Dynamics of income rank volatility:
Evidence from US and Germany

Louis Chauvel, **Anne Hartung**, **Flaviana Palmisano**
University of Luxembourg
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Abstract

The aim of this paper is to provide a new methodology to estimate and compare profiles of income volatility over time and across distributions. While most of the existing studies, focusing on income and earnings, obtain volatility estimates that can be affected by the variation of inequality over time. This paper proposes a framework that, based on appropriate measures of income ranks, can be used to estimate volatility in a variation-to-inequality neutral fashion. This methodological framework is applied to evaluate and compare profiles of rank volatility in Germany and US in the last three decades. It is shown that while poorer individuals are the most volatile in both countries and over the whole period, the volatility trend of the middle class clearly marks the difference between the volatility experiences of the two countries.

Keywords: income volatility, risk, inequality, middle-class.
JEL codes: D31, D63, J6.

1. Introduction

A disquieting fact, that motivated a renewed interest toward the understanding of the distributional dynamics, is the increase in inequality in many industrialized countries over the last decades. At the same time, a large number of studies show that individual earnings or income instability also increased significantly, especially in US, contributing to exacerbate inter-individual disparities (Gottschalk and Moffit 2002, 2009; Comin and Rabin 2009; Nichols 2010; Shin and Solon 2011; Dynan et al. 2012; Bania and Leete 2009).

Most of these studies use aggregate measures of instability, such as alternative estimators of transitory variance and volatility, and are based on individual earnings or income. This paper differs from the existing literature with respect to these two features.

First, this paper focuses on the instability of the rank of individuals in the overall household income distribution as opposed to the instability of (male) labor earnings. While earning instability is exclusively related to labor market dynamics, the broader concept of income instability also reflects the role of the welfare state and family dynamics in absorbing negative events and income shocks. In fact, income or earning instability can be costly for individuals. Its degree of undesirability depends on the individual risk preferences, risk pooling possibilities, and the level of insurance against the risks of income losses. Social protections as well as transfers of income and labor supply within the household provide such income-smoothing insurance. Therefore, instability based on household income appears to be a more appropriate methodological choice, as compared to approaches based on individual earnings (Jenkins 2011; Dynan et al. 2012).

A point is in order here. In this paper, we do not simply focus on income instability, we focus on income rank instability. In fact, in periods of sustained variations in inequality, income volatility can be seriously affected by the structural changes that occur in the distributions. For instance, comparing the composition of the income movements in Belgium, Germany and the US, Van Kerm (2004) shows that, despite the different level of income inequality in the investigated countries, the major component of these income movements is 'exchange mobility',[1] which refers to the change of one's position in the income pecking order.

In our paper, instability is measured through the concept of income volatility (see on this Shin and Solon 2011). Although a few other studies look at the volatility of household income in US and Germany, this is the first contribution to explore such rank based volatility in these countries and to provide an interpretative framework for their different experiences of volatility.

Second, this paper focuses on disaggregate measures of volatility, as opposed to aggregate measures. The latter, in fact, may arise to be unsatisfactory as aggregate volatility may hide countervailing volatility and volatility trends across the distribution. Jensen and Shore (2008) using data on US show that a systematic rise in volatility of incomes over time for the population at large cannot be found when decomposing the change in average volatility. Their

[1] It accounts for 67-76%[1] of the income movements between 1985 and 1997.

argument is that the increase of average volatility was largely driven by a sharp rise of the very volatile incomes.

We share these views and we argue that an alternative method to evaluate volatility from a microeconomic perspective needs to be based on profiles of volatility rather than on aggregate measures. Following these arguments and thus complementing other studies, we argue that such profiles and their trends can be asymmetric, that is they may affect differently different parts of the distribution. A reason for this is that, for instance, institutions affect the stability of income careers and job flexibility usually hits more those individuals placed lowest in the income distribution.

Some contributions have started to explore in detail individual heterogeneity in this trend. For instance, Dynan et al. (2012) find that volatility in US rose in the early 1970s as well as in the late 2000s, and that this widening of the income distribution was a phenomenon especially related to the changes in the tails of the distribution. In view of the comparably high volatility of family incomes at the extremes of the income distribution, Hardy and Ziliak (2013), allude even to a "wild ride" at the top and at the bottom. Such heterogeneity finds also support in the work by Bania and Leete (2009), who show that US volatility was highest for lower income households and their instability increased steeper than for other groups during the 1990s. This is in line with Gottschalk's (1997) findings that in US the probability of staying in the lowest quintile was lower than the probability of remaining in the top quintile.

However, most of these studies are either based on income levels rather than income ranks or they consider large categories, such as quintiles, which ignore intra-group volatility. Hence, differently from previous contributions, in this paper we investigate heterogeneity of volatility across the distribution adopting a methodology, which is based on a specific function of an individual's income rank: the logit of the rank (hereafter logitrank).

Note that many appealing features characterize our measurement framework. First, our framework does not require the estimation of a formal model of income dynamic. In this paper, we measure volatility using the magnitude of the change in income ranks rather than isolating the transitory components of those changes. In this respect, our methodology can be considered as a complementary approach to the Gottschalk-Moffitt procedure. Using a "descriptive approach", such as the one used in this paper, does not allow to disentangle transitory shocks from permanent one. However, it avoids the results to depend on the underlying assumptions about the income generating process, as it is the case using the Gottschalk-Moffitt procedure (see on this Shin and Solon (2011), Dynarski and Gruber (1997), Cameron and Tracy (1998), Congressional Budget Office (2007), and Dynan et al. (2008)).

A second appealing feature is that, as explained before, specifically focusing on income may provoke the estimates of volatility to be affected by the structural changes of an economy such as variations of inequality. Our methodological framework instead, based on income ranks, allows to assess "net" volatility, that is the part of volatility not due to structural changes in the distribution.[2]

[2] Note that the use of rank in volatility analyses may also be motivated on the base of the positional goods framework or Easterlin's theory of relative utility (Easterlin 1974), whose relevance has been largely confirmed

Moreover, as is well known in the literature, the use of income to measure instability requires non-trivial estimation procedures, that are necessary to reduce the bias produced by the particular treatment of the data (the top-coding, for instance). In addition, using ranks allows to use a stable benchmark as opposed to the use of incomes, which are not stable over time.

Last, we do not use income rank *per se*, but we use the logit transformation of the income ranks. In addition to solve the estimation problems at the boundaries, this transformation allows to establish a proportionality between income rank volatility and volatility of a particular definition of relative income that we call the medianized income, that is, an individual's income divided by the median income of the distribution.

We then apply this framework to evaluate income rank volatility and its trends in Germany and US between the 1984 and the 2009. To this aim, we use the Cross National Equivalent File (CNEF), a dataset containing harmonized data on these countries. We show that over the whole period: (i) the poorer experience much higher volatility than the richer in both countries; (ii) the poor are less volatile in Germany than in the US; (iii) the volatility gap between the poor and the rich tends to decline in Germany as opposed to the US; (iv) the volatility trend of the middle class clearly marks the difference between the volatility experiences of the two countries; (v) while volatility increased consistently in US for the lower-middle class, it decreased consistently in Germany for the upper-middle class.

Hence, the contribution of our paper to the existing literature is two-fold. The first is methodological, as we provide a new measurement framework to evaluate distributional profiles of volatility, which is based on the notion of income rank. In doing so, we are able to observe volatility trends net of structural changes. The second is empirical, as applying our methodological framework, we provide new insights on the volatility trends that took place in Germany and US in the last decades.

The rest of the paper is organized as follows. Section 2 introduces the methodological framework. Section 3 presents the results of our empirical analysis. Section 4 concludes.

2. The methodological framework

In this section, we propose a framework that can be used to investigate (changes in) income rank volatility along the income distribution.

Let a society's income distribution at time t be represented by the cumulative distribution function (cdf) $F: R_+ \rightarrow [0,1]$. Hence, $F(y_t) = P(\tilde{y}_t \in R_+ : \tilde{y}_t \leq y_t)$, that is the cdf returns the probability $p_t \in [0,1]$ of observing income less or equal to $\tilde{y}_t$ in that society at time t. The rank of individuals in this society will then be defined by $F(y_t)$. Thus, these individuals' ranks at time t are defined by their continuous relative position (rank) between 0 and 1.

Let, then, express the logit transformation of the income rank, the logitrank, as follows:

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by recent research. It has been largely shown that higher positions in an income distribution, rather than the absolute income or one's position compared to a reference wage, leads to utility gains (see Clark et al. 2008, 2009; Alpizar et al. 2005).

$$logit(p_t) = ln\left(\frac{p_t}{(1-p_t)}\right) \quad (1)$$

This transformation of our variable of interest allows for a better estimation of the volatility experienced by the individuals placed at the tails of the distribution. In fact, note that the use of the rank *per se* implies that while those individuals placed at the middle of the distribution can move in two directions (up or down), those placed at the bottom (top) of the distribution can only move in one direction, up (down). The logit transformation, instead, allows overcoming this drawback.

In addition, the logitrank approximates the Champernowne-Fisk distribution (CF) such that:

$$\ln(\bar{\bar{y}}_t) = \alpha * logit(p_t) \quad (2)$$

where α measures the degree of inequality, understood as the stretching out of the distribution, and $\bar{\bar{y}}_t$ is the so-called medianized income; that is, letting $Me(y_t)$ be the median income of the distribution (see on this Chauvel 2014, Dagum 2006, Fisk 1961):[3]

$$\bar{\bar{y}}_t = \frac{y_t}{Me(y_t)}. \quad (3)$$

In other words, $logit(p_t)$ is proportional, and thus an equivalent measure, to the log of the medianized income.

Table 1 reports the conversion between logitranks and percentile ranks. For instance, a magnitude of -2 relates to quantile .119, then close to the first decile, a magnitude of 2 relates to an income 2.7 times higher than the median.

Table 1. Magnitudes of logitrank and percentile rank.

Logitrank	-5	-4	-3	-2	-1	0	1	2	3	4	5
Rank	0.007	0.018	0.047	0.119	0.269	0.500	0.731	0.881	0.953	0.982	0.993

Note that CF, on which the methodology introduced here is based, is one of the many statistical laws used to model incomes.[4] The use of CF as a first approximation to income

[3] In a 212-samples comparison Chauvel (2014) shows that this relation approximates more than roughly the empirical distributions in terms of level of living (post-tax and transfer income per consumption unit).
[4] Although GB2 provides a better fit of the income distribution, CF represents a simple framework that can be used to capture changes in local inequality.

distributions is motivated by at least three main reasons. First, being characterized by two parameters ($Me(y_t)$ and α), CF results to be very parsimonious, with appropriate Pareto-type power-tails at both extremes. This parsimony is notable, and the coefficient α plays a remarkable role in the measurement of inequality since its value corresponds to the Gini coefficient. Second, it is a very simplified GB2. While CF is much less flexible than GB2, it does share some important features, such as power-tails. Third, CF produces income distributions that are solidly grounded in mathematical expressions. Here CF is at a crossroads of different theoretical traditions, nevertheless its formula remains very simple.

In microeconomics, GB2 (and, as a consequence, CF) can be seen as a result of Parker's neoclassic model of firm behavior (Parker, 1999). A number of other theoretical constructions, such as stochastic processes of income attainment, yield the same distribution. In a proposal from the field of finance, Gabaix (2009) considers stochastic models based on geometric Brownian motion that can generate this type of distribution. A number of different fields of research thus confirm the importance of the CF.

We then use logitranks to describe and visualise income rank volatility and changes in income ranks volatility over time. Let denote the average logitrank of individuals between two periods as follows:

$$\overline{logit(p)} = \frac{logit(p_t)+logit(p_{t+1})}{2}. \tag{4}$$

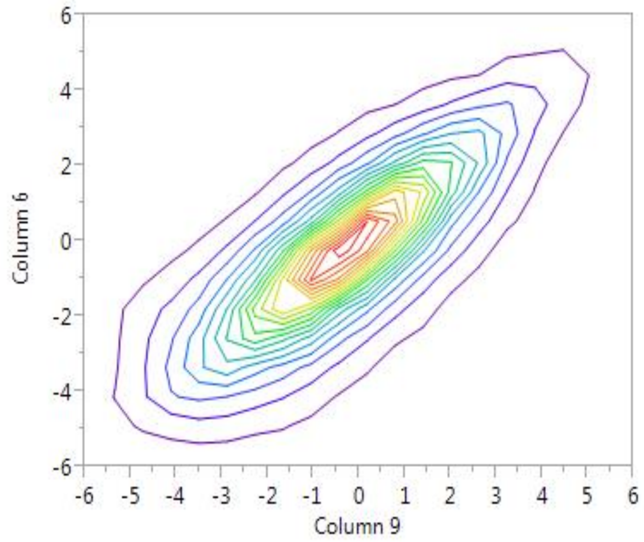
Eq. (4) is also interpretable as the intertemporal position of an individual in the society. Let then $\delta(p_t) = logit(p_t) - logit(p_{t+1})$ be the change in the logitrank between initial and final period. Plotting the logitrank of all individuals at two different points in time, we rely on contour plots using kernel density estimation as illustrated in Figure 1. Figure 1 allows to detect cases of complete stability or absence of volatility, arising when every individual has the same rank in t and $t+1$. In this case, each individual would be placed on the diagonal.

We measure individual volatility as the standard deviation of $logit(p_t)$ and $logit(p_{t+1})$, which reflects the intensity/magnitude of moves or the instability of a position. Individual income rank volatility is then defined as follows:

$$v\left(\overline{logit(p)}\right) = \sqrt{\frac{1}{2}\sum_{t=1}^{2}\left(logit(p_t) - \overline{logit(p)}\right)^2}\ . \tag{5}$$

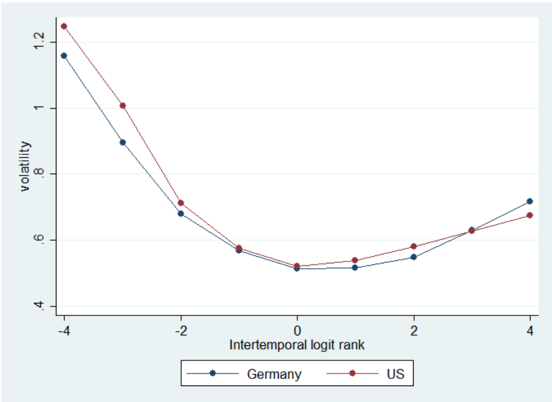
Plotting eq. (5) against each $\overline{logit(p)}$ we get the volatility profiles as the one reported in Figure 2. This a graphical tool that provides very intuitive information on the extent of income rank volatility across the distribution.

Figure 1. Logitrank volatility measured across two time points



Note: The contours refer to density isoquants. x_{t0} denotes the *logitrank* at year 0, x_{t2} denote the *logitrank* at year 2, h denotes the hierarchy, that is the average *logitrank* over the two periods, c denotes the change in the *logitrank* between the two points in time.

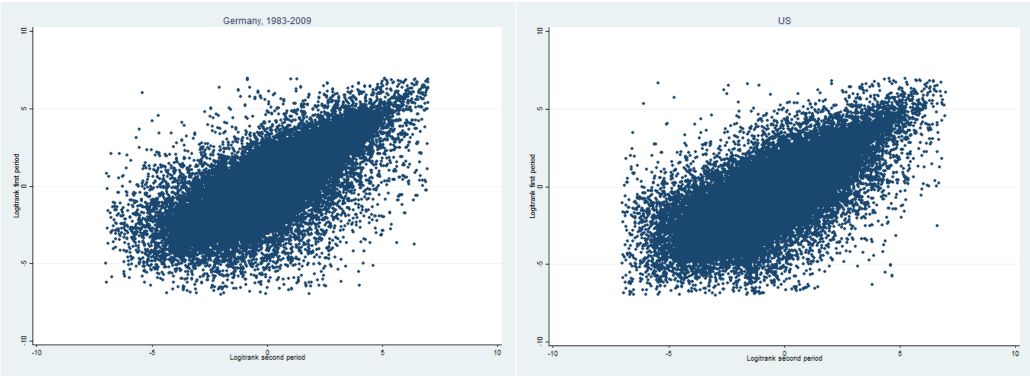
Figure 2. Average two-year volatility in Germany and US 1983-2009. Observed (left) versus estimated (right) curve



Source: Authors' calculation based on CNEF.

It is worth noticing that some of the statistical properties related to the use of the logitrak can be very useful in the context of volatility analysis. Figure 3 plots the period-to-period volatility of logitranks for both Germany and US over the whole period considered. It shows that the cloud of observations is anisotropic rather than a bi-normally distributed cloud. The plot suggests that there is less variation in ranks over the two time points at the top of the income distribution (upper right corner), as observations are clustered closer to the diagonal, than at other parts of the income distribution. Please note that contrary to the results of Gottschalk and Spolaore (2002), Germany's contours of the period-to-period volatility of logitranks do not lie in those of the US as we plot rank-based volatility instead of income-based mobility. However, also our results seem to indicate that Germany has a more equal distribution and less large rank changes in view of the slimmer shape or less deviations from the diagonal.

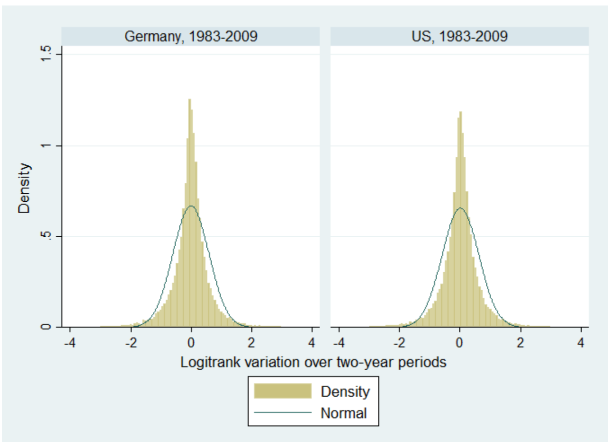
Figures 3. Distribution of logitrank in Germany and US, 1983-2008.



Source: Authors' computations based on CNEF.

This is better captured in Figure 4 reporting the density of the change in the logitrank of individuals over a two-year period. Indeed, Figure 4 shows that this change is very far from being normally distributed. Logitrank variations take much more extreme values than in the normal hypothesis. We detect here a typical Lévy alpha-stable distribution that belongs to the general family of stable distributions. A general stable distribution can be described by four parameters: an index of stability or characteristic exponent $\theta \in (0; 2]$ ($\theta = 2$ for a normal distribution and $\theta < 2$ for a leptokurtic distribution), a skewness parameter $\pi \in [-1; 1]$, a scale parameter $\rho > 0$ and a location parameter $\sigma \in R$ (Nolan 2009, Umarov et al. 2010). Here, θ is close to 1.3. Leptokurtic non-normal stable distributions are also known as stable Paretian distributions. These heavy-tailed distributions are common in the statistics of finance and assets volatility analysis.

Figure 4. Logitrank change over two periods in Germany and US, 1983-2009



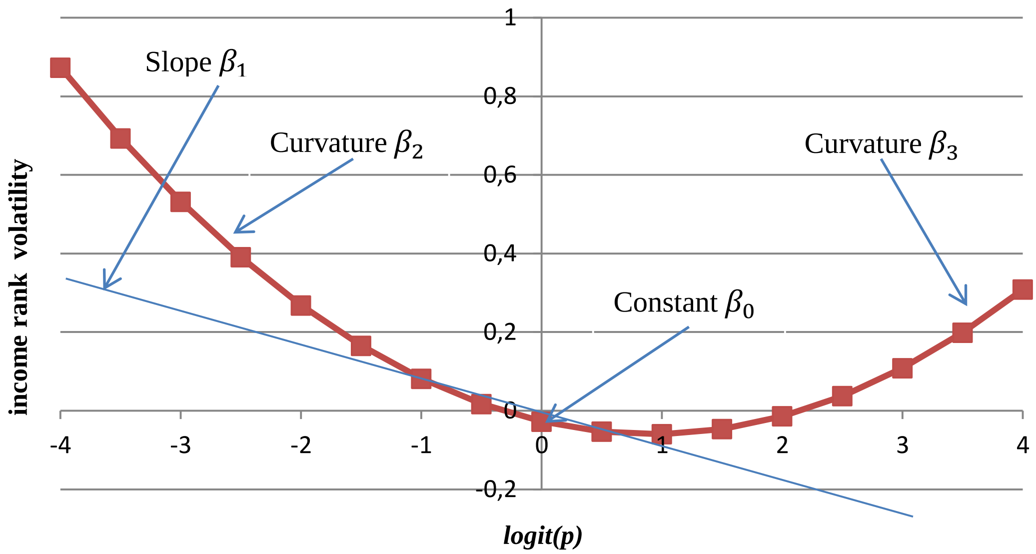
Source: Authors' computations based on CNEF.

These properties and the particular shape of the volatility profiles shown in Figure 2 make it possible to estimate a volatility profile as follows:

$$\ln\left(v\left(\overline{logit(p)}\right)\right) = \beta_0 + \beta_1 \overline{logit(p)} + \beta_2 \overline{logit(p)}^2{}_{|logit(p)<0} + \beta_3 \overline{logit(p)}^2{}_{|logit(p)>0} + \varepsilon \quad (6).$$

In order to account for dissymmetry, we collapse the curvature in two parameters below and above the median (logitrank=0), β_2 and β_3. Figure 5 illustrates the estimated volatility profile using eq. (6).

Figure 5. Profile of volatility



As shown in Figure 5, estimating profiles of volatility through eq. (6) is very useful as it allows capturing relevant information related to the income dynamics under analysis. These are the constant β_0 that catches volatility near the median, the slope β_1 that denotes the degree to which volatility is higher (or lower if $\beta_1 < 0$) at the top, and β_2 that expresses the degree of increase of volatility at the extremes of the distribution ($\beta_2 > 0$ provides a U shaped curve of volatility). Therefore, the variation of the three parameters gives interpretable information on the increase or decrease of volatility in specific parts of the distribution. A positive change in β_0 implies that volatility increases for all; a rise in the value of β_1 means more volatility for the richest individuals; last, higher values of β_2 and β_2 mean more volatility for extreme values (and relatively higher stability at the median level). As a result, we can better understand in which part of the distribution income rank volatility increases or decreases more in the distribution.

It is important to stress here the relevance of using logitranks for the measurement of volatility. Adopting this procedure, we simply transform the empirical quantile function of any distribution in its vertical projection. In case of panel analysis on two or several years, the logitrank transformation consists in the reshaping of the empirical distributions on an invariable reference distribution of shape defined in eq. (1). This implies that logitrank based volatility absorbs all the structural transformations to retain the sole exchange mobility (see Jenkins and Jantti 2015 on the relevance of focusing on rank rather than on income in the measurement of mobility). Hence, this approach allows performing meaningful comparisons over time and across countries, as we focus on pure volatility.

Second, descriptive statistics and more elaborated models based on logitrank as a dependent variable are meaningful since, according to the Champernowne-Fisk model, logitranks are just a simple linear transformation of the log of incomes. Therefore, working with the former is equivalent to working with the latter, but with the difference that logitrank variations are depurated from structural changes. Furthermore, as for any other ranking strategy, we dispose of a fixed point - relative position with respect to the rest of the distribution - that we cannot find when incomes are used.

Our approach also shares other appealing technical features. First, existing studies on income or earnings volatility using ranks often examine quintile or decile transitions over varying time periods (see among others Gottschalk 1997). However, this method is not able to differentiate the magnitude of the change in income ranks: it treats, for instance, changes from the 19^{th} to the 21^{st} percentile in the same way as transitions from the 1^{st} to the 39^{th} percentile. Our framework, based on a continuum of ranks, is able to account for distance concerns.

A further advantage of using our framework is that, differently from the use of the log of income, is not dominated by small changes in the level of income near zero that usually lead to huge or infinite changes when the log of income is used. Differently from the use of percentile ranks, instead, our framework does not give a greater importance to moves at the middle (when you get richer from fractional rank $p = .99$, big moves in $ln(y)$ has small effects in terms of variation of ranks).

3. Profiles of income rank volatility in Germany and US

3.1. The data

Our empirical analysis is based on the panel component of the Cross National Equivalent File (CNEF). The CNEF was designed at Cornell University to provide harmonized data for a set of eight country-specific surveys representative of the respective resident population. For the present paper, we consider all the waves between 1983 and 2009 for Germany (SOEP) and the US (PSID).

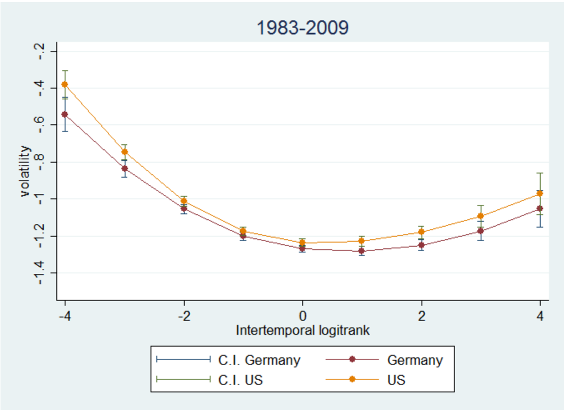
The unit of observation is the individual. Our data cover all persons aged 20 to 65. We restrict our US sample to Black and White Americans and exclude other ancestries such as Asians and Hispanics. We discard East Germany from the German sample due to its later start. The measure of living standards is disposable household income, which includes income after transfers and the deduction of income tax and social security contributions. Incomes are expressed in constant 2005 prices and are adjusted for differences in household size, using the square root of the household size. Individual volatility is measured over a two-year period.

We use sample weights to compute all estimates with standard errors obtained through 500 bootstrap replications.

3.2.Results

Figure 6 reports the estimated profiles of income rank volatility for Germany and US over the whole period considered. In both countries, the volatility profile appears to be U-shaped. This particular shape is not a natural result of our method. It denotes a higher average change at the bottom and the top of the distribution as compared to the change at the middle of the distribution.

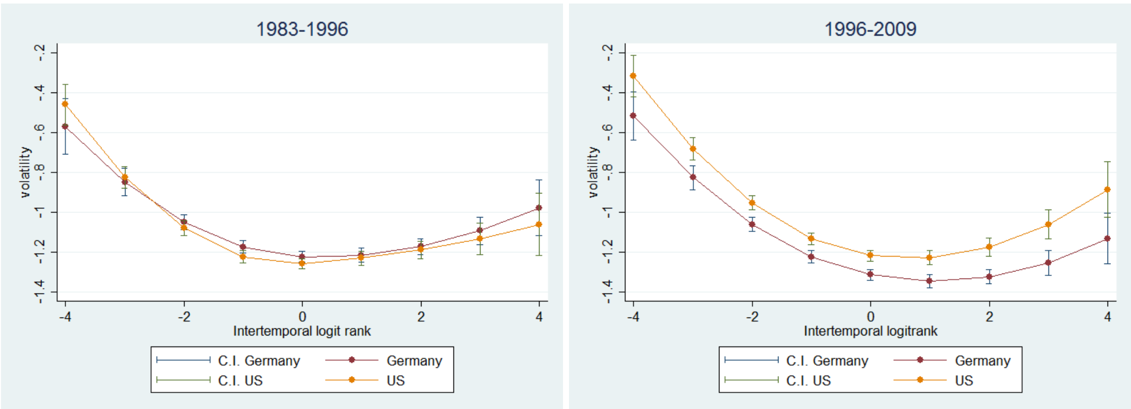
Figure 6. Logitrank volatility. Germany versus US, 1983-2009.



Source: Authors' computations based on the CNEF.

The U curvature that characterizes all the volatility profiles in Figure 6, 7, and 8 is however asymmetric and witnesses that, in both countries, the middle-income classes have been and are more stable and that the top income classes have been, and are still, more stable than the bottom, but more fluid (or unstable) than the middle.

Figure 7. Logitrank volatility. Germany versus US, 1983-1996 (left) and 1996-2009 (right).



Source: Authors' computations based on the CNEF.

Although these similarities, the countries under analysis have experienced quite different volatility history. If we consider the whole period, 1983-2009 (Figure 6), US appears to be the most volatile, although the difference with Germany is never statistically significant. This result corroborates previous studies focused on household income volatility, which have shown that incomes in US tend to experience larger fluctuations and are thus more "risky" than in Germany in a similar period (see, among others, Gottschalk and Spolaore 2002, Van Kerm 2004).

However, more insights on volatility in these two countries can be gained by exploring if and how such profile has changed over time. We, then, also focus on the trend of volatility from which some striking features arise. In the first period, 1983-1996 (Figure 7, left panel), the poorer in US are more volatile than in Germany, while for the rest of the distribution Germany seems to be more volatile, although the dominance in both cases is never statistically significant. This finding is also consistent with previous studies showing that Western Germany used to be a relatively mobile society before the reunification, even more mobile than the US (see, among others, Bayaz-Ozturk et al. (2014) and Maasoumi and Trede (2001)).

Compared to the first period, the volatility picture changes substantially in the second period, 1996-2009 (Figure 7, right panel). For every part of the distribution, US is more volatile than Germany. The difference between these countries is always statistically significant with the exception of the individuals ranked at the very bottom and top of the distribution. This result derives from the different experience of volatility in the two countries (Figure 8). In fact, in the US, the whole distribution and most importantly the lower middle-class become more volatile. In Germany, instead, the whole distribution, with the exception of the very poor (although not significantly), experiences less volatility than before. In particular, for the upper middle class volatility significantly decreases between the two periods.

The estimated coefficients of equation (6) that are reported in Table 2, and visualized in Figure 5, 6, and 7, are also very insightful. With respect to β_0, which refers to the extent of volatility at the median of the income distribution, US and Germany reverse their positions in the two periods investigated. While Germany seems to be more volatile near the median than the US in the period before 1996, the US becomes more volatile and Germany much less in the period after 1996 compared to before (lower values).

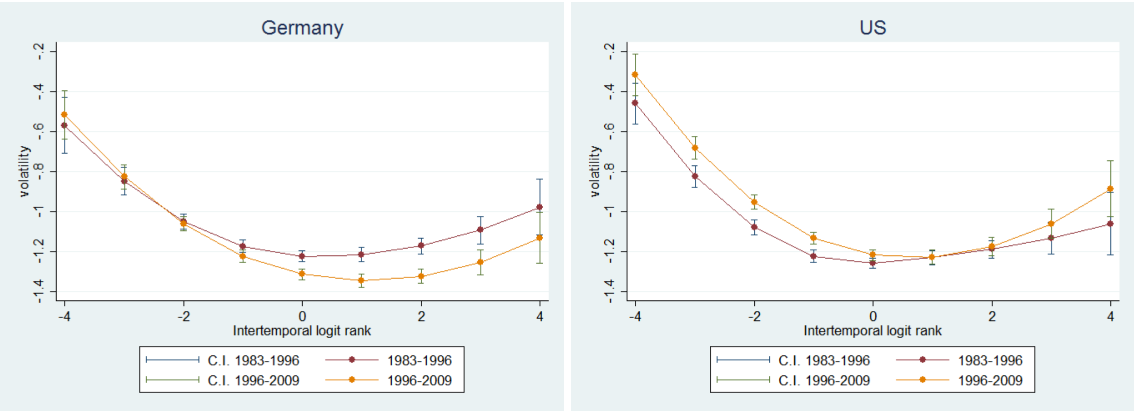
Regarding β_1, the general slope or the first order gradient of income volatility differences between the bottom and the top, Table 2 shows – with the exception of the US in the earlier period – in each case a negative coefficient. In other words, the income rank volatility is on average higher among the poor than among the rich. Looking at the size of the coefficients, a large change over time seems to occur at this level, the balance between the rich and the poor. Both countries see a reduction of this coefficient, which is stronger in US.

Last, β_2 and β_3 inform us on the weight of volatility at the tails of the distribution. These second order coefficients describe the (quadratic shape of the) curvature at the bottom and the top respectively, i.e. the deviation of the extremes from the volatility compared to the median. It appears that the first coefficient, β_2, is always higher in US than in Germany, implying that

US show higher volatility at the bottom compared to the median than Germany. By contrast, the second coefficient, β_3, is higher in Germany in the first period while higher in US in the second period, although it is lower in US if the whole period is considered. This shows that volatility at the top, as compared to that at the bottom, increases in the two countries, but it increases more in US.

Overall, these results confirm that the volatility gap between the poor and the rich tends to decline in Germany as opposed to the US and that the volatility trend of the middle class clearly marks the difference between the volatility experiences of the two countries.

Figure 8: Logitrank volatility. 1983-1996 versus 1996-2009, Germany (left) and US (right).



Source: Authors' computations based on the CNEF.

Although the aim of this section is purely illustrative and any analysis on the determinant of these trends is outside the scope of this paper, some hypothesis on this seems to be reasonable.

First, increasing volatility in US has traditionally been explained by increasing work incentives and labor market flexibility, leading to a reduction in welfare and job security. That the increase was concentrated in the lower-middle part of the population should raise concerns about whether consumption and well-being in that portion of the population has been adversely affected, particularly given the high likelihood of liquidity constraints for this income group and the imperfect public social insurance available to them.

As for Germany, the changes in the structure of earnings in West Germany after the reunification can explain the particular change in volatility that we find here. These changes together with a decrease in the mobility of labor earning may be key explanation of the reduction in volatility for the upper-middle part of the distribution in Germany.

The differences in the trend of US and German income rank volatility can, instead, be explained by differences in the effectiveness of government taxes and transfers and in government policies and welfare system in general in order to smooth out income volatility. Indeed, the US government seems to be less effective with respect to redistribution than Germany.

Table 2. Estimated coefficients of the volatility profiles.

	Overall (1983-2009)		First period (1983-1996)		Second period (1996-2009)	
	US	Germany	US	Germany	US	Germany
β_1	-.0112	-.0332	.0198	-.0103	-.0395	-.0546
(s.e.)	(.0135)	(.0126)	(.0186)	(.0174)	(.0172)	(.0173)
[c.i.]	[-.0375, .0152]	[-.0580, -.0084]	[-.0166, .0562]	[-.0444, .0237]	[-.0733, -.0057]	[-.0885, -.0206]
β_2	.0507	.0372	.0548	.0382	.0464	.0360
(s.e.)	(.0048)	(.0052)	(.0066)	(.0075)	(.0063)	(.0069)
[c.i.]	[.0412, .0601]	[.0271, .0474]	[.0418, .0678]	[.0235, .0529]	[.0340, .0587]	[.0225, .0496]
β_3	.0193	.0219	.0074	.0180	.0306	.0250
(s.e.)	(.0057)	(.0054)	(.0082)	(.0075)	(.0073)	(.0074)
[c.i.]	[.0081, .0305]	[.0113, .0326]	[-.0087, .0234]	[.0032, .0327]	[.0163, .0450]	[.0105, .0394]
β_0	-1.2359	-1.2705	-1.2577	-1.2229	-1.2178	-1.3137
(s.e.)	(.0098)	(.0098)	(.0130)	(.0137)	(.0133)	(.0131)
[c.i.]	[-1.2551, -1.2167]	[-1.2896, -1.2514]	[-1.2831, -1.2323]	[-1.2499, -1.1960]	[-1.2438, -1.1918]	[-1.3395, -1.2880]

Source: Authors' computations based on the CNEF.

4. Conclusions

In this paper, we have proposed a new methodological framework to evaluate and compare volatilities and their trends over time. This framework has departed from the main idea that a better understanding of volatility can be obtained by looking at the extent of this phenomenon in different parts of the distribution. Moreover, it has endorsed the view that income rank volatility may provide additional information that would helpfully complement the rooted consensus among scientists on the extent of income instability. In fact, although there is a huge literature that emphasizes the relevance, in different domain, of individuals relative position in the income distribution, rank volatility has not yet been deeply explored. In order to better capture the movements at the tails of the distribution, we have used the logit transformation of the rank. This transformation satisfies a number of additional statistical properties and makes our methodology an appealing tool in the domain of volatility analysis.

We have applied our framework to evaluate and compare the dynamic of income rank volatility in Germany and US in last two decades. Using the CNEF, we have shown that, in general, the poorer experience more volatility than the richer. The upper middle class households, on the contrary, have been and are getting more stable with respect to their income rank. Hence, aggregate volatility is mostly driven by changes at the bottom of the distribution. This volatility gap is however higher in the US than in Germany. Last, we have shown that while this trend appears to be increasing in US, it is quite stable in Germany.

This work can be extended in a number of directions. From a methodological perspective, it would be interesting to extend such framework to the analysis of exchange mobility. From a more empirical point of view, a potential application of our work would be the European context. When more recent data will be available, a relevant aspect to investigate would be whether or not the U-shape of the income rank volatility profiles has been affected by the recent financial crisis and whether there have been national specificities in this process. Such analysis would also allow understanding if and how the austerity policy introduced in some countries has affected individual instability.

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