t.[26]

4 Steady State Analysis

This section describes the steady state equilibria of the model. Whilst much of the following simply repeats Kiyotaki and Moore (1997), we also calculate the comparative static which has most bearing upon the economics of climate change and the transition to a zero-carbon economy, namely that to maximize the investment flow directed towards zero carbon resources, it might be necessary to allow some investment in high carbon infrastructure. Any assumptions that are not in the text can be found in Appendix A.2.

Proposition 1 *Given $\tau = a^H - a^L$, there exists a continuum of steady state equilibria,*

[25] This is obtained by solving for b_t the flow-of-funds constraint in (5), where (5a) applies to γ entrepreneurs and (5b) to $1-\gamma$, with $x_t = ck_{t-1}$.

[26] Note that equations (15) and (16) are very similar to the equations of motion derived in Kiyotaki and Moore (1997). The debt renegotiation mechanism of Cordoba and Ripoll (2004) does not affect these equations of motion under perfect foresight - since adverse shocks and hence debt renegotiation do not occur under perfect foresight. We return to the debt renegotiation mechanism of Cordoba and Ripoll (2004) in Section 5 where its incorporation will allow the economy to recover from very large exogenous shocks imposed at time t, through an instantaneous adjustment at time t^+, with the economy thereafter following the perfect foresight, risk free path.

($q^\star$, $K^\star$, $B^\star$), with associated $u^\star$, indexed by $\gamma \in [0,1]$, where[27]

$$\left(\frac{B}{K}\right)^\star = \frac{\phi\lambda - \phi + a^L + \frac{\gamma\tau}{1+m}}{R-1} \tag{17a}$$

$$u^\star = \frac{1}{R}\Psi'\left(\frac{\bar{K}-K^\star}{m}\right) = \frac{R-1}{R}q^\star \tag{17b}$$

$$u^\star = \frac{\pi\left(a^L + \frac{\gamma\tau}{1+m}\right) - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)}. \tag{17c}$$

Given Assumptions E and F, the values for $(B/K)^*$ and u^* in Equations (17a) and (17c) are positive. It is clear that, for any value of γ, this steady state is unique: the assumptions on the savers' production function make the middle term of equation (17b) decreasing (and continuous) in K, while the expression for $u^\star$ in the right hand side of equation (17c) is given by a constant. Thus, given Assumption D, the two expressions for u^* cross only once.[28] In Appendix A.5, we show that we can refer the interested reader to Kiyotaki and Moore (1995) for the analysis of the stability of the system.

Equation (17a) says that in steady state the net of tax amount of tradable output produced by the entrepreneur, $a^L K^\star$, together with the transfer from the government, $\frac{\gamma\tau}{1+m}K^\star$, is used to repay the interest on the debt, $(R-1)B^\star$, and to replace the amount of the investment good that has depreciated in the period, $\phi(1-\lambda)K^\star$. As a result, the scale of operation of the entrepreneurial sector neither increases nor decreases.

Figure 1 provides a visual representation of different scenarios. The horizontal axis shows demand for assets from the entrepreneurs from left to right and from the savers from right to left. Since the market for assets clears, the sum of the two demands is equal to $\bar{K}$. The vertical axis consists of the net of tax marginal product of assets, which is constant at $a^i + c$ for entrepreneurs but decreasing with asset usage for savers.

Were the debt enforcement problem absent, without the carbon tax the economy would be able to reach the first best allocation, E_{FB}, in which the entirety of the aggregate entrepreneurs' asset holdings is used with the high carbon investment good. In this scenario, entrepreneurs are not constrained in the amount they can borrow. Thus, the marginal products of the two sectors are identical. In contrast, in the constrained economy too much of the asset is left in the hands of the savers and entrepreneurs have higher marginal product than savers.

It is easy to see[29] that an equilibrium with no tax on high carbon production, $E_H^\star$, provides a larger share of assets to the entrepreneurs, $K_H^\star$, compared to an equilibrium

[27]See Appendix A.3 for derivation.

[28]As Kiyotaki and Moore (1997), we focus only on these interior steady state equilibria. Note, however, that in Kiyotaki and Moore (1997), there are other two steady states.

[29]In Appendix A.4, we derive two special cases of the steady state presented in the text.

where the carbon tax, $\tau > a^H - a^L$, is so high that the entrepreneurs use only low carbon investment goods, $E_L^\star$. As a consequence, output, investment, borrowing and consumption are higher in the untaxed than in the taxed steady state equilibrium. The tax lowers the productivity of the entrepreneurial sector which not only earns less revenue with respect to the untaxed economy but also has lower net worth. Thus, in general, entrepreneurs can borrow, invest and produce less. To clear the market, the demand for assets by the savers must be higher in the taxed world, which requires a lower user cost. But a lower user cost is associated with a lower asset price and thus with a lower net worth of the constrained sector, which translates into less collateral. Less collateral means lower investment and production, and so on in a vicious circle.

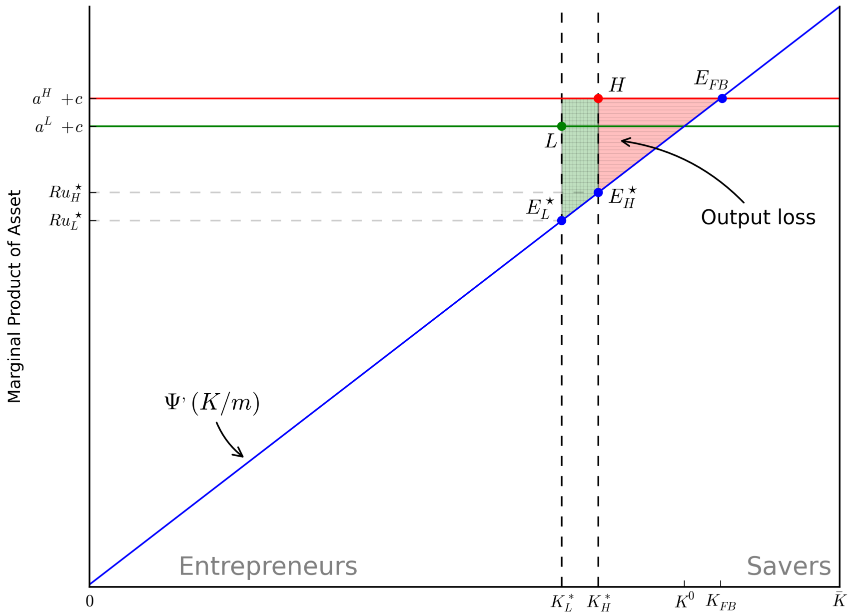

Figure 1: Comparison of steady states

The area of the triangle $HE_H^\star E_{FB}$ gives the output loss of the untaxed equilibrium relative to first best, while the remaining shaded area indicates the further output loss caused by the presence of the carbon tax, $\tau > a^H - a^L$.

When $\tau = a^H - a^L$, the amount of the asset in the hands of the entrepreneurs, $K^\star(\gamma)$, is a monotonically increasing function of the share of assets used in conjunction with high carbon investment goods, γ. If $\gamma = 0$, the economy reverts to the steady state equilibrium under $\tau > a^H - a^L$, and $K^\star(0) = K_L^\star$. On the contrary, with a carbon tax $\tau = a^H - a^L$, but with full usage of high carbon investment goods, $\gamma = 1$, $K^\star(1)$ is smaller than $K_H^\star$ since the presence of the carbon tax lowers the net worth of the entrepreneurial sector with respect to the untaxed high carbon steady state.

4.1 Implications for Climate Policy

We now show that, with $\tau = a^H - a^L$ (i.e. equal after tax productivities), then under certain conditions, the relationship between the proportion of high carbon production, γ, and the absolute value of zero carbon investment is not monotonic. Indeed, the higher the share of entrepreneurs using high carbon production and investing in high carbon investment goods, the higher the net productivity of the asset, and the higher the tax revenues and so the per capita transfer. This means that entrepreneurs have higher net worth and so can hold more of the asset. Since the asset is more productive in the hands of the entrepreneurs, its value increases. This potentially allows the entrepreneurs who are using low carbon production and investing in low carbon investment goods to borrow more, invest more and produce more. Crucially we show that this non-monotonic relationship is due to the presence of credit frictions.

In general, low carbon investment is given by $Z_t^L = \pi(1-\gamma)\phi(K_t - \lambda K_{t-1})$. In steady state this value is given by $Z^{L\star} = \pi(1-\gamma)\phi(1-\lambda)K^\star$. To obtain $K^\star$ as a function of parameters of the model, we follow Kiyotaki and Moore (1997) and impose the following linear structure for the user cost

Assumption C $\qquad u(K) \equiv K - \nu$

This allows us to express the steady state value of aggregate entrepreneurs' asset holdings as[30]

$$K^\star = \frac{\pi\left[a^L + \frac{\gamma(a^H - a^L)}{1+m}\right] - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} + \nu.$$

Therefore, investment in zero carbon infrastructure can be expressed as

$$Z^{L\star} = \pi(1-\gamma)\phi(1-\lambda)\left\{\frac{\pi\left[a^L + \frac{\gamma(a^H - a^L)}{1+m}\right] - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} + \nu\right\}. \tag{18}$$

Differentiating equation (18) with respect to γ gives

$$\frac{\partial Z^{L\star}}{\partial \gamma} = \pi\phi(1-\lambda)\left\{\frac{\pi[1-2\gamma]\frac{(a^H - a^L)}{1+m} - \pi a^L + \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)} - \nu\right\} \tag{19}$$

and by setting equation (19) equal to zero we obtain the level of γ which maximizes $Z^{L\star}$

$$\hat{\gamma} = \underset{\gamma}{\operatorname{argmax}}\, Z^{L\star} = \frac{-\nu\pi\lambda - (1-\lambda)(1-R+R\pi)(\nu - \phi) + \pi\frac{(a^H - a^L)}{1+m} - \pi a^L}{2\pi\frac{(a^H - a^L)}{1+m}}.$$

It is then easy to see that under certain conditions (e.g. a high enough difference between the productivities of the two technologies, or a high fraction of entrepreneurs with respect

[30] Assumption C implies $K^\star = u^\star + \nu$, where $u^\star$ is given by equation (17c).

to savers), $Z^{L\star}$ increases for low levels of γ before starting to decrease, as shown by the solid line in Figure 2.

We now want to show that this result is a consequence of the presence of the credit constraint. Consider an economy in which there are no debt enforcement problem so that capital can be optimally allocated. In such an allocation the marginal products of the two technologies would be equalized and the asset price would be given by the discounted gross return from using the entrepreneurs' technology, $q^0 = (a^L + c)/(R - 1)$. It follows that $u^0 = (a^L + c)/R$ and, given Assumption C, $K^0 = (a^L + c)/R + v$. Therefore, without the inefficiency caused by the presence of borrowing constraint, investment in zero carbon investment goods would be given by the following relationship

$$Z^{L0} = \pi(1-\gamma)\phi(1-\lambda)\left\{\frac{a^L + c}{R} + v\right\}$$

which is increasing in $1 - \gamma$, the proportion of asset used in conjunction with zero carbon investment goods, as shown by the dashed line in Figure 2. Since the carbon tax equalizes the private return from using the asset in either high or zero carbon entrepreneurial technologies, which is optimally set equal to the returns from using the asset in the savers' technology, it is clear that the proportion of high carbon technologies cannot affect the amount of the asset overall that is devoted to entrepreneurial technologies. Therefore, in steady state, the flow of low carbon investment is monotonically decreasing in the high carbon share, γ.

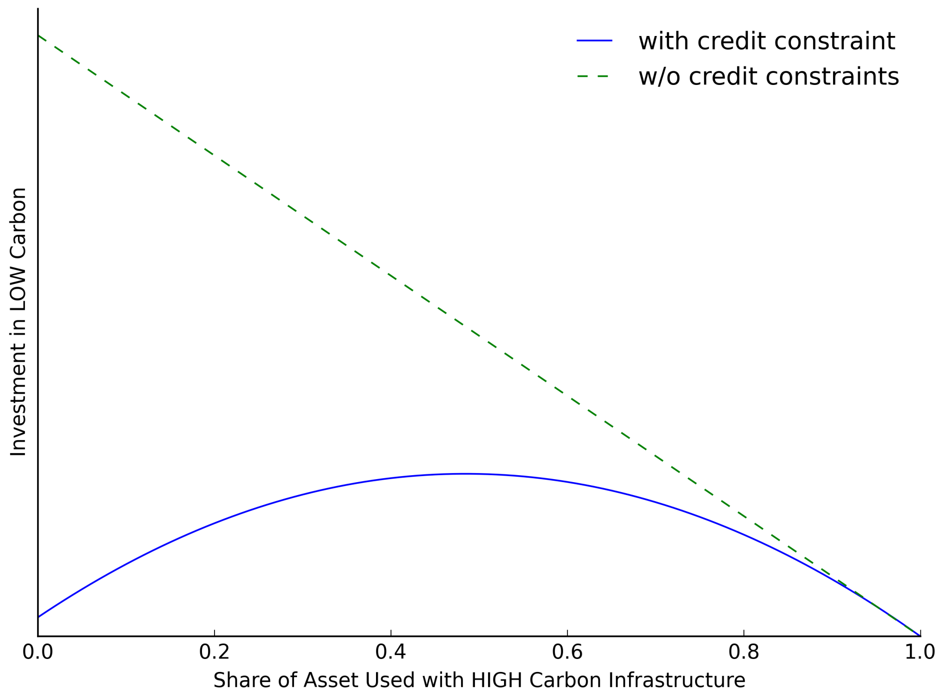

Figure 2: Absolute investment in low carbon resources as a function of γ

To the extent that the policy target is to maximise investment in low carbon infrastructure, this result shows that the optimal policy may be counter-intuitive: we may get more low carbon investment if we allow high carbon investment to continue.

5 Dynamic Simulations

Now that we have developed the analytic framework, in this section we turn to the issue of the Carbon Bubble. Here we imagine that the present stock of high carbon investment goods is more than sufficient to exceed some carbon budget. Therefore the social planner has banned any further investment in high carbon investment goods, and it remains to be decided how much of the initial stock of high carbon investment goods is also "written-off by dictat". The social planner has some legislative instrument which forces the entrepreneurs to leave a given quantity of the high carbon investment goods unused. The dynamics of the model are solved for using numerical simulation of the forward shooting method. Parameter values and details of the algorithm are given in Appendix A.6 but the rough approach is to guess the discontinuous change in asset price following the shock and iterate the economy forward through time to see if it converges back to steady state. If the price eventually explodes (tends to zero), the initial guess is revised downward (upward). This "guess and check" procedure is repeated until the asset price is arbitrarily close to the steady state in the long run.

Suppose that the economy has been running for a long time with $\tau = a^H - a^L$ so that at time $t = 0$ it is in steady state i.e. $K_0 = K^\star$ and $B_0 = B^\star$ with a certain γ.[31] At $t = 0$, the social planner realizes that the total carbon production, S, must be limited to a certain amount $\bar{S}$, namely the total carbon budget. The carbon production in period t, is linear in the amount of high carbon investment goods, Z_t^H, used in output production at t. Since we can choose units, let this amount equal Z_t^H for simplicity. New investment in high carbon goods is banned, and the existing stock depreciates at rate $1-\lambda$, therefore the cumulative emissions for any choice of the initial level of high carbon goods, Z_0^H, is given by

$$S = \sum_{t=0}^{\infty} Z_t^H = \frac{Z_0^H}{1-\lambda}. \tag{20}$$

Inspired by the Carbon Tracker Initiative (2011, 2013), we assume that for the planner to limit total carbon production to $\bar{S}$, 80% of the stock of the entrepreneurs' high carbon investment goods at $t = 0$ must remain unused and that thereafter only investments in

[31] In what follows we use $\gamma = 0.8$. U.S. Energy Information Administration (2011) estimates that about 21% of world electricity generation was from renewable energy in 2011.

low carbon resources are allowed i.e.

$$\bar{S} = \frac{0.2\gamma K_0}{1-\lambda}.$$

Figure 3 gives an overview of the responses of the economy to implementing $\bar{S}$ at $t = 0$. It shows movement in $K/K^\star$, $Y/Y^\star$, $B/B^\star$, $q/q^\star$, and $I/I^\star$ i.e. the ratios of entrepreneurs' asset holdings, total output, investors' debt, asset price, and aggregate investment, to their respective decarbonised steady state values.

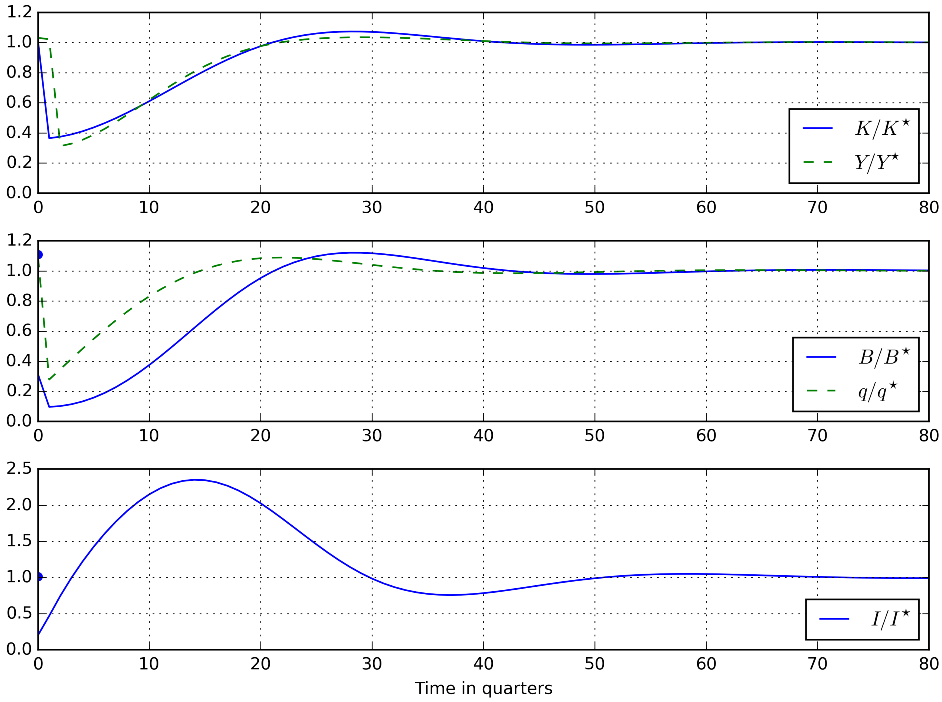

Figure 3: The burst of the bubble

As soon as assets are written-off, asset prices collapse by approximately 75%. Without the debt renegotiation mechanism of Cordoba and Ripoll (2004), the economy would collapse following a shock of this magnitude, and never return to steady state. With the Cordoba and Ripoll's (2004) mechanism, the entrepreneurs renegotiate their debt repayments down to the new value of the collateral and the economy can return to steady state.[32] However, even with this mechanism, the impact on the value of collateral is so severe that aggregate entrepreneurs' borrowing immediately falls[33] by 91% while aggregate entrepreneurs' asset holdings drop by 64%. Since most of the fixed asset is then employed

[32]More details are given in Appendix A.6.

[33]In the middle panel of Figure 3, $B/B^\star$ starts from the post renegotiation value at $t = 0^+$, when debt has been renegotiated down to approximately 28% of the debt in steady state at $t = 0$, while the scatter represents the pre-shock value, i.e. the ratio of the steady state value of debt before the write-off of high carbon assets, $B_0^\star$, to the decarbonised steady state value, $B^\star$.

in the low productivity sector, output collapses and at $t = 2$ it is 70% lower than its steady state value. The asset is now so unproductive and so cheap that entrepreneurs with investment opportunities start acquiring it and investing in low carbon resources, while the high carbon investment goods progressively depreciate and have effectively disappeared by period 200 (50 years). Note that despite the huge initial loss of productive capacity, aggregate investment falls sharply at the outset, before ramping up to replace the lost high carbon infrastructure.[34] The economy takes approximately 20 periods to recover and another 30 periods to stabilize around the new decarbonized steady state.

In the next subsections, we consider two possible additional actions for the planner that mitigate some of the enormous welfare loss associated with writing-off the high carbon assets. The first consists of a tax funded transfer of entrepreneurs' debt; the second involves deceiving the market on the usable amount of high carbon infrastructure stocks.

5.1 Tax Funded Transfer of Investors' Debt

The entrepreneurial sector is credit constrained, and following the imposition of climate policy, it is burdened with excessive debt relative to its assets. Perhaps the planner can achieve a better outcome if the burden of this debt is shifted to an economic actor who is not credit constrained. Specifically, we consider the planner themself taking over some of the debt, and funding the debt repayments through lump sum taxes.

At $t = 0$ the social planner announces $\bar{S}$, forcing the entrepreneurs to leave 80% of their stock of high carbon investment goods unused and allowing only investments in low carbon resources. Following the shock, entrepreneurs and savers renegotiate the level of debt down to the new value of the collateral. After the renegotiation takes place, the social planner then takes some debt from the entrepreneurs, B_0^G, which it funds by raising a constant per capita tax, τ^G, over $T = 40$ periods.[35] This implies

$$B_0^G = \sum_{t=1}^{T} (1+m)\,\beta^t \tau^G = (1+m)\,\tau^G \frac{\beta}{1-\beta}\left(1-\beta^T\right).$$

Then, in all periods $1 \leq t \leq T$, the social planner receives tax income of $(1+m)\tau^G$ from entrepreneurs and savers, repays the accumulated debt to the savers, RB_{t-1}^G, and raises

[34] In the lower panel of Figure 3, the line represents investment in low carbon asset weighted over its decarbonised steady state level. However, the scatter represents the ratio of the steady state level of investment at $t = 0$, which consists of both low and high carbon assets, to the decarbonised steady state level, which by definition is composed by investment in zero carbon asset alone.

[35] The choice of 40 periods (10 years) is relatively arbitrary. A less arbitrary choice would have been the issue of perpetuities, but this would have changed the steady state, which is problematic since we are running a numerical rather than analytic analysis. 40 periods was chosen as a specific term because, from Figure 3, the economy has roughly stabilised around the steady state by this time. And when comparing with the real world, 10-year debt is fairly common.

new debt from the savers, B_t^G, according to

$$B_t^G = (1+m)\,\tau^G \frac{\beta}{1-\beta}\left(1-\beta^{T-t}\right).$$

The social planner chooses the value of B_0^G (or equivalently, the value of τ^G) to maximise discounted utility. In Figure 4 one can see how discounted utility varies with the percentage of the entrepreneurs' debt that is taken over by the public sector. This shows that, given parameters and the write-off of 80% of the high carbon investment goods, it is welfare enhancing for the government to recapitalise investors: there is a clear optimal policy when choosing how much of the entrepreneurs' debt to take over.[36]

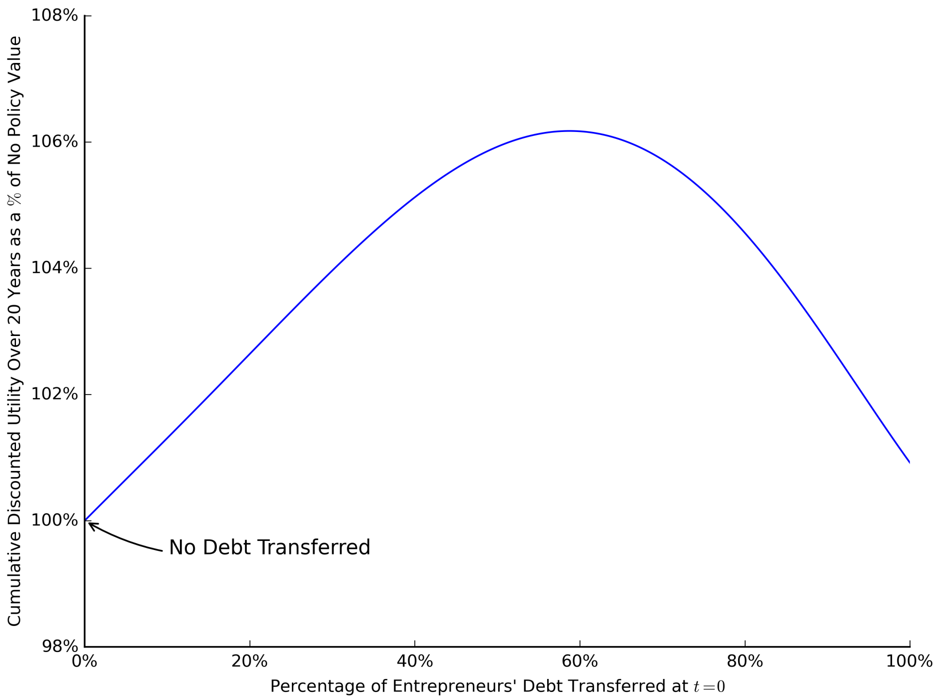

Figure 4: Welfare against percentage of entrepreneurs' debt transferred at $t = 0$

Since the optimal debt's transfer is around 55% under our parameters, Figure 5 gives an overview of the responses of the economy to implementing this optimal policy at $t = 0$. Following the policymaker's announcement, asset prices decrease by approximately 36%. While this still represents a severe jump, it is milder than in the case with no debt transferred. As a consequence of the decrease in asset prices, entrepreneurs renegotiate

[36]As a robustness check, we conduct the same analysis for the case without write-off of the high carbon investment goods at $t = 0$. We find that this policy is welfare destroying for any percentage of entrepreneurs' debt taken over by the government higher than 5%. Moreover, the optimal transfer is very small, around 2%, and the increase in cumulative discounted utility over 20 years is less than 0.2%. Thus, we conclude that almost all of the optimal tax / debt transfer that we calculate is due to the Carbon Bubble issue.

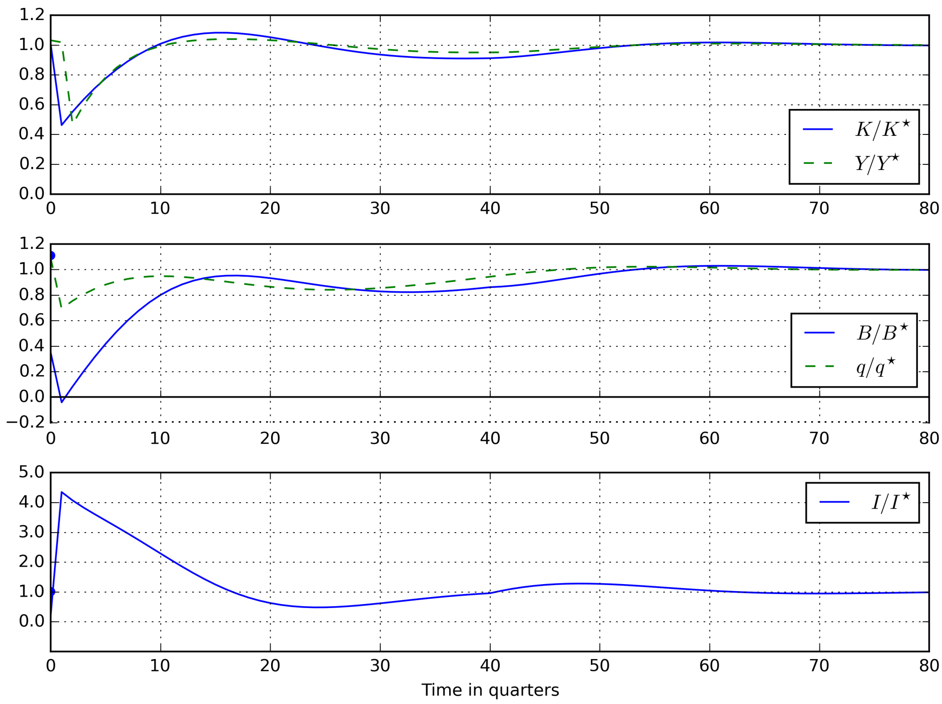

Figure 5: Transferring entrepreneurs' debt

their debt down to 70% of the steady state value. Further, the transfer of 55% of this debt from investors to social planner pushes aggregate entrepreneurs' borrowing down to approximately 32% of the initial steady state value.[37] Since entrepreneurs' desired asset holdings fall by 54% between $t = 0$ and $t = 1$, but price is kept higher than would be in the no debt transfer scenario, entrepreneurs return cash to the savers. As a consequence, aggregate entrepreneurs' debt turns slightly negative at $t = 1$. The decrease in output at $t = 2$ is 55%. The entrepreneurs' asset holdings are higher than they would be without the debt transfer, and their borrowings are much lower. Therefore the entrepreneurs are substantially richer and much more able to invest in replacement zero carbon stocks: aggregate investment in zero carbon infrastructure rockets. After 40 periods (10 years), the constant tax is no longer in place and the economy can converge back to the new decarbonized steady state. Over the course of 20 years, the cumulative investment in zero carbon infrastructures is approximately 3% higher, and the social welfare, in units of cumulative discounted consumption flow (since agents are risk neutral), is 6% higher (split 10% for entrepreneurs, and 4% for savers).

[37]This is showed as the initial value of the $B/B^\star$ line in the middle panel of Figure 5. The scatter indicates the original steady state value over the decarbonised steady state value. An interested reader can find more details in Appendix A.7.

5.2 Deception

Here we consider a different possible action for the planner: the planner can vary the amount by which it tells the market that current high carbon infrastructure stocks are usable.

We suppose that the planner can announce a different cumulate emissions target, S_{MAX}. Whatever the announced S_{MAX}, Z_0^H is determined by Equation (20). For $S_{MAX} > \bar{S}$, the economy's actual carbon budget, $\bar{S}$, is used at some time T. When this happens, $\bar{S}$ is revealed to all agents and the entrepreneurs are compelled to leave unused their remaining high carbon investment goods: high carbon production is abruptly banned in a desperate attempt to avoid catastrophic climate change and consequential societal collapse.

Why would the social planner want to announce $S_{MAX} \neq \bar{S}$? In a canonical growth or business cycle model, the social planner does not have any incentive to lie: stating an $S_{MAX} > \bar{S}$ would cause a welfare destroying discontinuity in consumption across the period in which $\bar{S}$ is revealed. In this model, on the contrary, overstating the actual carbon budget limits the fall in asset values and thus the decrease in the value of the collateral. This allows higher investment in low carbon technology, and potentially generates enough productive capacity between period 0 and T, when $\bar{S}$ is revealed, to counterbalance the loss in utility from the discontinuity in consumption at T.

Figure 6 presents the simulation for an S_{MAX} consistent with 55% of the high carbon investment goods being discarded at $t = 0$ (as opposed to $\bar{S}$ which is consistent with 80% being discarded).

This immediately causes the asset price to decrease by approximately 47% while aggregate entrepreneurs' borrowing falls by 69% and aggregate entrepreneurs' asset holdings fall by 44%. The decrease in output at $t = 2$ is 40%. To replace the written-off of high carbon investment goods, and the depreciation of those not written-off, investment in low carbon resources increases steadily. After approximately 23 periods (almost 6 years), the total carbon budget is used up. At this time, a little more than 75% of aggregate entrepreneurs' asset holdings are already dedicated to low carbon investment goods so that, when the remaining high carbon resources must be left unused, an alternative productive capacity already exists. This limits the magnitude of the recession which results - although it remains severe: the contemporaneous effect is to reduce asset values by 28%, aggregate entrepreneurs' asset holdings and borrowing by 26% and 40%, and output (one period after) by 18%. The decrease in entrepreneurs' asset holdings means that investment in low carbon resources also initially decreases. Eventually, the economy stabilizes around the new decarbonized steady state.

However, the social planner's deception is valuable in social welfare terms: over 20 years the cumulative investment flow in low carbon investment goods is approximately

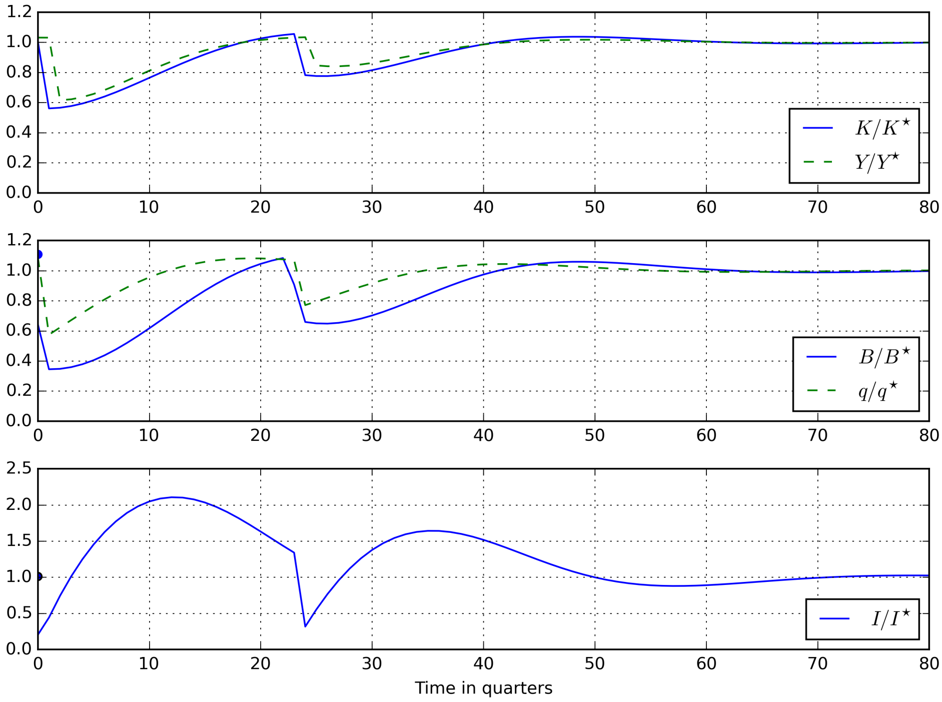

Figure 6: Dishonest social planner

2% higher in the scenario with the dishonest planner than in the scenario with the honest planner; and the social welfare is 5% higher (2% for entrepreneurs and 7% for savers).[38]

Figure 6 shows the simulation for an S_{MAX} consistent with 55% of the high carbon investment goods being discarded at $t = 0$, since this is approximately the optimal initial write-off under our parameters. Figure 7 shows how welfare varies with this percentage written-off. This shows that the policymaker has a clear optimal policy when faced with this dilemma. Write-off too much initially and the large asset price falls will overly depress output and investment over the period when there is still a carbon budget to utilise. However, if not enough is written-off then the problem of the Carbon Bubble is simply deferred until the carbon budget is exhausted.

6 Conclusions

This paper analyses the effects of the credible implementation of climate change targets in an economy characterized by collateral constraints. To do this, we consider a simple extension of the full model from Kiyotaki and Moore (1997), augmented with the debt renegotiation mechanism of Cordoba and Ripoll (2004). We allow for two investment

[38] Figures 3, 5, and 6 show that the economy has converged back to the neighbourhood of the steady state well before period 80, at which point the utility flows in each scenario are the same. As a consequence, the figures for impact on welfare are larger if we take a shorter view.

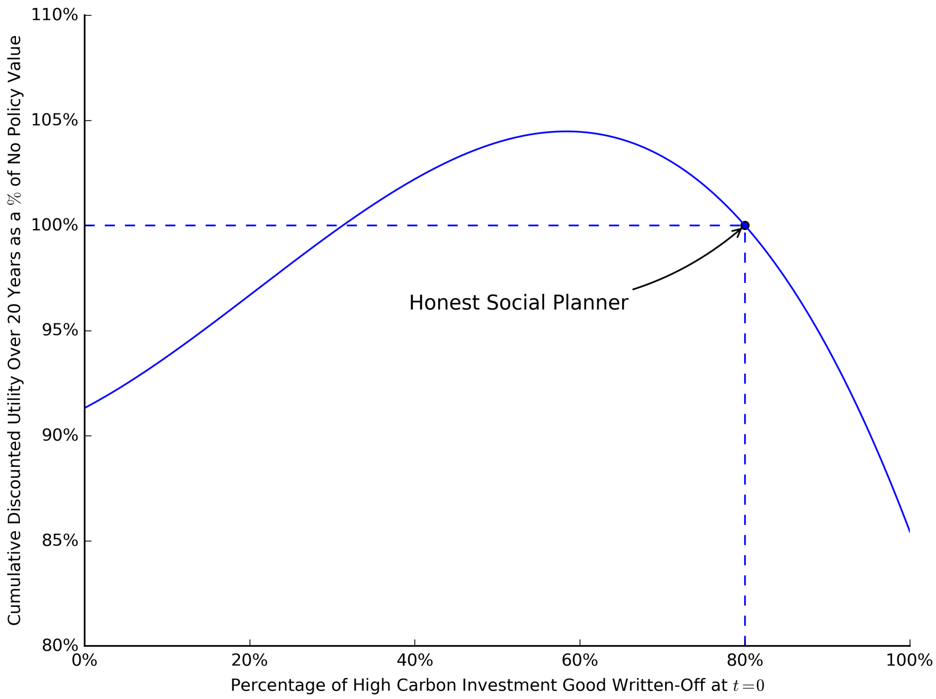

Figure 7: Welfare against percentage of high carbon goods written-off at 0

goods representing high carbon and low carbon infrastructure. This framework allows us to model, for the first time in the economics of climate change, the so-called "Carbon Bubble".

The Carbon Bubble, or the enforced write-off of carbon intensive assets, is an issue which has been introduced by the Carbon Tracker Initiative (2011) report as a warning to investors: as an individual, you should avoid an exposure to something bad that may happen. The reasoning behind this warning is that climate science mandates a climate policy response. This policy response will have an impact on financial markets and investors, that these markets and investors must take as exogenous. So far, no-one seems to have made the link back from this impact on financial markets to the appropriate climate policy response. In order to make this link, the financial accelerator mechanism which underlies much of the analysis of the 2007-09 financial crisis, must be considered. This is the first paper, to our knowledge, that has linked the Carbon Bubble phenomenon to the financial accelerator mechanism. It is therefore the first paper which can modify the climate policy advice coming from the economics of climate change literature, in light of the Carbon Bubble phenomenon.

We take as given that climate science mandates a severe climate policy response, such that society has a limited "carbon budget" relative to its ability to emit carbon pollution, and note that any welfare maximising outcome is likely to exhibit high investment levels in alternative zero carbon productive capacity over the period in which we still use carbon emitting productive capacity. We show that the presence of financial frictions can have the counter-intuitive consequence that the investment rate in low carbon resources is maximized if we allow high carbon investment to continue. This is because the presence of the more productive high carbon infrastructures increases the value of the assets. Since these assets are used as collateral, this means that the entrepreneurs who are investing in low carbon infrastructure can borrow more, and hence invest more.

We also consider the social planner's problem in facilitating the transition from an high carbon economy to the carbon-free era, taking the statements about the carbon budget in the recent reports by the Carbon Tracker Initiative (2011, 2013), as given. At $t = 0$, the social planner learns that, to avoid the collapse of civilization, the economy will be able to use only one-fifth of the current stock of high carbon investment goods. Investment in new high carbon infrastructures is thus forbidden, and the planner must enforce the write-off of 80% of the existing stock. We then experiment with allowing two strategies for the planner with which they can mitigate the welfare losses caused by this asset write-off.

The first strategy is for the public sector, which is assumed not to be credit constrained, to take over the debt obligations of the credit constrained entrepreneurs, which it repays from lump sum taxation. We find that it is always welfare enhancing for the public sector to take over some amount of the debt burden, and the optimal proportion (given the parameters we use) is large. While the burst remains severe, the improvement in the net asset position of entrepreneurs allows them to invest more in assets and low carbon infrastructures that would be the case without debt reallocation, driving the economy out of the recession faster, in spite of the presence of the lump sum tax.

The second strategy is for the planner to dishonestly announce a larger carbon budget than is the case. This causes a smaller recession with investment levels holding up better than would be the case under the true carbon budget. When the true carbon budget is exhausted, its existence is revealed and all usage of high carbon infrastructures must cease, causing a second recession. From the planner's point of view, this second burst causes a welfare destroying discontinuity in agents' consumption. However, between the announcement of the policy at $t = 0$, and the point at which the true carbon budget is revealed, economic activity is higher than it would have been given an honest announcement, and so investment in replacement zero carbon infrastructures has also been greater. When the economy must switch to the zero carbon technology, it has an alternative productive capacity already available which limits the reduction in output and consumption. Again we find that it is optimal for the planner to behave dishonestly and announce a carbon budget greater than the true carbon budget: so doing results in higher lifetime discounted utility, less output loss and more investment in low carbon resources.

These policy experiments show that the balance sheet effects of writing down high carbon assets on investment rates in zero carbon replacement infrastructures cannot be ignored in any rational climate policy analysis. The "global balance sheet" will be used to fund the zero carbon infrastructures which must be built to replace our fossil fuel based economy, and a naive, sudden full bursting of the carbon bubble could throw the economy into a deep recession, depriving green technology of investment funds right when they are most needed. Thus, even if the fossil fuels assets really should be written-off if disastrous global warming is to be avoided, it is likely to be sub-optimal to do this naively. The policy response to the threat of climate change must pay cognisance to the impact that it will have on investors' balance sheets.

References

Allen, M. R., D. J. Frame, C. Huntingford, C. D. Jones, J. A. Lowe, M. Meinshausen, and N. Meinshausen. 2009. "Warming caused by cumulative carbon emissions towards the trillionth tonne." *Nature*, 458: 1163 – 1166.

Carbon Tracker Initiative. 2011. "Unburnable carbon: Are the world's financial markets carrying a carbon bubble?"Technical report, Investor Watch.

Carbon Tracker Initiative. 2013. "Unburnable carbon 2013: Wasted capital and stranded assets."Technical report, Investor Watch.

Carney, M. 2014. "Letter to Joan Walley MP, Chair of House of Commons Environmental Audit Committee." http://www.parliament.uk/documents/commons-committees/environmental-audit/Letter-from-Mark-Carney-on-Stranded-Assets.pdf.

Cordoba, J.C., and M. Ripoll. 2004. "Collateral constraints in a monetary economy." *Journal of the European Economic Association*, 2(6): 1172 – 1205.

DeLong, J. B. 2011. "Understanding the lesser depression." Unpublished Essay.

EU Climate Change Expert Group. 2008. "The 2°C target: Background on impacts, emission pathways, mitigation options and costs."

Gerke, R., M. Jonsson, M. Kliema, M. Kolasa, P. Lafourcade, A. Locarno, K. Makarski, and P. McAdam. 2013. "Assessing macro-financial linkages: A model comparison exercise." *Economic Modelling*, 31: 253 – 264.

Hart, O., and J. H. Moore. 1994. "A theory of debt based on the inalienability of human capital." *Quaterly Journal of Economics*, 109(4): 841 – 879.

Hellwig, M. F. 2009. "Systemic risk in the financial sector: An analysis of the subprime-mortgage financial crisis." *De Economist*, 157(2): 129 – 207.

Hunter, J. D. 2007. "Matplotlib: A 2D graphics environment." *Computing in Science & Engineering*, 9(90): 90 – 95.

IPCC. 2014. "Climate change 2014."Technical report, Intergovernmental Panel on Climate Change.

Jaeger, C. C., and J. Jaeger. 2011. "Three views of two degrees." *Regional Environmental Change*, 11(1S): 15 – 26.

Kiyotaki, N., and J. H. Moore. 1995. "Credit cycles." Discussion Paper no. TE/95/285. London: London School Econ., Suntory-Toyota Internat. Centre Econ. and Related Disciplines, 1995; Working Paper no. 5083. Cambridge, Mass.

Kiyotaki, N., and J. H. Moore. 1997. "Credit cycles." *Journal of Political Economy*, 105(2): 211 – 248.

McGlade, C., and P. Ekins. 2015. "The geographical distribution of fossil fuels unused when limiting global warming to 2°c." *Nature*, 517: 187 – 190.

Mendoza, E. G. 2010. "Sudden stops, financial crises, and leverage." *American Economic Review*, 100(5): 1941 – 1966.

Nordhaus, W. D. 2008. *A question of balance*. London: Yale University Press.

Tirole, J. 1985. "Asset bubbles and overlapping generations." *Econometrica*, 53(6): 1499 – 1528.

U.S. Energy Information Administration. 2011. "International energy statistics, electricity generation."Technical report, U.S. Energy Information Administration.

van der Walt, S., S. C. Colbert, and G. Varoquaux. 2011. "The NumPy array: A structure for efficient numerical computation." *Computing in Science & Engineering*, 13(22): 22 – 30.

Weitzman, M. L. 2009. "On modeling and interpreting the economics of catastrophic climate change." *The Review of Economics and Statistics*, 91(1): 1 – 19.

A Appendix

A.1 Variables and Parameters Definition

In the text, lower case letters indicate variables for a representative entrepreneur or for a representative saver if followed by a prime symbol. Upper case letters are aggregate variables. Starred letters represent steady state equilibrium variables.

Variables and parameters definition

x	consumption
k	entrepreneur's asset holdings
$\bar{K}$	total supply of the asset
z^H	high carbon investment good
z^L	low carbon investment good
I	aggregate investment
b	debt
g	per capita government expenditure
y	output
a	tradable proportion of output
c	untradable proportion of output
u	user's cost of asset
q	asset price
R	gross interest rate
β	discount factor
τ	carbon tax rate
τ^G	tax funding the transfer of investors' debt
B^G	social planner's debt
$1-\lambda$	investment goods' depreciation rate
ϕ	output cost of investing
π	proportion of entrepreneurs with investment opportunity
m	savers' population size
γ	proportion of entrepreneurs using high carbon technology
S	cumulative emissions
$\bar{S}$	actual carbon budget
S_{MAX}	carbon budget announced by social planner

A.2 Assumptions

We specify here the relevant restrictions that we use to derive the steady state.

Assumption D $\quad \Psi'\left(\frac{\bar{K}}{m}\right) < \frac{\pi(a^L + \frac{\gamma\tau}{1+m}) - \phi(1-\lambda)(1+R\pi-R)}{\pi\lambda+(1-\lambda)(1-R+R\pi)} < \Psi'(0)$

This is included to avoid a corner solution i.e. to ensure that, in the neighbourhood of the steady state, both types of agent produce.

The following assumption says that the tradable output is at least enough to substitute the depreciated investment goods,

Assumption E $\quad a^L > (1-\lambda)\phi$

while this ensures that the probability of investment is not too small

Assumption F $\qquad \pi > \frac{R-1}{R}$

Both are used to ensure that the steady state values ($q^\star$, $K^\star$, $B^\star$) and the associated $u^\star$ are positive.

Moreover, to guarantee that the entrepreneur will not want to consume more than the nontradable output, we assume

Assumption G $\qquad c > \frac{1-\beta R\lambda(1-\pi)}{\beta R[\pi\lambda+(1-\lambda)(1-R+R\pi)]}\left(\frac{1}{\beta}-1\right)(a^L+\lambda\phi)$

Note that, since β and R are close to one, both Assumptions F and G are weak.

Finally, we avoid the explosion in asset prices with the following transversality condition

Assumption H $\qquad \lim_{s\to\infty} E_t(R^{-s}q_{t+s}) = 0$

A.3 Proof of Proposition 1

Noticing that the probability of having an investment opportunity is assumed independent through time and across entrepreneurs, we can derive the law of motion of aggregate entrepreneurs' asset holdings by assuming that a fraction $0 \leq \gamma \leq 1$ of entrepreneur's demand for assets relates to the high carbon technology. Since entrepreneurs' population has size normalized to 1, aggregate entrepreneurs' asset holdings can be written as

$$\begin{aligned} K_t =&(1-\pi)\lambda K_{t-1}+ \\ &+(1-\gamma)\frac{\pi}{q_t+\phi-\frac{q_{t+1}}{R}}\left[\left(q_t+\phi\lambda+a^L+\frac{\tau\gamma}{1+m}\right)K_{t-1}-RB_{t-1}\right]+ \\ &+\gamma\frac{\pi}{q_t+\phi-\frac{q_{t+1}}{R}}\left[\left(q_t+\phi\lambda+a^H-\tau+\frac{\tau\gamma}{1+m}\right)K_{t-1}-RB_{t-1}\right]= \\ =&(1-\pi)\lambda K_{t-1}+ \\ &+\frac{\pi}{q_t+\phi-\frac{q_{t+1}}{R}}\left\{\left[q_t+\phi\lambda+a^L+\frac{\tau\gamma}{1+m}\right]K_{t-1}-RB_{t-1}\right\} \end{aligned} \tag{A.1}$$

while aggregate entrepreneurs' borrowing is given by

$$\begin{aligned} B_t =&\gamma\left\{RB_{t-1}+q_t(K_t-K_{t-1})+\phi(K_t-\lambda K_{t-1})+\tau K_{t-1}-a^HK_{t-1}-\frac{G_t}{1+m}\right\}+ \\ &+(1-\gamma)\left\{RB_{t-1}+q_t(K_t-K_{t-1})+\phi(K_t-\lambda K_{t-1})-a^LK_{t-1}-\frac{G_t}{1+m}\right\}= \\ =&RB_{t-1}+q_t(K_t-K_{t-1})+\phi(K_t-\lambda K_{t-1})-\frac{G_t}{1+m}-a^LK_{t-1}. \end{aligned} \tag{A.2}$$

Equation (A.2) can be easily rearranged, with $B_{t-1}=B_t=B^\star$, $K_{t-1}=K_t=K^\star$ and $G_t=\gamma\tau K^\star$, into

$$\left(\frac{B}{K}\right)^\star = \frac{\lambda\phi-\phi+a^L+\frac{\tau\gamma}{1+m}}{R-1}.$$

By dividing the steady state counterpart of equation (A.1) by $K^\star$ and substituting $q^\star\frac{R-1}{R}=u^\star$ from equation (17b), we obtain

$$1-(1-\pi)\lambda = \frac{\pi}{\phi+u^\star}\left[u^\star\frac{R}{R-1}+\phi\lambda+a^L+\frac{\tau\gamma}{1+m}\right]-\frac{R\pi}{\phi+u^\star}\left(\frac{B}{K}\right)^\star.$$

By plugging in the expression for $(B/K)^\star$, and rearranging we obtain (17c).

A.4 Two Particular Steady States

We now present two particular cases of the steady state derived in the text. We start by considering the case in which the carbon tax is not present. As a consequences, all entrepreneurs prefer the high carbon investment good so that in equilibrium $\gamma = 1$.

Proposition 2 *Given $\tau = 0$, there exists a unique interior steady state equilibrium, ($q_H^\star$, $K_H^\star$, $B_H^\star$), with associated $u_H^\star$, where*

$$\left(\frac{B_H}{K_H}\right)^\star = \frac{\phi\lambda - \phi + a^H}{R-1}$$

$$u_H^\star = \frac{1}{R}\Psi'\left(\frac{\bar{K} - K_H^\star}{m}\right) = \frac{R-1}{R}q_H^\star$$

$$u_H^\star = \frac{\pi a^H - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)}.$$

Consider now the case in which the constant tax rate, τ, is such that the after-tax productivity of the high carbon technology is lower than the productivity of the low carbon technology i.e. $a^L > a^H - \tau$. In this scenario, the entrepreneurs will always prefer the low carbon technology and in steady state the economy is completely decarbonized.

Proposition 3 *Given $\tau > a^H - a^L$, there exists a unique interior steady state equilibrium, ($q_L^\star$, $K_L^\star$, $B_L^\star$), with associated $u_L^\star$, where*

$$\left(\frac{B_L}{K_L}\right)^\star = \frac{\phi\lambda - \phi + a^L}{R-1}$$

$$u_L^\star = \frac{1}{R}\Psi'\left(\frac{\bar{K} - K_L^\star}{m}\right) = \frac{R-1}{R}q_L^\star$$

$$u_L^\star = \frac{\pi a^L - \phi(1-\lambda)(1-R+R\pi)}{\pi\lambda + (1-\lambda)(1-R+R\pi)}.$$

A.5 Stability

In this section we follow Kiyotaki and Moore (1995, Appendix) to linearise the model around the steady state in order to examine the dynamics. The procedure requires using the laws of motion of aggregate entrepreneurs' asset holdings in (15) and borrowing in (16), together with the asset market equilibrium condition in (12), to find (K_t, B_t, q_{t+1}) as function of (K_{t-1}, B_{t-1}, q_t).

By combining equations (11) and (12), we find $q_{t+s} = R\big(q_{t+s-1} - u(K_{t+s-1})\big)$ and then substitute this value in equation (15). Together with (16), we now have the following system of "transition equations" for $s \geq 1$:

$$\begin{aligned}
q_{t+s} =& Rq_{t+s-1} - Ru(K_{t+s-1}) \\
B_{t+s} =& q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - \lambda K_{t+s-1}) + RB_{t+s-1} - aK_{t+s-1} + \\
& - \frac{\gamma\tau}{1+m} K_{t+s-1} \\
K_{t+s} =& (1-\pi)\lambda K_{t+s-1} + \\
& + \frac{\pi}{\phi + u(K_{t+s})} \left[\big(q_{t+s} + \phi\lambda + a^L\big) K_{t+s-1} - RB_{t+s-1} + \frac{\gamma\tau}{1+m} K_{t+s-1} \right].
\end{aligned}$$

Consider taking a first order Taylor series expansion to this system around the steady state,

$$\begin{aligned}
\frac{q_{t+s} - q^\star}{q^\star} \approx & \left. \frac{\partial q_{t+s}}{\partial q_{t+s-1}} \right|_{SS} \frac{q^\star}{q^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \left. \frac{\partial q_{t+s}}{\partial K_{t+s-1}} \right|_{SS} \frac{K^\star}{q^\star} \frac{K_{t+s-1} - K^\star}{K^\star} = \\
=& R \frac{q_{t+s-1} - q^\star}{q^\star} - Ru'(K^\star) \frac{K^\star}{q^\star} \frac{K_{t+s-1} - K^\star}{K^\star} = \quad \text{using } u^\star = q^\star \left(1 - \frac{1}{R}\right) \\
=& R \frac{q_{t+s-1} - q^\star}{q^\star} - (R-1) \frac{u'(K^\star)K^\star}{u(K^\star)} \frac{K_{t+s-1} - K^\star}{K^\star}
\end{aligned}$$

$$\begin{aligned}
\frac{B_{t+s} - B^\star}{B^\star} \approx & \left. \frac{\partial B_{t+s}}{\partial B_{t+s-1}} \right|_{SS} \frac{B^\star}{B^\star} \frac{B_{t+s-1} - B^\star}{B^\star} + \left. \frac{\partial B_{t+s}}{\partial q_{t+s-1}} \right|_{SS} \frac{q^\star}{B^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \\
& + \left. \frac{\partial B_{t+s}}{\partial K_{t+s-1}} \right|_{SS} \frac{K^\star}{B^\star} \frac{K_{t+s-1} - K^\star}{K^\star} = \\
=& \left[R + (q^\star + \phi) \frac{\partial K_{t+s}}{\partial B_{t+s-1}} \right] \frac{B_{t+s-1} - B^\star}{B^\star} + (q^\star + \phi) \frac{\partial K_{t+s}}{\partial q_{t+s-1}} \frac{q^\star}{B^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \\
& + \left[-\left(q^\star + \lambda\phi + a^L + \frac{\gamma\tau}{1+m} \right) + (q^\star + \phi) \frac{\partial K_{t+s}}{\partial K_{t+s-1}} \right] \frac{K^\star}{B^\star} \frac{K_{t+s-1} - K^\star}{K^\star}
\end{aligned}$$

$$\begin{aligned}
\frac{K_{t+s} - K^\star}{K^\star} \approx & \left. \frac{\partial K_{t+s}}{\partial q_{t+s-1}} \right|_{SS} \frac{q^\star}{K^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \left. \frac{\partial K_{t+s}}{\partial B_{t+s-1}} \right|_{SS} \frac{B^\star}{K^\star} \frac{B_{t+s-1} - B^\star}{B^\star} + \\
& + \left. \frac{\partial K_{t+s}}{\partial K_{t+s-1}} \right|_{SS} \frac{K^\star}{K^\star} \frac{K_{t+s-1} - K^\star}{K^\star} = \\
=& \left[\frac{R\pi K^\star}{\phi + u(K^\star)} - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial q_{t+s-1}} \right] \frac{q^\star}{K^\star} \frac{q_{t+s-1} - q^\star}{q^\star} + \\
& - \left[\frac{\pi R}{\phi + u(K^\star)} + \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial B_{t+s-1}} \right] \frac{B^\star}{K^\star} \frac{B_{t+s-1} - B^\star}{B^\star} + \\
& + \left\{ (1-\pi)\lambda + \frac{\pi\big(q^\star + \phi\lambda + a^L + \frac{\gamma\tau}{1+m}\big)}{\phi + u(K^\star)} + \frac{\pi K^\star}{\phi + u(K^\star)} \frac{\partial q_{t+s}}{\partial K_{t+s-1}} \right. \\
& \left. - \frac{K^\star(1 - \lambda + \pi\lambda)}{\phi + u(K^\star)} u'(K^\star) \frac{\partial K_{t+s}}{\partial K_{t+s-1}} \right\} \frac{K_{t+s-1} - K^\star}{K^\star}.
\end{aligned}$$

From the last approximation, it follows that

$$\begin{aligned}\frac{\partial K_{t+s}}{\partial q_{t+s-1}} &= \frac{R\pi K^\star}{\phi+u(K^\star)} - \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial q_{t+s-1}} = \\ &= \frac{R\pi K^\star}{\phi+u(K^\star)}\left[1+\frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\right]^{-1}\end{aligned}$$

$$\begin{aligned}\frac{\partial K_{t+s}}{\partial B_{t+s-1}} &= -\frac{\pi R}{\phi+u(K^\star)} - \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial B_{t+s-1}} = \\ &= -\frac{R\pi}{\phi+u(K^\star)}\left[1+\frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\right]^{-1}\end{aligned}$$

$$\begin{aligned}\frac{\partial K_{t+s}}{\partial K_{t+s-1}} =&(1-\pi)\lambda + \frac{\pi(q^\star+\phi\lambda+a^L+\frac{\gamma\tau}{1+m})}{\phi+u(K^\star)} + \frac{\pi K^\star}{\phi+u(K^\star)}\frac{\partial q_{t+s}}{\partial K_{t+s-1}} + \\ &- \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\frac{\partial K_{t+s}}{\partial K_{t+s-1}} = \\ =&\left[1+\frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\right]^{-1} \\ &\left[(1-\pi)\lambda + \frac{\pi(q^\star+\phi\lambda+a^L+\frac{\gamma\tau}{1+m})}{\phi+u(K^\star)} + \frac{\pi K^\star}{\phi+u(K^\star)}\frac{\partial q_{t+s}}{\partial K_{t+s-1}}\right].\end{aligned}$$

The system can be expressed more compactly as

$$\begin{pmatrix}\hat{q}_{t+s}\\ \hat{B}_{t+s}\\ \hat{K}_{t+s}\end{pmatrix} = J\begin{pmatrix}\hat{q}_{t+s-1}\\ \hat{B}_{t+s-1}\\ \hat{K}_{t+s-1}\end{pmatrix}$$

where an hatted variable indicates proportional deviation from the steady state and J is the Jacobian in elasticity form. An element of the Jacobian is indicated with J_{mn} $m, n = (q, B, K)$, so that J_{mn} is the derivative of m_{t+s} with respect to n_{t+s-1} times $n^\star/m^\star$, i.e. $J_{mn} = \frac{\partial m_{t+s}}{\partial n_{t+s-1}}\frac{n^\star}{m^\star}$. More specifically,

$$J_{qq} = R \qquad J_{qB} = 0 \qquad J_{qK} = -(R-1)\frac{u'(K^\star)K^\star}{u(K^\star)}$$

$$J_{Bq} = (q^\star+\phi)\frac{\partial K_{t+s}}{\partial q_{t+s-1}}\frac{q^\star}{B^\star} = (q^\star+\phi)J_{Kq}\frac{K^\star}{q^\star}\frac{q^\star}{B^\star} = (q^\star+\phi)J_{Kq}\frac{K^\star}{B^\star}$$

$$J_{BB} = R + (q^\star+\phi)J_{KB}\frac{K^\star}{B^\star} \qquad J_{BK} = \left[-\left(q^\star+\lambda\phi+a^L+\frac{\gamma\tau}{1+m}\right) + (q^\star+\phi)J_{KK}\right]\frac{K^\star}{B^\star}$$

$$J_{Kq} = \frac{R^2\pi}{\phi+u(K^\star)}\frac{u^\star}{R-1}\left[1+\frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\right]^{-1}$$

$$J_{KB} = -\frac{R\pi}{\phi+u(K^\star)}\left[1+\frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\right]^{-1}\frac{B^\star}{K^\star}$$

$$J_{KK} = \left[1 + \frac{K^\star(1-\lambda+\pi\lambda)}{\phi+u(K^\star)}u'(K^\star)\right]^{-1}$$
$$\left[(1-\pi)\lambda + \frac{\pi\left(q^\star+\phi\lambda+a^L+\frac{\gamma\tau}{1+m}\right)}{\phi+u(K^\star)} - R\pi\frac{u^\star K^\star}{\phi+u(K^\star)}\frac{u'(K^\star)}{u(K^\star)}\right].$$

By renaming the variables accordingly, we can refer the interested reader to Kiyotaki and Moore (1995, Appendix) for the analysis of the stability of the system around the steady state.

A.6 Parameter Values and Shooting Algorithm

Following Kiyotaki and Moore (1997), we impose $u(K) \equiv K - \nu$, where ν and $\bar{K}$ are chosen so that, in the steady state without tax, entrepreneurs use 66% of total assets and the elasticity of the residual supply of assets to entrepreneurs is 10%. The rest of the parameters reflect the ones used in Kiyotaki and Moore (1997) and can represent a quarterly model.

Parameters Values

R	1.01	λ	0.975	π	0.1	ν	4.92
$\bar{K}$	8.2	ϕ	20	a^H	1	a^L	0.9
γ	0.8	m	1	c	1	constant	0

The simulations are obtained using the shooting algorithm. By using the laws of motion of aggregate entrepreneurs' asset holdings in (15) and borrowing in (16), together with the asset market equilibrium condition in (12), we can find (K_t, B_t, q_{t+1}) as function of (K_{t-1}, B_{t-1}, q_t). From equations (11) and (12), we find $q_{t+1} = R(q_t - u(K_t))$. We now impose $u(K_t) \equiv K_t - \nu$: the previous becomes $q_{t+1} = R(q_t - K_t + \nu)$. The next step is to substitute this value in equation (15) and solve for K_t. We then have the following system of "transition equations" that we can iterate:[39]

$$q_{t+s} = R(q_{t+s-1} - K_{t+s-1} + \nu) \tag{A.6a}$$
$$B_{t+s} = q_{t+s}(K_{t+s} - K_{t+s-1}) + \phi(K_{t+s} - \lambda K_{t+s-1}) + RB_{t+s-1} + \\ - a^L K_{t+s-1} - \frac{\gamma_t\tau}{1+m}K_{t+s-1} \tag{A.6b}$$
$$K_{t+s} = \frac{1}{2}\left[\nu - \phi + (1-\pi)\lambda K_{t+s-1}\right] + \frac{1}{2}\Bigg\{\left[\phi - \nu - (1-\pi)\lambda K_{t+s-1}\right]^2 + \\ + 4\Bigg[(\phi-\nu)(1-\pi)\lambda K_{t+s-1} + \pi K_{t+s-1}\left[q_{t+s} + \phi\lambda + a^L\right] + \\ - \pi R B_{t+s-1} + \pi\frac{\gamma_t\tau}{1+m}K_{t+s-1}\Bigg]\Bigg\}^{0.5}. \tag{A.6c}$$

When 80% of the stock of the entrepreneurs' high carbon investment goods must remain unused, the amount of entrepreneurs' investment good, after depreciation, is reduced to $[0.2\gamma + (1-\gamma)]\lambda K_{t+s-1}$. When this shock hits, equation (A.6a) does not hold because

[39] Note that γ has a time subscript here. In the simulation, the social planner has banned investment in high carbon infrastructure, therefore depreciation implies that the share of land used with the high carbon stock will change over time and eventually go to zero.

the asset price jumps in response to the shock and entrepreneurs experience a loss on their asset holdings. In the original Kiyotaki and Moore's (1997) model, a shock of the magnitude we are interested in would throw the economy out of the basin of attraction of the interior steady state. To prevent this, we follow Cordoba and Ripoll (2004) and allow for renegotiation of the debt. Analytically, when debt can be renegotiated, debt repayments RB_{t+s-1} are pushed down to the market value $q_{t+s}K_{t+s-1}$ of the collateral. This makes the downturn less severe and allows the economy to converge back to the steady state.

Given the transversality condition in Assumption H, we know that $q_T = q^\star$ for large T. But since equations (11) and (12) define the asset price variation as a function of K_t, we can project the asset values back from steady state. So the rough ideas is to guess the initial variation in asset price given the shock and then iterate the economy forward through time to see if it converges again to the steady state. If the level of asset price eventually explodes, the initial guess is revised downward; if it is forever smaller then the initial guess is revised upward. This "guess and check" procedure is repeated until the asset price is close to the steady state (i.e. within the arbitrary level of tolerance).

When we allow the social planner to take over a fraction x of debt from the entrepreneurs, the following additional changes are required in the transition equations. Between the period in which the shock is announced and the following period, the value of the entrepreneurs' debt is further reduced to $(1-x)q_{t+s}K_{t+s-1}/R$. If the transfer of entrepreneurs' debt is funded with a constant tax τ^G over T periods, for T periods we add τ^G in the right hand side of (A.6b) and subtract $4\pi\tau^G$ in the right hand side of (A.6c) (inside the square root). Additionally, the budget constraint of the saver now includes debt repayments and new debt from the social planner, RB^G_{t+s-1} and B^G_{t+s}. While this does not directly influence the transition equations, it changes the consumption of the savers in each period, thus influencing the social welfare level reached by the economy. Finally, at $t = T+1$, there is no tax any more and the social planner holds no debt, so for $t \geq T+1$, the system of transition equations in (A.6) holds.

A.7 Comparing the Optimal Debt Transfer and the No Transfer Scenarios

In this section, we delve deeper on the consequences of allowing the social planner to take over some portion of aggregate entrepreneurs' debt. In particular, in Figure A.1 we compare the scenario where this policy tool is not available to the government with the case in which the proportion of debt transferred is 55% (which is approximately optimal under our parameters).

This shows that the entrepreneur's position is improved by the debt transfer relative to the no debt transfer case. In particular, let us focus on the graph for $B/B^\star$. The scatter represents the common starting point, $B^\star_0/B^\star$, but the line starts from the post-renegotiation and post-transfer values, $B_{0^+}/B^\star$, since, between $t=0$ and $t=1$, debt is firstly renegotiated and then, if allowed, transferred to the social planner. Thus, while it is true that entrepreneurs' debt is lower at $t=1$ under the optimal debt-transfer, the post-transfer value at $t=0^+$ is slightly higher.

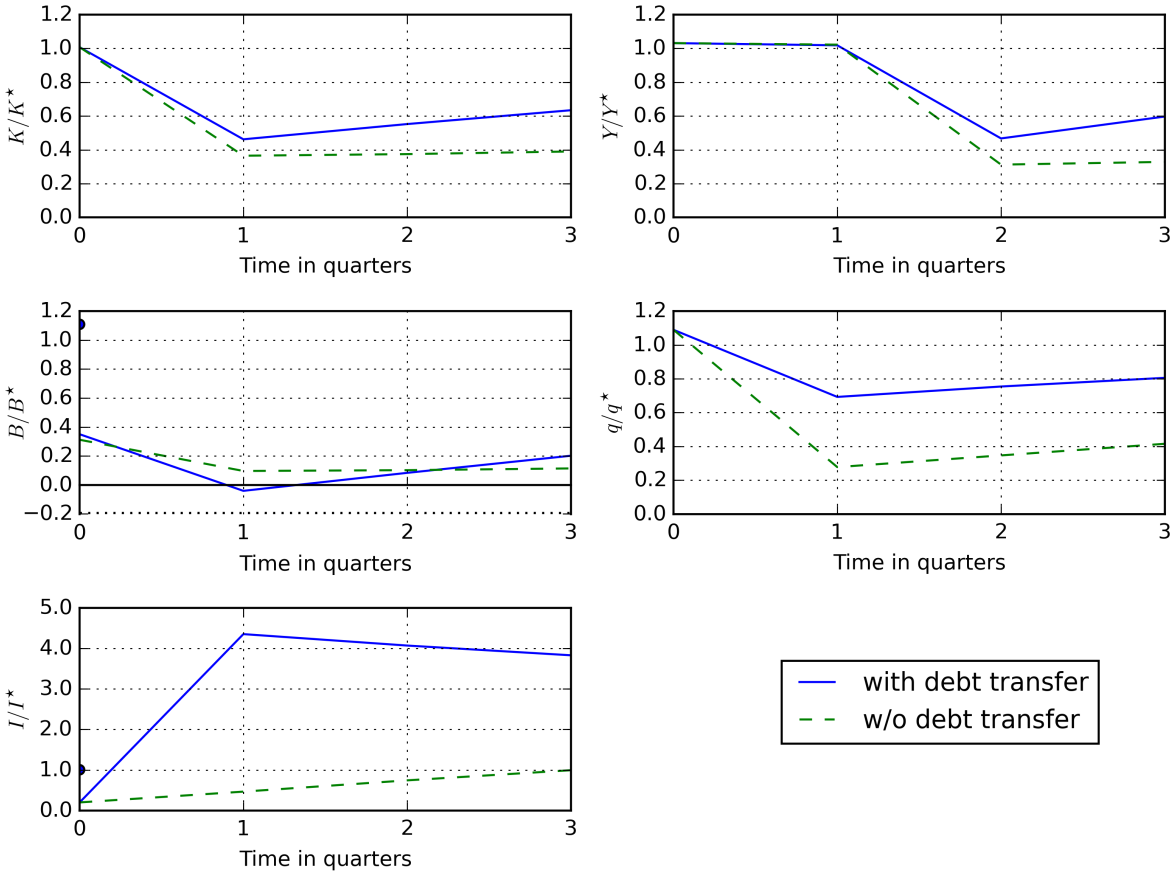

Figure A.1: The immediate effects of the transfer of investors' debt