Inequality, Informality and credit market imperfections

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Abstract

This paper develops a micro-founded macroeconomic modeling framework to investigate the relationship between informality and income distribution. We show that multiple equilibria may rise if credit markets are imperfect and that there is a non-divisible entry cost in the formal economy. The theoretical analysis demonstrates that in the steady state, low levels of inequality are negatively correlated with high informality; conversely, high inequality exacerbates informality. This finding supports the hypothesis of an optimal rate of inequality that minimizes the informal economy in comparison to other inequality levels. However, for ordinary levels of income distribution, changes in the level of inequality have only a slight effect on the informality ratio. We calibrate the model using data on the US economy to estimate the level of inequality that minimizes the informality ratio.


Keywords: Informal economy; Income Inequality; Formal entry cost; Credit market imperfections.

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1. Introduction

The theoretical and empirical literature on the informal economy has extensively investigated its determinants. An enormous amount of work has been done to develop the patterns of choice that explain decisions of informality. The majority of empirical studies show that informality is positively correlated with income inequality, tax burden, strictness of regulation, and entry costs of starting formal business activity, and it is negatively correlated with enforcement and the quality of economic and legal institutions.\(^1\)

In this paper, we develop a micro-founded macroeconomic modeling framework to investigate the relationship between the informality ratio and income distribution. Although there is prevailing evidence finding that income inequality positively affects the informal economy (see, for example, Rosser et al., 2000, 2003; Ahmed et al., 2007; Chong and Gradstein, 2007), the debate on the nature of this issue remains unsettled. Chong and Gradstein (2007) state that although several theoretical papers generate informality in equilibrium (e.g., Acemoglu, 1995; Acemoglu and Verdier, 1998; Loayza, 1996), none of these papers focuses on inequality. That is, they develop a model in which income inequality, in conjunction with institutional quality, is a determinant of informality in equilibrium. The economic intuition behind their model is that, under financial market imperfections, when protection of property rights in the formal sector is weak, poor individuals are at a disadvantage relative to the rich when operating in the formal sector, “hence, [they] find it beneficial to move into the informal sector, where although less productive, they are able to fully retain their production output” (Chong and Gradstein, 2007, p.160). However, part of the literature notes that the sign of the relationship between the informality and inequality is hard to predict by macro-econometric analyses. Eilat and Zinnes (2002) argue that the informality can affect income distribution through several channels, some increasing inequality and some decreasing it. Concerning the negative correlation between the informal sector and income inequality, Eilat and Zinnes (2002) state that if informal activities are associated with anti-competitive conduct, it may transfer economic surplus from consumers to equity owners, increasing inequality. In contrast, if informal activities provide employment to those with lower income, a “tax-free” informal sector may have a positive effect on income distribution. Consistent with this, the authors find evidence of a non-statistically significant relationship between the size of the informal economy and the Gini coefficient in Transition countries.

We aim to contribute to this literature by defining a model, inspired by Galor and Zeira’s (1993) theoretical framework, in which, other than taxes, the worker’s choice to operate in the formal or informal economy is also characterized by (1) the indivisible entry cost of starting a formal sector business; (2) credit market imperfections; and, at the aggregate level, (3) self-employed workers’ choices to operate formally or informally. Informality depends on self-employed workers’ choices to operate formally or informally because there is a segment of workers (i.e., employees) that do not have the chance to work in the informal economy.

Concerning the first hypothesis, several studies examine the relevance of entry cost in the agent’s choice to work in the formal or informal economy. Most of this literature regards the burdensome entry regulations

\(^1\) See Schneider (2011) for a recent survey of this literature.
as one of the most relevant causes that prevents small firms from becoming formal (e.g., de Soto 1989). This hypothesis has been empirically validated by Djankov (2002), who finds that, at a cross-country level, countries with more burdensome entry regulations have larger informal sectors. Auriol and Warlters (2005) argue that a crucial determinant of micro-enterprises’ informality in developing countries is the existence of high fixed costs involved in becoming formal. Ulyssea (2010) concludes that the magnitude of entry costs into the formal sector is a major determinant of the size of the informal sector. In particular, he finds high entry costs are associated with higher informality and worse labor market indicators in Brazil. However, recent analyses based on micro-data find weaker evidence between registration cost and informality. For instance, Bruhn (2011) observes that in Mexico, the reduction of registration costs increases the number of formal firms, but this increase comes largely from new entries rather than formalization of existing informal firms. Conversely, Kaplan et al. (2011), analyzing the same Mexican reform of registration costs, conclude that burdensome registration regulations may not be the most important barrier to firm creation or firm formalization.

Regarding the second hypothesis, a growing body of literature investigates the role of credit market imperfection and financial development to explain the size of informality. Considering the issue from an empirical perspective, Dabla-Norris et al. (2008) find that financial constraints tend to induce informality among small firms but not among large ones. Leino (2009) observes significant differences in loan use and other less common forms of financing for microenterprises; i.e., although 70 percent of formally registered micro-entrepreneurs use a bank account for their business, only 38 percent of entrepreneurs in the informal sector do. La Porta and Shleifer (2008) and Beck et al. (2010) find that the informal economy is negatively associated with some proxies of the access to finance services. Similarly, Blackburn et al. (2012) and Capasso and Jappelli (2013) conclude that by increasing the opportunity cost to operate informally, financial market development reduces the size of the informal sector.

With reference to the third hypothesis – i.e., the size of informal economy strongly depends on the size of self-employment – it is based on the economic intuition that there is a share of workers (i.e., employees) who lack a chance to work in the informal economy. This assumption is not unprecedented in the literature. For instance, Loayza and Rigolini (2011) measure informal employment as the percentage of the active labor force that is self-employed. Their hypothesis is based on the empirical evidence that in most developing countries there is a strong association between self-employment and informal activity. With reference to advanced economies, Bordignon and Zanardi (1997), Dell’Anno (2007), Dell’Anno et al. (2007) and Dell’Anno and Solomon (2008) find that a significant positive correlation between the size of informality and self-employment exists; therefore, self-employment emerges as one of the most relevant causes of the size of the informal economy.

It is worth discussing the definitions of informality. In contrast to national account statistics, in economic research, the adjectives ‘informal’, ‘shadow’, and ‘unofficial’ are often used synonymously with terms such

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2 See Capasso and Jappelli (2013) for a recent survey.
as ‘economy’, ‘sector’, and ‘employment’. However, these labels refer to distinct statistical aggregates.3 In short, the conceptual framework of the 15th International Conference of Labour Statisticians (ICSE-1993, see ILO, 1993) classifies the status of employment with reference to the distinction between “paid employment” jobs (i.e., employee) on one side and self-employment jobs (i.e., employers, own-account workers, members of producers’ cooperatives, contributing family workers) on the other. We preserve this classification, assuming that informality can exist only among self-employed workers. Accordingly, we assume that the workforce involves three categories of workers: Voluntary Informal workers (VI), who are informal self-employers (i.e., they do not pay taxes and administrative entry costs but encounter higher financial costs and earn a lower income than formal business activities); Voluntary Formal workers (VF), who correspond to the ICSE-1993 definition of self-employment and receive a higher income than VI workers (i.e., formality premium); and Involuntary Formal workers (IF), who include all employees corresponding to the definition of paid employment. For the sake of simplicity, there is no unemployment or workers with multiple jobs, hence, the total workforce is given by the sum of involuntary and voluntary (formal and informal) workers.

In sum, the paper contributes to the existing literature in two ways. First, we fill a gap in the literature concerning the effect of inequality on informality by considering jointly credit market imperfections, indivisible administrative costs and the structure of the workforce. Second, we provide theoretical arguments to the empirical literature that finds ambiguous findings on the relationship between income inequality and the informal economy. In particular, a positive (negative) correlation will occur if income inequality is higher (lower) than the optimal rate of inequality that minimizes the informality ratio.

The paper is organized as follows. Section 2 describes the model framework and theoretical findings. In section 3, we calibrate the model of the U.S. economy to estimate the steady state values of the informality ratio. Section 4 reports the general conclusions.

2. The model

2.1. Notation and Assumptions

Following Galor and Zeira’s (1993) model framework, we consider a small open overlapping-generations economy in which economic activity extends over infinite discrete time. The economy is populated by a continuum of consumer-entrepreneurs who live for two periods. At each date t, \( L_t \) workers are born who are alive at dates t and \( t+1 \); they are referred to as members of generation t. At the same date t, there also live \( L_t \) agents of generation \( t-1 \), in their last period of life. The model implies that each individual has one parent and one child.

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3 See ILO-WEIGO (2013) for a full treatment of the differences among these concepts. In particular, the Resolution concerning Statistics of Employment in the Informal Sector, adopted by the Fifteenth International Conference of Labour Statisticians in 1993, defined the informal sector as “all unregistered or unincorporated enterprises below a certain size, including: micro-enterprises owned by informal employers who hire one or more employees on a continuing basis; and own-account operations owned by individuals who may employ contributing family workers and employees on an occasional basis.” (ILO, 2002, p. 12).
We assume that there exist two types of occupation: self-employed workers who have the opportunity to
decide to be formal or informal worker and paid employees who do not have this opportunity: therefore, they
are inevitably formal workers. Accordingly, in the economy, three types of agents co-exist: Voluntary
Informal; Voluntary Formal (i.e., the official size of the self-employed) and Involuntary Formal (i.e.,
employees). Fixing the amounts of own-account workers and employees of generation \( t \) equal to
\[ \text{VI}_t + \text{VF}_t = \text{VF}_+ \text{L}_t = n \text{ and } \text{VF}_+ \text{L}_t = \text{m}, \]
respectively, the total population size in every time period is
equal to the total workforce of both generations, i.e., \( 2(m+n) \). The economic implication of this
simplification is that the structure of the workforce (i.e., self-employment vs. paid employment) is assumed
to be constant.

2.2. Self-employment

Assume that own-account workers have identical utility functions and differ only in their initial endowment
inherited from their parent \( (b_i) \). Members of generation \( t \) maximize an indirect utility function that is defined
in terms of their terminal (second period of life) wealth \( v_t(w_{i+t}) \), with \( v_t'(w_{i+t}) > 0 \) and \( v_t''(w_{i+t}) < 0 \) subjected
to the lifetime budget constraint \( c_{i+t} + b_{i+t} \leq w_{i+t} \). The agent’s endowment is given by
\[ w_{i+t} = R(e, p, b_i, r) - C(e, f, i, i) \]
where \( e \) are earnings; \( p \) is an “extra” economic value associated with the
status of formality (hereinafter formality premium); \( r \) is the lending interest rate; \( e \) accounts for entry costs
to work formally; \( f \) are deductible financial costs; \( t \) are income and capital taxes; and \( i \) is the borrowing
interest rate.

Two crucial assumptions distinguish our economy: credit markets are imperfect, and an indivisible fixed cost
is required to access the formal economy. Regarding the former, we assume that workers can lend unlimited
funds at the international capital market interest rate \( r \), but the interest rate for borrowers is \( i = r + z \). The
net interest spread \( (z) \) reflects both the moral hazard on the side of borrowers (i.e., the monitoring cost for
lenders to guarantee the reimbursement of debt) and the informal workers’ inability to include a formality
premium \( (p) \) as tangible collateral for the loan. With regards to the second key assumption, workers who
decide to work in the formal sector have to pay a fixed indivisible entry cost \( e \).

Two types of altruism between parents and offspring are considered:

1) Altruism through consumption. Worker’s consumption in the first period is pre-determined by the
parent’s wealth, which means that the consumption of old workers at time \( t \) is a convex combination of

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\(^4\) This factor accounts for the difference in income between formal and informal activity due to constraints on the latter
through marketing policy; size of firm; ability to make financial transactions; expected fine when the taxpayer is caught
cheating; and expected values of unemployment, maternity, sickness benefits, ability to tap business assistance
programs, subsidies to production, etc. This hypothesis is empirically supported, for instance, by Krstic and Sanfey
(2011, 179). They show that “informal employees earn significantly less than those in the formal sector, controlling for
a range of other variables, and informality plays an increasingly important role in explaining earnings inequality”.

\(^5\) In this cost, we include not only public cost (e.g., registration and administrative fees to obtain public authorizations)
but also fixed business taxes different from income and capital taxes or costs of private nature as remunerations for
safety work, employment, legal issues, tax advisers, etc.
offspring and own consumption: \( c_i = \gamma c_{i\text{young}} + (1 - \gamma)c_{i\text{old}}, \) where \( \gamma \) is a parameter that influences the split between parent and offspring consumption. Assuming that offspring consumption is an intrinsic part of household consumption helps to keep the model simple because worker resources in the first period are saved and invested in the capital market at the gross interest rate \( r \) for the second period of life.\(^6\)

2) **Altruism through inheritance.** Parental transfers allow interdependence in occupational decisions across dynasties. Intuitively, the bequest of old self-employed workers of generation \( t-1 \) affects the occupational choice of workers of generation \( t \). These young workers maximize their own utility by maximizing the individual’s second-period wealth; thus, fixing their bequest, they affect the decisions of workers in generation \( t+1 \), and so on.

As anticipated above, voluntary formal workers should earn a gross income \( (e^f) \) in each period, which is higher than the voluntary informal workers’ earnings \( (e^i) \). Accordingly, any formal self-employed benefit from an additional source of wealth in each period of life increases taxable income by \( p \). For the sake of simplicity, the earnings of voluntary informal and formal workers are also assumed to be constant over time. The government levies a flat tax on income. The taxable income of taxpayers is defined as the earnings due to their formal activity and the interest on savings less financial expenses and entry costs. We assume that capital and profit taxes are levied by proportional tax rates \( t_c \) and \( t_p \), respectively.

### 2.2.1. Worker’s timeline

In the first period of life, a (young) self-employed worker decides whether to work in the formal or informal sector in both periods of life. The worker cannot shift from informal to formal occupations between the two periods nor work in the formal and informal economies at the same time.

If he chooses to work in the formal economy during the first period, he must pay an indivisible entry cost, and taxes on income, and in both periods of life, he receives gross earning \( e^f = e^i + p \). If a (young) own-account worker chooses to work in the informal sector, he receives \( e^i \) in both periods and pays tax only on capital income. In the second period of life, all workers allocate their wealth between consumption by their family \((c_{i,1})\) and their bequest \((b_{i,1})\).

With regards to the timing of monetary transactions, payments are settled towards the end of periods; however, to avoid the unrealistic circumstance that entry costs can be paid by worker’s income at time \( t \), we assume that only parental transfer can be used to pay \( e \).

In other words, voluntary formal workers are split in two groups: those with initial endowment \( b_i \) more than the entry cost and the others with an inheritance less than \( e \). The first group saves the excess of parental

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\(^6\) This is a heroic assumption, but it is sometimes used in overlapping generations models (Cassar, 2007). In this setup, the economic interpretation is that the value of \( \gamma \) depends on young’s preferences independently of his/her occupation choice. E.g., it depends on the standard of living of his/her family, which is another way to consider the level of young consumption as a function of parent’s income. However, to avoid an over-parameterization of the model, we simply consider the level of consumption during the first period of the workers’ lives to be exogenous.
transfer for the second period of life at the lender interest rate \( r \). The second group of young formal workers covers the difference between \( \varepsilon \) and \( b_i \) by borrowing at the interest rate \( i = r + z \).

2.2.2. Model Specification

Following the literature, we assume that individuals have identical log-linear utility functions with a parameter \( \alpha \) that influences the splitting between worker’s consumption and bequest:

\[
    u = \alpha \log c_{t+1} + (1 - \alpha) \log b_{t+1}
\]

(1)

and the budget constraint is \( c_{t+1} + b_{t+1} \leq w_{t+1} \) where \( \alpha \in [0,1] \). Maximizing the utility function, we obtain

\[
    c_{t+1} = \alpha w_{t+1}
\]

(2)

\[
    b_{t+1} = (1 - \alpha) w_{t+1}
\]

(3)

That is, the indirect utility function of members of generation \( t, v_t \), increases monotonically in their second period wealth, \( w_{t+1} \):

\[
    v_t = \log w_{t+1} + \mu
\]

(4)

where \( \mu = \alpha \log \alpha + (1 - \alpha) \log (1 - \alpha) \).

Defining the coefficient to convert the capitalized income of the first period of life from gross to net as \( \beta_1 = 1 + r(1 - t_s) \) and the after-tax formal worker’s income before deductions for entry and financial cost as \( \beta_2 = (\varepsilon' + p)(1 - t_n) \), we formalize the lifetime agent’s endowment for the three groups of workers of our economy. The voluntary informal workers’ wealth (or simply Informal) is

\[
    w_{t+1}^{i'} = \varepsilon' + \beta_1 \left( b_i + \varepsilon' \right)
\]

(5)

The voluntary formal workers’ wealth with an initial endowment that exceeds entry cost (or simply Formal lenders) is

\[
    w_{t+1}^{f'} = \beta_2 + \beta_1 \left[ b_i + \beta_2 - \varepsilon (1 - t_s) \right]
\]

(6)

The voluntary formal workers’ wealth with entry cost exceeding their inheritance (or Formal borrowers) is

\[
    w_{t+1}^{f'} = \beta_2 - (\varepsilon - b_i)(r + z)(1 - t_n) - (\varepsilon - b_i)(1 + (r + z)) + \beta_1 \left[ b_i + \beta_2 - \varepsilon (1 - t_s) \right]
\]

(7)

2.2.3. Short and Long run analyses of occupation choice in self-employment

A self-employed worker of generation \( t \) who receives an inheritance \( b_i \) will decide to work in the formal sector if \( v(w_{t+1}^{i'}) = w_{t+1}^{i'}(b_i) > w_{t+1}^{i'}(b_i) = v(w_{t+1}^{i'}) \). By excluding the state of the world in which all workers are

\[^7\] The log or Cobb–Douglas specification for preferences has been popularly used in the related literature on income inequality for the sake of tractability (e.g., Benabou, 1996; Galor and Moav, 2004; Glomm and Ravikumar, 1992; Zilcha, 2003; McDonald and Zhang, 2012).
formal or informal, we derive the bequest at which own-account workers are indifferent between working in the informal and formal sectors in the short run by setting (5) equal to (7):

$$\phi \equiv b_t = \frac{(1+\beta_1)(e' - \beta_2) + e[1+\beta_2(1-t_e)]}{1+\beta_1} > 0$$  \hspace{1cm} (8)

Assuming that $p, e, \epsilon \in [0, \infty]; \alpha, \tau_e, \tau_t \in [0,1]; r, z \in [0,5]$, we find that there exists a positive separating equilibrium ($\phi$) if voluntary informal workers’ earnings are higher than after-tax formal workers’ income before deductions for entry and financial costs. Otherwise, when $e' < \beta_2$, $\phi > 0$ only if $\epsilon > (1+\beta_1)(\beta_2 - e')/[1+\beta_2(1-t_e)] + (1-t_e)\beta_1$.

Three categories of self-employed workers live in this economy: workers with $b_t < \phi$ who prefer to be Informal; agents with $\phi \leq b_t < \epsilon$ who find it profitable to borrow and work in the formal sector, i.e., Formal borrowers; and workers with $b_t \geq \epsilon$ who are Formal lenders.

Formally, substituting (5), (6) and (7) in (3), we obtain a piecewise linear function (eq. 9 - Figure 1):

$$b_{ret} = \begin{cases} 
(1-\alpha)[(1+\beta_1)e + \beta_1b_t] & \text{if } 0 \leq b_t < \phi \\
(1-\alpha)[\beta_2 + \beta_1(\beta_2 - \epsilon(1-t_e)] - \epsilon[1+(r+z)(2-t_e)] + (1-t_e)\beta_1b_t & \text{if } \phi \leq b_t < \epsilon \\
(1-\alpha)[\beta_2 + \beta_1(\beta_2 - \epsilon(1-t_e)] + \beta_1b_t & \text{if } \epsilon \leq b_t 
\end{cases}  \hspace{1cm} (9)$$

**Figure 1:** Thresholds level of bequest in Short Run – Self-employment

In this framework, the distribution of income in period $t$ directly determines the segmentation of the old generation in period $t+1$ between the voluntary formal and voluntary informal workers. Following Galor and

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8 This approach is consistent with the view of informality as a voluntary, equilibrium choice, as proposed in Loayza (1996), Maloney (2004) and Loayza and Rigolini (2011).
Zeira (1993), let the distribution of inheritance at time \( t \) be \( D_t(b_t) \); thus, \( \int_{0}^{\infty} D_t(b_t) \, db_t = L_t = n \), where \( L_t \) is the number of the old self-employed workers in period \( t \). It follows that the fractions of the old generation in period \( t+1 \) that choose to work in the informal and formal economies are \( l_{t+1}^i = \int_{0}^{\infty} D_t(b_t) \, db_t \) and \( l_{t+1}^f = \int_{0}^{\infty} D_t(b_t) \, db_t \), respectively. We will confine our analysis to the steady state; thus, in the long run, the evolution of the bequest is determined by the sequence of \( \{b_t\}_{t=0}^{\infty} \). Fixing \( b_{t+1} = b_t \) in (9), the long run stable \( (\bar{b}^i, \bar{b}^f) \) and unstable \( (g) \) equilibria are:

\[
\bar{b}^i = \frac{(1-\alpha)(1+\beta_i)}{1-(1-\alpha)\beta_i} \epsilon^i
\]

\[
\bar{b}^f = \frac{(1-\alpha)\beta_i + \beta_t[\beta_i - \epsilon(1-\tau)]}{1-(1-\alpha)\beta_t}
\]

\[
g = \frac{(1-\alpha)\epsilon \left[ 1 + (r + z)(2-t_s) + (1-t_s)\beta_t \right] - (1-\alpha)\beta_i - \beta_2 - \beta_1}{(1-\alpha)(r + z)(2-t_s) + \beta_i - \alpha(1+\beta_i)}
\]

These solutions allow the inference of own-account choice to be a formal or informal worker by looking at the level of bequest. To obtain multiple equilibria as shown in Figure 2, the parameters’ domains should be constrained as follows:

\[
\begin{align*}
\epsilon \in & \left[ (1+\beta_t)\epsilon \left[ (1-\alpha)r(2-t_s) + (1-\alpha)(1+\tau) + (1-\alpha)(2-t_s)z \right] - p(1-t_s)\beta_t(1-\alpha-1) \right] \left(1-\alpha\right) \left(1+\beta_t\right) \\
\alpha \in & \left[ r(1-t_s) \beta_i - \frac{1}{r(t_s - (3-t))} \left( 2 + (2-t_s)z \right) + 1 \right]
\end{align*}
\]

**Figure 2:** Thresholds levels of bequest in Long Run Equilibria – Self-employment

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9 The domains are derived to hold the following constraints: (1) slopes of dynamic path at \( \bar{b}^i, \bar{b}^f \) should be lower than 1 and at \( g \) greater than unit; (2) the levels of bequest have to be ranked as: \( \phi < g < \epsilon < b^i \) and (3) parameters on taxes and credit market should be constrained over realistic domains.
In the long run, self-employment is concentrated in two groups: Formal (richer) workers, for whom the bequest is equal to \( \bar{b}' \), and poor generations, among whom self-employed workers are informal with a bequest equal to \( \bar{b}'' \). The unstable equilibrium \( g \) is the critical value of bequest that determines the long run size of the informality ratio. The first result of a *ceteris paribus* analysis of the dynamic property of our model is that the informal sector increases endogenously when the economy transitions from short to long run equilibria. Based on this evidence, while self-employed workers who inherit between \( \phi \) and \( g \) find it optimal to work formally, they will transfer to their offspring a bequest lower than their inheritance; hence, their descendants will work as informal workers as soon as parent transfers decline to \( \phi \). That is, the bequests of these dynasties continue to decline to converge to the stable long run equilibria \( \bar{b}'' \).

### 2.3. Paid employment

Employees are involuntary formal workers, and because of the nature of their occupation, they do not have to pay the entry cost (\( \varepsilon = 0 \)). Accordingly, we assume that for these generations, *altruism* works only through consumption, i.e., there are no bequests (\( \alpha = 1 \)). That is, the steady state lifetime employee’s wealth is

\[
\begin{align*}
\ell c_t = \ell c_{t+1} = \ell w_f' = \ell w_{f+1}' = \ell w_f' = (\ell + p)(1-\ell r)[2 + r (1-\ell r)] = \beta_2 (1 + \beta_1)
\end{align*}
\]

### 2.4. Aggregate equilibria for the two types of occupation

In this section, we compute the aggregate level of informal and formal economy for own-account workers and wage earners to outline the relationship between income distribution and the size of the informal economy.

#### 2.4.1. Self-employed worker’s income

By extending Galor (2011), we determine the level of income per capita of voluntary formal workers. Assume that the public sector is in balance in each period and tax revenue finances only public transfer
payments; hence, we obtain a measure of GDP by using the income approach. Taking into account that in the long run, no formal workers run into debt, the income per capita of the voluntary formal \( \left( v_{yf} \right) \) and voluntary informal \( \left( v_y \right) \) workers are the average incomes of the old and young self-employed living in the same \( n \) households:

\[
v_{yf} = \frac{2(e' + p) + (e' + p + \bar{b} - \varepsilon)r}{2}
\]

\[
v_{y} = \frac{2e' + (e' + \bar{b})r}{2}
\]

Let the distribution of inheritance at time \( t \) be \( D_t(b_t) \); then, the critical level of bequest \( (g) \) determines the long-run composition of the labor force. Thus \( \lim_{t \to +\infty} \int_0^t D_t(b_t) db_t = \bar{I}^\prime \) and \( \lim_{t \to +\infty} \int_g^\infty D_t(b_t) db_t = \bar{I} \) are the fractions of informal and formal workers in each generation, respectively. Accordingly, an increase of \( g \) alters the informality ratio. Figure 3 shows this outcome by assuming a lognormal distribution of bequests.

**Figure 3:** Lognormal Distribution of Bequests – Self-employment

Considering that the own-account workforce in each period is \( 2n \), the GDP yielded by voluntary formal and informal workers are:

\[
^\prime Gdp' = 2n\bar{I}^\prime \left( v_{yf} \right) = n\left[ 2(e' + p) + (e' + p + \bar{b} - \varepsilon)r \right]\int_0^\infty D_t(b_t) db_t
\]

\[
^\prime Gdp = 2n\bar{I} \left( v_y \right) = n\left[ 2e' + (e' + \bar{b})r \right]\int_0^\infty D_t(b_t) db_t
\]

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10 \( v_{yf} = e' + p \), \( v_{yf} = (e' + p + \bar{b} - \varepsilon)r \), \( v_{yf} = e' \) and \( v_{yf} = e' + (e' + \bar{b})r \).
2.4.2. Wage earner’s income

The per capita income of involuntary formal workers \( \bar{y}' \) is the average income of the old, young, and young workers, \( \bar{y}'_{\text{young}} = e' + p \), living in the same \( m \) households; hence, we obtain

\[
\bar{y}' = \frac{(e' + p)(2 + r)}{2}.
\]  \hfill (18)

Considering that the number of employees in each period is \( 2m \), the extent of the involuntary formal economy is

\[
\bar{Gdp}' = 2m(\bar{y}') = m(e' + p)(2 + r)
\]  \hfill (19)

2.5. Aggregate Informal and Formal Economy

Considering that the population in each period is \( L = 2(n + m) \), the size of informal economy is \( (17) \), and the formal economy is given by \( (16) \) and \( (19) \), i.e., \( Gdp' = V Gdp' + F Gdp' \); then the informality ratio is given by

\[
\frac{V Gdp'}{F Gdp'} = \frac{\text{income per capita VI}}{\text{income per capita IF}} \left[ \frac{2e' + (e' + \bar{b}')r}{(e' + p)(2 + r)} \right] \int_{b'_l}^{b'} D_l(b_i) \, db_i \left[ \frac{M}{n} + \frac{2(e' + p + \bar{b}' - e)\varepsilon}{\text{income per capita VF}} \right] \int_{b'_l}^{b'} D_v(b_i) \, db_i
\]  \hfill (20)

where \( m/n \) is the reciprocal of (formal and informal) self-employed workers to the number of (formal) wage earners; hence, it depends on the structural characteristics of the labor market in the economy and is unaffected by the agent’s choice to work formally or informally.

This formulation emphasizes that changes in the informality ratio can be disentangled by distinguishing two types of effects: a “size effect” that depends on separating equilibrium \( g \) and determines the number of informal and formal self-employed workers; and an “income effect” that depends on the earnings of informal and formal self-employed workers (i.e., pooling equilibria \( \bar{b}' \) and \( \bar{b}' \)). That is, the variations on the steady-state informality ratio are due to interactions between changes of income per capita of formal and informal workers and the sizes of two types of employment (i.e., self- and paid employment).

In view of this result, by assuming that the distribution of workers’ wealth (see equation 3) can be approximated by the income distribution and that the latter follows an unimodal distribution (e.g., Lognormal), the theoretical model suggests that in the steady state, the sign of the relationship between the informality ratio and inequality depends on the “size effects”. Thus, we extrapolate the following propositions:

**Proposition 1. Inequality and Informal Workforce**

A concave upward curve exists between income inequality and the informal workforce \( (\bar{I}') \).
The economic intuition behind this result is that, from one side, in the presence of credit market imperfections, a high inequality generates a large share of young self-employed with a low level of inheritance. This event strongly increases the financial cost of the bank loan required by young self-employed workers to pay entry costs. Accordingly, high inequality increases the cost of formal business activity; hence, it boosts voluntary informal self-employment. On the other side, an excessively low inequality is associated with excessive redistribution, and in turn excessive income tax burden. In this situation, a higher tax rate on income in the formal sector makes working in the formal sector less attractive relative to working in the untaxed informal sector; hence, it increases the size of informal self-employment.  

Proposition 2. Inequality and formal workforce

A concave downward curve exists between income inequality and the formal workforce ($\bar{I}^f$).

Considering that $VF=1-VI$, the economic interpretation is straightforward.

Lemma 1. Inequality and informality workforce ratio

A concave upward curve exists between income inequality and the ratio between the informal and formal workforce $n\int_0^\infty D_i(b)b\left(m+n\int_0^\infty D_i(b)b\right)$.

With reference to the total effect of inequality on the informality ratio, considering that income distribution does not affect the income per capita of workers (i.e., $d^\prime(v')/d(Gini)=d^\prime(v')/d(Gini)=d^\prime(v')/d(Gini)=0$), from Lemma 1, we conclude the following:

Lemma 2. Income Inequality and Informality ratio

In the steady state, a concave upward curve exists between income inequality and the ratio between informal and formal GDP.

3. Inequality and Informality: a simulation analysis for the USA

3.1. Calibration

In this section, we calibrate the model to match the U.S. economy. Our main aim is to illustrate the effect of income inequality on the informality ratio by disentangling the dynamics of income and sizes for the three types of workers (i.e., $VF, VI, IF$) considered in our simplified economy.

11 This result confirms, with a different economic motivation, Valentini’s (2009) conclusion about the indefiniteness of the sign of correlation between inequality and informality. In fact, Valentini (2009) argues that there are no reasons to suppose that a growth in unrecorded income is uniform along income distribution. In particular, the sign of this correlation depends on the predominant nature of the informal income, i.e., if the informal income is higher for poorer individuals, a positive relationship could exist between the size of informal employment and income inequality, or vice versa.
Concerning the parameterization of long-run propensity to bequeath (α), we refer to Gale and Scholz’s (1994) estimates based on data from the 1983-86 Survey of Consumer Finances. They find that intended transfers – such as gifts from parents to adult children living in a separate household – plus bequests account for at least 51 percent of U.S. net worth.¹² Accordingly, we fix α = 0.51.

For the estimate of U.S. credit market parameters, we use the average from 2000 to 2010 of data extracted from World Development Indicators (WDI). In particular, the estimate of the spread between lending and borrowing interest rate (z) are fixed equal to double the average risk premium on lending.¹³ We consider a double value because loans allocated to less-established business activities have a relatively higher default risk; it then seems more realistic that the risk premium on interest rate charged by banks on loans to young worker should be higher than the average risk premium (z = 2 * 0.0325). For the lending interest rate (r), we use the average of the lending interest rate adjusted for inflation as measured by the GDP deflator (i.e., real interest rate; WDI, series code: FR.INR.RISK). As a consequence, the borrowing interest rate (i) is equal to 9.9%, which is a realistic annual percentage rate for high-risk business loans.¹⁴

For calibration of tax rates, we use the standard long-term capital gains tax rate in the USA (tₜ = 0.15). For the tax burden on formal worker’s income (tₓ), we calculate the average from 2000 to 2010 of U.S. “Income tax plus employee and employer contributions less cash benefits, single persons, 100% of average earnings” extracted from OECD Taxing wages 2014 database, hence tₓ = 0.30.¹⁵

Estimates of income of informal workers (e'), formality premium (p) and entry cost (ε) are not available; therefore, we fix these values to hold the constraints on the parameter domains necessary to generate multiple equilibria, as well as setting formal gross worker’s income (e') equal to the average (2000-2010) of formal GDP per capita at constant 2005 international $ (i.e., e' = e + p = $41,555; WDI series code: NY.GDP.PCAP.PP.KD). As a result, we fix p = $25,455 and e' = $16,100 and ε = $34,000.

Concerning bequest distribution, as mentioned above, we obtain a feasible distribution of Dₐ(bₐ) by assuming that the distribution of workers’ wealth can be approximated by the income distribution, i.e., W_r(w_r) = Y_r(y_r), where Y_r follows a lognormal distribution Y_r(y_r) = Λ(μ_r, σ_r) with μ_r and σ_r, being the

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¹² They also estimate that parental transfers for educational expenses account for an additional 12 percent of net worth. We did not add this percentage because these transfers occur at time t and can be considered part of the young’s educational consumption. According to the model assumption, the offspring’s consumption at time t is an intrinsic part of household consumption, therefore they cannot be included in the parent’s bequest at time t+1 and used to pay the entry cost for young workers from time t+1.

¹³ “Risk premium on lending” (WDI, series code: FR.INR.RISK) is defined by WDI as the interest rate charged by banks on loans to private sector customers minus the “risk free” treasury bill interest rate at which short-term government securities are issued or traded in the market.

¹⁴ WDI reports data on the lending rate (series code: FR.INR.LEND). It is defined as “the bank rate that usually meets the long- and medium-term financing needs of the private sector. This rate is normally differentiated according to creditworthiness of borrowers and objectives of financing.” In the decade before the last economic crisis (1998-2007) the average is 6.8%. Accordingly, considering that self-employed and small firms are usually classified as high risk business loans, an estimate of the U.S. annual percentage rate (APR) of 9.9% seems to be a realistic average value for this kind of business activity.

¹⁵ These calibration values of capital and income tax rates are also jointly consistent with the WDI statistics of the total amount of taxes and mandatory contributions payable by businesses after accounting for allowable deductions and exemptions as a share of commercial profits. The average for the available data (period 2005-2010) is equal to 0.46.
mean and standard deviation, respectively, of the associated normal distribution, while we denote $m_\gamma$ and $\lambda_\gamma$ as the mean and standard deviation of the non-logarithmized sample values.\(^{16}\)

Taking into account that the self-employed worker’s budget constraint (eq. 3) assumes a distribution of bequests as a linear transformation of $Y_i(y_i)$, we obtain $D_i(h_i) = \Lambda(\mu_\gamma + Ln(1-\alpha_\gamma),\sigma_\gamma = \sigma_\gamma' \gamma)$ This transformation demonstrates that the workers’ bequest distribution is a shifted log-normal with parameters that can be derived from observable data. Specifically, following Aitchison and Brown (1963), let $\Phi^{-1}$ denotes the inverse of the cumulative distribution of a standard normal variable; we derive the standard deviation and mean of the associated normal distribution of income per capita distribution from the Gini index (i.e. $\sigma_\gamma = \sqrt{2}\Phi^{-1}[(Gini + 1)/2]$) and real GDP per capita (i.e. $\mu_\gamma = Ln(m_\gamma) - \sigma_\gamma^2/2$). Hence, substituting the parameter that influences the splitting between self-employer worker’s consumption and bequest ($\alpha=0.51$), the distribution of bequests is given by

$$D(h) \sim \Lambda(\mu_\gamma, \lambda_\gamma, \alpha_\gamma = 0.1529)$$

where $m_\gamma$ is assumed to be equal to the formal real GDP per capita (i.e., $e' = $41,555) and “Gini” is the estimated index of Gini for USA in the 2000 – the only available value over the period 1995-2014 in the WDI database (i.e., $Gini = 0.4081$).

The ratio between $IF$ employment (i.e., paid employment) and the sum of $VF$ and $VI$ employment (i.e., self-employment) cannot be directly observed; hence, we need to use information from available data and theoretical results to obtain an estimate of $m/n$. In particular, we estimate from WDI data the average over the period 2000-2010 of (formal) self-employed on the total formal workers: $VF/(VF+IF) = 0.0731$; hence, the ratio between paid-employment on total formal workers is easily estimated as $IF/(VF+IF) = 0.9269$. We combine these statistics with a result of our theoretical model. Specifically, substituting calibration values to predict the critical level of bequest (i.e., $g = $33,203) and substituting this value in the ratio between formal self-employment and total self-employment (i.e., $\bar{I}' = VF/(VF+VI) = 1-\int_0^\infty D(h)b(h) dh$), we obtain $\bar{I}'=0.1529$. Accordingly, the ratio between paid employment and self-employment can be calculated as:

$$\frac{m}{n} = \frac{IF}{VF+VI} = \frac{VF+IF}{VF} \frac{IF}{VF+IF} \frac{VF}{VF+VI} = 0.0731^{-1} * 0.9269 * 0.1529 = 1.939$$

(22)

As a result, the size of paid employment is expected to be about twice as much as the total (formal and informal) self-employment. Table 1 summarizes the calibration values used in the simulation.

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\(^{16}\) The following relationship exists between these parameters: $\mu_\gamma = \ln\left(m_\gamma^2/\sqrt{m_\gamma^2 + \lambda_\gamma^2}\right)$ and $\sigma_\gamma = \sqrt{\ln\left(1+\lambda_\gamma^2/m_\gamma^2\right)}$. 

15
### Table 1 – Calibration Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Propensity to bequeath</td>
<td>51%</td>
</tr>
<tr>
<td>$r$</td>
<td>Interest rate for lenders</td>
<td>3.4%</td>
</tr>
<tr>
<td>$z$</td>
<td>Spread between lending ($r$) and borrowing ($i$) interest rates</td>
<td>6.5%</td>
</tr>
<tr>
<td>$t_\pi$</td>
<td>Tax rate on (formal) worker’s income</td>
<td>30%</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Tax rate on capital income</td>
<td>15%</td>
</tr>
<tr>
<td>$e^f$</td>
<td>Earnings of formal worker</td>
<td>$41,555</td>
</tr>
<tr>
<td>$e^i$</td>
<td>Earnings of informal worker</td>
<td>$16,100</td>
</tr>
<tr>
<td>$p$</td>
<td>Formality premium: $p = e^f - e^i$</td>
<td>$25,455</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Fixed indivisible entry cost</td>
<td>$34,000</td>
</tr>
<tr>
<td>$Gini$</td>
<td>Gini index</td>
<td>0.4081</td>
</tr>
<tr>
<td>$VF/(VF+IF)$</td>
<td>Formal self-employment / Total formal employment</td>
<td>7.31%</td>
</tr>
</tbody>
</table>

In conclusion, substituting $\tilde{b}^i$ from (10), $\tilde{b}^f$ from (11), $g$ from (12), the distribution of bequests $D(h_j)$ from (21), the ratio between involuntary formal worker and total self-employment $(m/n)$ from (22) and using calibrated values of Table 1 in the equation (20), we can simulate the effects of inequality on the steady state value of the informality ratio.

#### 3.2. Simulated results

Once we have calibrated the model and fixed the index of Gini equal to 0.4081, we can estimate the steady state values of formal wage earner’s gross income per capita ($y^f = $42,261), informal and formal self-employed worker’s income per capita ($y^i = $16,923; $y^f = $40,264) and the percentage of informal workforce in self-employment ($\overline{T}^i = 0.8471$). Figure 4 shows the relationship between the (formal and informal) workforce in self-employment and income inequality, i.e., propositions 1 and 2.

![Figure 4: Inequality and Workforce composition in Self-Employment](image-url)
Accordingly, including income effects, we are able to estimate the effect of variations of income inequality on the steady state value of informal production in the formal U.S. economy. Specifically, with a Gini index of 0.4081, the informality ratio is 16.2%.\textsuperscript{17}

Figure 5 shows the nonlinear relationship between inequality and the informality ratio over the whole range of the Gini index (i.e., Lemma 2).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Inequality and Informality ratio}
\end{figure}

The relationship is concave upward, therefore low (high) levels of inequality exerts a negative (positive) correlation with the informality ratio. It also demonstrates the existence of an optimal rate of inequality that minimizes the informality ratio in comparison to other inequality levels (i.e., if $Gini=0.51$ then $Gdp^f/Gdp^i=15.1\%$).

4. Conclusion

In this paper, we attempt to accommodate different approaches proposed in literature by modeling an appropriate characterization of workers’ decision to operate formally or informally in a parsimonious way. Following Rosser et al. (2003), Dessy and Pallage (2003) and Chong and Gradstein (2007), we build a model in which inequality affects the sizes of informality in equilibrium. Looking at Djankov et al. (2002), Straub (2005) and Antunes and Cavalcanti (2007), we assume that workers choose between formal and informal activities by trading off higher entry costs, tax obligations and gross income in the formal sector against higher financial costs in the informal sector. In this sense, we argue that the expense of starting a formal sector business, the inability to tap formal credit channels and, more generally, the various types of public assistance programs available to the formal sector are the main incentives and costs to operate in the informal sector mainly for small firms and self-employed workers.

\textsuperscript{17} The empirical estimates of the ratio between informal and formal economy for the 2000 provided by Schneider et al. (2010) and Alm and Embaye (2013) are 8.7% and 14%, respectively.
Following Galor and Zeira’s (1993) framework, we develop an overlapping-generations model to investigate the relationship between inequality and informality. In the proposed model, multiple equilibria may arise if credit markets are imperfect and there is a non-divisible entry cost in formal economy. Thus, under self-financing constraints at an individual level, income distribution affects the relative size of the informal aggregate production in both the short and long run.

The theoretical analysis demonstrates that in the steady state the relationship between inequality and informality is concave upward, therefore for countries with low levels of inequality is expected a negative correlation with informality; conversely, high inequality exacerbates informality. This finding supports the hypothesis of an optimal rate of inequality that minimizes the informal economy in comparison to other inequality levels. However, for ordinary levels of income distribution, changes in the inequality have only slight effects on the informality ratio.

In the second step of our analysis, the model is calibrated to match the U.S. economy. Considering the observed value of the Gini index in 2000, we estimate that the steady state level of the U.S. informality ratio is equal to 16.2%. In terms of policy implications, our model predicts that, if we increase the actual index of Gini to the optimal rate of inequality to minimize the informal economy (i.e., Gini = 0.51), the level of informality may decrease by approximately 1 percentage point (i.e., 15.1% of formal GDP).

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18 For instance, for USA it occurs for values of Gini index ranging between 0.4 and 0.7.
Reference


