

## **SIE-EACES session**

*“Labour Markets and Crises: Comparative Evidence and Policy Implications”*

### **Analysis of Regional Unemployment in Russia and Germany: Spatial-Econometric Approach**

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This paper analyzes regional unemployment in Russia and Germany in 2005-2010 and addresses issues of choosing the right specification of spatial-econometric models. The analysis is based on data of 75 Russian and 370 German regions showed that for Germany the choice of the spatial weighting matrix has a more significant influence on the parameter estimates than for Russia. Presumably this is due to stronger linkages between regional labor markets in Germany compared to Russia. The authors also proposed an algorithm for choosing between spatial matrices and demonstrated the application of this algorithm on simulated Russian data. The authors found that 1) the deviation of the results from the true ones increases when the spatial dependence between regions is higher and 2) the matrix of inverse distances is more preferable than the boundary one for the analysis of regional unemployment in Russia (because of the lower value of the mean squared error). The authors are also planning to apply the proposed algorithm for simulated data of Germany. These results allow accounting the spatial dependence more correctly when modeling regional unemployment which is very important for making right regional policy.

*Spatial econometric modeling, spatial weighting matrix, regional unemployment, Russian regions, German regions*

## **Introduction**

Modeling regional unemployment is essential for analyzing the effectiveness of economic policy in labor markets. Due to the fact that regional labor markets have nominal borders, we can propose that local shock in one labor market influences labor markets of other regions, i.e. there is a spatial interaction between regions. In order to analyze this kind of interactions one uses spatial econometric approach. This approach takes into account the mutual influence of geographical objects, which makes him different from linear regression models.

The importance of taking into account the spatial structure in estimated models is widely discussed in the literature. For example, if one ignores spatial lag in the estimated model, then the ordinary least squares estimates become biased and inconsistent [10].

Spatial aspects of regional unemployment were analyzed in many studies (e.g. [3;11;21]). One of the first authors, who studied unemployment taking into account the mutual allocation of regions in space, was Molho [24]. He analyzed male unemployment in 280 regions in UK and was one of the first who revealed the importance of spatial effects for labor markets. Niebuhr [26] analyzed spatial interactions of regional unemployment levels in Europe from 1986 till 2000, as a result detected the tendency of regions to cluster by the unemployment level.

Usage of spatial econometric models assumes presupposes facing a specification problem, which does not differ much from the specification problems in classic regression models. However, in case of spatial data these problems appear more intensively and become more difficult to solve [29]. First, spatial models require spatial weighting matrix, which reflects spatial dependence between observations [2]. Second, spatial effects are difficult to distinguish from common shocks and trends, spatial clusterization and unobserved spatial heterogeneity. Third, some aspects of spatial models specifications has not become popular among the applied researchers yet. Plumper and Neumaer in their paper [29] denote that in order to avoid bias in coefficient estimates one should properly model time dynamics, trends, common shocks and spatial weighting matrix.

Despite the rapid development of spatial econometric methods, issues of the choice of the spatial matrix are still open. The models which use wrong spatial structure are misspecified [14], that leads to the biases in the coefficient estimates and wrong inferences. Therefore, the issue of estimates' sensitivity to the choice of the weighting matrix remains one of the most discussed issues in spatial econometrics literature. Some researchers criticize econometric models for the sensitivity of the results to the choice of the weighting matrix [4],

others claim that sensitivity of the results to the choice of the weighting matrix is a myth [20]. In many cases the results of the estimation in some sense are sensitive to the choice of the weighting matrix, and that gives researchers some benefits in possibility of getting preferred results [29]. Bell and Bockstael [7] in their paper while comparing maximum likelihood estimation and generalized method of moments using microdata employ several types of weighting matrices, based on an existence of a common border and geographical distances. They find that the choice of the weighting causes more differences in estimates than the choice of the estimation method. Stakhovych and Bijmolt [30] study the consequences of choosing the wrong matrix with the help of Monte Carlo simulations, basing the procedure of the choice of the weighting matrix on statistical criteria. The most popular criteria, which is used for the choice of the weighting matrix is the value of the maximum likelihood function. The probability of the choice of the wrong matrix using this criteria increases if the spatial dependence is weak. However, the consequences of the wrong choice are minor because in case of weak spatial dependence the estimates are close to true coefficients. The bias in coefficient estimates and spatial coefficient in case of the wrong weighting matrix increases as the spatial interaction between regions strengthens.

There exist two different approaches to form the weighting matrix in the literature: it can be derived from the existing data or be exogenous [6]. In the first case the matrix is the result of the estimation on given data (e.g. [1;8;16]), however the second approach is more popular. Exogenous matrices types: contiguity matrix, matrix of inverse distances to some power; matrix based on the lengths of the common borders; matrix of k nearest neighbors and others [6], [14]. Among these matrices the most popular ones are the contiguity matrix, which reflects the presence or absence of the common border between regions/countries, matrix of inverse distances, based on geographical distance between objects [5].

Ideally the choice of the weighting matrix should base on theory, but usually economic theory doesn't provide the guidelines for the choice of the weighting matrix. Due to the fact that the nature of spatial dependence is not known a priori, and there exist no empirical test for the choice of the weighting matrix, one of the ways to solve the problem is the robustness check of the results to the weighting matrix type [13]. If one can show that the results with different weighting matrices stay close to each other, then one can rely on the estimation results.

In current paper we analyze regional unemployment in Russia and Germany using spatial econometric models in terms of importance of the choice of the right weighting matrix and the necessity of accounting for spatial structure in the regression model. The analysis of

the sensitivity of estimation results to the choice of the weighting matrix bases on real data and is performed using two different data sets with different aggregation levels.

Firstly, we check whether the parameter estimates differ when estimating spatial models of regional unemployment with different weighting matrices. When estimating regional unemployment for 75 Russian regions we do not observe big differences in coefficient estimates, whereas for 370 German regions we observe significant differences in estimates between models with different weighting matrices as well as in comparison to models without accounting for spatial structure. Possible explanation for that could be different level of data aggregation. The more detailed the regional structure, the stronger regional labor markets are connected to each other. So, when spatial dependence is stronger the role of the weighting matrix becomes more important.

In the second part of the paper we check the robustness of the estimates to the weighting matrix type with the help of simulated data. We find that when estimating spatial econometric models for regional unemployment in Russia, the matrix of inverse distances is more preferable, than the contiguity matrix. Choosing the matrix of inverse distances leads to lower biases on coefficient estimates. In addition, we conclude that the stronger the spatial relationship, the higher the bias in coefficient estimates when choosing the wrong weighting matrix as well as when not accounting for the spatial data structure.

The paper is organized as following. Next part is devoted to the analysis based on real data. We present main spatial econometric models, derive formulas for the coefficient estimates bias, provide the data description and present the analysis of the main results. The third part of the paper consists of analysis based on simulated data and conclusions about the obtained results. The last part concludes.

## ANALYSIS OF REAL DATA

### Core Spatial Econometrics Models

There are three main approaches to model spatial dependence in spatial econometric models. Spatial structure might be incorporated into the regression model by including spatial lag (WY), might be taken into account through spatial weighting of the exploratory variables (WX), or might be presented in error term ( $\varepsilon = \lambda W\varepsilon + \nu$ ). In current study we estimate several specifications of the spatial econometric models using panel data.

Spatial Autoregressive Model (SAR):

$$y_{it} = \rho \sum_{j=1}^N W_{ij} y_{jt} + X_{it} \beta + \mu_i + \sum_{k=2006}^{2010} \gamma_k d_k + \varepsilon_{it}, \quad (1)$$

where  $\rho$  is spatial correlation coefficient,  $W_{ij}$  is an  $ij$ -th element of the weighting matrix,  $X$  - is a matrix of explanatory variables,  $\beta$  is a vector of estimated coefficients,  $\mu_i$  are an individual effects,  $\gamma_k$  are time effects,  $\varepsilon_{it} \sim iid(0, \sigma^2)$ .

Spatial Durbin Model (SDM):

$$y_{it} = \rho \sum_{j=1}^N W_{ij} y_{jt} + X_{it} \beta + \sum_{j=1}^N W_{ij} X_{jt} \theta + \mu_i + \sum_{k=2006}^{2010} \gamma_k d_k + \varepsilon_{it}. \quad (2)$$

Spatial Durbin Model includes spatial lag of explanatory variables.

Spatial Error Model (SEM):

$$y_{it} = X_{it} \beta + \mu_i + \sum_{k=2006}^{2010} \gamma_k d_k + \varepsilon_{it}, \quad \varepsilon_{it} = \lambda \sum_{j=1}^N W_{ij} v_{jt} + v_{it}. \quad (3)$$

Spatial Autocorrelation Model (SAC):

$$y_{it} = \rho \sum_{j=1}^N W_{ij} y_{jt} + X_{it} \beta + \mu_i + \sum_{k=2006}^{2010} \gamma_k d_k + \varepsilon_{it}, \quad \varepsilon_{it} = \lambda \sum_{j=1}^N W_{ij} v_{jt} + v_{it}. \quad (4)$$

### Formula For The Estimation Bias In Spatial Econometric Models

With the following derivations we demonstrate that the coefficients estimates appear to be biased in case of misspecification. For simplicity we consider the case of cross-sectional data. Let the data generating model be the following:

$$y = \rho W y + X \beta + \varepsilon. \quad (5)$$

There exist several estimation methods of spatial econometric models. In current paper we use maximum likelihood estimation. Maximum likelihood function for the model with spatial lag looks as follows [19;27]:

$$\ln L = -\frac{n}{2} \ln(\pi \sigma^2) + \ln |I_n - \rho W| - \frac{\varepsilon^T \varepsilon}{2 \sigma^2}, \quad (6)$$

$$\varepsilon = y - \rho W y - X \beta. \quad (7)$$

$$\rho \in [\min(w)^{-1}, \max(w)^{-1}]. \quad (8)$$

where  $w$  is an  $N \times 1$  eigenvector, which includes eigenvalues of a matrix  $W$ . Using necessary conditions of extremum of function of many variables we obtain parameter estimates  $\beta$  and  $\sigma^2$  depending on  $\rho$ :  $\beta = (X^T X)^{-1} X^T (I_n - \rho W) y$ ,  $\sigma^2 = (y - \rho W y - X \beta)^T (y - \rho W y - X \beta) n^{-1}$ . After substituting these expressions in (6), we obtain one-dimensional optimization problem for the parameter  $\rho$ . Using the solution of this problem we can calculate values  $\hat{\beta}$  and  $\hat{\sigma}_2$ . Unfortunately, estimator for the parameter  $\rho$  cannot be expressed analytically, but can be solved only numerically (details in [2]). The estimate of  $\hat{\beta}$  is unbiased and consistent. The

consistency of the maximum likelihood estimates for the spatial lag model is examined by Lee [18].

However, in those cases when model estimation is based on a wrong weighting matrix, the estimate of the coefficient  $\beta$  is biased. It can be shown by simple calculations (see Appendix). Formula for the bias of estimate when using the wrong weighting matrix:

$$E(\hat{\beta}) = \beta + (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})[(I - \rho W)^{-1} - (I - \tilde{\rho}\tilde{W})^{-1}]X\beta.$$

Formula for the bias of estimate when spatial lag is ignored:

$$E(\hat{\beta}) = \beta + \rho(X'X)^{-1}X'W(I - \rho W)^{-1}X\beta.$$

Second terms in these formulas represent the bias of an estimate of a coefficient  $\beta$ .

Thus, when estimating spatial econometric models by maximum likelihood approach using wrong weighting matrix or ignoring spatial lag leads to the bias in the vector of coefficient estimates  $\beta$ , and the magnitude of a bias is proportional to the coefficient  $\rho$ .

The formulas for the bias are quite cumbersome. Moreover there are no analytical formulas for the spatial correlation coefficient  $\rho$ . So, it is quite difficult to determine the direction of a bias. Therefore, following other authors [30] we compare different parameter estimates using real data and evaluate the magnitude of a bias using simulated data.

### **Data and Variables**

We employ data on regional unemployment in Russia provided by the Federal Statistics Service. Data on regional labor markets in Germany is provided by the Federal Statistical Office of Germany (Statistisches Bundesamt) and by the Statistical office of Baden-Württemberg (Arbeitskreis Volks-Wirtschaftliche Gesamtrechnungen der Länder). In both cases we use panel data from 2005 till 2010 for 370 regions of Germany (NUTSIII aggregation level) and for 75 regions of Russia.

Level of unemployment is affected by a great variety of different regional factors. One reason of the existing differentials in unemployment levels between regions is the sectoral structure. For example, manufacturing sector is characterized by higher level of unemployment, than services (see [22]). The significance of the sectoral structure was detected in many studies (see e.g. [28], [15], [21]). More detailed review of papers which discuss this phenomena is written by Elhorst [12].

Next factor, which is important for determining the level of unemployment, is the age structure of a labor force. In those regions, where young population prevails level of unemployment is higher than in those regions with higher share of old population (e.g [28], [24]).

Along with the age structure education is an important factor. People with higher education have more chances to find a job in other regions, and therefore migrate more often than those with lower educational level. Therefore, when there is a high share of people with higher education unemployment level converges to its equilibrium faster, and level of unemployment goes down ([3]).

Negative relationship between gross regional product and unemployment (known as Okun's law) is established in many studies ([9;12;17;23]). Hence, we include this indicator also in the current study.

Thus, as exploratory variables for unemployment we employ indicators reflecting sectoral structure of a region, indicators of the age structure of the labor force, indicators of education and gross regional product per capita. The list of the exploratory variables is presented in table 1.

In current study we employ two types of weighting matrices: contiguity matrix and matrix of inverse distances. Diagonal elements of the matrix are zeros. Element  $w_{ij}$  of the contiguity matrix equals 1, if regions  $i$  and  $j$  have a common border and 0 otherwise. Element  $w_{ij}$  of the matrix of inverse distances reflects the distance as a crow between regional centroids (for Germany) and road distances between the regional centers (for Russia). Matrices are row normalized.

### **Results of the real data estimation**

Results of the models estimation for Germany and Russia are presented in tables 5 and 6. We can notice that the results of estimation (coefficient estimates and their significance) for Germany (Table 5) differ within the same model specification depending in the weighting matrix. Therefore, we can conclude that weighting matrix as well as accounting for spatial dependence matters when modeling regional unemployment in Germany.

Estimation results for Russia (Table 6) do not show meaningful differences in estimated coefficient estimates and their significance between models with different weighting matrices and within the same model specifications.

The difference between results for Russia and Germany is explained by the fact that 75 Russian regions occupy significantly bigger area, than 370 German regions. When aggregation level is high spatial interaction of labor markets, represented for example by the interregional labor mobility, is less important than in case of a more detailed regional division. Thus, because of the weaker intensity of interregional spatial interaction of unemployment in Russia, the type of the weighting matrix might be not so crucial as in case of modeling regional unemployment in Germany.

## ANALYSIS WITH SIMULATED DATA

Which spatial matrix is more preferable to use when estimating spatial econometric models? Below we provide an algorithm which allows to answer this question with the help of data simulation.

Amount of Russian regions is significantly smaller than the amount of German regions. Therefore we need considerably less time for computations on Russian data compared to German data, and decide to demonstrate the algorithm using only Russian data.

Knowing true estimates of a data generating model we are able to find out the direction and magnitude of an estimation bias. We analyze the bias of the spatial correlation coefficient  $\rho$  and coefficients  $\beta$  according to the strength of spatial dependence, i.e. of spatial correlation coefficient  $\rho$ . In order to do that we estimate models for each value of  $\rho \in [0.1; 0.9]$  with a step 0.1.

We simulate data basing on real values of three variables, whose coefficients appeared to be significant in the first part of the study (share of government services, share of nongovernment services, share of people younger than the working age in the whole population), on the coefficient estimates obtained while estimating spatial autoregressive model (SAR) in the previous section (Table 5). We also employ real weighting matrices (contiguity and matrix of inverse distances), that characterize spatial interaction of regional labor markets.

Errors are simulated according to the normal distribution with zero mean and unit variance. Individual fixed effects are generated according to Mundlak's paper [25], i.e. are modeled depending on exploratory variables. Time effects are generated by the normal distribution and have zero mean and variance equal to 2.

Values of a Y variable are generated according to the spatial autoregressive model and contiguity weighting matrix. Then we estimate obtained data (variable Y, given variables X) by maximum likelihood three times: using contiguity matrix, using inverse distance matrix, and without accounting for spatial dependence. Similar procedure is applied to the data generated according to the inverse distance matrix. Table 1 describes the simulation design.

*Table 1*

### Simulation Design

Spatial correlation coefficient	$\rho \in [0.1; 0.9]$ with step 0.1	9
Types of weighting matrices	Matrix of inverse distance, contiguity matrix	2
Total number of unique simulations	$9 \cdot 2$	18
Number of replications		10000
Total number of simulations		180 000

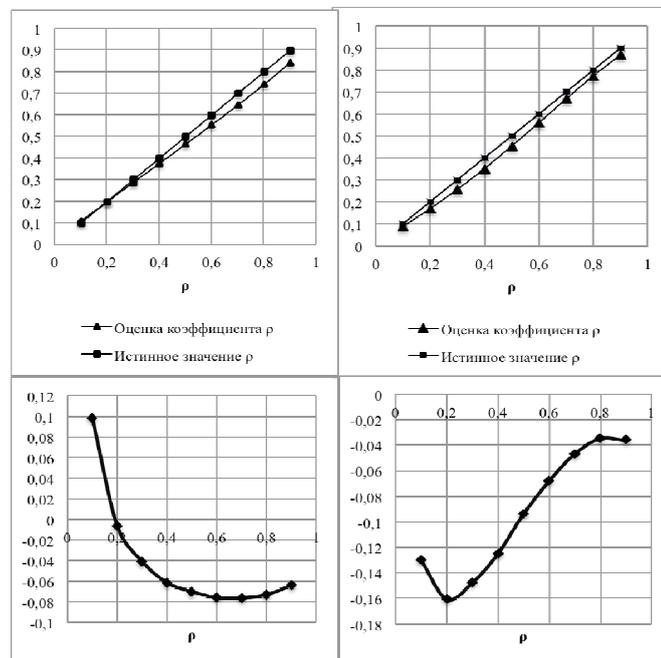
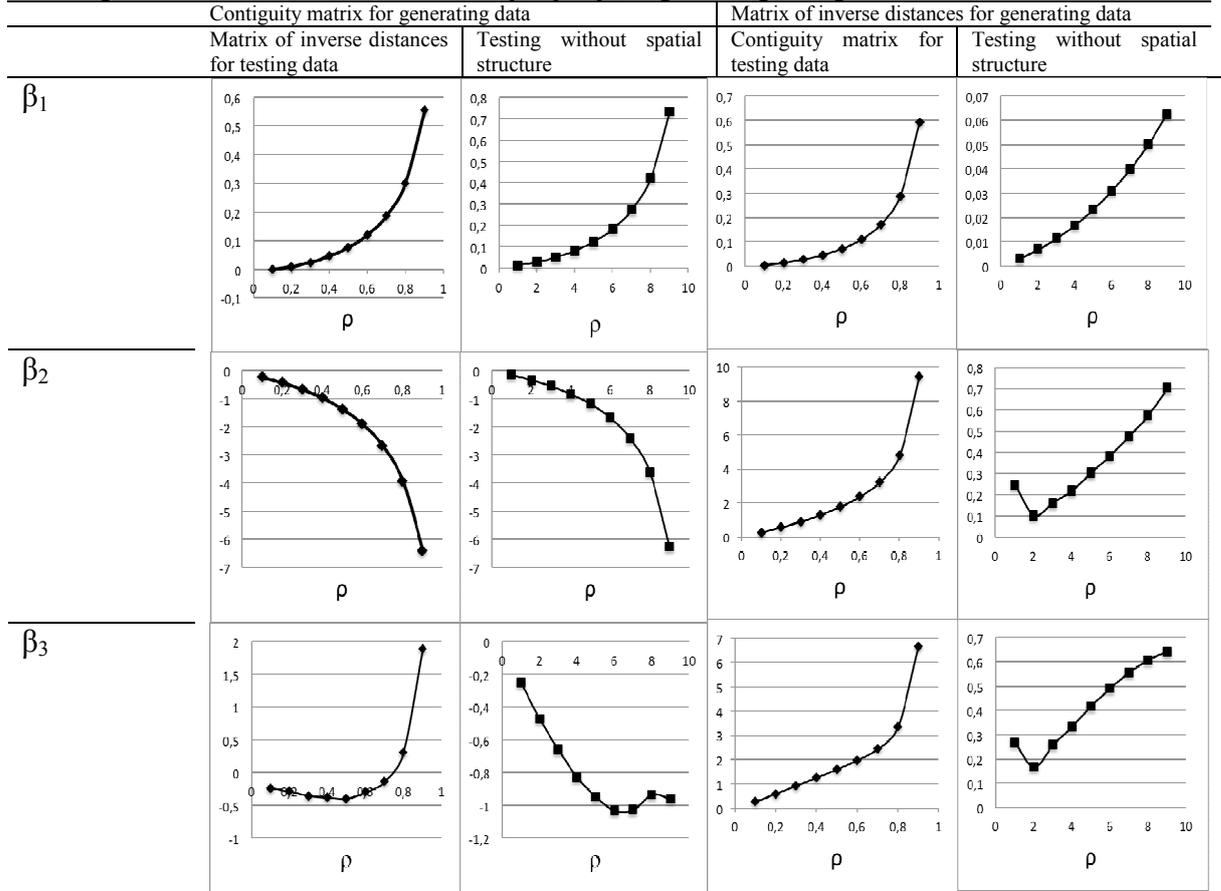
### **Results of the analysis on simulated data**

Table 2 shows average relative bias of coefficient estimates  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$  according to the spatial correlation coefficient  $\rho$ . For example, when Y is generated according to the spatial model with contiguity weighting matrix and the coefficient of spatial correlation equal to 0.8, but the model is estimated with assumption of a matrix of inverse distances, then the coefficient  $\beta_1$  is on average 30%-upward biased with respect to the true coefficient value. The absolute value of a bias in parameters  $\beta$  increases when coefficient  $\rho$  grows. This result is consistent with the result of the previous study [30] and reaffirms the importance of the weighting matrix choice increasing with rising strength of spatial interregional dependence.

Picture 1 represents the coefficient estimates of a spatial correlation coefficient  $\rho$  and its average relative bias dependent on spatial correlation coefficient in the generating data model. In both cases for every true value  $\rho$  the estimate is slightly downward biased with respect to the true value. This results is also consistent with the previous study [30].

Table 2

**Average relative bias of coefficients  $\beta_1, \beta_2, \beta_3$  depending on spatial correlation coefficient**



**Picture 1 Spatial parameter estimates ( $\rho$ ) and its average relative bias**

Notes. Graphs on the left represent the case when data generating model exploit contiguity matrix, but matrix of inverse distances is used for testing data. Graphs on the right represent the case data generating model exploit matrix of inverse distances, but contiguity matrix is used for testing data. Upper graphs show the estimates of the coefficient  $\rho$  as well as its true value. Lower graphs show average relative bias of estimates depending on the true values.

Table 3

<b>MSE of spatial parameter <math>\rho</math></b>			
		W for testing data	
		Contiguity W	W of inverse distances
W for generating data	Contiguity W	0,0077	0,0253
	W of inverse distances	0,0215	0,0220
	Total	0,0146	0,0237
<b>MSE of regression parameter <math>\beta_1</math></b>			
		W for testing data	
		Contiguity W	W of inverse distances
W for generating data	Contiguity W	0,0075	0,0077
	W of inverse distances	0,0082	0,0001
	Total	0,007863	0,0040
<b>MSE of regression parameter <math>\beta_2</math></b>			
		W for testing data	
		Contiguity W	W of inverse distances
W for generating data	Contiguity W	0,2770	0,9938
	W of inverse distances	1,8621	0,0298
	Total	1,0695	0,5118
<b>MSE of regression parameter <math>\beta_3</math></b>			
		W for testing data	
		Contiguity W	W of inverse distances
W for generating data	Contiguity W	35,23	28,9908
	W of inverse distances	151,2275	17,5000
	Total	93,2321	23,2453

Similar to the results of the paper [30], for four cases (contiguity matrix is used for generating and testing data; contiguity matrix – for generating data, matrix of inverse distances - for testing; matrix of inverse distances – for testing and generating data; matrix of inverse distances – for generating data, contiguity matrix – for testing data) we calculate mean squared error of coefficient estimates (Table 3). Mean squared errors of estimates of the coefficients  $\beta_1, \beta_2, \beta_3$  are lower in case when matrix of inverse distances is used.

Therefore, we conclude that when modeling regional unemployment in Russia with factors used in the current study, matrix of inverse distances, which reflects the interaction between all regions is more preferable than contiguity matrix. We plan to do similar calculations according to the above algorithm for German data, however it might take significant computational effort.

## CONCLUSION

Therefore, the current study shows that for the analysis of regional unemployment with the help of spatial econometric models, the choice of the weighting matrix is very important, especially when analyzing more detailed regional data. Besides possible bias of estimates appeared due to the usage of wrong weighting matrix or due to ignoring spatial structure increases with rising strength of spatial regional interaction. We find that when modeling regional unemployment in Russia with factors used in the current study, matrix of inverse

distances, which reflects the interaction between all regions is more preferable than contiguity matrix.

These results allow to model regional unemployment more effectively and obtain more precise coefficient estimates. It is important for understanding the possible influence of regional policy to labor markets, and for predicting unemployment level that we plan to do in further studies basing on the current paper results.

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## APPENDIX

### Derivation of the bias in case of choosing the wrong weighting matrix

Let  $Y = \rho WY + X\beta + \varepsilon_1$  be the data generating model, and  $Y = \tilde{\rho}\tilde{W}Y + X\tilde{\beta} + \varepsilon_2$  be an estimated model. Then transforming these expressions into

$Y = (I - \rho W)^{-1}(X\beta + \varepsilon_1)$  и  $Y = (I - \tilde{\rho}\tilde{W})^{-1}(X\tilde{\beta} + \varepsilon_2)$ , we obtain

$$\begin{aligned}\hat{\beta} &= (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})Y = (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})(I - \rho W)^{-1}(X\beta + \varepsilon_1) = \\ &= (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})[(I - \tilde{\rho}\tilde{W})^{-1} + (I - \rho W)^{-1} - (I - \tilde{\rho}\tilde{W})^{-1}](X\beta + \varepsilon_1) = \\ &= (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})(I - \tilde{\rho}\tilde{W})^{-1}(X\beta + \varepsilon_1) + \\ &+ (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})[(I - \rho W)^{-1} - (I - \tilde{\rho}\tilde{W})^{-1}](X\beta + \varepsilon_1) = \\ &= \beta + (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})(I - \tilde{\rho}\tilde{W})^{-1}\varepsilon_1 + (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})[(I - \rho W)^{-1} - (I - \tilde{\rho}\tilde{W})^{-1}]X\beta + \\ &+ (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})[(I - \rho W)^{-1} - (I - \tilde{\rho}\tilde{W})^{-1}]\varepsilon_1.\end{aligned}$$

Expression for the mean of an estimation coefficient:

$$E(\hat{\beta}) = \beta + (X'X)^{-1}X'(I - \tilde{\rho}\tilde{W})[(I - \rho W)^{-1} - (I - \tilde{\rho}\tilde{W})^{-1}]X\beta.$$

### Derivation of the bias in case of ignoring spatial structure of the data

Let  $Y = \rho WY + X\beta + \varepsilon_1$  be the data generating model, and be an estimated model. Then transforming these expressions into

$Y = (I - \rho W)^{-1}(X\beta + \varepsilon_1)$ , we obtain

$$\begin{aligned}\hat{\beta} &= (X'X)^{-1}X'Y = (X'X)^{-1}X'(I - \rho W)^{-1}(X\beta + \varepsilon) = \\ &= (X'X)^{-1}X'(I + \rho W + \rho^2 W^2 + \rho^3 W^3 + \dots)(X\beta + \varepsilon) = \\ &= (X'X)^{-1}X'(X\beta + \varepsilon) + (X'X)^{-1}X'\rho W(I - \rho W)^{-1}(X\beta + \varepsilon) = \\ &= \beta + (X'X)^{-1}X'\varepsilon + (X'X)^{-1}X'\rho W(I - \rho W)^{-1}X\beta + (X'X)^{-1}X'\rho W(I - \rho W)^{-1}\varepsilon.\end{aligned}$$

Expression for the mean of an estimation coefficient:

$$E(\hat{\beta}) = \beta + \rho(X'X)^{-1}X'W(I - \rho W)^{-1}X\beta$$

Table 4

<b>Variables description</b>	
<b>Germany</b>	<b>Russia</b>
Sectoral structure	
Employment share in agricultural sector	Share of the raw materials in gross value added
Employment share in production industry	Share of manufacturing in gross value added
Employment share in manufacturing industry	Share of government services (such as education, health care, administration of government) in gross value added
Employment share in construction	Share of nongovernment services (hotels, transport, telecommunication, financial services, real estate, other community and social services) in gross value added
Employment share in trade, hotels and restaurants, transport	
Employment share in financial business, renting, business services	
Employment share in public and private service provision	
Age structure	
Share of young people (15-25) in the population	Share of population younger than working age in whole population
Share of old people (55-65) in the population	Share of population older than working age in whole population
Education	
Share of employed people with only school education	
Share of employed people with university education	Share of employed people with higher education
GRP	
GRP per capita in prices of year 2005	GRP per capita in prices of year 2005 with accounting of purchasing power of the population

## Results of regional unemployment estimation for Germany

Variables	SAR	SAR	FE	SDM	SDM	SEM	SEM	SAC	SAC
	(direct effects) Contiguity W	(direct effects) W of inverse distances	Without W	(direct effects) Contiguity W	(direct effects) W of inverse distances	Contiguity W	W of inverse distances	(direct effects) Contiguity W	(direct effects) W of inverse distances
<b>Sectoral structure</b>									
Agricultural	-25.41* (13.70)	-12.58 (9.983)	-13.44 (13.04)	-25.31* (13.61)	1.247 (10.41)	-36.84*** (13.37)	-9.200 (11.28)	-18.34 (12.56)	-9.154 (9.673)
Production industry	30.87** (15.34)	18.34 (11.18)	21.93* (12.17)	32.89** (15.08)	14.73 (11.41)	17.63 (12.27)	15.24 (10.29)	32.46** (14.24)	13.52 (10.66)
Manufacturing	-38.27*** (11.16)	-14.49* (8.136)	-11.67 (8.319)	-36.13*** (10.39)	-18.04** (7.954)	-20.14** (8.565)	-13.89** (7.039)	-37.70*** (10.11)	-13.83* (7.739)
Construction	9.790 (13.39)	4.166 (9.719)	6.859 (11.19)	15.26 (13.07)	-2.626 (10.10)	2.474 (11.48)	3.404 (9.495)	12.19 (12.27)	2.761 (9.344)
Trade, hotels, transport	26.85** (11.99)	17.51** (8.698)	3.467 (10.23)	29.59** (11.63)	9.769 (8.758)	13.18 (10.47)	12.76 (8.614)	29.63*** (11.08)	10.96 (8.311)
Financial services	-23.85* (12.21)	-0.956 (8.830)	24.97** (10.15)	-11.45 (11.87)	-5.390 (8.802)	-23.28** (10.53)	-2.059 (8.670)	-12.25 (11.19)	-2.531 (8.502)
Service provision	31.83*** (11.65)	16.50* (8.424)	24.28** (10.13)	35.21*** (11.12)	11.46 (8.514)	13.82 (10.43)	12.84 (8.609)	32.18*** (10.71)	10.92 (8.083)
Share of young people	43.89*** (3.756)	53.98*** (3.272)	65.59*** (3.048)	50.02*** (3.321)	27.55*** (3.840)	54.75*** (3.258)	58.99*** (2.904)	27.40*** (3.107)	51.10*** (3.784)
Share of older people	-12.20*** (2.665)	7.663*** (1.950)	18.21*** (3.587)	3.350 (3.235)	7.382** (3.065)	-15.54*** (3.811)	15.14*** (3.264)	-2.307 (2.235)	12.17*** (3.249)
Share of employed with school education	36.20*** (5.172)	3.856 (3.750)	-1.193 (4.115)	31.69*** (4.649)	6.364* (3.737)	18.56*** (4.525)	2.349 (3.605)	31.50*** (4.528)	4.111 (3.856)
Share of employed with higher education	-14.38** (7.118)	0.789 (5.093)	22.84*** (6.537)	-0.955 (6.980)	4.172 (8.124)	-18.53*** (7.060)	8.698 (5.793)	-5.598 (6.345)	1.463 (5.610)
GRP	-63.03*** (12.83)	-22.40** (9.371)	-36.96*** (11.14)	-69.27*** (11.74)	-3.595 (12.55)	-28.53** (11.59)	-26.19*** (9.495)	-49.04*** (11.17)	-24.59** (9.878)
<b>WX</b>									
Agricultural				19.77 (24.94)	260.8* (136.6)				
Production industry				48.20** (23.46)	214.3 (149.2)				
Manufacturing				-39.46** (15.52)	-192.1** (92.25)				
Construction				39.33* (21.98)	-240.8* (131.4)				
Trade, hotels, transport				37.16* (20.49)	156.2 (123.5)				
Financial services				25.80 (20.49)	-5.262 (118.6)				
Service provision				48.22** (20.25)	152.9 (119.8)				
Share of young people				-58.00*** (4.910)	51.60** (21.73)				
Share of older people				-11.03** (4.488)	-47.95*** (13.98)				
Share of employed with school education				54.97*** (8.076)	81.24*** (30.54)				
Share of employed with higher education				-28.08*** (10.85)	275.4*** (57.30)				
GRP				-65.00*** (20.04)	557.7*** (64.85)				
Time effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Да
$\rho$	0.575*** (0.0150)	0.968*** (0.00966)		0.527*** (0.0197)	0.850*** (0.0499)			0.796*** (0.0233)	0.940*** (0.0233)
$\lambda$						0.779*** (0.0175)	0.986*** (0.00454)	-0.725*** (0.0340)	0.910*** (0.0356)
$\sigma_e^2$	0.488*** (0.0149)	0.261*** (0.00785)		0.426*** (0.0130)	0.241*** (0.00724)	0.473*** (0.0153)	0.271*** (0.00816)	0.420*** (0.0124)	0.296*** (0.00745)
F-stat (WX=0)				27.26	19.65				
P-value				0.0000	0.0000				
Total observations	2,220	2,590	2,220	2,220	2,220	2,220	2,220	2,220	2,220
Number of regions	370	370	370	370	370	370	370	370	370

## Results of regional unemployment estimation for Russia

Variables	SAR (direct effects) Contiguity W	SAR (direct effects) W of inverse distances	FE Without W	SDM (direct effects) Contiguity W	SDM (direct effects) W of inverse distances	SEM Contiguity W	SEM W of inverse distances	SAC (direct effects) Contiguity W	SAC (direct effects) W of inverse distances
<b>Sectoral structure</b>									
Raw materials	0.0254 (0.0229)	0.0210 (0.0221)	0.0118 (0.0289)	0.00499 (0.0228)	-0.00236 (0.0218)	0.0268 (0.0273)	0.0154 (0.0258)	0.0176 (0.0205)	0.0215 (0.0222)
Manufacturing	-0.00368 (0.0327)	0.0168 (0.0318)	0.0209 (0.0339)	-0.00587 (0.0321)	0.0315 (0.0315)	-0.00583 (0.0312)	0.00932 (0.0298)	-0.00470 (0.0306)	0.0214 (0.0324)
Government services	0.136*** (0.0508)	0.119** (0.0490)	0.122* (0.0628)	0.160*** (0.0613)	0.140** (0.0607)	0.121** (0.0519)	0.0880 (0.0545)	0.112*** (0.0401)	0.130*** (0.0481)
Nongovernment services	0.120*** (0.0392)	0.118*** (0.0379)	0.117*** (0.0434)	0.105*** (0.0389)	0.0837** (0.0382)	0.109*** (0.0406)	0.105*** (0.0386)	0.0945*** (0.0365)	0.120*** (0.0380)
GRP	-0.000275 (0.000477)	0.000455 (0.000475)	0.000698 (0.000640)	0.000712 (0.000523)	0.00106** (0.000521)	-0.000562 (0.000546)	0.000486 (0.000569)	0.000261 (0.000412)	0.000484 (0.000467)
Share of employed people with higher education	-0.0251 (0.0295)	-0.0380 (0.0286)	-0.0350 (0.0314)	-0.0297 (0.0305)	-0.0350 (0.0289)	-0.0223 (0.0284)	-0.0364 (0.0274)	-0.0420 (0.0280)	-0.0401 (0.0289)
People older than working age in population	0.185 (0.161)	0.0246 (0.156)	0.379 (0.268)	0.720** (0.341)	0.154 (0.307)	0.371** (0.168)	0.226 (0.216)	0.0338 (0.114)	-0.00700 (0.146)
People younger than working age in population	1.356*** (0.277)	1.002*** (0.270)	0.932*** (0.348)	0.519 (0.394)	0.613* (0.336)	1.499*** (0.314)	0.898*** (0.331)	0.920*** (0.223)	1.011*** (0.260)
<b>WX</b>									
Raw materials				0.0519 (0.0547)	-0.195 (0.208)				
Manufacturing				-0.0558 (0.0567)	0.333** (0.158)				
Government services				0.0772 (0.0821)	0.709*** (0.197)				
Nongovernment services				0.0720 (0.0728)	0.0669 (0.197)				
GRP				-0.00104 (0.000930)	0.000533 (0.00193)				
Share of employed people with higher education				-0.0431 (0.0461)	-0.0651 (0.166)				
People older than working age in population				-0.722** (0.363)	-0.584 (0.512)				
People younger than working age in population				1.293** (0.553)	4.593*** (1.274)				
Time effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$\rho$	0.294*** (0.0521)	0.680*** (0.0709)		0.213*** (0.0574)	0.331** (0.142)			0.704*** (0.0465)	0.730*** (0.0751)
$\lambda$						0.279*** (0.0591)	0.754*** (0.0651)	-0.702*** (0.0876)	-0.241 (0.255)
$\sigma_e^2$	1.518*** (0.102)	1.383*** (0.0927)		1.456*** (0.0974)	1.346*** (0.0899)	1.551*** (0.104)	1.393*** (0.0936)	1.356*** (0.0930)	1.644*** (0.0928)
F-stat. (WX=0)				2.93	2.53				
P-value				0.0035	0.0110				
Total observations	450	450	450	450	450	450	450	450	450
Number of regions	75	75	75	75	75	75	75	75	75

Note. Standart errors in parenthesis \*, \*\*, \*\*\* —5, 1, 0.1%—significance levels.