Antitrust Enforcement, Commitments and Remedies

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Abstract: *This paper analyses the optimal setting of remedies and commitments in antitrust enforcement against unilateral practices, a tool introduced with the Modernization reform in 2003 that is now intensively used both ny the European Commission and the National Antitrust Authorities. We model the issue as a signalling game where nature determines whether a practice is socially harmful or beneficial, the firm decides to act, the Authority decides whether to open the case, colect costly information, close the case, accept a remedy or proceeds upon the information gathered. We show that commitments should be used in more harmful cases where the Competition authroity hardly finds evidence. Commitments have an anticompetitive effect of softening deterrence and a procompetitive effect of making prosecution more effective.*

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1 Introduction

The modernization reform in 2003 has introduced some new tools that the competition autorities (CA) can use in prosecution unilateral practices. Firms that are involved in a case can offer commitments that, if acceptedby the CA, become compulsory remedies. Commitments usually refer to adopting, or abstaining from adopting, certain practices, and are intended to limit the anticompetitive effects of the firm's behavior, restoring competition. While appealing to shorten a case and save resources, obtaining practical effects in the market, commitments are offered and accepted while the CA has now a full understanding of the effects of the practices involved. Hence, the firm can exploit its superior information. The incentive of the CA to accept the commitments, in turn, rests on the possibility of saving time and resources, closing the case in a short deadline and with some practical effects at work.

Since the regulation 1/2003 has been introduced commitments have become a very common tool used by the European Commission and the national CA quite often, raising a debate on the opportunity of such a widespread adoption. Those that have a critical view on this tool suggest that they create an incentive for firms to attempt dubious practices, having the escape way of offering commitment if the CA opens the case, thereby waiving the fines. The ultimate effect, then should be that of weakening deterrence. On the other side, those in favor of this new instrument claim that commitments dramatically shorten the duration of antitrust case and obtain a certain and ready result, as compared to long, costly and uncertain outcomes if the case is run until the final decision.

Although commitments are today a very common tool, surprisingly there is no theoretical analysis, to the best of our knowledge, to frame the problem into a model that allows delivering answers regarding the optimal way to administer commitments. In this paper we propose a signalling game where nature determines whether a practice is socially harmful or beneficial, the firm decides whether to act or not, the CA decides whether to open the case, collecting costly evidence, or not and, upon gathering information, whether to close the case, accept commitments or proceed to a second phase where further inspection are run before the final decision.

We show that commitments should be used only when the practice is presumably socially harmful, when it is particularly damaging and when gathering information is hard. By offering commitments the firm cashes in, as retained extra profits, the saving in administrative costs that the CA can obtain by accepting the commitments and closing the case after a short period. The efficiency in enforcement, in turn, must be traded off with the increased incentives to undertake practices even when socially harmful, the weakening deterrence effect.

The paper is organized as follows: section 2 presents the set up and section 3 develops the analysis.

2 Set-up

In line with the rules introduced with Regulation 1/2003, and in particular art. 7 and 9, we analyze the enforcement of art. 102 against the abuse of a dominant position. If the competition authority (CA) has stated its competitive concern in the Statement of Objection at the end of the first phase of the procedure, the firm may submit commitments that, if accepted, become binding and the authority closes the case with no infringement decision. If, however, the authority rejects the commitments, the procedure continues. If the CA does not prove the infringement, it clears the case, whereas if the CA establishes the illegal behavior, it fines the firm and obliges it to undertake remedies that affect both the private and social effects of the practice. The basic trade-off in the use of commitments rests on a reduction in the time and (social and private) costs of the procedure versus an early decision taken on more incomplete information.

More formally, the setting we analyze is described as follows.

Time 0

i) Nature draws the firm's type $\theta \in \{G, B\}$, that is whether the action the firm may undertake (a business practice) is welfare decreasing (B) or welfare enhancing (G), with $\Pr(\theta = G) = \lambda < 1$. The firm observes its type θ at a cost γ while the CA does not, and only knows the probability λ. The cost γ is uniformely distributed in $[0, \Pi]$ independently of the firm's type θ.

ii) The type-θ firm decides to undertake the action with probability α_θ. If it acts, when the type is good ($\theta = G$) private (Π) and social (W) benefits are positive, that is $\Pi_G = \Pi > 0$ and $W_G = W > 0$, while when the type is bad ($\theta = B$), private and social effects diverge, $\Pi_B = \Pi > 0$ and $W_B = -L$, with $L > 0$ being the social loss

Time 1 **(Phase** 1**)**

i) The CA is informed with probability $\mu \in (0, 1)$ that a firm is acting and, upon observing the action, the CA, updates its prior λ to λ_1 according to the equilibrium probabilities $\widehat{\alpha}_\theta$ that the θ firm acts. Then, the CA decides whether to open the case ($\delta_1 = 1$) or not ($\delta_1 = 0$).

ii) Conditional on opening the case, the CA receives a signal $\sigma_1 \in \{\theta, \emptyset\}$ incurring a cost κ_1. The firm observes the signal. The signal reveals the true type θ with probability $\rho_1 \in (0, 1)$ and no information with probability $1 - \rho_1$.

iii) If the signal is informative ($\sigma_1 = \theta$) the CA learns the firm's type θ and imposes the remedy $R_1 \in [0, 1]$ and the fine $F_1 \in \{0, F\}$, where 0 is the fine if the firm is not proven guilty and $F = \frac{\rho_2 \Pi}{1-\rho_2}$ is the level of fine, set in the norm, that the CA has to apply if the firm is convicted[1]. Then, $\Pi_\theta(R_1, F_1) = (1 - R_1)\Pi - F_1$ and $W_\theta(R^1) = (1 - R^1)W_\theta$.

iv) If the signal is not informative ($\sigma_1 = \emptyset$) the firm offers a commitment $C \in [0, 1]$ that produces the effects $\Pi_\theta(C) = (1-C)\Pi$ and $W_\theta(C) = (1-C)W_\theta$. The CA, then, has three options: closing the case ($\delta_2 = R_1 = F_1 = 0$); accepting the commitment, making it binding and closing the case ($\delta_2 = 0$, $R_1 = C$ and $F_1 = 0$), or going to Phase 2 ($\delta_2 = 1$).

Time 2 **(Phase** 2**)**

i) The CA receives a second signal $\sigma_2 \in \{\theta, \emptyset\}$ on the firm's type θ, incurring a cost κ_2. The signal reveals the true state of nature with probability $\rho_2 \in (0, 1)$ and no information with probability $1 - \rho_2$.

ii) At the end of the procedure, the CA either closes the case ($R_2 = F_2 = 0$) or imposes a remedy $R_2 \in (0, 1]$ and a fine $F_2 \in \{0, F\}$, such that $\Pi_\theta(R_2, F_2) = (1 - R_2)\Pi - F_2$ and $W_\theta(R_2) = (1 - R_2)W_\theta$.

[1] Hence, while in general the CA cannot commit, in our set up, to a predetermined policy and chooses ex-post the welfare maximising one, it has no discretion (it is committed) in applying fines. These latter has set in the norm and precribe not to fine an innocent firm and to apply F if the firm is convicted.

3 Analysis

Beliefs and updating

If the firm receives at time $t = 1, 2$ an informative signal, it updates its beliefs accordingly. To shorten notation, let $\lambda_t(\sigma_t = \theta) = \lambda_t(\theta)$. Then,

$$\lambda_t(G) = 1 \quad \text{and} \quad \lambda_t(B) = 0.$$

If, however, the signal received is uninformative, in a Perfect Bayesian Equilibrium (PBE) the CA updates its beliefs on the firm's type based on the equilibrium choices of a type-θ firm, as long as the observed choice is in the equilibrium set. In a (candidate) separating PBE, if at time t the CA observes the equilibrium strategy of the good type, then it updates its belief to $\lambda_t = 1$, whereas the posterior is $\lambda_t = 0$ after observing the equilibrium choice of the bad type. If the PBE, instead, is pooling and the signal is uniformative. the beliefs do not change when observing an equilibrium choice, i.e. $\lambda_t = \lambda_{t-1}$.

Moreover, we assume the CA adopts *passive beliefs* after observing an out-of-equilibrium choice: the CA assigns the same probability to either type having undertaken the out-of-equilibrium choice, and therefore does not update its beliefs, i.e. $\lambda_t = \lambda_{t-1}$.

Optimal policies. Let us construct the expected welfare and the optimal decision rule of the CA moving backward and starting from $t = 2$. Phase 2 is reached if the CA opens the case ($\delta_1 = 1$) and if, at $t = 1$, it does not impose a remedy, it does not close the case nor accepts the commitment. Then, the decision rule adopted at Phase 2 depends on the belief λ_2, which, in turn, are determined by the signal received σ_2. Notice that during Phase 2 the firm does not take any action that might signal its type. Hence, the belief λ_2 is updated only if the signal is informative ($\sigma_2 = \theta$), while $\lambda_2 = \lambda_1$ if $\sigma_2 = \emptyset$. The expected welfare at $t = 2$ is

$$EW_2 = \lambda_2 W - (1 - \lambda_2)L.$$

Hence, the decision rule is

$$\begin{aligned} R_2(\lambda_2) &= F_2(\lambda_2) = 0 \quad \text{if} \quad \lambda_2 \geq \frac{L}{W+L} = \overline{\lambda} \in (0,1) \\ R_2(\lambda_2) &= 1 \quad \text{and} \quad F_2(\lambda_2) = F \quad \text{if} \quad \lambda_2 < \overline{\lambda}. \end{aligned} \tag{1}$$

Notice that this rule covers also the case when the second signal is informative since $\lambda_2(G) = 1$ and $\lambda_2(B) = 0$.

At $t = 1$, if the CA has opened the case ($\delta_1 = 1$), it receives the signal σ_1. If it is informative, then $\lambda_1(G) = 1$ and $\lambda_1(B) = 0$. Therefore, the CA closes the case after the good signal and imposes full remedy and fine if the type is bad: $R_1(\lambda_1)$ and $F_1(\lambda_1)$ are set optimally at $R_1(1) = F_1(1) = 0$ and $R_1(0) = 1$ and $F_1(0) = F$.

If, however, the signal is not informative, the firm offers a commitment C that may represent a signal of its type. Then, the CA may update its beliefs

λ_1, based on the commitment. We can establish here a preliminary result that excludes the possibility of a separating equilibrium.

Lemma 1: *There exists no separating PBE in which the two types offer different commitments* $\widehat{C}_G \neq \widehat{C}_B$.

Proof. Suppose a PB separating equilibrium $\widehat{C}_G \neq \widehat{C}_B$ exists. Then, in equilibrium type θ selects a commitment $\widehat{C}_\theta$ and the CA updates its beliefs accordingly: $\lambda_1(\widehat{C}_G) = 1 = 1 - \lambda_1(\widehat{C}_B)$. Along the equilibrium path, the CA, then, closes the case when observing $\widehat{C}_G$ and imposes a full remedy and fine when observing $\widehat{C}_B$. Let us check if a deviation is convenient. Consider type $\theta = B$. If it deviates to $\widehat{C}_B$ it gets $\Pi > 0$ rather than $-F$ since the CA cannot detect the deviation and, possibly, apply a different decision rule. Hence, ittype B deviates. Then, no separating equilibrium exists. ■

Intuitively, in our setting offering the commitment is equally costly for either type, and therefore no single crossing condition can help separating the two types.

We are therefore left with a candidate pooling equilibrium in which both types offer the same commitment $\widehat{C}$. Notice that, the CA does not update its beliefs λ_1 in this case: if it observes the commitment $\widehat{C}$, it does not update λ_1 being it part of a pooling equilibrium. And it does not update λ_1 even after observing $\widetilde{C} \neq \widehat{C}$ under passive beliefs. Let us define the expected welfare given λ_1 as

$$EW_1 = \lambda_1 W - (1 - \lambda_1)L, \tag{2}$$

where $EW_1 \geq 0$ if $\lambda_1 \geq \overline{\lambda}$.

Since we are discussing the continuation game if the signal σ_1 is not informative, the CA can close the case, accepts the commitment or move to Phase 2. In this latter case we have to distinguish whether $\lambda_1 \geq \overline{\lambda}$ ($EW_1 \geq 0$), or $\lambda_1 < \overline{\lambda}$ ($EW_1 < 0$), since the policy in Phase 2 changes in the two situations. The following Lemmas cover the two cases.

Lemma 2: *Let*

$$\overline{\kappa}_2 \equiv (1 - \lambda_1)\rho_2 L$$

Suppose $\lambda_1 \geq \overline{\lambda}$. *Then, after an uninformative signal* $\sigma_1 = \emptyset$ *there exist pooling equilibria* $\widehat{C}_G = \widehat{C}_B = \widehat{C} \in [0, 1]$. *All the pooling equilibria yield the same outcome:*

- *if* $\kappa_2 \leq \overline{\kappa}_2$ *the* CA *goes to Phase* 2 ($\delta_2 = 1$) *for any* $\widehat{C} \in [0, 1]$*;*
- *if* $\kappa_2 > \overline{\kappa}_2$*, either* $\widehat{C} = 0$ *and the commitment is accepted* ($\delta_2 = 0$, $R_1 = \widehat{C}$, $F_1 = 0$)*, or* $\widehat{C} > 0$ *and the* CA *closes the case* ($\delta_2 = R_1 = F_1 = 0$).

Proof. Consider first the case of low costs in Phase 2, i.e. $\kappa_2 \leq \overline{\kappa}_2$. Along the equilibrium path, since both types offer the same commitment $\widehat{C}$ there is no update in beliefs. Moreover, if σ_2 is not informative no update occurs in Phase 2 either, that is $\lambda_2 = \lambda_1$. The expected welfare at time 1 upon receiving an uninformative signal $\sigma_1 = \emptyset$ and continuing to Phase 2 (i.e. $\delta_2 = 1$) when $\lambda_1 = \lambda_2 \geq \overline{\lambda}$ is:

$$\begin{aligned} EW_1(\delta_2 = 1) &= \lambda_1 W - (1-\lambda_1)(1-\rho_2)L - \kappa_1 - \kappa_2 = \\ &= EW_1 - \kappa_1 + (1-\lambda_1)\rho_2 L - \kappa_2 \end{aligned} \tag{3}$$

since the CA closes the case in Phase 2, i.e. $R_2 = 0$, if the signal is $\sigma_2 = G$ or $\sigma_2 = \emptyset$, whereas it imposes a full remedy $R_2 = 1$ and fine if the signal is $\sigma_2 = B$. Closing the case in Phase 1, instead, yields

$$EW_1(R_1 = 0) = EW_1 - \kappa_1. \tag{4}$$

Finally, accepting the commitment C gives

$$EW_1(R_1 = C) = (1-C)EW_1 - \kappa_1. \tag{5}$$

Hence, when $\kappa_2 \leq \overline{\kappa}_2$ we have for any $C \in [0,1]$

$$EW_1(\delta_2 = 1) \geq EW_1(R_1 = 0) \geq EW_1(R_1 = C) \tag{6}$$

given λ_1 and having observed C. The CA, therefore, prefers to proceed to Phase 2. In order to establish that the pooling equilibrium exists, let us check for possible deviations from $\widehat{C}$. Under passive beliefs, after observing a deviation $\widetilde{C} \neq \widehat{C}$, the CA does not update its posterior λ_1. Then, moving on to Phase 2 is still the optimal policy with respect to closing the case or accepting the commitment. Hence, there is no change in payoffs for the firm of either type, since the CA never accepts the commitment upon deviation. Consequently, there is no gain from deviation and the pooling equilibrium exists.

Consider next the case of high costs in Phase 2, i.e. $\kappa_2 > \overline{\kappa}_2$. For any $C \in [0,1]$,

$$EW_1(R_1 = 0) \geq \max\{EW_1(\delta_2 = 1), EW_1(R_1 = C)\}$$

and the CA closes the case upon observing $C > 0$, or it is indifferent between closing the case or accepting $C = 0$. Checking for deviations, the firm of either type obtaines the first best payoff Π along the equilibrium path and there is no possibility of gaining more by deviating, for any possible reaction of the CA, i.e. for any out of equilibrium beliefs and updating rule, including passive beliefs. ∎

Hence, when beliefs on the effects of the action are optimistic ($\lambda_1 \geq \overline{\lambda}$) the CA never accepts a commitment, and goes to Phase 2 when the administrative cost are low or closes the case directly at Phase 1 otherwise. We consider now the case of pessimistic beliefs.

Lemma 3: *Let*

$$\underline{\kappa}_2 \equiv \lambda_1 \rho_2 W$$

Suppose $\lambda_1 < \overline{\lambda}$ (pessimistic beliefs). Then, after an uninformative signal $\sigma_1 = \emptyset$

- *if* $\kappa_2 \leq \underline{\kappa}_2$ *there exist pooling equilibria* $\widehat{C}_G = \widehat{C}_B = \widehat{C} \in [0,1]$ *in which both types offer a commitment* $\widehat{C}$*, the* CA *rejects it and moves on to Phase 2.*

- *If* $\kappa_2 > \underline{\kappa}_2$ *there exists a unique pooling equilibrium in which the firm of either type offers*

$$\widehat{C}_G = \widehat{C}_B = \max\left\{0, \overline{C}\right\} \quad \text{where} \quad \overline{C} \equiv 1 - \frac{\kappa_2 - \underline{\kappa}_2}{L - \lambda_1(W+L)} \tag{7}$$

and the CA *accepts it.*

Proof. When $\lambda_1 < \overline{\lambda}$ the action is welfare decreasing according to the posterior λ_1. After an uninformative signal $\sigma_1 = \emptyset$ the CA can close the case, accept the commitment or proceed to Phase 2. As already argued, with pessimistic beliefs if the CA proceeds to Phase 2, it closes the case after an informative signal $\sigma_2 = G$ whereas it imposes full remedy and fine when receiving a bad or uninformative signal. The expected welfare if the CA goes to Phase 2 is then:

$$EW_1(\delta_2 = 1) = \lambda_1 \rho_2 W - \kappa_1 - \kappa_2. \tag{8}$$

The expected welfare if the CA closes the case in Phase 1 is

$$EW_1(R_1 = 0) = EW_1 - \kappa_1 \tag{9}$$

and the one if it accepts a commitment is

$$EW_1(R_1 = C) = (1 - C)EW_1 - \kappa_1. \tag{10}$$

Then, $EW_1(R_1 = 0) \leq EW_1(R_1 = C)$ i.e. accepting the commitment weakly dominates closing the case in Phase 1 where the equal sign holds when $C = 0$. Comparing commitment and going to Phase 2,

$$EW_1(\delta_2 = 1) \leq EW_1(R_1 = C) \quad \text{whenever} \quad C \geq 1 - \frac{\kappa_2 - \underline{\kappa}_2}{L - \lambda_1(W+L)} \equiv \overline{C}. \tag{11}$$

Since the feasible commitments are $C \in [0,1]$, we have to distinguish two cases. When the administrative cost in Phase 2 is low, i.e. $\kappa_2 \leq \underline{\kappa}_2$, then $EW_1(\delta_2 = 1) \geq EW_1(R_1 = 1)$. In this case, even the most generous commitment $C = 1$ is insufficient to induce the CA to accept it rather than going to Phase 2. This conclusion holds true even when the CA observes a deviation $\widetilde{C} \neq C$ since under passive beliefs the CA does not update λ_1 and still prefers to go to Phase 2. Then, we conclude that when $\lambda_1 < \overline{\lambda}$ and $\kappa_2 \leq \underline{\kappa}_2$ there exists an infinite number of pooling equilibria $\widehat{C}_G = \widehat{C}_B = \widehat{C} \in [0,1]$ that are payoff equivalent, in which both types offer $\widehat{C}$ and the CA rejects the commitment and proceeds to Phase 2.

When instead the costs are high, that is $\kappa_2 > \underline{\kappa}_2$, any commitment $C \geq \overline{C}$ would be accepted. Since the feasible commitments are $C \in [0,1]$, we have to analyse the candidate pooling equilibria $\widehat{C}_G = \widehat{C}_B = \widehat{C} \in [0,1]$. Consider first the case $\kappa_2 \in (\underline{\kappa}_2, \underline{\kappa}_2 - EW_1)$, where $EW_1 = \lambda_1(W+L) - L$, such that $\overline{C} > 0$. If $C < \overline{C}$ the CA, upon observing C, is willing to move to Phase 2. Let $E\Pi_\theta(C)$ be the expected profits when type θ offers commitment C. The good type $\theta = G$, then, obtains $E\Pi_G(C) = \rho_2\Pi - (1-\rho_2)F = 0$ being $F = \frac{\rho_2\Pi}{1-\rho_2}$ by assumption, whereas the bad type obtains $E\Pi_B(C) = -F$. Consider now a deviation to $\widetilde{C} = \overline{C}$ by either type. The CA, under passive beliefs, then, does not update λ_1 and accepts the commitment. The deviating firm, then, obtains

$$E\Pi_\theta(\overline{C}) = \frac{\kappa_2 - \underline{\kappa}_2}{L - \lambda_1(W+L)}\Pi > 0. \tag{12}$$

Hence, both types have an incentive to deviate and the candidate pooling equilibrium $\widehat{C} < \overline{C}$ does not exist. Consider next a candidate pooling equilibrium $\widehat{C} > \overline{C}$. Since along the equilibrium path the CA accepts the commitment, we have $E\Pi_\theta(C) \geq 0$. Any downward deviation $\widetilde{C} \in \left[\overline{C}, C\right)$ from the candidate equilibrium still induces the CA to accept the commitment since the beliefs are not revised. These deviations are profitable since they entail a lower commitment. Hence, no pooling equilibrium $C > \overline{C}$ exists. Finally, consider a candidate pooling equilibrium $\widehat{C} = \overline{C} > 0$. The CA accepts the commitment and the firm of either type makes positive profits. Any upward deviation would be accepted, reducing the profits. A downward deviation to $\widetilde{C} < \overline{C}$, instead, would make the CA moving to Phase 2, imposing full remedy $R_2 = 1$ and setting the fine $F_2 = F$ if the signal σ_2 is bad or uninformative. Then, the deviation is unprofitable for the good type since

$$E\Pi_G(\widetilde{C}) = \rho_2\Pi - (1-\rho_2)F = 0 < E\Pi_G(\overline{C}) = \frac{\kappa_2 - \underline{\kappa}_2}{L - \lambda_1(W+L)}\Pi,$$

being $F = \frac{\rho_2\Pi}{1-\rho_2}$ by assumption. Notice that $E\Pi_B(\overline{C}) > 0 > E\Pi_B(\widetilde{C}) = -F$. Hence, also the bad type never deviates from commitment $\overline{C}$. Hence, when $\lambda_1 < \overline{\lambda}$ and $\kappa_2 \in (\underline{\kappa}_2, \underline{\kappa}_2 - EW_1)$ there exists a unique pooling equilibrium $\widehat{C}_G = \widehat{C}_B = \overline{C} > 0$ in which the firm of either type offers $\overline{C}$ and the CA accepts it.

Finally, consider the case of very high administrative costs, $\kappa_2 \geq \underline{\kappa}_2 - EW_1$ such that $\overline{C} \leq 0$. In this case the firm can offer the minimum feasible commitment $\widehat{C} = 0$ that is accepted. Since this is the highest payoff the firm can obtain after an uninformative signal, it has no incentive to deviate offering a higher commitment, that would be accepted. Notice that this outcome is payoff equivalent to the CA closing the case ($R_1 = F_1 = 0$) after an uninformative signal ∎

Hence, when the beliefs are pessimistic the CA rejects the commitment and proceeds to Phase 2 whenever the administrative costs in Phase 2 are not too high, and otherwise it accepts the commitment. In this latter case, the firm offers the lowest commitment that induces the CA to accept it, cashing in in terms of retained profits the saving in administrative costs that the authority realises by not proceeding to Phase 2. For very high administrative costs, then, even a zero commitment, equivalent to closing the case, is accepted.

We move now at the beginning of Phase 1. Having observed that an action has been undertaken, the CA updates its prior λ on the probability that the action is welfare improving, taking into account the equilibrium probabilities $\widehat{\alpha}_\theta$ of acting by either type:

$$\lambda_1 = \frac{\lambda \widehat{\alpha}_G}{\lambda \widehat{\alpha}_G + (1-\lambda)\widehat{\alpha}_B}. \tag{13}$$

Then, the CA decides whether to open the case or not, where not opening the case ($\delta_1 = 0$) yields an expected welfare

$$EW_1(\delta_1 = 0) = EW_1. \tag{14}$$

Opening the case, instead, gives an expected welfare that depends on the level of $\lambda_1 \gtreqless \overline{\lambda}$ and on the level of administrative costs κ_1 and κ_2, since the optimal policies depend on whether the beliefs are optimistic or pessimistic, and on the size of the costs in Phase 1 and 2. The following Lemmas summarise the equilibrium policy outcomes for given probabilities of acting $\widehat{\alpha}_\theta$. To ease the exposition, we split the results distinguishing the case of optimistic and pessimistic beliefs. In the following Lemma we identify the optimal policy regimes when beliefs are optimistic.

Lemma 4 *Suppose $\widehat{\alpha}_\theta \geq 0$, $\theta = G, B$, implying $\lambda_1 \in [0,1]$ according to (13). Let*

$$\overline{\kappa}_t = (1-\lambda_1)\rho_t L$$

for $t = 1, 2$. *When the beliefs are optimistic, i.e. $\lambda_1 \geq \overline{\lambda}$ the equilibrium outcomes of the pooling equilibria are characterised as follows:*

Regime a) *When*

$$\kappa_2 \leq \frac{\overline{\kappa}_1 + (1-\rho_2)\overline{\kappa}_2}{1-\rho_2} - \frac{\kappa_1}{1-\rho_2}, \quad \kappa_2 \leq \overline{\kappa}_2. \tag{15}$$

*the CA opens the case ($\delta_1 = 1$), if $\sigma_1 = G$ it closes the case ($R_1 = F_1 = 0$), if $\sigma_1 = B$ it imposes remedies and fines ($R_1 = 1$, $F_1 = F$), if $\sigma_1 = \emptyset$ the CA **proceeds to Phase** 2 ($\delta_2 = 1$) and closes the case if $\sigma_2 = G$ or $\sigma_2 = \emptyset$ whereas it imposes remedies $R_2 = 1$ and fines $F_2 = F$ if $\sigma_2 = B$.*

Regime b) *When*

$$\kappa_1 \leq \overline{\kappa}_1, \;\; \kappa_2 > \overline{\kappa}_2. \tag{16}$$

*the CA opens the case ($\delta_1 = 1$), if $\sigma_1 = G$ or $\sigma_1 = \emptyset$ it **closes the case** ($R_1 = F_1 = 0$), if $\sigma_1 = B$ it imposes remedies and fines ($R_1 = 1$, $F_1 = F$), never proceeding to phase 2 ($\delta_2 = 0$).*

Regime c) *When*

$$\kappa_2 > \frac{\overline{\kappa}_1 + (1-\rho_2)\overline{\kappa}_2}{1-\rho_2} - \frac{\kappa_1}{1-\rho_2}, \;\; \kappa_1 > \overline{\kappa}_1 \tag{17}$$

the CA ***does not open the case*** *($\delta_1 = 0$).*

Proof. Consider first regime a. If the signal in Phase 1 is not informative the CA is willing to proceed to Phase 2. The expected welfare if opening the case, then, is

$$\begin{aligned} EW_{1\backslash}(\delta_1 = 1) &= \lambda_1\rho_1 W + (1-\rho_1)\left[\lambda_1 W - (1-\lambda_1)(1-\rho_2)L - \kappa_2\right] - \kappa_1 = \\ &= EW_1(\delta_1 = 0) - \kappa_1 - (1-\rho_1)\kappa_2 + \overline{\kappa}_1 + (1-\rho_1)\overline{\kappa}_2. \end{aligned}$$

Then, setting $EW_1(\delta_1 = 1) \geq EW_1(\delta_1 = 0)$ gives the first inequality: opening the case is optimal even when $\kappa^1 > \overline{\kappa}^1$ if the saving in Phase 2 costs compensate the extra costs in Phase 1.

In regime b, instead, after an uninformative signal σ_1 the CA closes the case. Then, the relevant cost threshold is referred to Phase 1. The expected welfare if opening the case is:

$$\begin{aligned} EW_1(\delta_1 = 1) &= \lambda_1\rho_1 W + (1-\rho_1)\left[\lambda_1(W+L) - L\right] - \kappa_1 = \\ &= EW_1(\delta_1 = 0) - \kappa_1 + \overline{\kappa}_1 \end{aligned}$$

and the first inequality follows.

Finally, in regime c the administrative costs are too high, no matter how they can be compensated across Phases, to justify opening the case. ■

The case of pessimistic beliefs is analysed in the following Lemma, where we identify the optimal policy regimes.

Lemma 5. *Suppose $\widehat{\alpha}_\theta \geq 0$, $\theta = G, B$, implying* $\lambda_1 \in [0,1]$ according to (13). *Let*

$$\underline{\kappa}_t = \lambda_1\rho_t W$$

for $t = 1, 2$. *When the beliefs are pessimistic, i.e. $\lambda_1 < \overline{\lambda}$ the equilibrium outcomes of the pooling equilibria are characterised as follows:*

Regime a') *When*

$$\kappa_2 \leq \frac{\underline{\kappa}_1 + (1-\rho_2)\underline{\kappa}_2 - EW_1}{1-\rho_2} - \frac{\kappa_1}{1-\rho_2}, \quad \kappa_2 \leq \underline{\kappa}_2 \tag{18}$$

the CA opens the case ($\delta_1 = 1$), if $\sigma_1 = G$ it closes the case ($R_1 = F_1 = 0$), if $\sigma_1 = B$ it imposes remedies and fines ($R_1 = 1$, $F_1 = F$), if $\sigma_1 = \emptyset$ the CA ***proceeds to Phase*** *2 ($\delta_2 = 1$) and closes the case if $\sigma_2 = G$ whereas it imposes remedies $R_2 = 1$ and fines $F_2 = F$ if $\sigma_2 = B$ or $\sigma_2 = \emptyset$.*

Regime a") *When*

$$\kappa_2 \leq \frac{\underline{\kappa}_1 + (1-\rho_2)\underline{\kappa}_2 - EW_1}{1-\rho_2} - \frac{\kappa_1}{1-\rho_2}, \quad \kappa_2 \in (\underline{\kappa}_2, \underline{\kappa}_2 - EW_1) \tag{19}$$

the CA opens the case ($\delta_1 = 1$), if $\sigma_1 = G$ it closes the case ($R_1 = F_1 = 0$), if $\sigma_1 = B$ it imposes remedies and fines ($R_1 = 1$, $F_1 = F$), if $\sigma_1 = \emptyset$ it ***accepts the commitment*** $\overline{C}$ *and never proceeds to phase 2 ($\delta_2 = 0$).*

Regime b') *When*

$$\kappa_2 \geq \underline{\kappa}_2 - EW_1, \quad k_1 < \overline{\kappa}_1$$

the CA opens the case ($\delta_1 = 1$), if $\sigma_1 = G$ it closes the case ($R_1 = F_1 = 0$), if $\sigma_1 = B$ it imposes remedies and fines ($R_1 = 1$, $F_1 = F$), if $\sigma_1 = \emptyset$ it ***closes the case*** *($R_1 = F_1 = 0$) and never proceeds to phase 2 ($\delta_2 = 0$).*

Regime c') *When*

$$\kappa_2 \geq \frac{\underline{\kappa}_1 + (1-\rho_2)\underline{\kappa}_2 - EW_1}{1-\rho_2} - \frac{\kappa_1}{1-\rho_2}, \quad k_1 > \overline{\kappa}_1 \tag{20}$$

the CA ***does not open the case*** *($\delta_1 = 0$).*

Proof. Consider regime a'. If the signal in Phase 1 is not informative the CA is willing to proceed to Phase 2. The expected welfare if opening the case, then, are

$$\begin{aligned} EW_1(\delta_1 = 1) = \lambda_1\rho_1 W + (1-\rho_1)\left[\lambda_1\rho_2 W - \kappa_2\right] - \kappa_1 = \\ = \underline{\kappa}_1 + (1-\rho_1)\underline{\kappa}_2 - \kappa 1 - (1-\rho_1)\kappa_2. \end{aligned}$$

Then, setting $EW_1(\delta_1 = 1) \geq EW_1(\delta_1 = 0)$ gives the first inequality.

In regime a'', instead, after an uninformative signal the CA is willing to accept any commitment $C \geq \overline{C} > 0$ rather than going ahead to Phase 2, and the firm offers the minimum commitment $\overline{C}$. Since, by the very definition of $\overline{C}$, the CA obtains the same expected welfare when going on to Phase 2 or accepting the commitment after an uninformative signal, $EW_1(\delta_1 = 1, R_1 = \overline{C}) = EW_1(\delta_1 = 1, \delta_2 = 1)$ and the same inequality $\kappa_2 < \frac{\underline{\kappa}_1+(1-\rho_2)\underline{\kappa}_2-EW_1}{1-\rho_2} - \frac{\kappa_1}{1-\rho_2}$ guarantees that the CA prefers to open the case (and accept the commitment after an uninformative signal).

In regime b' the CA closes the case after an uninformative signal. Then,

$$\begin{aligned} EW_1(\delta_1 &= 1) = \lambda_1\rho_1 W + (1-\rho_1)EW_1(\delta_1 = 0) - \kappa_1 = \\ &= EW_1(\delta_1 = 0) + (1-\lambda_1)\rho_1 L - \kappa_1 \end{aligned}$$

and the inequality follows.

Finally, in regime c' the CA prefers not to open the case, due to the excessive administrative costs. ∎

We can now turn to the initial decision of the firm at time 0. A firm of type θ bears a cost $\gamma \in [0, \Pi]$ when observing the true state θ, uniformely distributed. Then, the net profit from undertaking the action, according to the firm's type θ and cost γ depend on the probability μ that the CA observes the action and on the policy regimes that, in this latter case, the CA will optimally implement. Then, the fraction of firms of type $\theta = G, B$ decides to undertake the action, anticipating that the policy regime will be $r = a, a', a'', b, b', c, c'$, is

$$\widehat{\alpha}_\theta^r = \frac{E\Pi_\theta^r}{\Pi}.$$

In the following proposition we establish the ranking of $\widehat{\alpha}_\theta^r$ across types and regions.

Lemma 6: *For $\mu \in (0,1]$ The fraction of firms of type $\theta = G, B$ that undertake the action anticipatin that the policy regime will be $r = a, a', a'', b, b', c, c'$ follows the following rankings:*

$$\begin{aligned} \widehat{\alpha}_G^r &\geq \widehat{\alpha}_B^r \\ \widehat{\alpha}_B^a &< \widehat{\alpha}_B^b < \widehat{\alpha}_B^c \\ \widehat{\alpha}_B^{a'} &< \widehat{\alpha}_B^{a''} < \widehat{\alpha}_B^{b'} < \widehat{\alpha}_B^{c'} \\ \widehat{\alpha}_G^{a''}(\overline{C} &= 1) = \widehat{\alpha}_G^{a'} & (21) \\ \widehat{\alpha}_G^{a''}(\overline{C} &= 0) = \widehat{\alpha}_G^{b'} & (22) \end{aligned}$$

Proof. With optimistic beliefs, the policy outcomes correspond to regime a, where, after an uninformative signal σ_1 the CA proceeds to Phase 2, regime b, where instead after $\sigma^1 = \emptyset$ the case is closed, and regime c where the case is not started. Let $E\Pi_\theta^r$ represent firm θ expected profits, gross of cost γ, under regime r. After rearranging we obtain $E\Pi_G^r = \Pi$ in all three regions $r = a, b, c$ and $E\Pi_B^r = E\Pi_G^r - \mu\Delta\Pi^r$ where

$$\begin{aligned} \Delta\Pi^a &= [\rho_1 + (1-\rho_1)\rho_2](\Pi + F) & (23) \\ \Delta\Pi^b &= \rho_1(\Pi + F) \\ \Delta\Pi^c &= 0. \end{aligned}$$

With pessimistic beliefs, instead, the relevant policy regimes are a', where after an uninformative signal in Phase 1 the CA proceeds to Phase 2, regime a'' where it accepts the commitment, regime b' where the CA closes the case after an uninformative signal, and regime c' where it does not even open the case. The corresponding profits of the good type, gross of the cost γ of becoming informed, after rearranging, are $E\Pi_G^{a'} = \Pi - \mu(1-\rho_1)(1-\rho_2)(\Pi + F) = \Pi[1 - \mu(1-\rho_1)]$ since $F = \frac{\rho_2 W}{1-\rho_2}$, $E\Pi_G^{a''}(\overline{C}) = \Pi\left[1 - \mu(1-\rho_1)\overline{C}\right]$, $E\Pi_G^{b'} = E\Pi_G^{c'} = \Pi$. Then, $E\Pi_G^{a''}(\overline{C} = 1) = \Pi[1 - \mu(1-\rho_1)] = E\Pi_G^{a'}$ and $E\Pi_G^{a''}(\overline{C} = 0) = \Pi = E\Pi_G^{b'}$. The profits of the bad type for $r = a', a'', b', c'$ are $E\Pi_B^r = E\Pi_G^r - \mu\Delta\Pi^r$ where

$$\begin{aligned} \Delta\Pi^{a'} &= [\rho_1 + (1-\rho_1)\rho_2](\Pi + F) \qquad (24) \\ \Delta\Pi^{a''} &= \Delta\Pi^{b'} = \rho_1(\Pi + F) \\ \Delta\Pi^{c'} &= 0. \end{aligned}$$

Since $\widehat{\alpha}_\theta^r = \frac{E\Pi_\theta^r}{\Pi}$ the inequalities and equalities follow. ■

The posterior probability at the beginning of Phase 1, when the firm anticipatesthe policy regime r, is

$$\lambda_1^r = \frac{\lambda E\Pi_G^r}{\lambda E\Pi_G^r + (1-\lambda)E\Pi_B^r} = \frac{\lambda E\Pi_G^r}{EE\Pi_G^r - \mu(1-\lambda)\Delta\Pi^r}. \qquad (25)$$

The following Lemma establishes that a necessary condition for a PBE to exist is that a positive fraction of the firms of both types undertake the action.

Lemma 7: *In a PBE corresponding to policy regime r, $\widehat{\alpha}_G^r \geq \widehat{\alpha}_B^r > 0$.*

Proof. Suppose that in a candidate equilibrium corresponding to region r type B never undertakes the action while the good type acts, that is $\widehat{\alpha}_G^r > \widehat{\alpha}_B^r = 0$. Then, upon observing the action, the CA updates the belief to $\lambda_1^r = 1$. The optimal policy, therefore, is not to open the case. But then the bad type has the incentive to undertake the action for any γ, obtaining $E\Pi_B^r = \Pi > 0$ deviating from the candidate equilibrium. ■

Since $E\Pi_B^r$ is a non increasing function of μ, we can identify, in each region, the maximum probability of monitoring the action that makes the expected profits of the bad type, gross of the costs γ, positive. Let us define:

$$\mu^r \text{ such that } E\Pi_B^r(\mu^r) = 0.$$

From the expressions of $E\Pi_B^r$, we can see that

$$\begin{aligned} \mu^a &= \frac{\Pi}{[\rho_1 + (1-\rho_1)\rho_2](\Pi + F)} < \mu^b = \frac{\Pi}{\rho_1(\Pi + F)} \\ \mu^{a'} &= \frac{\Pi}{(\Pi + F)} < \mu^{a''} = \frac{\Pi}{(1-\rho^1)\overline{C}\Pi + \rho_1(\Pi + F)} < \mu^{b'} = \frac{\Pi}{\rho_1(\Pi + F)} \end{aligned}$$

Then, since $\mu \in (0,1)$, Lemma 7 implies that a PBE corresponding to policy regime r may exist only if $\mu \in (0, \min\{\mu^r, 1\})$.

The following Lemma establishes that λ_1^r is an increasing function of μ. If the CA is more likely to discover that the action has been undertaken, then the bad type is more discouraged than the good type from undertaking it, leading to an upward shift in the posterior probability that the firm is of the good type once observed the action.

Lemma 8: $\lambda_1^r(\mu)$ *is an increasing function of* μ *with* $\lambda_1^r(0) = \lambda$. Moreover, $\lambda_1^a(\mu) > \lambda_1^b(\mu) > \lambda_1^c(\mu) = \lambda$ and $\lambda_1^{a'}(\mu) > \lambda_1^{a''}(\mu, \overline{C}) \geq \lambda_1^{b'}(\mu) > \lambda_1^{c'}(\mu) = \lambda$ where $\lambda_1^{a''}(\mu, \overline{C} = 0) = \lambda_1^{b'}(\mu)$.

Proof. Given (25) and Lemma 6,

$$sign \frac{\partial \lambda_1^r}{\partial \mu} = sign \left[-\mu \phi_r \frac{\partial E\Pi_G^r}{\partial \mu} + E\Pi_G^r \Delta \Pi^r \right] > 0$$

since $\frac{\partial E\Pi_G^r}{\partial \mu} \leq 0$. Considering the ranking of the posterior beliefs, using (25) and substituting the inequalities follow Moreover, since $E\Pi_G^{a''}(\overline{C} = 0) = \Pi = E\Pi_B^{b'}$ and $E\Pi_{a''}^B(\overline{C} = 0) = E\Pi_G^{a''}(\overline{C} = 0) - \Delta\Pi^{a''} = E\Pi_B^{b'} = E\Pi_G^{b'} - \Delta\Pi^{b'}$, then $\lambda_1^{a''}(\mu, \overline{C} = 0) = \lambda_1^{b'}(\mu)$. ∎

Since $\lambda_1^r(\mu) \geq \lambda$ and increasing in μ, when the prior is optimistic, that is $\lambda \geq \overline{\lambda}$, the posterior is optimistic as well for any $\mu > 0$, and only the policy regimes a, b, c can occur in equilibrium. When, instead, the prior is pessimistic, i.e. $\lambda < \overline{\lambda}$, and $\lambda_1^r(\min\{\mu^r, 1\}) > \overline{\lambda}$, then there exists a $\overline{\mu}^r$ such that $\lambda_1^r(\overline{\mu}^r) = \overline{\lambda}$ and, for $\mu \geq \overline{\mu}_r^r$, the posterior beliefs λ_1^r are optimistic. In this case, according to the value of μ, we may have both pessimistic ($\mu < \overline{\mu}^r$) or optimistic ($\mu \geq \overline{\mu}^r$) posterior beliefs and therefore, the policy regimes a, b, c in the former and regimes a', a'', b', c' in the latter case.

Before moving to the analysis of the equilibria we have to explicitly define the different regions. Indeed, in Lemma 4 and 5, we have analysed the policy choices at the beginning of time 1 for given $\widehat{\alpha}_\theta$ and λ_1, identifying the different policy regimes r accordingly. We can now define the regions κ^r in the (κ_1, κ_2) space corresponding to the policy regime r as defined in Lemma 4 and 5, taking into account that, in each of them, the expected profits $E\Pi_\theta^r$ and the equilibrium decision to undertake the action, $\widehat{\alpha}_\theta^r$, change, affecting the posterior λ_1^r and the thresholds. Let us define:

$$\begin{aligned}
\overline{\kappa}_t^r &= (1 - \lambda_1^r)\rho_t L \quad \text{for } t = 1, 2 \text{ and } r = a, a', a'', b, b', b'', c, c' \\
\underline{\kappa}_t^r &= \lambda_1^r \rho_t W \quad \text{for } t = 1, 2 \text{ and } r = a, a', a'', b, b', b'', c, c' \\
EW_1^r &= \lambda_1^r W - (1 - \lambda_1^r)L \quad \text{for } r = a, a', a'', b, b', b'', c, c'
\end{aligned}$$

For $\kappa_t \geq 0$, $t = 1, 2$ we can now define the regions according to the equilibrium values:

$$
\begin{aligned}
\kappa^{a} &= \left\{ (\kappa_1, \kappa_2) \,\middle|\, \kappa_2 \leq \frac{\overline{\kappa}_1^{a} + (1-\rho_2)\overline{\kappa}_2^{a} - \kappa_1}{1-\rho_2}, \quad \kappa_2 \leq \overline{\kappa}_2^{a} \right\} && (26)\\
\kappa^{b} &= \left\{ (\kappa_1, \kappa_2) \,\middle|\, \kappa_1 \leq \overline{\kappa}_1^{b}, \ \ \kappa_2 > \overline{\kappa}_2^{b} \right\} && (27)\\
\kappa^{c} &= \left\{ (\kappa_1, \kappa_2) \,\middle|\, \kappa_2 > \frac{\overline{\kappa}_1^{c} + (1-\rho_2)\overline{\kappa}_2^{c} - \kappa_1}{1-\rho_2}, \ \ \kappa_1 > \overline{\kappa}_1^{c} \right\} && (28)\\
\kappa^{a'} &= \left\{ (\kappa_1, \kappa_2) \,\middle|\, \kappa_2 \leq \frac{\underline{\kappa}_1^{a'} + (1-\rho_2)\underline{\kappa}_2^{a'} - EW_1^{a'} - \kappa_1}{1-\rho_2}, \quad \kappa_2 \leq \underline{\kappa}_2^{a'} \right\} && (29)\\
\kappa^{a''} &= \left\{ (\kappa_1, \kappa_2) \,\middle|\, \kappa_2 \leq \frac{\underline{\kappa}_1^{a''} + (1-\rho_2)\underline{\kappa}_2^{a''} - EW_1^{a''} - \kappa_1}{1-\rho_2}, \quad \kappa_2 \in \left(\underline{\kappa}_2^{a''}, \underline{\kappa}_2^{a''} - EW_1^{a''} \right) \right\} && (30)\\
\kappa^{b'} &= \left\{ (\kappa_1, \kappa_2) \,\middle|\, \kappa_2 \geq \underline{\kappa}_2^{b'} - EW_1^{b'}, \quad k_1 < \overline{\kappa}_1^{b'} \right\} && (31)\\
\kappa^{c'} &= \left\{ (\kappa_1, \kappa_2) \,\middle|\, \kappa_2 \geq \frac{\underline{\kappa}_1^{c'} + (1-\rho_2)\underline{\kappa}_2^{c'} - EW_1^{c'} - \kappa_1}{1-\rho_2}, \quad k_1 > \overline{\kappa}_1^{c'} \right\} && (32)
\end{aligned}
$$

We can observe that a PB pooling equilibrium corresponding to the policy regime r exists if:

- $(\kappa_1, \kappa_2) \in \kappa^r$;
- a positive fraction of both types undertake the action, a condition that holds if $\mu < \mu_r$
- $\lambda_1^r(\mu)$, as determined by the prior beliefs λ and the equilibrium fraction of type θ's firms acting, $\widehat{\alpha}_\theta^r$, is below or above the threshold $\overline{\lambda}$ as required by the definition of region r, which, in turn may imply a further restriction on the admissible values of μ;

Hence, we can state the equilibrium conditions in terms of the prior beliefs λ, the probability of monitoring μ and the administrative costs κ_1 and κ_2. The key point is that if a PB pooling equilibrium corresponding to the policy regime r exists, for given $W, L, \Pi, \lambda, \rho_1, \rho_2$ the expected profits $E\Pi_\theta^r$, the fraction of acting firms $\widehat{\alpha}_\theta^r$ and therefore the posterior λ_1^r are a monotone function of μ, the probability that the CA discovers the action. Then, if there exists a set of values of μ such that $\widehat{\alpha}_\theta^r$ are positive for both types, λ_1^r is consistent with the definition of region r (pessimistic or optimistic beliefs) and $(\kappa_1, \kappa_2) \in \kappa^r$, then the equilibrium is established. The following Proposition states the result.

Proposition 1: *If $(\kappa_1, \kappa_2) \in \kappa^r$ a PB pooling equilibrium corresponding to region $r = a, b, c$ exists in two cases: for any $\mu \in (0, \min\{\mu^r, 1\})$ if the prior beliefs are optimistic, i.e. $\lambda \geq \overline{\lambda}$, or if the prior beliefs are pessimistic, the posterior beliefs when $\mu = \min\{\mu^r, 1\}$ are optimistic ($\lambda_1^r(\mu = \min\{\mu^r, 1\}) > \overline{\lambda}$) and the probability of monitoring is sufficiently high ($\mu \in (\overline{\mu}^r, \min\{\mu^r, 1\})$).*

If $(\kappa_1, \kappa_2) \in \kappa^r$ a PB pooling equilibrium corresponding to region $r = a', a'', b', c'$ exists when the prior beliefs are pessimistic ($\lambda < \overline{\lambda}$) and $\mu \in (0, \min\{\overline{\mu}^r, \mu^r, 1\})$, such that the posterior beliefs are pessimistic ($\lambda_1^r(\mu) < \overline{\lambda}$).

Proof. Consider first the case of optimistic priors, i.e. $\lambda \geq \overline{\lambda}$. Since $\lambda_1^r(\mu)$ is increasing in μ with $\lambda_1^r(0) = \lambda$, the posterior beliefs are optimistic for any admissible value of $\mu \in (0, \min\{\mu^r, 1\})$. Then, only the olicy regimes a, b and c are admissible. For each of them the corresponding regions κ^r are well defined. They set restrictions on the administrative costs such that if $(\kappa_1, \kappa_2) \in \kappa^r$ the optimal policies are those described in Lemma 4 and a fraction $\widehat{\alpha}_\theta^r$ of firms of type θ act according to Lemma 6, inducing $\lambda_1^r(\mu) > \overline{\lambda}$.

Secondly, consider the case of pessimistic priors, i.e. $\lambda < \overline{\lambda}$. Since for $\mu \in (0, \min\{\mu^r, 1\})$ both types act with positive probability and $\lambda_1^r(\mu)$ is increasing in μ, for $\mu \in (\overline{\mu}^r, \min\{\mu^r, 1\})$ the posterior beliefs are $\lambda_1^r(\mu) > \overline{\lambda}$. Hence, regions a, b or c are admissible. Then, the previous arguments apply.

Turn now to the case of pessimistic beliefs and $\mu \in (0, \min\{\overline{\mu}^r, \mu^r, 1\})$. In this case both types act with positive probability, and $\lambda_1^r(\mu) < \overline{\lambda}$, i.e. the posterior beliefs are pessimistic. Then, the policy regimes a', a'', b' and c' are admissible. For each of them the corresponding regions κ^r are well defined. They set restrictions on the administrative costs such that if $(\kappa_1, \kappa_2) \in \kappa^r$ the optimal policies are those described in Lemma 5 and a fraction $\widehat{\alpha}_\theta^r$ of firms of type θ act according to Lemma 6, inducing $\lambda_1^r(\mu) < \overline{\lambda}$. ■

Once established the existence of PB pooling equilibria corresponding to the different regions, we can further characterise them in the space (κ_1, κ_2). Indeed, the regions κ^r depend on λ_1^r that changes from one to the other according to the ranking specified in Lemma 8, shifting the boundaries accordingly. As a consequence, since $\overline{\kappa}_t^r = (1 - \lambda_1^r)\rho_t L$, $\underline{\kappa}_t^r = \lambda_1^r \rho_t W$ and $EW_1^r = \lambda_1^r(W + L) - L$, we have

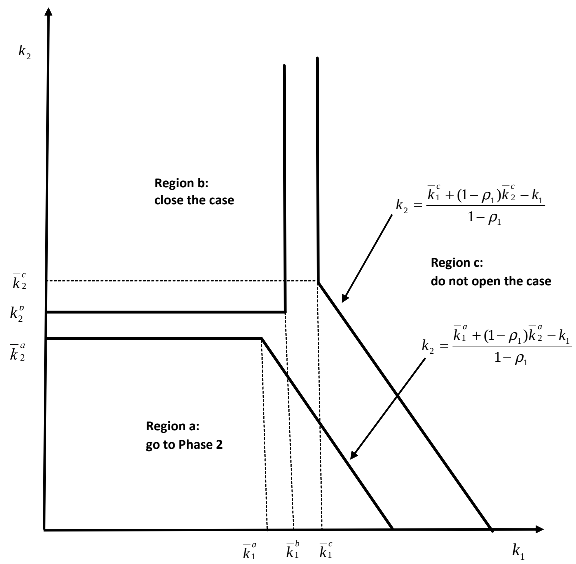
$$\begin{aligned} \overline{\kappa}_t^a &< \overline{\kappa}_t^b < \overline{\kappa}_t^c \\ \underline{\kappa}_t^{a'} &> \underline{\kappa}_t^{a''}(\overline{C}) \geq \underline{\kappa}_t^{b'} > \underline{\kappa}_t^{c'}. \end{aligned}$$

where $\underline{\kappa}_t^{a''}(\overline{C} = 0) = \underline{\kappa}_t^{b'}$. Moreover,

$$EW_1^{a'} > EW_1^{a''}(\overline{C}) \geq EW_1^{b'} > EW_1^{c'}$$

where $EW_1^{a''}(\overline{C} = 0) = EW_1^{b'}$.

Let us consider first how the regions a, b and c locate in the (κ_1, κ_2) space. Notice that region a's upper bound for $\kappa_1 \leq \overline{\kappa}_1^a$ is $\overline{\kappa}_2^a$ and $\kappa_2 = \frac{\overline{\kappa}_1^a + (1-\rho_1)\overline{\kappa}_2^a - \kappa_1}{1-\rho_1)}$ for $\kappa_1 > \overline{\kappa}_1^a$, whereas the lower bound of region b for $\kappa_1 \leq \overline{\kappa}_1^b$ is $\overline{\kappa}_2^b > \overline{\kappa}_2^a$. Hence, regions a and b do not touch. Analogously, region c is to the right of $\overline{\kappa}_1^c > \overline{\kappa}_1^b$ and $\kappa_2 = \frac{\overline{\kappa}_1^c + (1-\rho_1)\overline{\kappa}_2^c - \kappa_1}{1-\rho_1)} > \frac{\overline{\kappa}_1^a + (1-\rho_1)\overline{\kappa}_2^a - \kappa_1}{1-\rho_1)}$ and therefore does not touch region b nor a. Figure 1 represents the three regions.



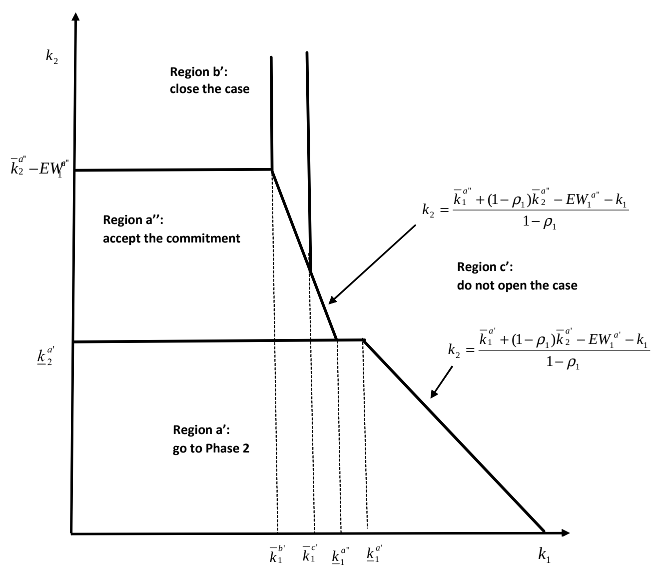
We turn now to regions $a', a''.b'$ and b'. It is important to notice that the boudaries of region a'' depend on the level of commitment $\overline{C}$, that affects the expected profits and the fraction of firms that undertake the action. In turn, the optimal level of commitment depends on κ_2, with $\overline{C} = 1$ for $\kappa_2 = \overline{\kappa}_2^{a''}$ and $\overline{C} = 0$ for $\kappa_2 = \overline{\kappa}_2^{a''} - EW_1^{a''} = \overline{\kappa}_2^{b'} - EW_1^{b'}$ according to Lemma 8.

Since $\overline{\kappa}_t^r$ shifts down moving from a' to a'', b' and c' the different regions overlap. We assume that, whenever for a given pair (κ_1, κ_2) two pooling equilibria exist corresponding to different policy regimes, the CA selects the one associated with the higher expected welfare.

Then, region a' is bounded above by $\underline{\kappa}_2^{a'}$ for $\kappa_1 \leq \underline{\kappa}_1^{a'}$ and by $\kappa_2 = \frac{\underline{\kappa}_1^{a'} + (1-\rho_1)\underline{\kappa}_2^{a'} - EW_1^{a'} - \kappa_1}{1-\rho_1}$ for higher κ_1. Consider next $\kappa_2 \in (\underline{\kappa}_2^{a''}, \underline{\kappa}_2^{a''} - EW_1^{a''})$ such that $\overline{C}$ goes down from 1 to 0, $E\Pi_{a'}^G$ goes up, λ_1^r decreases and $\underline{\kappa}_t^{a''} < \underline{\kappa}_t^{a'}$, $t = 1, 2$. Then, the curve $\kappa_2 = \frac{\underline{\kappa}_1^{a''} + (1-\rho_1)\underline{\kappa}_2^{a''} - EW_1^{a''} - \kappa_1}{1-\rho_1}$ is to the left of $\kappa_2 = \frac{\underline{\kappa}_1^{a'} + (1-\rho_1)\underline{\kappa}_2^{a'} - EW_1^{a'} - \kappa_1}{1-\rho_1}$ and becomes steeper. Indeed, let $K^{a''} = \underline{\kappa}_1^{a''} + (1-\rho_1)\underline{\kappa}_2^{a''} - EW_1^{a''}$. Then,

$$\frac{\partial K^{a''}}{\partial \kappa_2} = [\rho_1 W + (1-\rho_1)\rho_2 W - (W + L)] \frac{\partial \lambda_1^{a''}}{\partial \overline{C}} \frac{\partial \overline{C}}{\partial \kappa_2} > 0$$

since the term in squared brackets is negative, $\frac{\partial \lambda_1^{a''}}{\partial \overline{C}} > 0$ and $\frac{\partial \overline{C}}{\partial \kappa_2} < 0$. Hence, when κ_2 increases within region $\kappa^{a''}$, $K^{a''}$ increase as well, making the boundary steeper. Finally, at $\kappa_2 = \underline{\kappa}_2^{a''} - EW_1^{a''}$ we have, according to Lemma 8, $\lambda_1^{a''} = \lambda_1^{b'}$: hence, $\underline{\kappa}_2^{a''} - EW_1^{a''} = \underline{\kappa}_2^{b'} - EW_1^{b'}$ and $\overline{\kappa}_1^{a''} = \overline{\kappa}_1^{b'}$. Then, region a'' and b' are contiguous, as shown in Figure 2. Finally, since $\overline{\kappa}_1^{c'} > \overline{\kappa}_1^{b'}$ region c' locates to the right, as shown in the figure.



k_2
Region b': close the case
$\overline{k}_2^{a''} - EW_1^{a''}$
Region a'': accept the commitment
$k_2 = \dfrac{\overline{k}_1^{a''} + (1-\rho_1)\overline{k}_2^{a''} - EW_1^{a''} - k_1}{1-\rho_1}$
Region c': do not open the case
$\underline{k}_2^{a'}$
$k_2 = \dfrac{\overline{k}_1^{a'} + (1-\rho_1)\overline{k}_2^{a'} - EW_1^{a'} - k_1}{1-\rho_1}$
Region a': go to Phase 2
$\overline{k}_1^{b'}$ $\overline{k}_1^{c'}$ $\underline{k}_1^{a''}$ $\underline{k}_1^{a'}$
k_1