An Agent Based Macroeconomic Model with Social Classes and Endogenous Crises

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Preliminary Draft, July 2nd 2015

Abstract This paper proposes an agent based macroeconomic model in which income distribution and wealth accumulation depend on the role that agents play in productive activities, that is capitalists or workers. In this framework, social class dynamics underlie the endogenous process of firm formation. The focus is on the interplay between the evolution of social structure and macroeconomic dynamics and on then how business cycles and crises may endogenously emerge as the result of the interaction between financial and real factors underlying the process of capitalist production.

Keywords: heterogeneous interacting agents; social structure, macroeconomic dynamics, inequality, crisis.

JEL Classification Numbers: P10, D31, C63.

*The author would like to thank the participants at the 20th Workshop on Economics with Heterogeneous Interacting Agents (WEHIA), Sophia Antipolis (France), May 21-23, 2015, particularly Michele Catalano, Pascal Seppecher, Tania Treibich and Marco Valente. *Corresponding address*: Università Politecnica delle Marche, Faculty of Economics "'Giogio Fuà"', Department of Management, Piazzale Martelli 8, 60121 Ancona, Italy; e-mail: *alberto.russo@univpm.it*.

1 Introduction

This paper investigates the interplay between social structure and macroeconomic dynamics. In particular, the joint evolution of social classes, production and distribution is analyzed within a macroeconomic model with heterogeneous agents. The aim is to show how endogenous business cycles and crises may emerge from decentralized interactions among agents that self-organize in social classes along the process of capitalist production. As a result, highly unequal societies may develop even starting from perfect equality.

Inequality is an unavoidable feature of a capitalist economy, although its degree may vary depending on political choices. The distribution of income and wealth is related to how the society is organized and the associated level of inequality shapes macroeconomic evolution. In particular, increasing inequality may result in a lack of aggregate demand that, in turn, may reduce the profit rate of productive firms, with a consequent negative impact on bank profits due to the contagion of financial distress (e.g. bankruptcy chains); moreover, the decline of the profit rate may impact the "social relations of production", with further macroeconomic effects.

As for the methodology, the paper proposes a agent based macroeconomic model with an initial population of homogeneous agents that differentiate along time, thus becoming heterogeneous particularly due to the role they play in productive activities, e.g. capitalists or workers. As we will see below, for an agent, belonging to one or another social class depends on a wealth-related stochastic process which is also shaped by the evolution of the profit rate. Therefore, the model is characterized by an endogenous process of class division that, in turn, shapes the co-evolution of production and distribution. Moreover, social class dynamics are also at the root of the process of firm formation, though in a simplified setting in which one capitalists can hire one or more workers, giving rise to a firm. In general, then, this paper proposes a combination of the *old* theoretical tradition of Classical economists and the *new* methodology of agent based modelling.

The paper is organized as follows. In Section 2 the literature related with the topics investigated in the paper is presented and discussed around three main points: (i) macroeconomics with heterogeneous interacting agents, (ii) endogenous social classes and firm formation, (iii) stochastic processes and income/wealth distribution. Section 3 presents the model by explaining the agents' behavioural rules and the mechanisms governing their interaction, the role of the banking sector and that of policy makers. Model dynamics are described in Section 4: first of all, the initialization of the system is studied; then, the typical behaviour of the model is analyzed based on a single run of the model. Section 5 concludes.

2 Related literature

In this section we briefly review the literature on three main topics, on which the subsequent model is based, that the paper tries to merge in order to analyze the co-evolution of production, distribution and social structure in a macroeconomic setting with heterogeneous interacting agents.

2.1 Agent based modelling in macroeconomics

In recent years, there has been a blossoming of agent based macroeconomic models, especially in Europe. In what follows a few examples are provided regarding the most recent papers.

A large-scale model carried out in the *EURACE* European project is described in Dawid et al. (2013): this is an agent based stock flow consistent framework for macroeconomic analysis.[1] Within the same project other versions have been developed, as for instance Cincotti et al. (2010); this model has been used to analyze the impact of debt and deleveraging on macroeconomic performance (Raberto et al., 2012), the effectiveness of the banks' capital adequacy regulation (Teglio et al., 2012), and the role of macroprudential policies (Cincotti et al., 2012); moreover, the same agent based macroeconomic model has been employed to analyze the mortgages market and housing bubbles dynamics (Erlingsson et al., 2014). An empirically grounded agent based model of housing boom and bust and systemic risk has been proposed by Geanakoplos et al. (2012). An extension of the EURACE framework including housing market dynamics is proposed in van der Hoog and Dawid (2015).

Dosi et al. (2013) studies the relationship between income distribution and monetary/fiscal policies based on a previous agent based macroeconomic model that, combining Keynesian and Schumpeterian features, finds strong complementarities between factors influencing aggregate demand and drivers of technological change that affect both short-run fluctuations and long-term growth patterns (Dosi et al., 2010). This framework has been further extended in Dosi et al. (2015) to identify the most appropriate combination of fiscal and monetary policies in economies subject to banking crises and deep recessions.

Built on previous works as Russo et al. (2007) and Riccetti et al. (2013a), Riccetti et al. (2014b) proposes a macroeconomic model with heterogeneous agents, that directly interact in different markets according to a common decentralized matching mechanism, showing that business cycles as well as extended crises endogenously emerge due to the interplay between real and financial factors; the model has been employed to analyze the

[1] Basically, stock flow consistency works as a macroeconomic discipline for the model assuring that, for example, an asset for an agent is a liability for (at least) another agent and that flow dynamics are consistent with the accumulation of stocks; see for instance (Godley and Lavoie, 2007).

role of unemployment benefits (Riccetti et al., 2013c), the effects of the financialization of non-financial corporations (Riccetti et al., 2014a), the effectiveness of financial regulation (Riccetti et al., 2013b), the consequences of growing inequality (Russo et al., 2014), and the relationship between household debt, financial fragility and crisis episodes triggered by income and wealth distribution dynamics (Russo et al., 2015).

Combining some features of above papers, particularly the centrality of innovation dynamics in Dosi et al. (2013) and the importance of financial fragility and default contagion in Riccetti et al. (2014b), Caiani et al. (2015) develop an agent based macroeconomic model, in which both micro and macro variables are stock flow consistent, to analyze the behaviour of business cycle fluctuations and replicate a variety of micro-meso-macro stylized facts.

The above models present some recurrent characteristics, firstly the presence of heterogeneous, boundendly rational, agents in one or more sectors of the economy: there are households (sometimes just a representative one), one or two types of productive firms (final goods producers and capital goods producers – even only one of the latter – or downstream and upstream firms), a banking sector (that can be represented by a single agent in some cases or being composed of heterogeneous banks), and the policy makers (the government and the central bank). Heterogeneity may regard the size of firms as well as of banks and their balance sheet (implying different degrees of financial fragility), income and wealth distribution, goods' and machines' quality (for instance, various vintages), and so forth. In fact, there is a broad literature on mainstream models with heterogeneous agents – just to make an example, see Krusell and Smith (1998) – though in some *dsge* model the degree of heterogeneity is limited to two kinds of agents, e.g. patient and impatient households. The other central, and more peculiar, ingredient of agent based models is interaction: in particular, in these models agents *directly* interact (that is, not only indirectly, via the price system) with each other or with a subsample of potential partners (according to some matching mechanism). A very interesting case is that regarding "financial contagion", that is when the failure of an agent influences the financial condition of one or more connected agents, possibly leading to other defaults and then triggering a bankruptcy chain. For instance, Delli Gatti et al. (2010) show that the endogenous network of credit interlinkages between banks, downstream and upstream firms may further amplifies business cycle fluctuations due to bankruptcy avalanches, giving rise to a "network-based financial accelerator". Based on that, heterogeneous interacting agents (HIAs) models can be considered as a promising alternative to the neoclassical (Walrasian) microfoundation of macroeconomics (Gaffeo et al., 2007).

A particular aspect of agent based macroeconomic models regards the number and the role agents play in the economy. Usually, a fixed number of firms, workers and banks are assumed. This means that in many cases the structure of markets is given and the number of competitors in a market is fixed. Even if the number of agents varies, based on some

proxy of sector profitability, the *social structure* is exogenous. The present paper wants to overcome this limitation by assuming that each agent can potentially play different roles in productive activities; therefore, in each period of time, depending on the conditions we will analyze below, an agent can act as a capitalist or as a worker, thus resulting in an endogenous social class structure that affects economic performance and it is affected in turn. What the paper proposes, then, is a macroeconomic model with heterogeneous agents that self-organize in social classes.

2.2 Endogenous firm formation and social classes

According to Axtell (1999), firms emerge as cooperative teams under increasing returns in a population of agents that have preferences for both income and leisure time: within each group the output is divided into equal shares; each agent periodically adjusts its effort level to maximize utility non-cooperatively. Agents are allowed to join other firms or start up new firms when it is welfare-improving to do so. As a firm becomes large, agents have little incentive to supply effort, since each agent's share is relatively insensitive to its effort level, thus giving rise to free-riders; as free-riding becomes commonplace in a large firm, agents migrate to other firms and the large firm declines. Moreover, firm size distribution tends to a power law under a continuous process of firm entry and exit. This is an empirical fact that validates the main finding of the model.

One of the central assumption in the above paper is that firm's output is equally divided among team participants. This is not the usual distribution of the surplus created in a capitalist firm. Obviously, this is not the case because different agents have different roles and their remuneration is set accordingly. Moreover, when agents play different roles, the workers' supply of effort can be monitored by the entrepreneur (in a small firm) or by managers (for larger firms). However, large firms – that present dynamic advantages due to economies of scale – can exhibit a static inefficiency due to the increasing number of layers of managers needed to control people working in the firm (Ciarli et al., 2010). Why then is it convenient to manage a (large) firm instead of performing a set of private market buying-selling transactions that reproduce the same activity?

That is a matter of *power* (Bowles and Gintis, 2008) and, in particular, the "right to manage" productive activities. In a sense, a firm can be considered as a "command economy"(Coase, 1937) in which the manager has the right to decide (and control) what the workers will do. In a competitive economy, then, many transactions are not market exchanges via prices, when instead operations within firms. According to the Simon (1951)'s model, indeed, under certain conditions an employment contract is preferred to a sales contract, giving the manager the authority over workers to organize the production and maximize firm's profit.

Given the power relationship between the employer and the employee, next question

is which role an individual plays within the firm and on which factors this depends. According to Classical economists, the society is organized in classes that are characterized by a "conflict of interests" in the capitalist process of production and distribution. How do social classes emerge? Axtell et al. (2007) propose a multi-agent bargaining model in which *social norms* endogenously emerge based on decentralized interactions of many individuals. In particular, the presence of a distinguishing "tag" (e.g., light and dark, that is a tag which can be completely meaningless, so that different individuals are identical in competence) can affect agents' play when recognize a type or another as an opponent in the bargaining process; as a result, different bargaining behaviours can emerge among agents with respect to diverse partners and *classes* endogenously emerge.

However, being the manager or playing the capitalist role in productive activities is not a random outcome, nor it can be just the consequence of recognizing a "tag" (though this can be sufficient to generate different classes), when instead the consequence of material conditions of the individual as well as of the society. In particular, the individual choice of becoming a capitalist can be constrained by the amount of wealth an agent can invest in the productive activity: the larger the individual wealth the higher the probability to become a capitalist. The availability of external finance and credit rationing (Stiglitz and Weiss, 1981) may amplify this selection mechanism. Therefore, the crucial constraint to become an entrepreneur is the lack of capital, as suggested by Blanchflower and Oswald (1998). In other words, raising the necessary capital is the main problem in order to become an entrepreneur: if not enough capital is owned by an agent, and/or external finance is not available, the agent will play the worker role. According to Marx, the "social relations of production" in a capitalist economy are based on the employee-employer relationship and, according to Wright (2005, 2009), these relations can be considered as part of the microfoundations of macroeconomics: a small class of capitalist has the economic means to hire a large class of workers to be employed in productive activities performed in firms. The present paper proposes a simple mechanism of firm formation based on social class dynamics along these lines.

2.3 Stochastic processes and income/wealth distribution

"Since Pareto, it has long been known that the tail of distribution of income or wealth w universally obeys a power-law distribution $w^{-\alpha}$ for a constant α around $1.5 - 2.5$. A multiplicative process of wealth accumulation has been a standard explanation for the heavy tail" (Nirei and Souma, 2007, p. 440). For example, the Gibrat's "law of proportionate effect" is a multiplicative process which results in a lognormal distribution, if income's growth rates are stochastic and independent of the initial size (Sutton, 1997). By slightly modifying such a mechanism, for instance by introducing a "reflective barrier" to the multiplicative process (or something similar leading to a "Kesten process"), a power

law in the right tail may emerge.

Levy (2003) introduces a reflective lower bound to a multiplicative process, within a stochastic model of wealth accumulation by financial investments, in order to study under which conditions the system converges to the empirically observed Pareto wealth distribution (Pareto, 1897). Based on multiple simulations of the model, "it seems that any stochastic multiplicative wealth accumulation model which assumes even a mild degree of differential investment talent leads to a distribution of wealth which is inconsistent with the empirical Pareto distribution" (Levy, 2003, p. 56). The main result is that Levy (2003)'s model replicates the empirically observed Pareto distribution for the high-wealth range, under the condition that the abilities of agents are *homogeneous*. Indeed, when abilities are *heterogeneous*, the model no longer converges to a power law tail, suggesting that in this framework the high inequality of wealth distribution is mainly due to *chance* rather than *differential abilities*. "The result of this paper does not mean that *only* luck matters, and that any investment strategy is as good as any other. On the contrary, it means that one must apply his investment skills just in order to have a fair chance in the competition with other investors. Our findings suggest that because investors in the high-wealth range seem to have similar investment talents, at the margin it is only luck that differentiates between them" (Levy, 2003, p. 58). However, this model studies only top income dynamics for which the power law characterizes the "fat" right tail of the distribution. In other words, the author confined the analysis to wealth levels larger than $\hat{\psi}$, maintaining that the wealth of individuals in the high-wealth range typically changes due to capital investments, leaving out of the analysis labour income (which is typically an additive rather than a multiplicative process), that is the main factor affecting wealth accumulation at the middle-to-lower range.

An extension of the Levy (2003)'s model has been proposed by Nirei and Souma (2007) by including also the labour income in the stochastic process of wealth accumulation. They propose a two-factor model in which asset returns are generated by a *multiplicative* process, while wages depend on *additive* process. Thus, they extend Levy (2003)'s results by adding to the power law tail for the high-wealth range the exponential decay for the low-wealth range, in line with Dragulescu and Yakovenko (2001)'s findings. Empirically, "the data analysis of income distribution in the USA reveals the coexistence of two social classes. The lower class (about 97% of population) is characterized by the exponential Boltzmann-Gibbs distribution, and the upper class (the top 3% of the population) has the power-law Pareto distribution" (Banerjee and Yakovenko, 2010, p. 9).[2]

By assumption that an agent is either an employee or an employer, Russo (2014) proposes a model in which agents either gain a capital income (i.e., a profit as the

[2]An alternative approach has been proposed by Clementi et al. (2010) for which, instead of combining two different distributions, a (three-parameter) κ-generalized distribution is used to describe the whole range of income and wealth.

result of a multiplicative playing the capitalist role) or a labour income (i.e., a wage as the result of an additive process playing the worker role). As in Nirei and Souma (2007), this paper analyzes both the high-wealth range and the middle-low one; by considering *class division*, this model also studies the continuous inflows and outflows of agents from and to different classes. The aim is to discover the conditions, i.e. a large space of parameter combinations, under which richer individuals are more "powerful" than poorer ones in accumulating wealth, even starting equal initial endowments and homogeneous abilities. Moreover, this paper investigates the relation between class division and income/wealth inequality by applying a maximum likelihood estimation procedure to the wealth distribution for detecting the existence of a power law tail.

The present paper aims at extending the modelling framework proposed in Russo (2014) by plugging an endogenous social structure, that is a wealth-based class division process (subsection 2.3), into an agent based macroeconomic framework (subsection 2.1), where also the endogenous process of firm formation depends on social class dynamics (subsection 2.2).

3 The model

3.1 Model setup

In this section the structure of the model is presented. The economy is composed of N agents, a banking sector,[3] the government and the central bank. When referring to all the N agents the index i is used as a subscript, while the indexes k and j indicate an agent in the role of capitalist or worker, respectively.

The story begins with the government hiring some public workers by injecting an initial amount of public expenditure in the system (the central bank prints the money corresponding to government spending). At the beginning, all agents (including the banking system) have no income and no wealth; therefore, there are also no liabilities in the system (but for public debt financed by the money printed by the central bank). In each subsequent period, every agent has a probability to become a *capitalist* that depends on its relative wealth and the (past) average profit rate. This process gives rise to a social structure that shapes economic evolution (and vice versa).

In a monetary production system, as the simple economy we are describing, capitalists aim at making a *monetary profit* by hiring workers, producing and selling consumption goods to household and the public sector. In other words, we assume that production

[3]This is a simplifying assumption we would like to remove in the future, possibly considering heterogeneous banks.

regards only consumption goods and that only labour is used as an input.[4] The banking sector finances production (and possibly consumption) through loans and receive deposits from households. The central bank sets the policy rate (reacting to inflation and unemployment) and interacts with the banking sector through providing cash advances or receiving bank reserves. In order to keep under control the behaviour of public finances, tax rates evolve according to some fiscal rules. In what follows, the details of the model will be explained.

3.2 Behavioural rules and interaction mechanisms

3.2.1 Consumption

The desired consumption of agent i at time t is based on the following equation:

$$c_{i,t} = max\{c_{i,t-1},\ c_y y_{i,t-1} + c_\omega \omega_{i,t-1}\} \tag{1}$$

where y represents past (net) income and ω is accumulated (net) wealth, while c_y and c_ω are the propensity to consume out income and out of wealth, respectively; hence, agents calculate a consumption level based on both income and wealth, they compare it with past consumption, then choosing the largest one. This is a very simple mechanism to introduce *habit formation* in agents' consumption behaviour. According to this assumption, there are not differences in the consumption behaviour for agents belonging to different classes; indeed, differently from post-Keynesian models in which typically *capitalists* have a higher propensity to save than *workers*, we assume that the propensities to consume are constant and fixed across agents.[5] Nevertheless, consumption inequality can characterize model dynamics given income and wealth inequalities which endogenously emerge from agents' interaction and the social evolution of class structure.

3.2.2 Social classes

Agent i's probability to play as a capitalist at time t depends on (after consumption) relative wealth and an adjustment factor related to the (past) profit rate, according to the following rule:

- the first step is to calculate the variable bk for each agent at time t:

$$bk_{i,t} = 1 - \sqrt{(\hat{\omega}_t^{max} - \hat{\omega}_{i,t})/(\hat{\omega}_t^{max} - \hat{\omega}_t^{min})} + \kappa \pi_{t-1}^r. \tag{2}$$

[4]Obviously, the next step for modelling a more realistic capitalist system is to introduce physical *capital* in the production process. For now only working capital enters production.

[5]This does not mean that the post-Keynesian assumption about different saving propensities is not (empirically) founded for aggregative models. In this paper the aim is instead to stress the role of income and wealth heterogeneity, even assuming homogeneous propensities for the two social classes. To make the post-Keynesian assumption should amplifies the results we will discuss below.

where $\hat{\omega}_t^{max}$ and $\hat{\omega}_t^{min}$ are, respectively, the maximum and the minimum value of the wealth distribution at time t, $\hat{\omega}_{i,t}$ is the agent i's wealth, π_{t-1}^r is the past rate of profit, and $\kappa > 0$ is a parameter;

- then, a new variable $\bar{bk}_{i,t}$ is computed by imposing 0 and 1 as the lower and upper bound of $bk_{i,t}$, respectively.
- the individual variable $rn_{i,t}$ is set by picking a number at random from a uniform distribution $U(0,1)$.
- finally, if $\bar{bk}_{i,t} > rn_{i,t}$, then agent i is a *capitalist* at time t.

For the sake of simplicity, it is assumed that such a stochastic rule is performed in each period of time; in principle, then, an agent could continuously change her role as time elapses.[6] It is worth to note that, even in such a simplified framework, if playing as capitalist tends to result in higher incomes (than those gained by acting as a worker), thus wealth accumulation may lead to lower social mobility and the emergence of a restricted and relatively stable capitalist class, because of a wealth-based reinforcement mechanism.

3.2.3 Production plans and demand expectations

Capitalist k invests a fraction of her (after consumption) wealth, $\hat{\omega}_{k,t}$, in the productive activity with the aim of obtaining a final wealth, $\omega_{k,t+1}$, larger than the initial one, $\omega_{k,t}$, by producing and selling commodities, $q_{k,t}$. According to the Marxian scheme $\omega_{k,t} - q_{k,t} - \omega_{k,t+1}$, $\omega_{k,t+1}$ should turn out to be larger than $\omega_{k,t}$ when the goods market closes. Therefore, agent k's capital invested at time t in her firm is equal to

$$a_{k,t} = \alpha \, \hat{\omega}_{k,t} \tag{3}$$

where $0 < \alpha < 1$ is a parameter.[7] The amount of wealth $(1 - \alpha) \, \hat{\omega}_{k,t}$ is retained by the capitalist as a *buffer* that can be used as a collateral against the credit provided by the banking sector.

Based on demand expectations and labour demand, either self-financing production or a demand for bank loans will emerge. In particular, demand expectations depend on the aggregate quantity of goods sold in the previous period, Q_{t-1}, and each capitalist expects to face a market share that depends on her invested wealth, $\hat{\omega}_{k,t}$, relative to total wealth invested; the latter is proxied by the total wealth invested in the previous period, $\hat{\Omega}_{t-1}$,

[6] By contrast, one could assume that an agent plays a role until a certain condition holds: for instance, until a firm operates with a positive net worth, the agent financing that firm continues to play the capitalist role; then, the exit condition could be the firm default.

[7] A possible extension is to consider heterogeneous attitudes to invest own capital in productive activities, that can be related to different levels of risk aversion.

augmented by the inflation rate, $\dot{p}_{t-1}$; accordingly, the quantity of goods the capitalist k expects to sell in period t is:

$$q^d_{k,t} = Q_{t-1} \cdot \{\hat{\omega}_{k,t}/[\hat{\Omega}_{t-1}\ (1 + \dot{p}_{t-1})]\} \tag{4}$$

This means that agents have *naive* expectations as for aggregate demand; moreover, they make a guess about the market share by assuming that this is proportional to the ratio between own capital and the total capital invested in productive activities (where total capital is proxied by its past values updated by considering a growth rates of the price level equal to the past inflation rate). Given that agents playing as capitalists in a certain period do not know in advance the number of competitors, the behaviour described in Equation 4 does not prevent them to make systematic errors in forecasting market shares.

In order to produce the quantity of goods capitalists expect to sell, they have to assume workers to be employed in the production process. Assuming that production is a linear function of the number of workers to be hired by each firm, firm k's labour demand is given by[8]

$$l^d_{k,t} = \lceil q^d_{k,t}/\phi \rfloor \tag{5}$$

where $\phi > 0$ is a parameter.[9]

Given the labour demand and the price of productive inputs, i.e. the wage rate w, capitalist k computes the total financing of production as follows:

$$ab_{k,t} = l^d_{k,t} \cdot w_{t-1}(1 + \dot{w}_{t-1}) \tag{6}$$

where it is assumed that expected value of the wage rate at time t is given by the past wage rate, w_{t-1}, updated by considering the past wage inflation rate, $\dot{w}_{t-1}$.

3.2.4 Self-financing and bank credit

Given total financing and the amount invested by each capitalist, production can either be (fully) self-financed or (partially) depend on bank credit. Therefore, if the capital invested by the *k-th* capitalist, that is $a_{k,t}$, is smaller than total financing, $ab_{k,t}$, then capitalist k's demand for credit is given by

[8]The discrete number of workers to be searched on the labour market is calculated by rounding the result in Equation 5.

[9]Here another possible extension can be made by allowing for heterogeneous productivity, for instance related to firm- and/or worker-specific characteristics. This could also allow to consider a more complex social structure with capitalists, high-skill workers and low-skill workers. Finally, once allowed for the separation between the ownership and the management of a firm, also a class of managers could be included in an extended framework. The present model is a first step towards this challenging task.

$$b^d_{k,t} = ab_{k,t} - a_{k,t} \tag{7}$$

Otherwise, the capitalist is able to self-finance production and then no credit is demanded to the banking sector. Moreover, if capitalist k's invested capital is larger than total financing, then the difference is deposited in the banking sector.

For those capitalists requiring external finance, i.e. when $b^d_{k,t} > 0$, the credit provided by the banking sector is set as follows:

$$b_{k,t} = min\{b^d_{k,t},\ \beta[(1-\alpha)\hat{\omega}_{k,t}]\} \tag{8}$$

where $0 < \beta < 1$ is a parameter which represents the loan-to-value ratio. Therefore, the effective credit a firm k can receive from the bank is constrained by the availability of a *collateral* that is represented by the capitalist k's wealth non invested in the firm; in case of default, the bank receives that amount and an endogenous *recovery rate* (RR) can be computed. Depending of the value of RR, the bank has to write down a "bad debt" in its balance sheet (as we will further analyze below).

The bank sets the interest rate on the loan provided to the capitalist k at time t by charging a *risk premium* which depends on firm k's leverage, according to this equation:

$$r_{k,t} = \bar{r}_t + \sqrt{1 + \rho\ b_{k,t}/a_{k,t}} - 1 \tag{9}$$

where $\rho > 0$ is a parameter and $\bar{r}_t$ is the policy rate set by the central bank (see below for details).

3.2.5 Labour market

When the labour market opens, the government hires a fraction $0 < g < 1$ of the population as public workers, among them not selected to play the role of capitalists at time t. The remaining workers and capitalists meet in the labour market according to a decentralized matching mechanism.
In each period the following interaction protocol takes place:

- a randomized list of unemployed workers is set;
- the first agent on the list, say the agent j, selects all the firms that have enough money to pay her desired wage $w^d_{j,t}$ and chooses one of those at random, say the firm k; thus worker j gains the wage $w^d_{j,t}$ and the firm k's wage bill is updated accordingly; in the case there are no feasible matchings, i.e. if agent j's desired wage $w^d_{j,t}$ is larger than firms' available funds for hiring workers, then the worker j remains unemployed at time t;

- the second agent on the list performs the same activity and so on, until the last agent on the randomized list;
- at the end of the process, the unemployment rate can be computed, also including agents that, though selected as capitalists, were unable to hire workers in the labour market; each firm ends up with certain number of workers, $l_{k,t} \leq l^d_{k,t}$, that is equal or smaller than labour demand (thus unfulfilled vacancies can result from the matching process); the *k-th* firm's wage bill $wb_{k,t}$ is given by the sum of wages paid to hired workers.

Workers pay a proportional tax on wage given by the tax rate t_w. Unemployed people receive a benefit from the government: $ub_{j,t} = \sigma w_t$, where w_t is the average paid wage at time t, and $\sigma > 0$ is a parameter.

When the labour market closes, workers update their desired wage based on (past) inflation and employment status; for the generic worker j:

$$w^d_{j,t+1} = \begin{cases} w_{j,t} \cdot (1 + \eta \cdot max\{0, \dot{p}_t\} + \eta \cdot U(0,1)), & \text{if } i \text{ employed at time } t \\ w_{j,t} \cdot (1 + \eta \cdot max\{0, \dot{p}_t\} - \eta \cdot U(0,1)), & \text{if } i \text{ unemployed at time } t \end{cases} \tag{10}$$

where $U(0,1)$ is a uniformly distributed random number between 0 and 1, and $\eta > 0$ is a parameter governing the speed of adjustment in the adaptive rule of behaviour. As for capitalists, they also update their desired wage for the next period by only considering inflation; then, for the generic capitalist k: $w^d_{k,t+1} = w^d_{k,t} \cdot (1 + \eta \cdot max\{0, \dot{p}_t\})$.[10]

3.2.6 Production, price setting and corporate profits

Firms produce homogeneous consumption goods by using only labour as input, i.e. the workers $l_{k,t}$ hired in the labour market, according to a linear production function with productivity, $\phi > 0$, constant across firms and along time:

$$q_{k,t} = \phi \, l_{k,t} \tag{11}$$

Capitalists set the selling price according to the following adaptive rule:

$$p_{k,t} = max\{p_{t-1} \cdot [1 + \eta \cdot U(0,1) \cdot EDI_{t-1}], \ (wb_{k,t} + r_{k,t} b_{k,t})/q_{k,t}\} \tag{12}$$

where p_{t-1} is the price level in the previous period and EDI is an index of excess aggregate demand calculated as follows:

$$EDI_t = \frac{Q^d_{t-1} - Q^s_{t-1}}{Q^d_{t-1} + Q^s_{t-1}} \tag{13}$$

[10]It is also assumed that agents do not consider price adjustment in case of deflation.

where Q^d and Q^s represent the aggregate demand and the aggregate supply, respectively.

Market shares for different firms are inversely related to prices, according to the following equation:

$$ms_{k,t} = [1/(K_t - 1)] \cdot [1 - (p_{k,t}/p_t)(1/K_t)] \tag{14}$$

where K_t is the number of capitalists at time t.

Based on market shares, the maximum (monetary) value of revenues for each firm can be computed as follows:

$$re\bar{v}_{k,t} = AD_t \cdot ms_{k,t} \tag{15}$$

where AD_t is the (nominal) aggregate demand at time t.

Therefore, firm k's profit is equal to

$$\pi_{k,t} = min\{p_{k,t}q_{k,t},\ re\bar{v}_{k,t}\} - wb_{k,t} - r_{k,t}b_{k,t} \tag{16}$$

on which (if positive) capitalists pay a proportional tax based on the tax rate t_π. The first term of Equation 16 means that a short side rule applies given that either (i) the firm is able to sell its produced output at price set in Equation 12 or (ii) it is rationed on the demand side and then its (monetary) revenue is the one computed in Equation 15. It is assumed that produced output is composed of perishable goods that can be eliminated at no cost. Moreover, in case (i), given that the monetary flow directed to the firm is larger than the monetary value of produced output, the difference represents forced saving.

3.2.7 Saving and wealth

The main source of saving is given by the remaining part of agents' income after consumption. There are also two other components: (i) the part of invested wealth that capitalists do not employ in the productive activity (for instance, when the number of hired workers is below the labour demand); (ii) forced saving (for instance, due to goods market mismatch).

Voluntary or involuntary saving is deposited in the bank which remunerates depositors by applying a markdown, ψ, on the policy rate. Let D_t represent the sum of deposits at time t, the interest on saving is given by $\bar{r}(1 - \psi)D_{i,t}$. Agents pay a proportional tax on interest on saving based on the tax rate t_s.

At the end of each time period, agents update their wealth by adding to past wealth the income they gained either as worker (net wage or the unemployment benefit) or capitalist (net profit or unemployment benefit), the income deriving from saving and dividends (if

distributed by the bank; more details on this below). in some cases, a default can result if wealth becomes negative. In the case of a firm going bankrupt, a non-performing loan arises that impacts the bank's capital. The defaulted agent will start next period with zero wealth.

3.2.8 Banking sector

The profit of the banking sector is given by interest on firm loans, interest on government bills (see below for more details), interest on reserves at the central bank minus interest on deposits, interest on cash advances provided by the central bank, and non-performing loans.

The bank pays a proportional tax on positive profits based on the tax rate t_π. If financially sound, the banking sector distributes a fraction of net profit to agents, proportionally to their wealth. The percentage of profit to be distributed is computed by means of the following adaptive rule:

$$d_{t+1} = \begin{cases} min\{1,\ d_t + \delta \cdot U(0,1)\}, & \text{if } \omega_t^b > AD_t \text{ and } \omega_t^b > (1+\dot{p}_{t-1})\omega_{t-1}^b \\ max\{0,\ d_t - \delta \cdot U(0,1)\}, & \text{otherwise} \end{cases} \tag{17}$$

where $\delta > 0$ is a parameter.

At the end of the time period, the banking sector updates its net worth by summing up current profit (net of tax and distributed dividends). In the unlikely case in which bank's net worth turns to be negative, the government intervenes through a bank bailout.

As for the relationship between the banking sector and the central bank, if the assets (i.e., firm loans and government bills) are larger than liabilities (i.e., deposits and net worth), then the difference is deposited at the central bank as a bank reserve (on which the bank receives an interest given by applying a markdown to the policy rate, $\bar{r}_t$); in the opposite case, the banking system receives cash advances from the central bank (on which the bank pays $\bar{r}_t$).

3.2.9 Policy makers

The central bank either provides money (cash advances) or receives reserves depending on the banking sector's balance sheet and sets the policy rate $\bar{r}_t$ according to a Taylor-like rule:

$$\bar{r}_t = max\{0,\ \zeta\bar{r}_{t-1} + (1-\zeta)[\theta'(\dot{p}_{t-1} - \bar{\dot{p}}) + \theta''(\bar{u} - u_{t-1})]\} \tag{18}$$

where $0 < \zeta < 1$ and θ' and θ'' represent, respectively, the weight of inflation and unemployment in the monetary policy rule.

Government deficit is given by public expenditure for buying goods, payment of wage to public workers, unemployment benefits, interest on government bonds, in the extreme case of private bank defaults, bailout expenses; to these expenditures, revenues from taxing wages, profits and interest have to be subtracted. The public sector can issue government bonds on which it pays an interest whose spread on the policy rate, r_t^s, depends on the public debt-to-gdp ratio, that is μ, as follows:

$$r_t^s = \gamma(\sqrt{1+\mu_t} - 1) \tag{19}$$

where $\gamma > 0$ is a parameter.

Tax rates evolve according to the following fiscal rules that aim at controlling the dynamics of both public deficit and debt over nominal gdp. Let consider the tax rate on wage, t_w:

$$\tau_t^w = \begin{cases} min\{1,\ \tau_t^w + \delta \cdot U(0,1)\}, & \text{if } \mu_t > \bar{\mu} \text{ or } \nu_t > \bar{\nu} \\ max\{0,\ \tau_t^w - \delta \cdot U(0,1)\}, & \text{otherwise} \end{cases} \tag{20}$$

where ν_t is the public deficit-to-gdp ratio, while $\bar{\mu}$ and $\bar{\nu}$ represent, respectively, the threshold for public debt and deficit over gdp. The same holds for the other tax rates.[11]

3.2.10 Aggregate variables

Aggregate variables are obtained by summing up individual variables: for instance, $Q_t = \sum_k q_{k,t}$ is aggregate production at time t. The price level P_t is given by the weighted average of prices set by single firms (the weight being produced output). The interest rate on loans r_t is also a weighted average across firms. Moreover, we compute other statistics, as for instance the Gini index for consumption, income and wealth, in order to analyze the dynamic behaviour of the economic system.

4 Dynamics

Model dynamics are studied by means of computer simulation. First of all, the typical behaviour of the model is described by presenting and commenting the output of a single run of the baseline scenario. Parameter values for the baseline model are presented in Table 1 Afterwards, multiple simulations are performed in order to collect statistics about model dynamics when starting from different seeds of the pseudo-number generation process and then computing averages and dispersion measures of model variables across repetitions. Next step is a sensitivity analysis for investigating the impact on model dynamics of changing one parameter at time (thus keeping unchanged the other

[11] Evidently, an alternative to this assumption is to consider an endogenous adjustment of public expenditure both for hiring public workers and buying goods.

Table 1: Parameter setting

N	number of agents	1000
κ	class formation parameter	0.25
ϕ	firm productivity	100
c_y	propensity to consume income	0.7
c_ω	propensity to consume wealth	0.3
δ	additive adjustment	0.05
η	multiplicative adjustment	0.3
α	wealth buffer parameter	0.5
β	loan-to-value ratio	0.75
ρ	interest rate parameter	0.1
ψ	markdown	0.8
g	% of public workers	1/3
σ	unemployment benefit parameter	0.1
$\bar{v}$	public deficit threshold	0.1
$\bar{\mu}$	public debt threshold	1
γ	spread parameter	0.1
ζ	Taylor rule (TR) parameter	0.8
θ'	inflation weight in the TR	2
θ''	unemployment weight in the TR	1
$\bar{\dot{p}}$	inflation target	3%
$\bar{u}$	unemployment target	10%

parameters). Finally, a Monte Carlo analysis is performed to explore model dynamics under different configurations of the parameter space.

4.1 Initial conditions

At the beginning, i.e. for $t = 0$, there are N agents with neither income nor wealth or liabilities. The first step of the simulation is given by the government that hires some public workers by spending money which is printed by the central bank. Therefore, there is an initial public debt whose counterpart is the central bank's money. Indeed, the aggregate demand for the first period of time is composed of only public spending, as shown in the bottom-left panel of Figure 1. After this initial injection of public resources, from the second period on, agents become to be selected to play the capitalist or worker role, and a capitalist class emerges which finances the endogenous formation of firms that, in turn, assume workers (upper-right panel of Figure 1). Then, production takes place and starts growing, as we can observe in the upper-left panel of Figure 1. The bottom-right panel of Figure 1 shoes the evolution of the real wage (as compared with the constant productivity).

Figure 1: Simulations: 1000 agents, initialization phase

4.2 A single run

Figure 2 presents some simulation results for 200 periods of time, from $t = 201$ to $t = 300$, after the first 100 periods of the initialization phase described in Figure 1. Aggregate production exhibits business cycle fluctuations (upper-left panel of Figure 2). The unemployment rate oscillates between 20% and 30%; an opposite oscillation between 80% and 90% is that of the wage share (upper-right panel of Figure 2). The numerosity of the capitalist class varies along business cycles but its average is quite stationary around 15% (bottom-left plot of Figure 2). A relevant component in explaining this behaviour is given by the movement of the corporate profit rate; also the banking sector's profit rate oscillates and, in some cases, it presents negative values due to both negative interest and non-performing loans (bottom-right panel of Figure 2).

Other simulations results are presented in Figure 3: the upper-left panel shows a measure of inequality both for income and wealth;the average firms' leverage is described in the upper-right panel; in the bottom-left panel the policy rate and the average interest rate on loans are shown; finally, the bottom-right panel displays the delinquency rate, i.e. the ratio between non-performing loans and total loans.

The evolution of both public deficit and debt is presented in the upper-left panel of Figure 4. This evolution depends on the endogenous change of tax rates, as shown in the

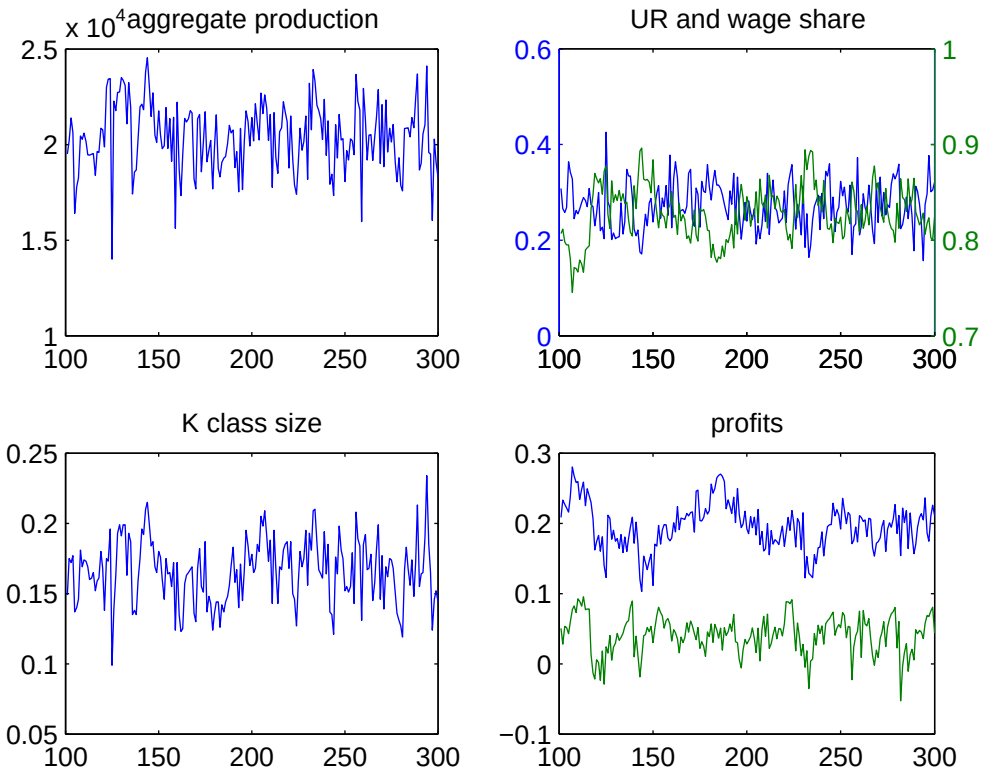

Figure 2: Simulations: from t=101 to T = 300

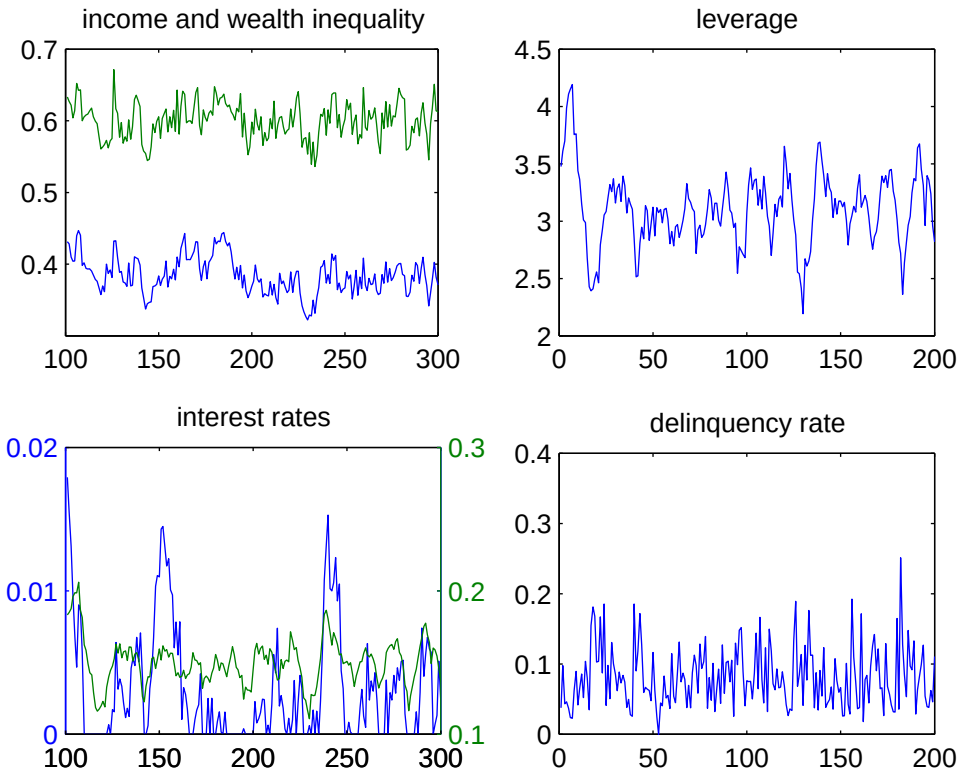

Figure 3: Simulations: from t=101 to T = 300

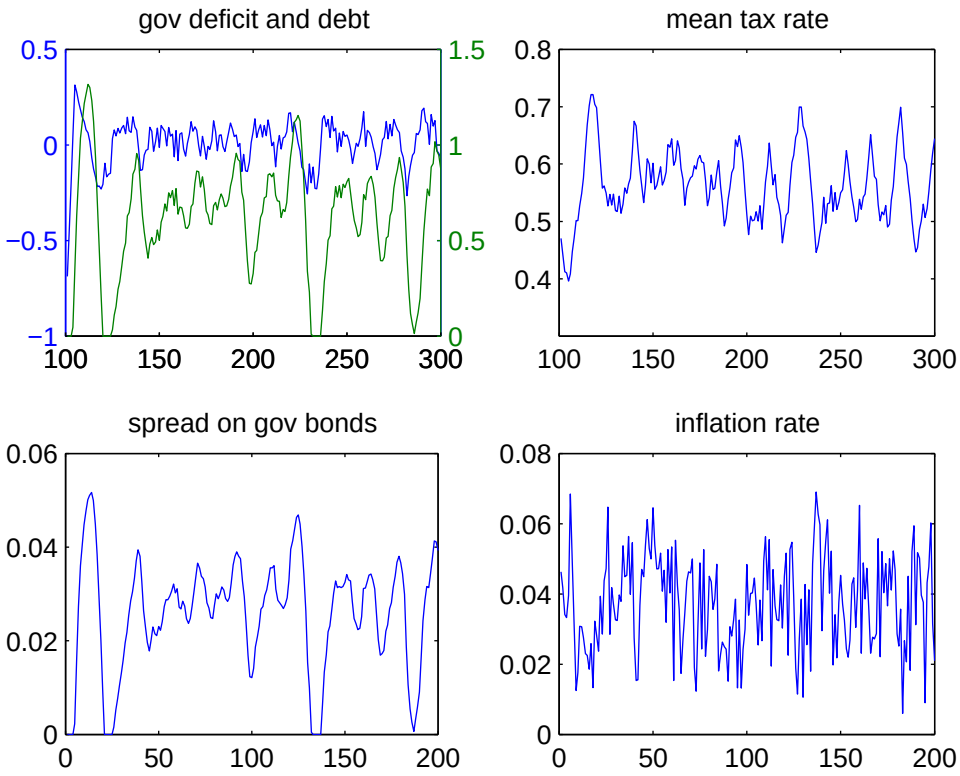

Figure 4: Simulations: from t=101 to T = 300

upper-right panel of the same figure. The spread on government bonds is displayed in the bottom-left panel, while the botto-right panel shows the inflation rate.

Finally, Figure 5 shows that there is a positive relationship between the time an agent spends playing as a capitalist and the real wealth accumulated along simulation time. Being a capitalist leads to gaining a profit that, if larger enough, further increases the accumulated wealth, thus increasing the likelihood of playing as a capitalist in the next period. This does not exclude the possibility of wealth disruption due to negative profits and firm defaults. However, in the average, playing as a capitalist results in a higher real wealth at the end of the simulation time.

5 Conclusions

This paper proposes an agent based a macroeconomic framework with endogenous social structure. The main novelty with respect to the related literature is that, based on social class dynamics, firms endogenously emerge as a relationship between employers and employees in a decentralized labour market. A stochastic process based on both individual wealth (relative to the whole wealth distribution) and the corporate profit rate determines if an agent plays as a capitalist or a worker in each period of time. Simulation

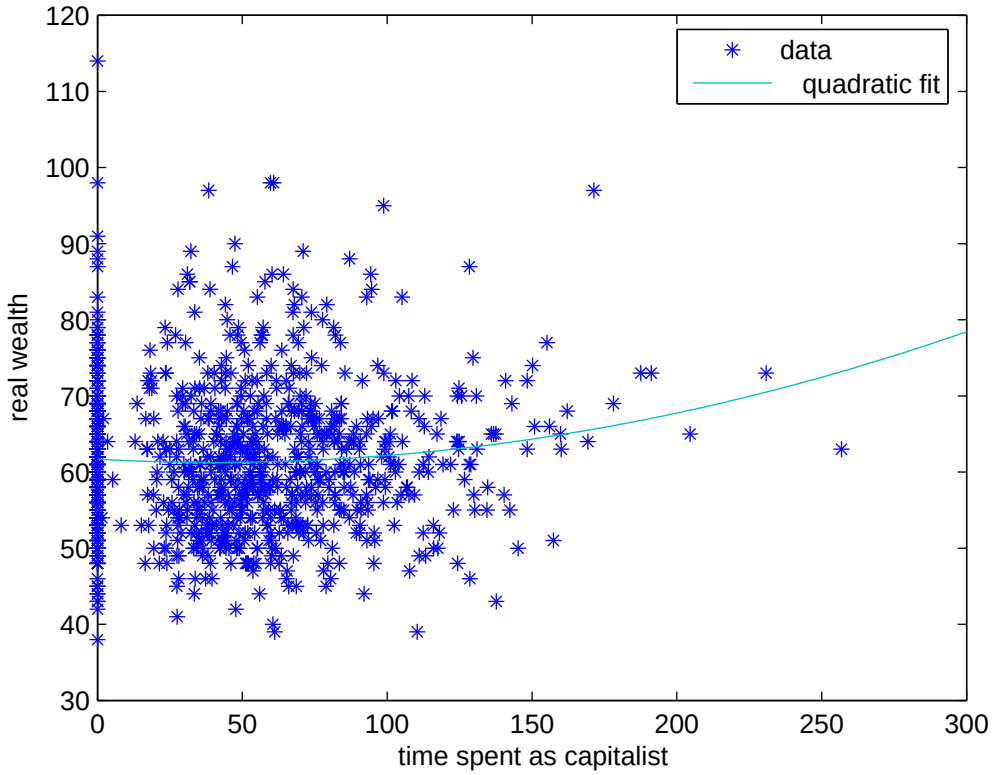

Figure 5: Time spent as capitalist vs accumulated wealth

results show that being a capitalist in one period rises (in the average) the probability to be again a capitalist in the next period. Macroeconomic results are compatible with the typical behaviour of a (stylized) capitalist system. Multiple simulations, sensitivity analysis and Monte Carlo simulations have to be performed to assess the robustness of results and explore the behaviour of the system under alternative configurations of the parameter space. After that, some policy experiments will be performed. This is only a first step towards a complex quest, that is to further extend the present framework with only *variable* capital by introducing also *fixed* capital.

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