Cartel Stability, Mark-Up Cyclicality and Product Differentiation*

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Abstract

Based on seminal Rotemberg and Saloner (1986) contribution, we build a theoretical framework featuring a Bertrand duopoly with stochastic demand and product differentiation, where the analysis of cartel stability under partial collusion points towards procyclical pricing. According to the intensity of demand cyclicality, this can produce a procyclical mark up or - at least - render it less countercyclical than expected, with relevant consequences, for example, on the transmission mechanism of fiscal policy. In particular, the non-monotone relationship between demand and the intensity of collusion, if products are not significantly differentiated, is a useful indicator for an antitrust agency, and can be used as evidence against colluding firms.

JEL Codes: C73, E60, L13

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1 Introduction

In this paper we present a theoretical framework based on strategic interaction able to rationalize the existence of pro-cyclical pricing depending on the degree of product differentiation. While doing so, we also emphasize the consequences of our results on the size of the output effect resulting from aggregate demand shock, which can be of some interest for the macroeconomic literature studying the size and magnitude of government spending multipliers (Woodford, 2011; Christiano *et al.*, 2011; Auerbach and Gorodnichenko 2012, 2013). Our benchmark model is the one originally proposed by Rotemberg and Saloner (1986).

We set out by offering a brief summary of their analysis, reconstructing the countercyclical behavior of prices in a simple repeated duopoly game with homogeneous goods, in which demand is subject to random shocks affecting the vertical intercept of the demand function and the cartel sets the monopoly price after observing the demand state. Then, we extend the model encompassing the presence of product differentiation and the possibility for firms to collude on virtually any price between the monopoly and the Nash equilibrium one.

Our contribution indeed consists in (i) characterising the maximum degree of collusion (i.e., the highest collusive price) that can be sustained in a stochastic environment, given time preferences and product differentiation; and (ii) to illustrate the emergence of procyclical pricing emerging under a sufficiently high degree of product differentiation, if firms' degree of collusion falls short of the pure monopoly price.

More precisely, our results indicate that under partial collusion (at any price between the noncooperative level and the pure monopoly one), (i) procyclical cartel pricing obtains if a unilateral deviation does not throw all other firms out of the market, or product differentiation is sufficiently high; and (ii) conversely, countercyclical pricing should be observed if products are very close substitutes. While case (i) makes a cartel a hard nut to crunch for an antitrust agency, because one can argue that prices are going up precisely thanks to a demand stimulus, case (ii) portrays a situation in which a decrease in prices is revealing of an underlying collusive activity and the antitrust authority might take this as an instrument to be used against firms suspected of collusive behaviour. Put differently, a price decrease following a positive demand shock does not convey good news, because it is exactly the opposite of what one should expect from well-behaving firms competing in prices *à la* Nash.

The rest of paper proceeds as follows. Section 2 spells out the potential relevance of our research question for the macroeconomic literature on government spending multipliers. Section 3 recalls the Rotemberg and Saloner's framework, generalized in stochastic demand framework. In section 4 we provide a framework able to bridge the two positions. Section 5 concludes.

2 A macroeconomic view of the consequences of mark-up cyclicality

Recent theoretical contributions on government spending multipliers (Hall, 2009; Woodford, 2011) highlight the importance of price mark-up's cyclical behaviour for the transmission mechanism of fiscal policy. Previous literature (Galì *et al.,* 2005; Galì, 2005) had already stressed that an exogenous reduction in the aggregate inefficiency wedge (price or wage mark-up) amplifies the effects of a government spending stimulus on aggregate demand, and *vice versa*. However, by relating mark-up movements to the business cycle, it is possible to investigate analytically the relationship between government spending multipliers and the degree of pro/countercyclicality of mark-up. In a stylized sticky prices macroeconomic model, Hall (2009) shows that if we define $\mu(y) = y^{-\omega}$ as the price mark-up, parameter ω indicating its sensitivity to the income level y and g the level of government spending, then:

$$sign\left[\frac{\partial\left(\frac{dy}{dg}\right)}{\partial\omega}\right] = sign\left(\omega\right) \tag{1}$$

Considering that the government spending multiplier is positive, the above equation means that if the mark-up is countercyclical ($\omega > 0$) then the higher the sensitivity to aggregate demand ($\omega \uparrow$), the higher the government spending multiplier $\left(\frac{dy}{dg} \uparrow\right)$. On the other hand, if mark-up is procyclical ($\omega < 0$), a more pronounced cycle elasticity ($\omega \uparrow$)lowers the expansionary effects of government purchases on output $\left(\frac{dy}{dg} \downarrow\right)$.

What does economic literature have to say about the direction of mark-up cyclicality?

Theoretical literature has mainly focused on countercyclicality,[1] by taking two alternative roads that we could label "the macroeconomic view" and "the industrial organization view".

As to the former, the traditional explanation has centered on nominal rigidities: if prices are sticky, an increase in aggregate demand - assuming flexibility of some elements of marginal costs - results in a mark-up reduction (Goodfriend and King 1997; Rotemberg and Woodford 1999; Woodford 2003).

The industrial organization view focuses instead on firms' strategic interaction in a non-competitive environment. Rotemberg and Saloner (1986) argue that oligopolies are likely to behave more competitively when demand rises, especially when price is the strategic variable. Under these circumstances, in fact, the benefit from deviation is larger, and the punishment is diminished because it will be implemented when the expansionary demand shock will have already been absorbed. As a results, price/marginal cost ratio declines as aggregate demand increases. Haltiwanger and Harrington (1991) extend the analysis to allow for time-varying firms'expectations on future demand, by relaxing the assumption of *i.i.d.* demand shocks so to induce serial correlation in the cycle. They highlight potential asymmetries in collusive pricing behavior across different state of the business cycle, as their findings show that collusion is more difficult during recessions than during booms[2].

[1] Lindbeck and Snower (1987) and Bils (1987) achieve mark-up countercyclicality by establishing a positive relationship between aggregate demand and elasticity of demand. Edmond and Veldkamp (2009) assign the central role to income distribution: during booms, income shifts towards the lower tail of the distribution, featured by higher-elasticity consumers.

[2] Fabra (2006) shows that this result can be overturned if firms'capacities are sufficiently

In fact, establishing a relation between future and current demand induces asymmetries between the opportunity costs of engaging in price wars according to the direction of demand. Under falling demand, the forgone collusive profits are on a intertemporally decreasing path, so the incentive to collude is lower and likely to remain so. On the other hand, in a period of increasing demand, the traditional Rotemberg and Saloner result is mitigated by the fact that joint-maximizing profits are going to be higher in the future. Along this path, Bagwell and Staiger (1997) develop a theory of collusive pricing in a framework where aggregate demand alternates stochastically between slow and fast growth states, and where the transition is governed by a Markov process. They find that the cyclical behaviour of collusive prices depends crucially on correlation of demand growth rates through time and the expected duration of boom and recessions. Particularly, collusive prices are procyclical in presence of positive demand correlation through time, and countercyclical otherwise. Furthermore, the amplitude of the collusive pricing is larger when the recession has a longer expected duration or - conversely- when the boom has a lower length. Those two contributions stress that the qualitative and quantitative dimensions of collusive pricing can differ according to the state of the business cycle. Such an asymmetry in the intensity of the collusion is directly related to the mark-up cyclical behaviour and thus - as we will argue - to the size of government spending multipliers. Therefore, those results might provide an explanation for multipliers' asymmetries over the business cycle, recently emphasized by the empirical macroeconomic literature (Canzoneri *et al.*, 2011, Auerbach and Gorodnichenko 2012)[3].

More recently, a new strand of literature combines traditional general equilibrium macro models with an industrial organization approach, emphasizing the procyclicality of entry in determining mark-up countercyclicality,

small. Along the same idea, Knittel and Lepore (2010) show that if the marginal cost of capacity is high enough, prices in booms are generally lower than prices in recession.

[3]Rotemberg and Woodford (1992) develop the idea of mark-up countercyclicality in a dynamic general equilibrium setting, finding that the model's empirical performances are closer to actual postwar US data than the corresponding predictions of the perfectly competitive model.

through the competition effect (Ghironi and Melitz 2005; Jaimovich and Floetotto 2008; Etro and Colciago 2010).

However, how empirically robust is the evidence about mark-up countercyclicality?

Although a considerable number of contributions points towards countercyclicality,[4] empirical literature on mark-up cyclical behaviour is not unambiguous. Donowitz *et al.* (1986, 1988) find evidence on procyclicality in the US; Chirinko and Fazzari (1994) use a dynamic factor model to estimate markups, finding that they are procyclical in nine of the eleven 4-digit industries they analyze. Updating Bils (1987) analysis - in favor of countercyclicality - with more recent and richer data, Nekarda and Ramey (2010) find that all measures of markups are either procylical or acyclical.

Hall (2009) provides a simple first-cut test for cyclicality by noting that the mark-up can be expressed as the ratio between the elasticity of output with respect to labor input $\left(\frac{\partial Y}{\partial L}\frac{L}{Y}\right)$ and the share of labor compensation over nominal income $\left(\frac{WL}{PY} = s\right)$. In fact, since by the envelope theorem property, a cost-minimizing firm equalizes the marginal cost of increasing output across all possible margins for varying production, we can express marginal cost as:

$$MC = \frac{W}{\frac{\partial Y}{\partial L}} \tag{2}$$

As gross mark-up ($= \mu$) is defined as the ratio between price index and marginal costs,and multiplying and dividing by $\frac{Y}{L}$, then:

$$\mu = \frac{\frac{\partial Y}{\partial L}\frac{L}{Y}}{s} \tag{3}$$

If the production process is approximated by a Cobb-Douglas $Y = L^{\alpha}K^{1-\alpha}$, then the numerator of (3) is α and can be considered relatively stable over time. Thus, the countercyclicality of the mark-up μ requires the procyclicality of labor share s.

[4]See Martins et al (1996) on OECD, Chevalier *et al.* (2003) on the US, Portier (1995) on France.

Table 1 reports the correlation between real detrended GDP and labor share (computed as annual average wage times employed population over nominal output) in five major OECD economies from 1990 to 2009.

Table 1: Correlation between detrended GDP and labour share

COUNTRY	**CORRELATION**
US	0.61
UK	0
FRANCE	-0.12
GERMANY	-0.52
ITALY	-0.49

With the exception of US there seems to be a negative rather than positive correlation between labor share and the business cycle. Or no correlation at all, as in United Kingdom. Therefore Hall's rule-of-thumb test fail to result in unambiguous evidence in favor of mark-up countercyclicality for the countries in our sample. As discussed in this section, this has relevant consequences for the size of government spending multipliers: in that case in fact, the expansionary effect of a government spending stimulus is decreasing in the mark-up's sensitivity to the business cycle. In the following sections we will build up a theoretical framework able to account for the existence of a procyclical mark-up.

3 The theoretical status-quo

We set out by briefly summarizing first Rotemberg and Saloner's (1986) model of countercyclical pricing with homogeneous goods, assuming that firms collude (if they can do so) along the frontier of industry profits. Then, we extend the model to accomodate the presence of product differentiation, allowing at the same time firms to collude at the highest price level compatible with their time preferences.

The following is a simplified version of Rotemberg and Saloner's (1986) setup, in which we focus on the behaviour of a cartel formed by two firms, without further loss of generality.[5] As in their paper, we consider a market for a homogenous good, over an infinite horizon. Time t is discrete, with $t = 0, 1, 2, ...\infty$, and the demand function at any time t is $p_t = \sigma_t - q_{1t} - q_{2t}$, with $q_{it} \geq 0$ being firm i's output. Firms have identical technologies represented by the cost function $\Gamma_i = cq_i$, with vertical intercept (or *reservation price*) $\sigma_t > c \geq 0$. To ease the exposition throughout the paper, and again w.l.o.g., we set $c = 0$. The profit function of the individual firm thus coincides with revenues[6], $\pi_i = pq_i$. The reservation price σ_t is stochastic, and in each period can take one of two values, $a > b > 0$, with probabilities $\mathfrak{p}(a) = m$ and $\mathfrak{p}(b) = 1 - m$, respectively, with $m \in [0, 1]$.

The supergame unravels following the rules of Friedman's (1971) *perfect folk theorem*, whereby any unilateral deviation from the collusive path is punished by a permanent reversal to the Nash equilibrium of the constituent stage game forever (the so-called *grim trigger strategy*). In the present setting, product homogeneity entails that the per-period profits at the Bertrand-Nash equilibrium are nil. As in Rotemberg and Saloner (1986), suppose firms (i) set prices after having observed the state of demand (either a or b), and (ii) collude at the monopoly price in each period. Later, we will come to the case of partial collusion.

At any t, monopoly price $p_t^M = \sigma_t/2$ delivers the individual expected cartel profit:

$$E\pi^C = \frac{m\pi^M(a) + (1-m)\pi^M(b)}{2} = \frac{ma^2 + (1-m)b^2}{8} \tag{4}$$

where superscript C stands for *cartel* and $\pi^M(\sigma_t) = \Pi^M(\sigma_t)/2$, $\sigma_t = a, b$ is the per-period symmetric share of monopoly profits accruing to each firm in the cartel phase, given the realization of the demand state σ_t. If a firm

[5] This exposition relies on (and slightly generalises) the simplified version of the countercyclical pricing model in Tirole (1988, pp. 248-250). It is formally equivalent to (but more manageable than) the linear model adopted in Rotemberg and Saloner (1986, p. 396, expressions (8-9)).

[6] Due to this assumption, cyclical properties of mark-up and pricing are identical.

contemplates the possibility of deviation, the best option is to do so in a period of high demand, so that slightly undercutting the monopoly price grants the cheating firms full monopoly profits in that period, delivering deviation profits $\pi^D = a^2/4$. As already explained above, such deviation at any t is punished by driving profits to zero from $t+1$ to doomsday through the adoption of the marginal cost pricing rule at the Bertrand-Nash equilibrium. Assuming firms share identical time preferences measured by a symmetric and time-invariant discount factor $\delta \in (0,1)$, the stability of price collusion requires δ to meet the following necessary and sufficient condition:

$$E\pi^C \sum_{t=0}^{\infty} \delta^t \geq \pi^D \Leftrightarrow \frac{a^2 \left[1-\delta\left(1+m\right)\right] + b^2 \left(1-m\right)\delta}{8\left(1-\delta\right)} \geq 0 \tag{5}$$

which is satisfied by all

$$\delta \geq \frac{a^2}{a^2\left(1+m\right) + b^2\left(1-m\right)} \equiv \delta^*, \tag{6}$$

with

$$\frac{\partial \delta^*}{\partial a} = \frac{2ab^2\left(1-m\right)}{\left[a^2\left(1+m\right) + b^2\left(1-m\right)\right]^2} > 0 \, \forall \, m \in [0,1) \, . \tag{7}$$

Property (7) indicates that the critical threshold of the discount factor stabilizing full collusion increases with the good state. This is one of the elements leading to the (by now classical) interpretation of this model, according to which firms should collude less if demand gets higher, as the size of the market ensures high profits anyway, and this suggests the idea of countercyclical pricing. This argument is reinforced if one examines the perspective of activating some degree of partial collusion at the highest price $p^* \in \left(0, p^M\right)$ sustainable if δ is lower than δ^*.

From the above exposition we can draw the following:

Lemma 1 *Under demand uncertainty and product homogeneity, any positive shock on demand increasing the level of the reservation price in the best state makes price collusion more difficult to sustain, all else equal.*

The foregoing analysis, which sums up Rotemberg and Saloner's (1986), is based on two assumptions:

A1 Firms sell perfect substitutes, i.e., the good is homogeneous

A2 If firms collude, they do so along the frontier of monopoly profits. Otherwise, they play the one-shot Bertrand-Nash equilibrium forever.

These are very special assumptions, and one may be induced to wonder whether they yield general conclusions - in this case, concerning the cyclical pattern of a cartel's pricing behaviour. In the remainder, we are going to relax both A1 and A2, in search of a supergame configuration in which procyclical pricing arises. We will present a more general model where some degree of product differentiation is present, and then illustrate why and how firms may collude on some price level between the monopoly price and the Nash equilibrium one.

4 Product differentiation and partial collusion

The standard approach in industrial organization theory has traditionally devoted a large amount of attention to the bearings of product differentiation on the intensity and stability of implicit collusion.[7] Accordingly, we consider a market where two single-product firms offer differentiated products over discrete time $t = 0, 1, 2, 3, ...\infty$. At any t, the inverse demand function for variety i is (see Spence, 1976; and Singh and Vives, 1984, *inter alia*):

$$p_{it} = \sigma_t - q_{it} - sq_{jt} \tag{8}$$

where $\sigma_t = a, b$, $a > b$, and $s \in [0, 1]$ is the symmetric degree of substitutability between any pair of varieties. If $s = 1$, products are completely homogeneous; if instead $s = 0$, strategic interaction disappears and firms are

[7]The material appearing in this section is a compact exposition of a large debate. See Deneckere (1983), Majerus (1988), Chang (1991, 1992), Rothschild (1992), Ross (1992), Lambertini (1997) and Albæk and Lambertini (1998), *inter alia*.

independent monopolists over n separated markets. The direct demand function to be used under Bertrand behaviour obtains by inverting the system (8):

$$q_{it}(\sigma_t) = \frac{\sigma_t}{1+s} - \frac{p_{it}}{1-s^2} + \frac{sp_{jt}}{1-s^2}. \tag{9}$$

Function (9) is defined for all $s \in [0,1)$, while in $s = 1$ products become homogeneous and therefore we are back to the initial model. As in the latter, firms share the same technology, summarized by the cost function $C_i = cq_i$, wit marginal cost c being normalized to zero for the sake of simplicity. Therefore, for a given realization of σ_t, per-period individual profits are $\pi_{it}(\sigma_t) = p_{it}q_{it}(\sigma_t)$. Throughout the game, firms also share the same intertemporal preferences measured by the constant discount factor $\delta \in (0,1)$.

We set out characterising the features of the one-shot game and derive the per-period Bertrand-Nash profit, a task which can be quickly worked out. Given σ_t, the first order condition for the maximization of $\pi_{it}(\sigma_t)$ w.r.t. p_{it} is

$$\frac{\partial \pi_{it}(\sigma_t)}{\partial p_{it}} = \frac{\sigma_t(1-s) - 2p_{it} + sp_{jt}}{1-s^2} = 0 \tag{10}$$

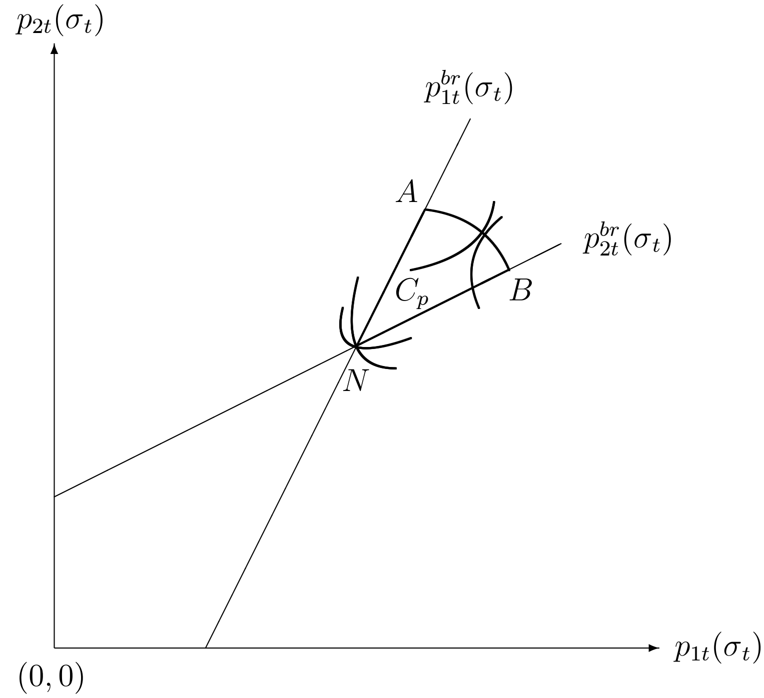
whereby the symmetric Nash equilibrium price is $p_t^N(\sigma_t) = \sigma_t(1-s)/(2-s)$, while individual Nash equilibrium profits are $\pi^N(\sigma_t) = \sigma_t^2(1-s)/\left[(1+s)(2-s)^2\right]$, both falling to zero in the homogeneous good case.

From (10), one gets the best reply function:

$$p_{it}^{br}(\sigma_t) = \frac{\sigma_t(1-s) + sp_{jt}}{2} \tag{11}$$

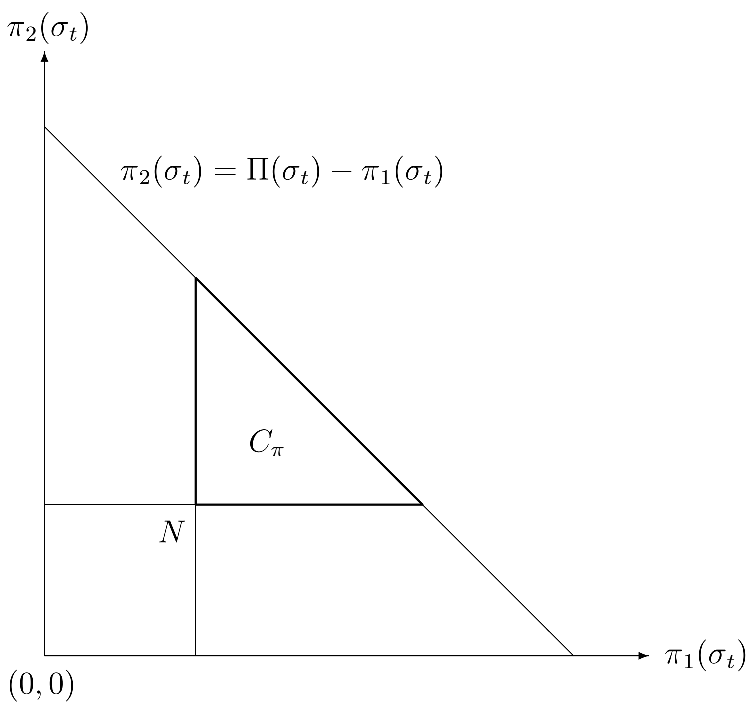
which is positively sloped for all positive values of s, since $\partial p_{it}^{br}(\sigma_t)/\partial p_{jt} = s/2$. Hence, the map of best reply functions in the price space appears as in Figure 1. The intersection point N singles out the noncooperative Nash equilibrium of the constituent one-shot game, and we can also identify the two isoprofit curves measuring the symmetric Nash equilibrium profits π^N.

Figure 1 An embarassment of riches: collusion in the price space



What draws our attention, though, is the area C_p lying north-east of the intersection of best replies, and identified by the segments NA and NB of the best replies themselves, together with the curve AB. The latter is the geometrical locus of all the tangency points between pairs of isoprofit curves, each belonging to one of the two firms. That is, AB is the Pareto-efficient curve along which the sum of firms' profits is equal to monopoly profits. In order to stabilise collusion along AB, firms' time preferences must satisfy the stability condition $\delta \in [\delta^*, 1)$. If this doesn't happen, should they necessarily play the one-shot Nash equilibrium forever? No, as the only situation where this surely happens is that in which $\delta = 0$, implying that firms are unable to detect the existence of any future periods and consequently at any t they play as if the game were one-shot. For any (even slightly) positive value of δ, firms are able to raise prices above the Nash equilibrium one and set up a cartel featuring some degree of partial collusion, corresponding to a generic point inside area C_p to the north-east of N but below curve AB. The closer to δ^* is δ, the closer firms will get to AB. This gives rise to what Tirole defines an *embarassment of riches*, since the infinitely repeated game yields infinitely many equilibria, all of them (except the fully noncooperative one) being characterised by some degree of collusion, so that the theory of repeated games is "too successful in explaining tacit collusion" (Tirole, 1988, p. 247).

Figure 2 An embarassment of riches: collusion in the profit space



The same point can be grasped by looking at the profit space, as in Figure 2. Here, area C_π is the triangle in bold between the Nash equilibrium point N and the linear frontier of monopoly profits, and there exists a one-to-one correspondence between any given point in C_π and its equivalent in C_p, as well as between any point of AB and its equivalent point along the bold segment of the frontier of monopoly profits.

The perfect folk theorem yields the following condition, that must be satisfied in order for the collusive path to be stable:

$$E\pi^C \sum_{t=0}^{\infty} \delta^t \geq \pi^D + E\pi^N \sum_{t=1}^{\infty} \delta^t, \tag{12}$$

where, unlike the previous model,

$$E\pi^N = \frac{\left[ma^2 + (1-m)\, b^2\right](1-s)}{(1+s)\,(2-s)^2} \geq 0 \tag{13}$$

for all $s \in [0,1]$ is the profit generated by the Bertrand-Nash equilibrium of the constituent game. For any $s \in [0,1)$, the presence of imperfect substitutability between the two product varieties requires computing the deviation profits π^D anew, as unilateral deviation does not necessarily imply that the cheated firm is driven out of business. Here, the optimal deviation from cartel pricing may take two alternative forms, depending on the value of s. If s is sufficiently low (i.e., differentiation is high enough), deviation by firm i takes place along its best reply, and is given by

$$p^D\left(p^C\right) = \frac{a\,(1-s) + sp^C}{2} \tag{14}$$

where $p^C \in \left(p^N(a), a/2\right]$ is the cartel price to which firm j sticks; $p_i^D\left(p^C\right)$ delivers profits

$$\pi^D\left(p^C\right) = \frac{\left[a\,(1-s) + sp^C\right]^2}{4\,(1-s^2)} \tag{15}$$

to the deviating firm. Otherwise, if s is sufficiently close to one (i.e., differentiation is low enough), the deviation price

$$p^{D'}\left(p^C\right) = \frac{p^C - a\,(1-s)}{s} \tag{16}$$

drives the cheated firm's demand function to zero, and deviation profits amount to

$$\pi^{D'}\left(p^C\right) = \frac{\left(a - p^C\right)\left[p^C - a\,(1-s)\right]}{s^2}. \tag{17}$$

It is easily ascertained that $p^{D'}\left(p^{C}\right) = p^{D}\left(p^{C}\right)$ and $\pi^{D'}\left(p^{C}\right) = \pi^{D}\left(p^{C}\right)$ at $\underline{p}^{C} = a\left[2 - s\left(1 - s\right)\right] / \left(2 - s^{2}\right) \geq a/2$ for all $s \in \left[\sqrt{3} - 1, 1\right]$. Accordingly, we may formulate the following:

Lemma 2 *For all $s \in \left[\sqrt{3} - 1, 1\right]$, the optimal deviation from collusive price p^{C} is $p^{D'}\left(p^{C}\right)$, and the cheating firm stands alone on the market. For all $s \in \left[0, \sqrt{3} - 1\right)$, the optimal deviation from collusive price p^{C} is $p^{D}\left(p^{C}\right)$, and duopoly persists as the cheated firm's market share remains strictly positive.*

Therefore, four relevant cases are to be examined:

- firms' time preferences allow for full collusion at the pure monopoly price, and product differentiation is high enough to allow the cheated firm to operate on the market with positive market share and profits in the deviation period;

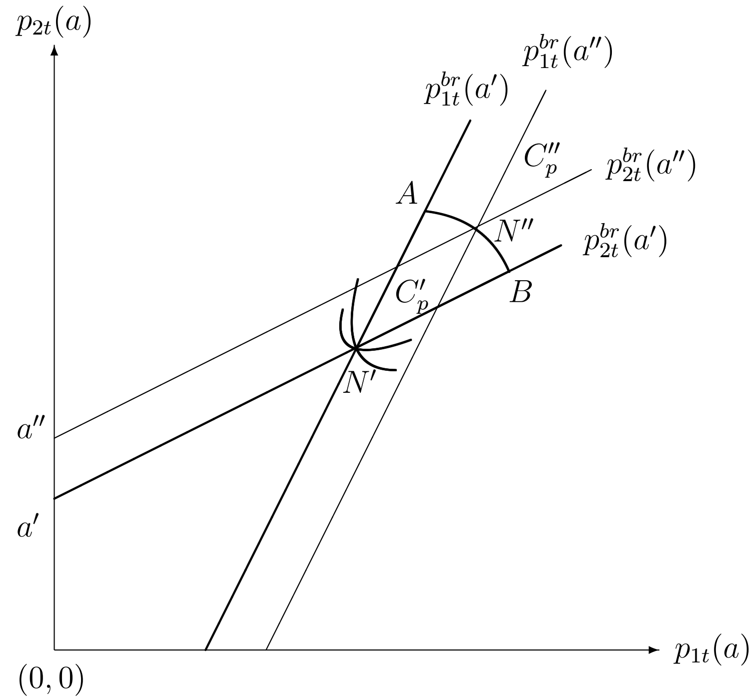
- firms' time preferences allow for full collusion at the pure monopoly price, but product differentiation is low enough to cause the cheated firm's market share and profits to fall to zero in the deviation period;

- firms' time preferences only allow for some degree of partial collusion, and product differentiation is high enough to allow the cheated firm to operate on the market with positive market share and profits in the deviation period;

- firms' time preferences only allow for some degree of partial collusion, but product differentiation is low enough to cause the cheated firm's market share and profits to fall to zero in the deviation period.

The relevance of partial collusion, and its role in generating procyclical pricing behaviour, can be appreciated by looking at Figure 3, where the effects of a positive demand shock are outlined in the price space.

Figure 3 Example: the emergence of procyclical pricing



Here, the vertical intercept increases from a' to a'', causing a parallel shift of the two best replies and a consequent north-east shift of the collusive region, from C_p' to C_p''. Now suppose that, in correspondence of the initial condition, firms were setting the monopoly price along AB, because their time preferences allowed them to do so. If, after the positive demand shock, they are unable to sustain the monopoly price at a'', still they might sustain some degree of collusion represented by a point in C_p''. In the worst scenario, if indeed they are altogether unable to collude, they will end up playing the Nash equilibrium price whose level is identified by point N'', *which belongs to AB*. If so, casual observation would suggest that the demand shock has had no bearings on the price level, although the structure of the game - summarised here by the map of best replies - entails that firms have abandoned collusion for a fully noncooperative behavior, but *this does not imply the emergence of countercyclical pricing*. Furthermore, if the shock drives N'' above AB (even slightly so), then firms' attitude switches from fully collusive in state a' to fully noncooperative in state a'', and yet market price increases, i.e., *pricing behaviour is procyclical*. And it is so, *a fortiori*, if firms succeed in building up and sustain some degree of collusion in C_p'''.

4.1 Best reply deviation and the persistence of duopoly

We are in the parameter region defined by $s \in \left[0, \sqrt{3}-1\right)$. Here, the deviation price is (14) and the resulting deviation profits are (15). We set out by taking a quick look at the stability condition for full collusion.

Monopoly price in state σ_t is $p_t^M = \sigma_t/2$, delivering expected per-firm cartel profits $E\pi^C = \left[ma^2 + (1-m)\, b^2\right] / \left[4\,(1+s)\right]$. The deviation price and profits in correspondence of the best demand state correspond to $p^D\left(p^M\right) = a\,(2-s)\,/4$ and $\pi^D\left(p^M\right) = a^2\,(2-s)^2 / \left[16\left(1-s^2\right)\right]$, respectively. The individual expected Bertrand-Nash profits in each period of the punishment phase are given by (13). As a result, collusive stability now requires

$$\delta \geq \frac{a^2\,(2-s)^2}{a^2\left[4m\,(1-s)-(2-s)^2\right]+b^2\,(1-m)^2\,(1-s)^2} \equiv \delta^*, \qquad (18)$$

with $\partial\delta^*/\partial a \propto (1-m)\,(1-s) > 0$ for all $m \in [0,1)$ and $s \in \left[0, \sqrt{3}-1\right)$.

As for partial collusion, define the expected partially collusive profit as $E\pi^C = \left[m\,(a-p^*(a))\,p^*(a) + (1-m)\,(b-p^*(b))\,p^*(b)\right] / (1+s)$ with $p^*(b) = b/2$, in such a way that the only unknown variable is the partially collusive price in the best state, $p^*(a)$. That is, we assume firms will charge the best collusive price in the worst state, and appropriately tune $p^*(a)$ so as to satisfy the stability condition (12), which we are about to construct step by step.

Using (14), the best deviation against $p^*(a)$ along the reaction function is $p^D(p^*(a)) = \left[a\,(1-s) + sp^*(a)\right]/2$. Then, the expected payoff in each period of the punishment phase is (13). From the stability condition (12), we obtain a single admissible solution:

$$p^*(a) = \frac{a(2-s)(1-s)\Theta - 2\delta(1-s)s\sqrt{\Psi}}{(2-s)\left[s^2(1-\delta)+4(1-s)\right]\left[(2-s)^2-\delta\left(4m(1-s)-(2-s)^2\right)\right]} \tag{19}$$

where $\Theta = [2\delta m + (2-s)(1-\delta)][s^2(1-\delta)+4(1-s)] > 0$ and

$$\Psi = a^2m^2\left[s^2(1-\delta)+4(1-s)\right]^2 + \tag{20}$$

$$4b^2(2-s)^2(1-\delta)(1-m)\left[(2-s)^2-\delta\left(4m(1-s)-(2-s)^2\right)\right] > 0.$$

Taking the partial derivative of $p^*(a)$ w.r.t. a, one can verify that

$$\frac{\partial p^*(a)}{\partial a} \propto a^2m^2\left[s^2(1-\delta)+4(1-s)\right]\left[(2-s)^2(1-\delta)+4m\right] +$$

$$4b^2(2-s)^4(1-\delta)(1-m)[2\delta m+(2-s)(1-\delta)]^2 > 0 \tag{21}$$

everywhere. Hence, if the cheated firm survives the deviation by the defecting firm, there emerges a picture reflecting the idea behind Figure 3, whereby partial collusion indeed is accompanied by procyclical pricing. On the basis of the foregoing analysis, we can state

Proposition 3 *If deviation from the collusive path does not grant monopoly power, then the maximum collusive price sustainable under stochastic demand conditions is monotonically increasing in the level of the best demand state.*

That is, if the profitability of a unilateral deviation is limited by the presence of a sufficiently high degree of product differentiation, collusion exhibits a well defined procyclical behaviour. Hence, antitrust agencies should suspect that the price increase following a positive demand shock is partly due to a more intense collusion among firms. Of course, it is also true that firms finding themselves under investigation could use this argument as an effective defensive tool, given the understandable difficulty on the part of the authority - which could be affected by incomplete information - to verify whether (and if so, how much) the price increase exceeds the reasonable amount that should be imputed to the demand increase at the fully noncooperative equilibrium.

4.2 Defecting to monopoly

The last step consists in investigating the case in which a unilateral deviation from the cartel price turns the deviator into a monopolist. Under full collusion, the only detail that has to be modified is the deviation price in correspondence of the best state, which causes the cheated firm's output to drop to zero. This is (16), ensuring the deviation profits (17) for all $s \in \left[\sqrt{3}-1, 1\right]$. The resulting stability condition is

$$\delta \geq \frac{a^2 (2-s)^2 \left[(1+s)s-1\right]}{a^2 \left[s^4 (m+1) + 4(4s-1) - s^2 (1+3s)\right] + b^2 (1-m) s^4} \equiv \delta^{*'}, \qquad (22)$$

with $\partial\delta^{*'}/\partial a > 0$ and $\partial\delta^{*'}/\partial m < 0$ for all $s \in \left[\sqrt{3}-1, 1\right]$, so that the picture remains much the same as we already know it, along the frontier of industry profits.

Now suppose firms' time preferences fall short of (22). If so, firms may activate the highest sustainable degree of partial collusion inside region $C_{p,\pi}$ in Figures 1-2. In such a case, they set $p^*(b) = b/2$ whenever demand is low, and solve the stability condition (12) w.r.t. $p^*(a)$, obtaining:

Lemma 4 *For all $s \in \left[\sqrt{3}-1, 1\right]$, the highest sustainable collusive price is*

$$p^{*'}(a) = \frac{a(2-s)\left[(1-\delta)(2-s(2s-1)) - \delta m s^2\right] - s\sqrt{\Gamma}}{2(2-s)\left[(1-\delta)(1+s(1-s)) - \delta m s^2\right]}$$

with $\Gamma > 0$ in the whole admissible parameter range.

Proof. See the Appendix. ■

The last step consists in differentiating $p^{*'}(a)$ w.r.t. a, defining $r \equiv a/b > 1$ and then solving $\partial p^{*'}(a)/\partial a = 0$ w.r.t. r, obtaining[8]

$$r = \frac{s(2-s)\left[(1-\delta)(2-s(2s-1)) - \delta m s^2\right]\sqrt{\delta\Upsilon}}{2\sqrt{(1-s)\,\Omega}} = \overline{r} \qquad (23)$$

with $\Upsilon \equiv (1-m)\left[(1-\delta)(1+s(1-s)) - \delta m s^2\right]$ and

$$\Omega = \left[(1-\delta)(1+s(1-s)) - \delta m s^2\right]\left[\delta m s^2 - (1-\delta)(2-s)^2(1+s)\right] \times$$

[8]The second solution can be disregarded as it is always negative.

$$\left[s^2\left(1-\delta\left(1-ms\right)\right)^2+4\left(1-\delta\right)\left(\left(1-\delta\right)\left(1-s\right)+\delta m\left(2s-1\right)\right)\right]. \tag{24}$$

Now, it can be shown analytically that $r \in \mathcal{R}^+$; moreover,

$$\lim_{m\to 1} r = 0 \text{ and } \lim_{m\to 0} r = \frac{s\left[2\left(1-s^2\right)+s\right]\sqrt{\delta}}{\left(2-s\right)\sqrt{\left(1-\delta\right)\left(1-s^2\right)}} > 1 \tag{25}$$

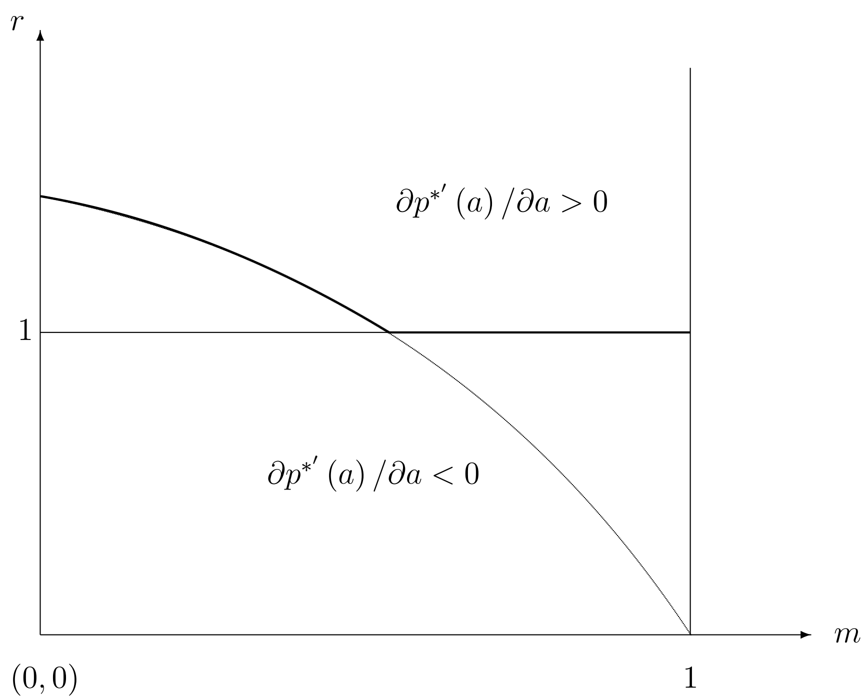
for all

$$\delta > \frac{\left(2-s\right)^2\left(1-s^2\right)}{s^2+4\left[1-s\left(1+s\left(1-s\right)\right)\left(1-s\right)\left(1-s^2\right)\right]} = \underline{\delta} \tag{26}$$

which is decreasing and concave in s, with $\underline{\delta} = 3\left(39+38\sqrt{3}\right)/937 \simeq 0.336$ in $s = \sqrt{3}-1$ and $\underline{\delta} = 0$ in $s = 1$. Consequently, the bahaviour of the highest collusive price $p^{*'}\left(a\right)$ can be characterised as follows:

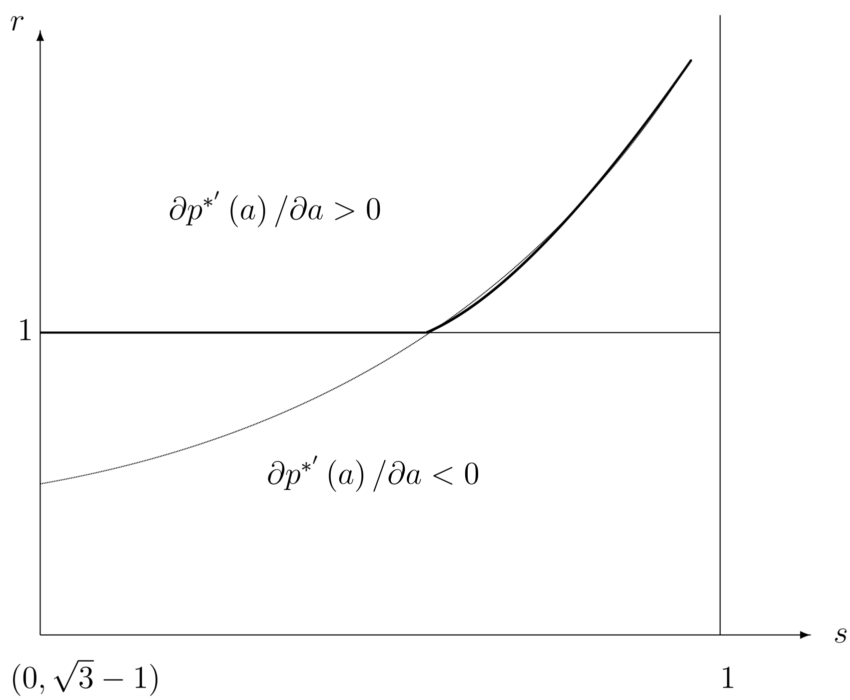
Lemma 5 $\partial p^{*'}\left(a\right)/\partial a > 0$ *for all* $r > \max\left\{1, \overline{r}\right\}$.

Figure 4 Collusive pricing in the space (m, r).



The opposite happens for all r between one and $\overline{r}$, if indeed $\overline{r} > 1$. In general, $\overline{r}$ is decreasing and concave in m (a fact that can be ascertained numerically), giving rise to a picture like the one reported in Figure 4, in which the sign of $\partial p^{*'}(a)/\partial a$ explicitly appears in each region. Above the upper envelope in bold we observe $\partial p_{-}^{*'}(a)/\partial a > 0$; if $\overline{r} > 1$ for at least some acceptable parameter values, then in such a region $\partial p^{*'}(a)/\partial a < 0$.[9] The fact that $\partial\overline{r}/\partial m < 0$ reveals that the region where collusive pricing is procyclical expands as the probability attached to the high demand state increases, to the extent that, beyond a certain value of $m \in (0,1)$, the high demand state is so likely to realise that procyclical pricing arises for *any* admissible $r > 1$, i.e., even if a is only slightly higher than b.

Figure 5 Collusive pricing in the space (s, r).



[9]Recall that in state b, by assumption, firms collude on the monopoly frontier.

An analogous exercise can be carried out in the space (s, r). This is done in Figure 5, which shows that countercyclical pricing emerges in the region in which $r \in (1, \overline{r})$ and the two product varieties are sufficiently close substitutes, i.e., in the left neighbourhood of the homogeneous good case considered in Rotemberg and Saloner (1986) which we have revisited above. In this regard, it is also worth observing that $\overline{r}$ asymptotically increases to infinity as s approaches 1 in the limit, in which case pricing is countercyclical irrespective of r. This property shows that indeed the case of perfect substitutes is a very special one, and is not representative of a generalised behaviour. In fact, if differentiation is large enough, then a procyclical pricing pattern is observed even if deviation grants a monopolistic position to the defecting firm.

Figure 6 Collusive pricing in the space (δ, r).

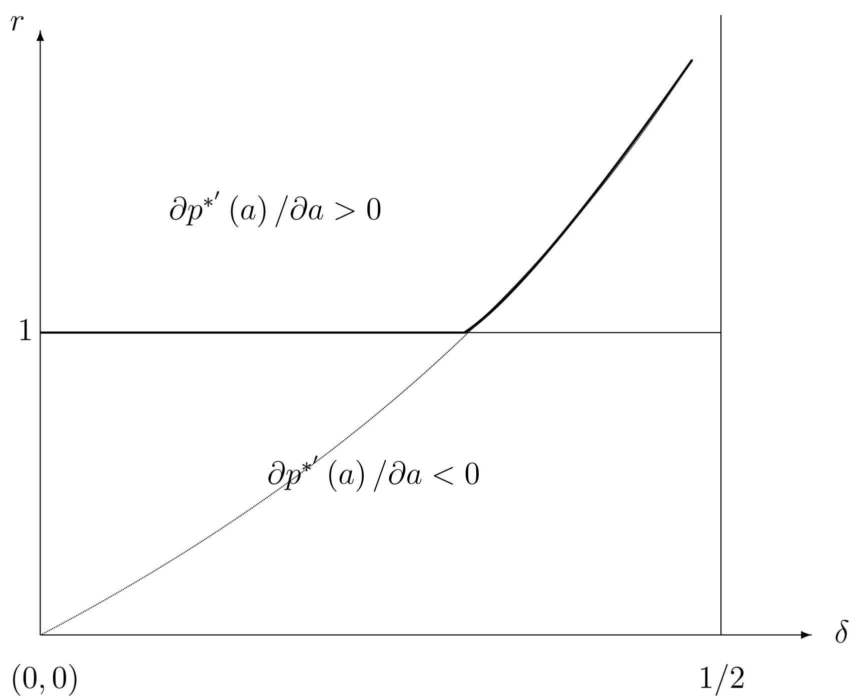


Figure 6, drawn in the space (δ, r), reflects the initial result highlighted in Lemma 1, where differentiation is assumed away and the relationship between

δ and a is positive. Here, with a limited degree of differentiation, we see that $\partial\overline{r}/\partial\delta > 0$ for all $r > 1$, this result being an obvious consequence of the fact that sign$\{\partial\overline{r}/\partial\delta\}$ =sign$\{\partial a/\partial\delta\}$ =sign$\{\partial\delta/\partial a\}$, the latter being positive. Hence, above (resp., below) the upper envelope defined by $\max\{1, \overline{r}\}$, we observe a procyclical (resp., countercyclical) pricing pattern.

Our final result can be therefore stated as follows:

Proposition 6 *If deviation from the collusive path grants monopoly power, then the maximum collusive price sustainable under stochastic demand conditions is monotonically decreasing (increasing) in the level of the best demand state if product differentiation is low (high) enough.*

The above Proposition delivers a relevant message, that can be spelled out in the following terms. To begin with, recall the fact that deviation makes the defecting firm a monopolist is made possible by the close (if not necessarily complete) substitutability between goods. Hence, if products are sufficiently similar, the model delivers a non-monotone reaction of the best collusive price to a positive demand shock. If - although limited - substitutability is still sufficiently low, we are back to the same picture yielded by Proposition 3, with analogous implications. If instead products are very similar to each other, then the highest collusive price reacts negatively to a positive demand shock. Hence, observing a decrease in price as a reaction to an increase in demand should alert an antitrust agency to the effect that these firms are implicitly colluding, and the authority could intervene and use this argument to nail them to their responsibilities.

5 Concluding remarks

We have used a repeated duopoly game to revisit the issue of cyclical pricing so as to reconcile Rotemberg and Saloner's (1986) results - about the emergence of countercyclical pricing under uncertainty - with the procyclical flavour traditionally associated with implicit collusion in the perfect certainty approach, which is a typical (although by no means univocal) feature of the debate on cartel behaviour in the theory of industrial organization.

The bottom line of our analysis is that the cyclical properties of firms' pricing behaviour are sensitive to the degree of product differentiation across product varieties, in such a way that pricing is procyclical whenever the cheated firm retains a positive market share during deviations (because products are weak substitutes), while instead countercyclicality indeed obtains provided that (i) the deviator becomes a monopolist and (ii) product differentiation is sufficiently low. In such a case, the negative reaction of prices to a positive demand shock indicates the presence of collusive behaviour and can be used by antitrust authorities.

Mark-up cyclical behaviour might therefore be a far more complicated issue than we previously thought, especially since - as we discussed in Section 2 - empirical evidence on the issue is far from being conclusive. The direction of markup cyclicality has relevant consequences on macroeconomic issues such as the real short term effects of fiscal policies. In imperfectly competitive settings, in fact, the size of the government spending multiplier is increasing function of mark-up countercyclicality but decreasing function of its procyclicality. Hence, determining the actual direction of pricing cyclicality in non-competitive settings becomes crucial for the analysis on the fiscal policy multiplier's actual size. Future research will be concerned with an empirical analysis attempting to link different sectors (featured by different degrees of product differentiation) with different cyclical properties of their average mark-ups.

Appendix

Proof of Lemma 4. From (12) one obtains the price pair

$$p_{\pm}^{*\prime}(a) = \frac{a(2-s)\left[(1-\delta)(2-s(2s-1)) - \delta m s^2\right] \pm s\sqrt{\Gamma}}{2(2-s)\left[(1-\delta)(1+s(1-s)) - \delta m s^2\right]} \tag{a1}$$

in which

$$\Gamma = a^2\left[s^2(1-\delta(1-ms))^2 + \right. \tag{a2}$$

$$4(1-\delta)((1-\delta)(1-s)-\delta m(2s-1))]$$
$$-b^2\delta s^2(1-m)\left[(1-\delta)(1+s(1-s))-\delta ms^2\right]$$

with

$$(1-\delta)(2-s(2s-1))-\delta ms^2>0, \tag{a3}$$

$$s^2(1-\delta(1-ms))^2+4(1-\delta)((1-\delta)(1-s)-\delta m(2s-1))>0 \tag{a4}$$

and

$$-\delta s^2(1-m)\left[(1-\delta)(1+s(1-s))-\delta ms^2\right]<0 \tag{a5}$$

for all δ and m in the unit interval and all $s\in\left[\sqrt{3}-1,1\right]$. In the same parameter region, one also has that (i) $\Gamma>0$ for all

$$\frac{a^2}{b^2}>\frac{\delta s^2(1-m)[(1-\delta)(1+s(1-s))-\delta ms^2]}{s^2(1-\delta(1-ms))^2+4(1-\delta)((1-\delta)(1-s)-\delta m(2s-1))} \tag{a6}$$

with the threshold on the r.h.s. of (a6) being always lower than one, so that $p_{\pm}^{*\prime}(a)$ is real. Then, to identify the correct solution, it suffices to verify that only $p_{-}^{*\prime}(a)$ indeed belongs to the interval $\left(p^N,a/2\right)$. This completes the proof.■

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