On multidimensional risk and inequality assessment

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Extended Abstract

Decision making under uncertainty presupposes the ability to rank random variables by associating a constant with the variable. The certainty equivalent can be defined as a mean value (Bonferroni (1924), De Finetti (1931), Kolmogorov (1930), Nagumo (1931)), that represents a sure amount for which a subject is indifferent to a risky position.

This concept can be linked either to a premium principle (Pratt (1964), Bühlmann (1970), Wang (1996)), or to a risk measure (Denneberg (1990), Föllmer and Schied (2002), Müller (2007)) or to an inefficiency measure (Debreu (1951), Sharpe (1966), Graaf (1977)), or to the measurement of multidimensional inequality (Atkinson (1970), Kolm (1976), Sen (1973)).

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Expected utility theory is the model used for decision making under uncertainty. It has been criticized on many aspects (Yaari (1987) and Machina (1982)), but it is still widely used, for instance in inequality theory, insurance, portfolio, health science.

Both Rothschild and Stiglitz (1970-1971) and Atkinson (1970), in their work, compare distributions of a single attribute, but, the former approach is concerned with relative riskiness, while the latter is about relative inequality.

Then, the generalizations to the multidimensional case have followed different ways. We mention the theory about multivariate risk aversion (Kihlstrom and Mirman (1974), Richard (1975)), that proposes an extention of the concept of risk aversion of Pratt (1964) and Arrow (1965), and the definition of multivariate risk premium, concept closely related to the certainty equivalent, used to compare the risky attitudes of different decision makers. Then, Kolm (1977) and Tsui (1995) propose a measure of inequality, that is the extension to the multivariate case of the equally distributed equivalent income of Atkinson (1970), and Atkinson and Bourguignon (1982) exploit the stochastic dominance of portfolio theory, to get a way to compare different distributions, not readily by means of an inequality index. Therefore, the former approach concentrates on the dispersion degree, while the latter is based on the positive dependence among risks.

The dependence structure of the variables can be captured in different ways. For instance, the ALEP definition of complementarity, that involves the mixed partial derivative of the utility function is also an index of correlation aversion, as introduced by Crainich et al. (2013), and it can be used as a choice criterion among different transfer principles.

Following Arrow-Pratt definition of risk aversion, we show that in the multivariate case we can ask for different conditions to compare the mixed partial derivatives of different utility functions, but a more risk averse agent in the single dimension is always characterized by greater correlation aversion. Moreover, after a concave transformation of the given utility function, the sign of the mixed partial derivative could change from positive to negative, conditioning the definition of ALEP complementarity. For these reasons, we consider an alternative approach for studying the concepts of risk aversion, more risk aversion and correlation aversion in the multidimensional case.

We generalize the one-dimensional definition of more risk aversion of Ross (1981) in

order to provide a multidimenional definition of more risk aversion and correlation aversion. Hence, it will be possible to introduce transfer principles based on Ross theory to compare different social welfare states from the inequality point of view. These comparisons will be shown to be reproduced through alternative formulations of the certainty equivalent in the multivariate case.

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