Money Laundering in a Bayesian DSGE model

Amedeo Argentiero* Carlo Andrea Bollino†

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Abstract

In this paper we build a three-sector DSGE model to measure money laundering. The economy is populated by a sunlight firm, an underground firm and a criminal firm. Two kinds of goods are produced: a legal good and a criminal good. The former can be produced in a perfectly competitive market regime either by the sunlight firm and the underground firm, whereas the latter is produced only by a monopolistic criminal firm. Money laundering is generated by the profits coming from criminal firms that need to hide their activity with the corresponding outcome. The demand side of the economy is populated by an infinite number of households with preferences defined over legal and criminal good consumption, public expenditure and labor services on a period-by-period basis. The Government collects taxation from the sunlight sector and fights tax evasion, criminal economy and money laundering through audit activity undertaken by public officers.

*University of Perugia, Department of Economics. Corresponding author: via Alessandro Pascoli 20, 06123 Perugia, Italy. Phone (+39)0755855293, fax (+39)0697254249, e-mail: amedeo.argentiero@unipg.it

†University of Perugia, Department of Economics and AIEE

The model is estimated for Italy, using Bayesian techniques from 1980 to 2014. Then we analyse the dynamic behavior of the model variables through impulse response functions and finally we generate through MCMC (Monte Carlo Markow Chains) methods a time series for money laundering, criminal economy and underground economy.

JEL codes: D58, K34, K42.

Keywords: Money Laundering, Criminal Economy, Underground Economy, Sunlight Economy

1 Introduction

Money Laundering is the process by which criminal organizations try to disguise the criminal origin of a sum and hence represents a sort of "by product" of criminal economy. The use of more sophisticated techniques to launder money has made it increasingly difficult for the authorities to find out this kind of activity. Therefore criminal activities have used money laundering over time as a good technique for risk diversification.

In this paper we construct a three-sector DSGE model made up of a sunlight sector, an underground sector and a criminal sector, in the spirit of Argentiero and Bollino (2015). However, in this context we assume that criminal sector operates according to a monopolistic market regime, whereas legal sectors (underground and sunlight) are perfectly competitive. The monopolistic market regime appears to be more realistic than perfect competition expecially for the criminal sector; in fact if profits were null in the long-term (that is the outcome of a perfectly competitive market), criminal economy and money laundering would be only short-term phenomenons able to disappear with an increasing competition in the market. Indeed, the presence of high entry costs makes criminal market not competitive.

A part of the profits coming from the criminal sector are laundered either through sunlight firms and underground firms. The sunlight firm is subject to distortionary taxation, whereas the underground firm evades taxation. The criminal good is produced only by the criminal firm, that evades any form of taxation in addition to violating the criminal law.

The economy is subject to stochastic uncorrelated technology shocks on total factor productivity on private sectors and public labor and to fiscal shocks on tax rates.

The demand side of the economy is populated by an infinite number of households with preferences defined over legal good consumption, criminal good consumption, public expenditure and labor services on a period-by-period basis.

The Government collects taxation from the sunlight sector and fights tax evasion, criminal economy and money laundering through audit activity undertaken by public officers. When detected, underground firms are subject to regular taxation and additional fine payments, whereas criminal firms and money laundering firms are forced to close, i.e. their workers are arrested and the capital confiscated. Nevertheless, because the endogenous probability of being detected is lower for a money laundering firm than for a criminal one, criminal sector chooses money laundering as a risk sharing activity.

We estimate the model with Bayesian techniques for Italy from 1980 to 2014, we analyse the dynamic behavior of the model variables through impulse response functions and finally we generate through MCMC (Monte Carlo Markow Chains) methods a time series for money laundering, criminal economy and underground economy. These data are compared with some official data and the main findings of the literature on unobserved economy.

2 The model structure

2.1 The firms

The supply-side of the economy is populated by three kinds of firms, the *sunlight* firm, the *underground* firm and the *criminal* firm, that produce two different goods: the legal good Y^l and the criminal good Y^c. The former can be produced either by the sunlight firm or the underground firm, whereas the latter is produced only by the criminal firm.

We assume a perfect competition market regime for legal sector, whereas criminal firms acts as a monopolistic producer.

The firms are owned by the households who earn profits in the form of dividends, as we will show in the next section.

Total production Y is allocated to these three sectors according to a percentage α for *sunlight* production, β for *underground* production and $1-\alpha-\beta$ for *criminal* production, so that the sum of the enterprises' shares of total output is equal to unity:

$$Y_t = \underbrace{(\alpha+\beta)\,Y_t}_{Y^l} + \underbrace{(1-\alpha-\beta)\,Y_t}_{Y^c} \tag{1}$$

Moreover, a share ι_t of criminal profits are laundered through legal economy:

$$m_t = \iota_t * E(\pi_t^c) \tag{2}$$

where m is money laundering and π^c are criminal profits. In particular, a share ϱ_t of money laundering belongs to the underground firms, whereas $1-\varrho_t$ is money laundering undertaken by the sunlight firms, i.e.:

$$\begin{aligned} m_t^s &= \varrho_t * m_t \\ m_t^u &= (1-\varrho_t) * m_t \end{aligned}$$

The production functions have constant returns to scale and use as inputs labor, n, capital, k and money laundering, m.

$$\begin{aligned} \alpha Y_t &= Y_t^s = \lambda_t^s\,(k_t^s)^{\gamma}\,(n_t^s)^{\varpi}\,(m_t^s)^{1-\gamma-\varpi} && (3) \\ \beta Y_t &= Y_t^u = \lambda_t^u\,(k_t^u)^{\delta}\,(n_t^u)^{\kappa}\,(m_t^u)^{1-\delta-\kappa} && (4) \\ (1-\alpha-\beta)\,Y_t &= Y_t^c = \lambda_t^c\,(k_t^c)^{\zeta}\,(n_t^c)^{1-\zeta} && (5) \end{aligned}$$

The superscripts s, u and c stand for sunlight, underground and criminal, whereas $\lambda_t^s, \lambda_t^u, \lambda_t^c$ represent total factor productivities (TFP) whose law of motion is described as follows:

$$\Lambda'_{t+1} = \Sigma * \Lambda'_t + \upsilon_t, \tag{6}$$

where $\Lambda_t = [\lambda_t^s, \lambda_t^u, \lambda_t^c, \lambda_t^{gu}, \lambda_t^{gc}]$ is a stochastic disturbance vector including TFP, υ_t is a vector of the shocks' innovations, λ_t^{gu} represents fiscal agent labor productivity and λ_t^{gc} is criminal police labor productivity; the autocorrelation coefficient matrix Σ and the covariance matrix Ξ are defined below:

$$\Sigma = \begin{bmatrix} \varphi_{\Lambda^s} & 0 & 0 & 0 & 0 \\ 0 & \varphi_{\Lambda^u} & 0 & 0 & 0 \\ 0 & 0 & \varphi_{\Lambda^c} & 0 & 0 \\ 0 & 0 & 0 & \varphi_{\Lambda^{gu}} & 0 \\ 0 & 0 & 0 & 0 & \varphi_{\Lambda^{gc}} \end{bmatrix} \text{ and } \Xi = \begin{bmatrix} \xi_{\Lambda^s} & 0 & 0 & 0 & 0 \\ 0 & \xi_{\Lambda^u} & 0 & 0 & 0 \\ 0 & 0 & \xi_{\Lambda^c} & 0 & 0 \\ 0 & 0 & 0 & \xi_{\Lambda^{gu}} & 0 \\ 0 & 0 & 0 & 0 & \xi_{\Lambda^{gc}} \end{bmatrix} \tag{7}$$

Notice that in this model firms differ in productive structure and in expected profit level. Profits crucially depend on the probabilities of being detected when a firm belongs to an unobserved sector.

Define a price vector for this economy as $\left[p_t^l, p_t^c, w_t^s, w_t^u, w_t^c, w_t^{gu}, w_t^{gc}, r_t^s, r_t^u, r_t^c, i_t^s, i_t^c\right]$ where p_t^l is the legal good price, p_t^c is the criminal good price, w_t^s, w_t^u, w_t^c, w_t^{gu}, w_t^{gc} are sunlight, underground, criminal, fiscal agent and criminal police wages, r_t^s, r_t^u and r_t^c are the rentals of sunlight, underground and criminal capital and i_t^s and i_t^u are sunlight and underground rentals for "dirty" money m_t^s and m_t^u. Normalizing the legal price p_t^l to unity, the normalized price vector supporting the equilibrium equals $\left[1, p_t^{c*}, w_t^{s*}, w_t^{u*}, w_t^{c*}, w_t^{gu*}, w_t^{gc*}, r_t^{s*}, r_t^{u*}, r_t^{c*}\right]$ where starred variables denote equilibrium values.

The *sunlight* firm's net profit structure reads as:

$$\pi_t^s = [\alpha (1 - \tau_s) Y_t - (1 + s) w_t^s n_t^s - r_t^s k_t^s - i_t^s m_t^s] \tag{8}$$

where τ_s is a distortionary tax rate on sales and $(1+s)$ are social security contributions, that are a percentage of the wages paid, n_t^s is the labor offered in the sunlight sector and k_t^s is the capital invested in the sunlight firm (Argentiero and Bollino, 2014, 2015).

The *underground* firm evades any form of taxation, and hence profits are higher than those of the *sunlight* firm, but only if it is not detected evading, that happens with an endogenous probability d_t, whose determinants will be discussed below; if, instead, the underground firm is detected evading, with probability $1 - d_t$, it is fined with the same amount of taxation as the *sunlight* firm **plus** a penalty factor, $\vartheta^u k_t^u$ with $0 < \vartheta^u < 1$, that is a fraction of the capital invested in the underground firm.

The underground firm's expected profit structure is:

$$\begin{aligned} E(\pi_t^u) \quad &= \quad d_t \underbrace{(\beta Y_t - w_t^u n_t^u - r_t^u k_t^u - i_t^u m_t^u)}_{\pi_{nd}^u} \\ &\quad + (1 - d_t) \underbrace{[\beta (1 - \tau_s) Y_t - (1 + s) w_t^u n_t^u - k_t^u (r_t^u + \vartheta^u)]}_{\pi_d^u} \end{aligned} \tag{9}$$

Notice that the first part of the sum identifies underground firm profits in the case of not detection (π_{nd}^u), whereas the second part represents profits whenever the underground firm is detected evading (π_d^u) (Argentiero and Bollino, 2014, 2015).

The *criminal* firm evades any form of taxation and violates penal law (i.e. the production of a criminal good) if it is not discovered, that happens with probability e_t; if, instead, the criminal firm is detected evading and committing criminal offenses, with probability $1 - e_t$, it is hit by a penal fine, $r_t^c k_t^c + w_t^c n_t^c$, equal to the entire value of labor and capital employed in the criminal production. This means, that, whenever a criminal firm is discovered, it is forced to close. Criminal labor detected is arrested and becomes a social cost for its rehabilitation, whereas criminal capital is confiscated and represents a positive resource for the Government. Therefore, criminal firm's expected profits can be expressed as:

$$
\begin{aligned}
E(\pi_t^c) \quad = \quad & e_t \left[\underbrace{p_t^c \left(1 - \alpha - \beta\right) Y_t + i_t^s m_t^s + i_t^u m_t^u - w_t^c n_t^c - r_t^c k_t^c}_{\pi_{nd}^c} \right] \\
& + (1 - e_t) * \underbrace{0}_{\pi_d^c}
\end{aligned}
\tag{10}
$$

Also in this case the first part of the sum identifies criminal firm profits in the case of not detection (π_{nd}^c), that include also the rentals coming from money laundering activity, whereas the second part represents profits whenever the criminal firm is detected (π_d^c).

2.2 The households

The demand-side of the economy is populated by an infinite number of infinitely-lived households with preferences defined over legal consumption C_t^l, with share η^l, criminal consumption C_t^c, with the share η^c, public investments G_t, with share $1 - \eta^l - \eta^c$, and labor services N_t. These latter are allocated to regular production (N_t^s), underground production (N_t^u), criminal production (N_t^c) , fiscal agent activity (N_t^{gu}) and criminal police activity (N_t^{gc}) on a period-by-period basis.

As we show in section 3.3 public investments represent the "profit" of the Government that aims at maximizing net fiscal revenues.

Each agent maximizes the expected value of an intertemporal utility function, i.e.:

$$E_0 \sum_{t=0}^{\infty} \rho^t U_t \left(C_t^l, C_t^c, N_t^s, N_t^u, N_t^c, N_t^{gu}, N_t^{gc}, G_t \right) \tag{11}$$

with ρ^t corresponding to the subjective discount factor.

We assume that there is an idiosyncratic cost in supplying labor in the unobserved sectors, $B^c > B^u > 1$ (similarly to Busato and Chiarini, 2004, and Argentiero et al., 2008). We rationalize these costs as the lack of social protection for those workers who decide to work in unobserved economy.

Let the period utility function assume the following form:

$$\begin{aligned} U_t &= \eta^l \frac{\left(c_t^l\right)^{1-q_1}}{1-q_1} + \eta^c \frac{\left(c_t^c\right)^{1-q_2}}{1-q_2} + (1-\eta^l-\eta^c) g_t + \\ &\quad - \frac{\left(n_t^s\right)^{1+\psi}}{1+\psi} - \frac{\left(n_t^{gu}\right)^{1+\nu}}{1+\nu} - \frac{\left(n_t^{gc}\right)^{1+\chi}}{1+\chi} - B^u \frac{\left(n_t^u\right)^{1+\omega}}{1+\omega} - B^c \frac{\left(n_t^c\right)^{1+\varkappa}}{1+\varkappa} \end{aligned} \tag{12}$$

There are three resource constraints in our model; the first two regard the allocation of labor, where, for the sake of simplicity and without any loss of generality, we normalize to one total labor, and capital services, i.e.:

$$1 = n_t^s + n_t^u + n_t^c + n_t^{gu} + n_t^{gc} \tag{13}$$

$$k_t = k_t^s + k_t^u + k_t^c \tag{14}$$

The third one is typically an intertemporal budget constraint, stating that the total flow of consumptions and investments, indicated with x_t^s, x_t^u and x_t^c cannot exceed disposable income, net of taxes[1]:

$$
\begin{aligned}
c_t^l + p_t^c c_t^c + x_t^s + x_t^u + x_t^c \quad \leq \quad & \left(1-\tau^n\right) w_t^s n_t^s + \left(1-\tau^k\right) r_t^s k_t^s + \\
& + d_t (w_t^u n_t^u + r_t^u k_t^u) + \\
& + \left(1-d_t\right) \left[\left(1-\tau^n\right) w_t^u n_t^u + \left(1-\tau^k\right) r_t^u k_t^u\right] + \\
& + e_t \left(w_t^c n_t^c + r_t^c k_t^c\right) + \\
& + \left(1-e_t\right) \left(1-\tau^n\right) w_t^c n_t^c + \left(1-\tau^n\right) w_t^{gu} n_t^{gu} + \\
& + \left(1-\tau^n\right) w_t^{gc} n_t^{gc} + \pi_t^s + \pi_t^u + \pi_t^c
\end{aligned}
\tag{15}
$$

where τ^n and τ^k are tax rates on wages and capital rents.

Capital accumulation constraints are:

$$x_t^s = k_{t+1}^s - (1-\Omega^s) k_t^s \tag{16}$$

$$x_t^u = k_{t+1}^u - (1-\Omega^u) k_t^u \tag{17}$$

$$x_t^c = k_{t+1}^c - (1-\Omega^c) k_t^c \tag{18}$$

where Ω indicates the rate of capital depreciation.

2.3 The Government

The Government maximizes an expected profit function given by the difference between the expected fiscal revenues and the audit costs paid to detect unob-

[1] For the sake of simplicity, we do not consider the share of social security contributions payed by the workers, that in general is much lower than the one payed by the firm.

served firms, as in Argentiero and Bollino (2015). However, the novelty of this framework is that we realistically assume two different kinds of audit authorities: fiscal agents and criminal police.

The former are responsible to prosecute underground firms whereas the latter detects criminal firms, confiscates their capital stocks and arrests their workers.

The audit costs $[C(A)]_t^i$ $(i = u, c)$ for detecting unobserved firms are given by the gross wages paid to public officers involved in the audit activity:

$$[C(A)]_t^u = n_t^{gu} w_t^{gu} (1+s) \tag{19}$$

$$[C(A)]_t^c = n_t^{gc} w_t^{gc} (1+s) \tag{20}$$

The output of this activity is given by the probability of being detected in the unobserved sectors according to the following functions:

$$1 - d_t = \frac{n_t^{gu}}{\mu_t^u + n_t^{gu}} \tag{21}$$

$$1 - e_t = \frac{n_t^{gc}}{\mu_t^c + n_t^{gc}} \tag{22}$$

These functions are concave and have a codomain $\mathbb{C}\,[0,1]$ indicating $\mu_t^u = \frac{1}{\lambda_t^{gu}}$ and $\mu_t^c = \frac{1}{\lambda_t^{gc}}$ the degree of concavity, that here represents a measure of efficiency in the audit activity (see the theoretical Appendix for a detailed discussion). The intuition is that the probability of being detected is a function of the Government resource commitment, given by the number of public employees involved in the audit activity, and of the productivity of the audit action (success in contrasting and eliminating evasion and criminal activities)[2].

[2] Chiarini et al. (2009) endogenize the probability of being detected in belonging to under-

The expected fiscal revenues, $E(R_t)$, given by the sum of fiscal revenues levied on sunlight firms and households' income taxation, R_t^s, expected fiscal revenues levied on underground firms and households' income taxation, $E(R_t^u)$ and expected criminal revenues from criminal capital confiscated net of rehabilitation costs of criminal workers are:

$$
\begin{aligned}
E(R_t) &= R_t^s + E(R_t^u) + E(R_t^c) \qquad (23) \\
&= \tau_s Y_t + s w_t^s n_t^s + \tau^n w_t^s n_t^s + \tau^k r_t^s k_t^s + \\
&\quad + \underbrace{(1-d)\left[\tau_s Y_t^u + s w_t^u n_t^u + \tau^n w_t^u n_t^u + \tau^k r_t^u k_t^u + \vartheta^u k_t^u\right]}_{E(R_t^u)} + \\
&\quad + \underbrace{(1-e)[r_t^c k_t^c - w_t^c n_t^c (1+s)]}_{E(R_t^c)}
\end{aligned}
$$

The Government, for a given level of tax rates and fines, chooses an optimal level of audit activity $(n_t^{gu}, n_t^{gc} > 0)$ which is able to detect unobserved activities:

$$
\max_{n_t^{gu}, n_t^{gc}} [E(G_t)] = E(R_t) - [C(A)]_t^i \quad (i = u, c) \qquad (24)
$$

The optimal levels of fiscal agents and criminal police generate the efficient level of the probability of being detected, $1 - d_t$ and $1 - e_t$.

ground economy, in a partial equilibrium framework, through a function that depends, as in our model, to the degree of efficiency in the audit activity at the denominator, but instead of considering the number of fiscal agents in the numerator, it takes the number of irregular workers.

3 Model estimation

3.1 Methodology and data

The inferential procedure we adopt to estimate the parameters, to simulate the time series for the key variables of the model and to analyze their dynamics is based on MCMC methods and, in particular, on the Metropolis-Hastings algorithm. This methodology belongs to the family of Bayesian estimation methods that are very common in the empirical macroeconomic literature (see among others Canova, 2007; Smets and Wouters, 2007). Moreover, because the model is non-linear in some equations, we have preliminarily linearized and solved it using the Blanchard and Khan (1980) algorithm. Next, we have built a multi-chain MCMC procedure based on 4 chains of size 100,000; the algorithm converges within 55,000 iterations to its expected value. This high number of iterations—together with the 95 percent confidence interval for the estimates—ensures the robustness of our results[3].

3.2 Calibration and prior distributions

The prior densities are consistent with the domain of the parameters.

Our calibration is based on quarterly data of Italian economy (1980:01-2014:01); the choice of Italy is related to a size of unobserved economy wider than other industrialized countries (Schneider, 2010).

The system of equations we use to compute the dynamic equilibria of the model depends on a set of 28 parameters. Twelve regard the household preferences $(q_1, q_2, \eta^l, \eta^c, \psi, \nu, \chi, \omega, \varkappa, B^u, B^c, \rho)$, eleven pertain to technology

[3] In detail, our estimation procedure is based on two steps. In the first step, we estimate the mode of the posterior distribution by maximizing the log posterior density function, which is a combination of the prior information on the structural parameters with the likelihood of the data. In the second step, we use the Metropolis-Hastings algorithm in order to draw a complete picture of the posterior distribution and compute the log marginal likelihood of the model. The convergence diagnostic is based on Brooks and Gelman (1998) method.

$(\gamma, \delta, \zeta, \varphi_{\Lambda^s}, \varphi_{\Lambda^u}, \varphi_{\Lambda^c}, \varphi_{\Lambda^{gu}}, \varphi_{\Lambda^{gc}}, \Omega^s, \Omega^u, \Omega^c)$ and the remaining five $(\tau^n, \tau^k, \tau^s, s, \vartheta^u)$ deal with fiscal and institutional features.

Because, to the best of our knowledge, we are not aware of other studies that include criminal economy in a DSGE model, we calibrate the parameters related to criminal sector following theoretical assumptions and stylized facts.

In table **1** we show all the parameters used in the model together with their calibrated values, their definitions and the sources where the data are taken from.

TABLE 3 ABOUT HERE

Parameters q_1 and q_2 represent the legal and criminal relative risk aversion coefficients, respectively. The former is set to 0.9, a value close to unity, that shows an high risk aversion of the legal sector, in accordance with the analysis of Busato and Chiarini (2004) and Censolo and Onofri (1993) for Italy. Instead, we assume a very low relative risk aversion coefficient (0.1) for the criminal sector, under the hypothesis that a criminal individual has a greater attitude toward risk for which, according to Becker (1968), the expected benefits of his choice are greater than the expected costs.

The share of legal consumption in the utility function η^l is calibrated with the average propensity to consumption (0.85) calculated on Istat (2014a) data, whereas the share of criminal consumption η^c is residually parametrized, as the difference $1 - \eta^l - \eta^g$ where η^g is the share of public consumption in the utility function. We calibrate this last parameter with the average ratio of public investments to GDP (0.05) using Istat (2014a) data. Hence η^c is set to a value of 0.10.

The parameters $\psi, \nu, \chi, \omega, \varkappa$ are the inverse of Frisch elasticities of labor supply regarding sunlight, underground, criminal economy, fiscal agents and criminal police. These measures are defined as the elasticity of labor supply with respect to wage for a given level of consumption marginal utility. Hence an high Frisch elasticity means more flexibility in the labor market. The calibration of these parameters is not straightforward. To be consistent with the literature, particularly with regards to unobserved economy, we calibrate these parameters using Busato and Chiarini (2004) procedure that is based on Cho and Cooley (1994). Hence, the calibrated values are able to match four empirical moments: the ratio of standard deviation of total output to the standard deviation of total consumption, the correlation between total output and total consumption, the correlation between underground production and total consumption and the correlation between regular production and total consumption[4]. Following this method, the resulting calibrated values are: $\psi = 3$, $\nu = 3.5$, $\chi = 3.6$, $\omega = 1.5$, $\varkappa = 2$. Note that the Frisch elasticities of unobserved economy are higher than the ones of regular economy: this means that in the absence of any form of regulation the workers are much more flexible in supplying labor than in a more regulated environment. With the same methodology we calibrate the idiosyncratic costs in supplying labor in the unobserved sectors, $B^c = 4$ and $B^u = 2$. Also the intertemporal subjective discount factor ρ is calibrated to 0.98, a typical value in this literature (Busato and Chiarini, 2004 and Censolo and Onofri, 1993).

The sunlight capital share γ is set consistently with the literature (Censolo and Onofri, 1993) to a value of 0.55, whereas for the unobserved sectors the corresponding values, δ and z, are assumed slightly lower ($\delta = 0.25$, $z = 0.2$) under the hypothesis that these firms are less capital intensive than the sunlight one, due to the fine linked to capital held that they have to pay in the case of detection.

The values of tax rates are calibrated with the corresponding implicit tax

[4] Because in the data there is not any evidence of criminal production we cannot compute another specific statistical moment able to take into account criminal consumption.

rates in Italy calculated on Istat (2014b) data: tax rate on capital rentals τ^k is set to 0.3, tax rate on labor τ^n is set to 0.4 and tax rate on sales τ^s equals to 0.25.

The rate for social security contributions s is set to 0.33, that is the Italian average legal rate for social security contributions paid by the firms calculated on Istat (2014b) data.

Following the real business cycle literature (King and Rebelo (1999)), we set high values for the persistence coefficients of sunlight, fiscal agent and criminal police total factor productivity $\varphi_{\Lambda^s}, \varphi_{\Lambda^{gu}}, \varphi_{\Lambda^{gc}}(0.9)$ and in the absence of data on unobserved total factor productivity we assume same values for $\varphi_{\Lambda^u}, \varphi_{\Lambda^c}$.

The sunlight capital depreciation rate Ω^s has been calibrated respectively to 0.025 following Busato and Chiarini (2004) and Censolo and Onofri (1993). Also in this case, because of the lack of any statistics on underground and criminal labor we make the assumption that unobserved capital depreciates at the same rate of regular one.

Finally, following Busato and Chiarini (2004), we set the penalty factor ϑ^u to 0.3, that is the surcharge on the standard tax rate that a firm must pay when detected employing workers in underground sector, according to the Italian Tax Law (Legislative Decree 471/97, Section 13, paragraph 1).

Because the model does not provide a description of the trends in the series, we focus on the cyclical component by using the Hodrick and Prescott filter[5].

3.3 Posterior distributions

TO BE ADDED

[5] The model generates time series at a quarterly frequency; after log linear transformation of the series, the trend is computed setting the smoothing parameter to 1600, as the standard value in literature. The cyclical component is obtained as the difference between the actual (raw) series and the computed trend component.

4 Model Dynamics

TO BE ADDED

5 Conclusions

This paper has derived a measure for unobserved economy and money laundering basing on a three-sector DSGE model theoretically consistent, both in its structure and in the parameter calibration, with the dynamics of the variables involved in the analysis.

The methodology adopted has the advantage to generate high frequency data for non observable quantities and to study their relationships with the rest of the economic system, basing on a theoretical macroeconomic model qualitatively consistent with the stylized facts.

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6 Tables

Table 1: **Parameters, calibrated values and data sources**

Parameter	Calibrated Value	Definition	Source
q_1	0.9	Legal relative risk aversion	A
q_2	0.1	Criminal relative risk aversion	B
η^l	0.85	Share of legal consumption	C
η^c	0.1	Share of criminal consumption	B
ψ	3	Inverse of Frisch elasticity for sunlight firms	D
ν	3.5	Inverse of Frisch elasticity for underground firms	D
χ	3.6	Inverse of Frisch elasticity for criminal firms	D
ω	1.5	Inverse of Frisch elasticity for fiscal agents	D
$\varkappa$	2	Inverse of Frisch elasticity for criminal police	D
B^u	2	Idyosincratic labor costs for undeground firms	D
B^c	4	Idyosincratic labor costs for criminal firms	D
ρ	0.98	Intertemporal subjective discount factor	A
γ	0.55	Sunlight capital share	A
δ	0.25	Underground capital share	B
ζ	0.2	Criminal capital share	B
φ_{Λ^s}	0.9	Persistence in sunlight TFP	E
φ_{Λ^u}	0.9	Persistence in undeground TFP	B
φ_{Λ^c}	0.9	Persistence in criminal TFP	B
$\varphi_{\Lambda^{gu}}$	0.9	Persistence in fiscal agents' TFP	E
$\varphi_{\Lambda^{gc}}$	0.9	Persistence in criminal police TFP	E
Ω^s	0.025	Depreciation rate for sunlight capital	A
Ω^u	0.025	Depreciation rate for undeground capital	B
Ω^c	0.025	Depreciation rate for criminal capital	B
τ^n	0.4	Tax rate on sunlight wages	F
τ^k	0.3	Tax rate on sunlight capital rents	F
τ^s	0.25	Tax rate on sunlight sales	F
s	0.33	Rate of social security contribution	F
ϑ^u	0.3	Rate of penalty factor for underground firms	A

Notes: Legend for the capital letters of the "Source" column: A=Busato and Chiarini (2004); B=Our assumption; C=Our calculations on Istat (2014a) data; D=Our calculations on Cho and Cooley (1994); E=King and Rebelo (1999); F=Our calculations on Istat (2014b) data.