The Role of Income Distribution in the Diffusion of Corporate Social Responsibility

Simone D’Alessandro† Domenico Fanelli‡

Abstract

The purpose of this paper is to investigate the link between CSR growth and income distribution. We present a general equilibrium model where social responsibility enters both firms’ and consumers’ decisions. The model admits one single equilibrium. However, depending on the values of parameters, different scenarios may arise, each characterized by a different diffusion of CSR. We study the conditions under which there exists a virtuous circle which ties increases in the diffusion of CSR to reductions in income inequality and vice versa. Under certain circumstances, any policy which promotes CSR diffusion induces a reduction in income inequality. By contrast, when such conditions are not satisfied, only redistributive policies may generate the virtuous circle.

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†Department of Economics, University of Pisa, Via Ridolfi 10, Pisa, Italy. Tel.: +39 050 2216333; Fax: +39 050 598040. E-mail address: s.dale@ec.unipi.it.
‡Department of Economics, Ca’ Foscari University of Venice, Cannaregio, 873 S. Giobbe - Venice, Italy. E-mail address: domenico.fanelli@unive.it.
1 Introduction

Corporate social responsibility (CSR) is defined as “[a] concept whereby companies integrate social and environmental concerns in their business operations and in their interaction with their stakeholders on a voluntary basis” (see Green Paper, 2001).

Research into CSR can be traced back to an important question in the political and economic debate: do firms have any kind of social responsibility beyond the maximization of profits? This question is at the core of two articles written by Friedman (1970) and Arrow (1973). Both economists claim that the role of firms is to maximize profits as long as it stays within the rules embodied in law and in ethical custom, while the role of institutions is to establish the rules to prevent or repair the socially inefficient outcomes due to the maximization of profits. Whatever the opinion of the two economists, nowadays firms engage in CSR. A firm that commits to CSR – *ethical firm* – voluntarily undertakes an action which goes beyond legal requirements. There are several forms of CSR depending on which stakeholder the action is communicated to: consumers, investors, institutions. In this paper we focus on *CSR addressed to consumers*.

A CSR action is usually costly and firms may find it convenient to invest in CSR because some consumers – *socially concerned consumers* – positively value CSR commitments of firms. This phenomenon is called ethical consumption. Many surveys and opinion polls show that the majority of consumers have a positive attitude towards CSR (see Environics International, 1999, MORI, 2000, The Co-operative Bank, 2007); however there appears to be a divergence between these findings and the volume of sales of ethical products: although in the last years there has been a substantial increase in investment in CSR activities in all OECD nations (see Paton and Siegel, 2005), ethical market is still a small proportion of the total annual household consumer spending (see, for instance, The Co-operative Bank, 2007, Tallontire et al., 2001). Not all the consumers who declare to be socially (or environmentally) concerned purchase ethical firms’ products. Indeed many studies show that income distribution is a crucial variable in determining ethical firms’ demand (see Starr, 2009, Livraghi, 2007 and D’Alessio
et al., 2007). These studies show that consumers that purchase ethical commodities usually have a medium-high level of income implying that only the richest share of socially concerned consumers can afford ethical commodities. This may explain the aforementioned divergence as a phenomenon related to both the price of ethical commodities and the income of consumers.

The purpose of this paper is to investigate the role of income distribution in the diffusion of CSR. Our main finding is that under certain circumstances there exists a virtuous circle which ties increases in the diffusion of CSR to reductions in income inequality. This result has strong policy implications since the public authority considers both CSR growth and inequality reduction as two crucial policy goals to obtain a sustainable economy (see, as an example, the Europe 2020 strategy).

Most contributions in economics on CSR go beyond the debate on whether firms should undertake a CSR action (see Friedman, 1970 and Arrow, 1973) and include CSR differentiation strategies of firms and social preferences of consumers in partial equilibrium models (see, amongst others, Arora and Gangopadhyay, 1995, Amacher et al., 2004, Alves and Santos-Pinto, 2008, Besley and Ghatak, 2007, Becchetti and Solferino, 2003, Bagnoli and Watts, 2003, Conrad, 2005, Davies, 2005, Mitrokostas and Petrakis, 2008, Rodriguez-Ibeas, 2007). These contributions are similar in their structure to traditional product differentiation models where firms offer different qualities of products at different segments of the market (see, for instance, Motta, 1993, Tirole, 1988). The majority of these contributions are two-stage duopoly games in which firms first decide their commitments to CSR and then compete over prices (or quantities). Commitments to CSR are costly and at increasing levels of CSR are associated higher production costs. In equilibrium, firms are able to separate the market and obtain different demand functions: the firm with the highest commitment to CSR serves the most socially concerned consumers while the firm with the lowest commitment to CSR serves the less concerned consumers, each firm obtaining positive market demands and profits. On the consumers’ side, most of these contributions assume that the entire population of consumers are socially (or environmentally) concerned in the sense that they are willing to pay a price premium for buying ethical firms’ products. Only Besley and Ghatak
(2007) and Rodríguez-Ibeas (2007) assume two types of consumers: socially concerned and traditional consumers. The group of traditional consumers does not give any value to the CSR attributes of products. This assumption is relevant because not all the consumers positively value CSR commitments of firms.

We follow this literature by assuming that some consumers are socially concerned, and that CSR is modeled as a variable cost that affects the prices of ethical firms. By contrast, we adopt a general equilibrium perspective. This approach allows us to go one step forward in understanding ethical consumption, that is, it allows us to investigate the relationship between CSR growth and income inequality. Such a relationship can not be properly analyzed in a partial equilibrium set-up. The role of income distribution in the diffusion of CSR, to the best of our knowledge, has not been yet analyzed, even if, as shown by Starr (2009), Livraghi (2007) and D’Alessio et al. (2007), it is a crucial variable in determining ethical firms’ demand.

We present a simple version of a general equilibrium model. The economy is divided into two sectors, the standard and the ethical. We refer to the latter as the sector where ethical firms operate. Moreover, a share of consumers are socially concerned. Hence, social responsibility is incorporated in the model both in production and consumption decisions. We make two assumptions. i) Traditional consumers consider ethical and standard products as perfect substitutes, while socially concerned consumers have nonhomothetic preferences: as the attainable level of utility increases, the preference for ethical products rises. As a result, an increase in income may induce socially concerned consumers to switch from standard to ethical products rather than consuming more of standard goods (see Livraghi, 2007, D’Alessio et al., 2007). ii) Only one group of workers receives a share of

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1In Besley and Ghatak (2007) and Rodriguez-Ibeas (2007) “traditional” consumers are respectively called “neutral” and “brown” consumers while “socially concerned” consumers are respectively called “green” and “caring” consumers.

2Applications of CSR to a general equilibrium set-up has not been deeply analyzed so far. Two examples in this direction are Allouch (2009) and Becchetti and Adriani (2004). In Becchetti and Adriani (2004), authors analyze a North-South model of trade, where a single consumption good is produced in the two countries. However, income distribution does not affect the equilibrium outcome in their model.

3This assumption is well documented in the marketing and the economics literature (see, for instance, Allenby and Rossi, 1991). Allenby and Rossi (1991) study a discrete
profits in addition to wages. This implies that consumers’ demands depend not only on preferences, but also on income distribution. Hence, we can investigate whether income inequality is a deterrent to CSR growth.

We assume that ethical sector devotes a percentage of its product to obtain an ethical certification which does not apply to the standard sector since ethical firms internalize a social cost which standard firms neglect. We also assume that the amount of product used to obtain the ethical certification is destroyed and hence it is not redistributed to any economic agent; the intuition behind this assumption is similar to that of “iceberg” cost à la Samuelson (1954). We may also assume that the share of product lost to internalize the social cost is distributed to a sector whose aim is to reduce negative production externalities suffered by consumers. However, such analysis would go beyond the purposes of our study, since our aim is to explain the link between income distribution and the expansion of the E-sector assuming both CSR growth and reduction of inequality as two crucial policy goal of public authority. Indeed, we do not justify such assumption in terms of welfare analysis even if it would be interesting for future research.

Under these assumptions the model alternatively admits, at equilibrium, different scenarios each characterized by a different extent of the ethical sector. The extent of each sector is given by the fraction of the total labor force employed in that sector. Preferences and the presence of two classes of income can produce four different cases: all consumers purchase ethical goods (this happens when the price of ethical goods is lower than the price of standard goods), only socially concerned consumers purchase ethical goods, only socially concerned consumers getting the share of profits purchase them, no one does it. The relation between the price of ethical goods and the two classes of income determines which of these situations emerges.

This result is relevant because the emergence at equilibrium of one scenario rather than another influences income inequality. We find that, under plausible conditions the increase in the size of the ethical sector is associated

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choice model where consumers have nonhomotetic preferences over different brands of the same product. They use a model with nonconstant marginal utility which produces a system of linear but rotating indifference curves. Rotating indifference curves allow utility maximizing choice behavior to exhibit switching from low to high quality brands due to income effects.
with a reduction of inequality. In this case there exists a virtuous circle between two policy goals: the expansion of the ethical sector and the reduction of inequality. Under such conditions any policy which promotes the diffusion of CSR induces a reduction of income inequality. By contrast, when such conditions do not apply, we show that only redistributive policies can promote both a reduction of inequality and an increase in CSR diffusion.

The next section introduces the main features of the model. Section 3 describes the assumptions on preferences and income distribution. In Section 4, we investigate the equilibrium configurations of the model. In Section 5, we give a brief description of the dynamics. In Section 6, we find the circumstances under which there exists the virtuous circle. In Section 7 we investigate the consequences of two kinds of policies that affect preferences for ethical consumption and income distribution. Section 8 concludes.

2 A General Equilibrium Model

The economy is divided into two sectors, the standard (S) and the ethical (E). In each sector a representative firm operates. The two representative firms produce a single good with two similar technologies. The ethical sector (hereafter, E-sector) devotes a percentage \( c \in (0, 1) \) of its product \( E \) to obtain an ethical certification which does not apply to the standard sector (hereafter, S-sector).\(^4\) We denote \( w_E \) and \( w_S \) as the wage of E and S-sector respectively. In both sectors, firms maximize profits. Profits are equally shared among a quota, \( \sigma \in (0, 1] \), of the labor force, \( L \), irrespective of the sector where they work. We define \( \gamma \) as the quota of workers employed in the S-sector, \( \gamma \equiv L_S / L \). Since we assume full employment in the economy, \( L = L_S + L_E \), the quota of workers employed in the E-sector \( L_E / L \) is given by

\(^4\)As we pointed out in the introduction, ethical firms must internalize a social cost which standard firms neglect. In our model the amount of product used to obtain the ethical certification is destroyed and hence it is not redistributed to any economic agent; this implies that, at equilibrium, the demand of ethical goods \( D_E \) is equal to the net supply of ethical goods \( E(1-c) \), where \( E \) is the supply of ethical goods and \( cE \) is the share of product that is lost to internalize the social cost of certification. The intuition behind this assumption is similar to that of “iceberg” cost à la Samuelson (1954). We may also assume that the share \( cE \) is distributed to a sector whose aim is to reduce negative production externalities suffered by consumers. However such analysis would go beyond the purposes of our study, since our aim is to explain the link between income distribution and the expansion of the E-sector.
We assume that the production in the two sectors follows a Cobb-Douglas technology. Hence the two production functions are

\[ S(\gamma) = B(\gamma L)^\beta, \quad (1) \]

where \( B > 0 \) and \( \beta \in (0, 1) \), and

\[ E(\gamma) = A[(1 - \gamma)L]^\alpha, \quad (2) \]

where \( A > 0 \) and \( \alpha \in (0, 1) \), in the S- and E-sectors respectively. Total profits are given by

\[ \Pi = \Pi_S + \Pi_E, \quad (3) \]

where, given (1) and (2)

\[ \Pi_S = p_S S(\gamma) - w_S \gamma L, \quad (4) \]

\[ \Pi_E = p_E (1 - c) E(\gamma) - w_E (1 - \gamma)L. \quad (5) \]

Profit maximization implies

\[ w_S = p_S S'(\gamma), \quad (6) \]

\[ w_E = p_E (1 - c) E'(\gamma), \quad (7) \]

where \( S' \) is \( \frac{dS}{dL} \) and \( E' \) is \( \frac{dE}{dL} \). We assume that labor is perfectly mobile; hence at equilibrium the wages in the two sectors must be equal, that is \( w \equiv w_E = w_S \). Defining the standard commodity as numeraire, \( p_S = 1 \), from (1), (2), (6) and (7), we get

\[ w = \beta B(\gamma L)^{\beta - 1}, \quad (8) \]

and

\[ p_E = \frac{S'(\gamma)}{(1 - c) E'(\gamma)} = \frac{\beta B[(1 - \gamma)L]^{1-\alpha}}{\alpha A(\gamma L)^{1-\beta}(1 - c)}. \quad (9) \]
From equations (1), (2), (3), (8) and (9) we obtain:

$$\Pi = \frac{B}{\alpha^\gamma} \gamma^{\beta-1} L^\beta [(1 - \gamma)\beta + \alpha \gamma - \alpha \beta].$$

(10)

## 3 Preferences and Income Distribution

We assume that there are two types of consumers, socially concerned and traditional. The share of socially concerned consumers is denoted by $\phi \in (0,1)$, while the share of traditional consumers is $1 - \phi$. Budget constraint of both types of consumers is

$$p_E q_E + q_S \leq y,$$

(11)

where $y > 0$ is the income of each consumer.

Traditional consumers are not interested in ethical aspects of product and consider goods produced by the two sectors as perfect substitutes (as products of the same quality); the utility of traditional consumers is then given by:

$$U_T(q_E, q_S) = q_E + q_S,$$

(12)

where $q_i$ is the quantity of the $i$–th sector demanded by each traditional consumer.

Differently from traditional consumers, socially concerned consumers consider ethical products as of higher quality and their preferences are non-homothetic: as the attainable level of utility for the consumers increases, ethical goods are highly valued. In particular, we assume that the indifference curves of the utility function are linear and rotating in slope as the level of utility increases (see Figure 1):

$$q_E = \frac{u}{2} - \frac{a}{2u} q_S,$$

(13)

where $0 < a < 4 \min y$ and $u$ is the attainable level of utility.\(^5\) This implies that as the budget constraint shifts out, socially concerned consumers will

\(^5\)When $a < 4 \min y$, we have that, if $p_E \leq p_S$, $MRS_{E,S} > \frac{p_E}{p_S}$ and socially concerned consumers find it convenient to spend entirely their income in buying ethical goods. Otherwise (when $a \geq 4 \min y$) we would have the case in which, for $p_E \leq p_S$, socially concerned consumers may find it convenient to spend entirely their income in buying standard goods.
switch from standard to ethical products rather than consuming more of standard goods (see Livraghi, 2007, D’Alessio et al., 2007). The utility of socially concerned consumers is given by:

\[ U_{SC}(q_E, q_S) = q_E + \sqrt{q_E^2 + aq_S}. \] (14)

Figure 1: Indifference curves for socially concerned consumers when \( a = 1. \)

By maximizing utility of each type of consumer subject to the budget constraint we obtain their demand functions. The quantities demanded of goods \( S \) and \( E \) by each traditional consumer are

\[ q_S = \begin{cases} 0, & \text{if } p E \leq 1, \\ y, & \text{if } p E > 1 \end{cases} \] (15)

and

As we will see throughout this section, there are two classes of income: a share \( \sigma \in (0, 1) \) of the population receives, besides their wages, an equal fraction \( \theta \) of total profits (the income of this class of laborers is then given by \( w + \theta \)), while the share \( 1 - \sigma \) receives only wage \( w \). This implies that \( \min y \equiv \min w \). By (8), we have that, for \( \gamma = 1 \), \( w(\gamma) \) is minimum and is given by \( w(1) = \frac{\beta B}{1-\sigma} \); hence \( \min y \equiv \min w = \frac{\beta B}{1-\sigma} \).
\[ q_E = \begin{cases} y, & \text{if } p_E \leq 1, \\ 0, & \text{if } p_E > 1 \end{cases} \quad (16) \]

The quantities demanded of goods \( S \) and \( E \) by each socially concerned consumer are

\[ q_S = \begin{cases} 0, & \text{if } 0 < p_E \leq 2\sqrt{\frac{y}{a}}, \\ y, & \text{if } p_E > 2\sqrt{\frac{y}{a}} \end{cases} \quad (17) \]

and

\[ q_E = \begin{cases} \frac{y}{p_E}, & \text{if } 0 < p_E \leq 2\sqrt{\frac{y}{a}}, \\ 0, & \text{if } p_E > 2\sqrt{\frac{y}{a}} \end{cases} \quad (18) \]

To define the aggregate demand functions for the two sectors we need to introduce income distribution. In particular, we assume that a share of the population \( \sigma \in (0, 1) \) receive, besides their wages, an equal fraction \( \theta \) of total profits\(^7\). From equation (10): \[ \theta = \frac{\Pi}{\sigma L} = \frac{B}{\alpha \sigma} (\gamma L)^{\beta - 1} [(1 - \gamma) \beta + \alpha \gamma - \alpha \beta], \quad (19) \]

A share \((1 - \sigma)\) of the labor force receives only wages. For the sake of argument, both workers employed in the \( S \)- and the \( E \)-sectors may receive a share of profits. Since \( w_E = w_S \), we obtain only two different income classes: \((1 - \sigma)L\) workers gets \( w \), while \( \sigma L \) gets \( w + \theta \) independently of the sector where they work. This implies that the aggregate demand functions in the two sectors are given by

\[ D_S = \begin{cases} 0 & \text{if } 0 < p_E \leq 1, \\ (1 - \phi)(w + \sigma \theta)L & \text{if } 1 < p_E \leq \Psi, \\ (w + \sigma \theta)L - \phi \sigma (w + \theta)L & \text{if } \Psi < p_E \leq \overline{\Psi}, \\ (w + \sigma \theta)L & \text{if } p_E > \overline{\Psi} \end{cases} \quad (20) \]

\(^6\)Thanks to the condition \( 0 < a < 4 \min(y) \), we have that \( p_S = 1 < 2\sqrt{\frac{7}{a}} \) for any \( y \).

\(^7\)A similar assumption on income distribution is in Bilancini and D’Alessandro (2008).
D_E = \begin{cases} \frac{(w+\sigma \theta)L}{p_E} & \text{if } 0 < p_E \leq 1, \\ \frac{\phi}{p_E}(w+\sigma \theta)L & \text{if } 1 < p_E \leq \Psi, \\ \phi \sigma \frac{(w+\theta)L}{p_E} & \text{if } \Psi < p_E \leq \overline{\Psi}, \\ 0 & \text{if } p_E > \overline{\Psi}. \end{cases} \tag{21}

where \( \Psi \equiv 2\sqrt{\frac{w}{a}} \) and \( \overline{\Psi} \equiv 2\sqrt{\frac{w+\theta}{a}} \), with \( p_S = 1 < \Psi < \overline{\Psi} \).

4 Excess demand and equilibria

At equilibrium a vector of prices \( p^* = \{p_S^*, p_E^*\} \) ensures that demand and supply in each sector are equalized, i.e. \( D_S = S(\gamma) \) and \( D_E = (1-c)E(\gamma) \).

Since \( \frac{\partial p_E(\gamma)}{\partial \gamma} < 0, \forall \gamma \in [0,1] \), in order to study the features of the equilibria, we can focus on the share of workers employed in the S-sector \( (\gamma) \), which indirectly measures the degree of the E-sector development.

In the S-sector, from equations (8), (10), (20) and (19), we can define \( D_S \) as a function of \( \gamma \):

\[
D_S(\gamma) = \begin{cases} 
D_{S1}(\gamma) & \text{if } 0 < p_E(\gamma) \leq 1, \\
D_{S2}(\gamma) & \text{if } 1 < p_E(\gamma) \leq \Psi(\gamma), \\
D_{S3}(\gamma) & \text{if } \Psi(\gamma) < p_E(\gamma) \leq \overline{\Psi}(\gamma), \\
D_{S4}(\gamma) & \text{if } p_E(\gamma) > \overline{\Psi}; 
\end{cases} \tag{22}
\]

where \( D_{S1}(\gamma) = 0 \); \( D_{S2}(\gamma) = (1-\phi)f(\gamma)(\beta + \gamma(\alpha - \beta)); \) \( D_{S3}(\gamma) = D_{S2}(\gamma) + f(\gamma)\alpha\beta\phi(1-\sigma); \) \( D_{S4}(\gamma) = f(\gamma)(\beta + \gamma(\alpha - \beta)); \) \( f(\gamma) = \frac{S'L}{\alpha \beta} = \frac{BL^\beta}{\alpha \gamma^{1-\beta}}. \) Furthermore, it is easy to prove that

\( \Psi \equiv 2\sqrt{\frac{w}{a}} \) and \( \overline{\Psi} \equiv 2\sqrt{\frac{w+\theta}{a}} \), with \( p_S = 1 < \Psi < \overline{\Psi} \).
\[
\frac{\partial D_{Si}(\gamma)}{\partial \gamma} \leq 0, \quad \forall \gamma \in [0, 1] \quad (27)
\]

with \(i = 1, 2, 3, 4\). The sign of the derivative of \(D_{Si}\) is important in the description of the system dynamics (see Section 5).

Let us define \(Z(\gamma) = D_S(\gamma) - S(\gamma)\) as the excess demand function in the S-sector. Given the shape of the demand function, \(Z(\gamma)\) is a piecewise continuous function

\[
Z(\gamma) = \begin{cases} 
Z_1(\gamma) & \text{if } 0 < p_E(\gamma) \leq 1, \\
Z_2(\gamma) & \text{if } 1 < p_E(\gamma) \leq \Psi(\gamma), \\
Z_3(\gamma) & \text{if } \Psi(\gamma) < p_E(\gamma) \leq \overline{\Psi}(\gamma), \\
Z_4(\gamma) & \text{if } p_E(\gamma) > \overline{\Psi}(\gamma); 
\end{cases} 
\quad (28)
\]

where \(Z_j(\gamma) = D_{S,j}(\gamma) - S(\gamma)\) with \(j = 1, 2, 3, 4\), and \(Z_1(\gamma) \leq Z_2(\gamma) \leq Z_3(\gamma) \leq Z_4(\gamma) \forall \gamma\). The market clears if \(Z(\gamma) = 0\). Each \(Z_j(\gamma)\) is equal to zero for the following values of \(\gamma\):

\[
\gamma^*_{Z_1} = 0; \quad (29)
\]

\[
\gamma^*_{Z_2} = \frac{\beta(1 - \phi)}{\alpha\phi + (1 - \phi)\beta}; \quad (30)
\]

\[
\gamma^*_{Z_3} = \frac{\alpha \beta \phi(1 - \sigma) + \beta(1 - \phi)}{\alpha \phi + \beta(1 - \phi)}; \quad (31)
\]

\[
\gamma^*_{Z_4} = 1. \quad (32)
\]

Hence, \(\gamma^*_{Z_1}\) is an equilibrium if and only if \(p_E(\gamma^*_{Z_1}) \leq 1\), \(\gamma^*_{Z_2}\) if and only if \(1 < p_E(\gamma^*_{Z_2}) \leq \Psi(\gamma^*_{Z_2})\), \(\gamma^*_{Z_3}\) if and only if \(\Psi(\gamma^*_{Z_3}) < p_E(\gamma^*_{Z_3}) \leq \overline{\Psi}(\gamma^*_{Z_3})\), and \(\gamma^*_{Z_4}\) if and only if \(p_E(\gamma^*_{Z_4}) > \overline{\Psi}(\gamma^*_{Z_4})\). From (29), (30), (31) and (32), it follows that \(0 \leq \gamma^*_{Z_1} \leq \gamma^*_{Z_2} \leq \gamma^*_{Z_3} \leq \gamma^*_{Z_4}\). Moreover, it is easy to verify that

(a) \(\gamma^*_{Z_1}\) can never be an equilibrium since, for \(\gamma = \gamma^*_{Z_1}\), it is \(p_E(\gamma) \to +\infty\) and hence condition \(p_E(\gamma^*_{Z_1}) \leq 1\) is never satisfied;

(b) \(\gamma^*_{Z_4}\) can never be an equilibrium since, for \(\gamma = \gamma^*_{Z_4}\), it is \(p_E(\gamma) \to 0\) and hence condition \(p_E(\gamma^*_{Z_4}) > \overline{\Psi}(\gamma^*_{Z_4})\) is never satisfied since \(\overline{\Psi}(\gamma^*_{Z_4}) > 0\).
A numerical illustration of the model is represented in Figure 2. The first graph shows the curves $p_E(\gamma)$, $p_S = 1$, $\Psi(\gamma)$, and $\overline{\Psi}(\gamma)$. The second graph displays the excess demand function in the S-sector, which is denoted by the thickest curve $Z(\gamma)$. The lowest curve $Z_1(\gamma)$ shows the case in which all the consumers purchase the ethical good, $p_E < 1$, curve $Z_2(\gamma)$ the case in which all socially concerned consumers purchase the ethical good, $1 < p_E(\gamma) \leq \Psi(\gamma)$, curve $Z_3(\gamma)$ the case in which only the socially concerned consumers who get the share of profits, $\theta$, purchase the ethical good, $\Psi(\gamma) < p_E(\gamma) \leq \Psi(\gamma)$, while the highest curve $Z_4(\gamma)$ the case in which no one purchases it, $p_E(\gamma) > \overline{\Psi}(\gamma)$. In the interval $[0, \tilde{\gamma}]$ the excess demand function assumes the value $Z_4(\gamma)$ (since $p_E(\gamma) \leq \Psi(\gamma)$); in the interval $(\tilde{\gamma}, \bar{\gamma}]$ the excess demand function assumes the value $Z_3(\gamma)$ (since $\Psi(\gamma) < p_E(\gamma) \leq \Psi(\gamma)$); between $(\tilde{\gamma}, \tilde{\bar{\gamma}}]$ the value $Z_2(\gamma)$ (since $1 < p_E(\gamma) \leq \Psi(\gamma)$); between $(\tilde{\bar{\gamma}}, 1]$ the value $Z_1(\gamma)$ (since $p_E(\gamma) \leq 1$). In this example the model admits one equilibrium: $\gamma^*_Z$.

Figure 3 is a numeral illustration in which the equilibrium is $\gamma^*_Z$, while, Figure 4 shows the case in which there exists a stable limit cycle. A stable limit cycle exists if and only if, given $\gamma_1, \gamma_2 \in [0, 1]$ and $\gamma_2 = \gamma_1 + \epsilon$, $\forall$ arbitrarily small $\epsilon > 0$, it holds that

i. $Z(\gamma_1) = Z_i(\gamma_1)$ and $Z(\gamma_2) = Z_j(\gamma_1)$, with $i > j$;

ii. $Z(\gamma_1) > 0$ and $Z(\gamma_2) < 0$.

Figure 4 clarify this result. In $\gamma^{**}$ the excess demand function jumps from a positive to a negative value. Although prices do not clear the markets, market forces tend to keep the relative extent of the two sectors around $\gamma^{**}$ – i.e. $\gamma^{**}$ is a stable limit cycle.\(^\text{10}\)

We summarize our findings in the following Proposition.

**Proposition 4.1.** The model always admits either an equilibrium or a stable limit cycle.

**Proof.** In our model, any $Z_i(\gamma)$, for $i = 1, 2, 3, 4$, is a decreasing function of $\gamma$, $Z(0) \geq 0$, $Z(1) \leq 0$, and the excess demand function is always defined in

\(^\text{10}\)In order to better explain this result, the dynamics of the system must be introduced. This is discussed in the next section.
Figure 2: The first picture shows the graph of $p_E(\gamma)$, $p_S = 1$, $\Psi(\gamma)$ and $\overline{\Psi}(\gamma)$. The interceptions between $p_E$ and $\Psi$ or $\overline{\Psi}$ or $p_S = 1$ determine the intervals of the excess demand function. The second picture shows the graph of the excess demand function – i.e. the thickest piecewise curve. Values of parameters are: $c = 0.25$, $\phi = 0.5$, $\sigma = 0.5$, $a = 1$, $\alpha = 0.8$, $\beta = 0.75$, $B = 6$, $A = 2$, $L = 100$. 
Figure 3: Graph of the excess demand function. A stable equilibrium on $Z_2(\gamma)$. Values of parameters: $c = 0.15$, $\phi = 0.5$, $\sigma = 0.5$, $\alpha = 1$, $\alpha = 0.8$, $\beta = 0.75$, $B = 6$, $A = 3.5$, $L = 100$.

all its domain. Given these properties, we have the following results. There always exists either an equilibrium or a stable limit cycle, since otherwise there is no way to obtain $Z(1) \leq 0$ starting from $Z(0) \geq 0$.

It is easy to prove that when either $\alpha \geq \beta$ or $\alpha < \beta$ and $\alpha \leq \frac{1}{2}$, there exists only one intersection between curves $\Psi(\gamma)$ and $p_E(\gamma)$, and between $\Psi(\gamma)$ and $p_E(\gamma)$; this implies that the model admits only one equilibrium (see Figures 2 and 3) or a stable limit cycle (see Figure 4); otherwise, curves $\Psi(\gamma)$ and $p_E(\gamma)$ might intersect either one or three times, and, in principle, the model may admit the existence of both an equilibrium and a stable limit cycle.\(^\text{11}\) Hence, the existence of multiple equilibria is excluded.

\(^{11}\)The equilibrium that emerges can be either $\gamma^*_{Z_2}$ or $\gamma^*_{Z_3}$. In our numerical simulations three intersections between curves $p_E$ and $\Psi$ only emerge when $\beta$ is very close to 1. This means that curves $Z_3$ and $Z_4$ are very close to each other. In such a case, $\gamma^*_{Z_3}$ is close to one, and it is not possible to obtain one equilibrium and a limit cycle.
Figure 4: Graph of the excess demand function. The double circle indicates the limit cycle. Values of parameters: $c = 0.15$, $\phi = 0.8$, $\sigma = 0.5$, $a = 1$, $\alpha = 0.7$, $\beta = 0.75$, $B = 4$, $A = 3$, $L = 100$.

5 Dynamics

Let us assume that at a certain instant $\gamma = \gamma_0$ and $Z(\gamma_0) > 0$, i.e. there is an excess of demand in the S-sector and an excess of supply in the E-sector. Since we defined the standard commodity as numeraire, market forces tend to reduce the relative price of the ethical goods, i.e. $p_E$ decreases. Since the price of the E-sector is decreasing in $\gamma$, the reduction in $p_E$ induces an increase in $\gamma$. The change in $\gamma$ modifies the distribution in the economy. However, from inequality (27), an increase in $\gamma$ implies a decrease in the demand of the S-sector. Hence, as expected, the reduction in the price of ethical goods induces an increase in the demand of the E-sector. This adjustment process continues until the relative price of ethical goods is such that $Z(\gamma) = 0$.

In other words, the univocal relation between $p_E$ and $\gamma$ allows us to consider the dynamics of the model in terms of $Z(\gamma)$ and $\gamma$. We capture the
movement of the system through the following dynamics:

$$\dot{\gamma}_t = h(Z(\gamma_t)),$$

(33)

where $t$ is the time index, $\dot{\gamma}_t \equiv \frac{d\gamma}{dt}$, $\frac{dh(Z)}{dZ} > 0$, and $\dot{\gamma}_t = 0 \iff h(0) = 0$, that is when the economy is at equilibrium. As we pointed out in Section 4, the model can admit two different equilibrium configurations\(^{12}\), hence initial conditions determine which equilibrium arises. Internal equilibrium, if it exists, is always locally stable since the derivative of each excess demand function with respect to $\gamma$ is always negative (see inequality (27)).

The basin of attraction of any equilibrium for $\gamma \in [0, 1]$ is given by the interval defined by the maximum $\gamma$ in which $Z(\gamma) < 0$ for any $\gamma < \gamma^*_2$; and by the minimum $\gamma$ in which $Z(\gamma) > 0$ for any $\gamma > \gamma^*_3$. If these two values do not exist, the boundaries are $\gamma = 0$ and $\gamma = 1$ respectively. For instance, in Figures 2 and 3 the basin of attraction of the equilibrium is always defined by the interval $[0, 1]$. A different basin of attraction can emerge only when both an equilibrium and a stable limit cycle exist.

Figure 4 shows the phase diagram of the model with the presence of a stable limit cycle around $\gamma^{**}$ – marked with a double circle. On the left of $\gamma^{**}$ there is an excess of demand in the S-sector, hence $\gamma$ tends to increase. By contrast, on its right side there is an excess of supply, hence $\gamma$ tends to decrease. This dynamics generates a stable limit cycle.

### 6 CSR growth and Income Inequality

Expansion of the E-sector affects income inequality in the economy since to different values of $\gamma$ different levels of wage and total profits are associated – see equations (8) and (10). This issue is relevant because i) the emergence of either $\gamma^*_2$ or $\gamma^*_3$ as equilibrium affects the degree of inequality in the economy; ii) policies on preferences and income distribution shape the demand in the two sectors, moving the equilibrium.

We define as **virtuous circle** a trajectory of $\gamma$ which associates an expansion of the E-sector (a reduction of $\gamma$) to a reduction of income inequality and viceversa. A central question of this chapter is to study under what\(^{12}\) $\gamma^*_2$ and $\gamma^*_3$. 

\(^{12}\)
conditions the described virtuous circle emerges. In order to investigate this issue, in Appendix A1 we compute the Gini Index for this economy, \(G(\gamma)\), as an index of income inequality.\(^{13}\) Then it holds that

\[
G(\gamma) = \frac{1 - \sigma}{2} \left( 1 - \frac{\alpha \beta}{L(\beta + \gamma(\alpha - \beta))} \right).
\]

(34)

Proposition 6.1 presents the results on the relation between the Gini Index and \(\gamma\).

**Proposition 6.1.** If \(\alpha > \beta\), then \(\frac{\partial G(\gamma)}{\partial \gamma} > 0\), for any \(\gamma \in [0, 1]\). Otherwise, \(\frac{\partial G(\gamma)}{\partial \gamma} \leq 0\), for any \(\gamma \in [0, 1]\).

*Proof.* From equation (34), it holds that

\[
\frac{\partial G(\gamma)}{\partial \gamma} = \frac{(1 - \sigma)\alpha \beta(\alpha - \beta)}{2L(\beta + \gamma(\alpha - \beta))^2}.
\]

(35)

This derivative is positive for \(\alpha > \beta\), while it is non-positive otherwise. \(\square\)

When the derivative of the Gini Index with respect to \(\gamma\) is positive, any expansion of the E-sector – that is a reduction in \(\gamma\) – reduces inequality in the economy. Proposition 6.1 proves that this result holds if and only if the share of product going to workers in the E-sector is higher than the corresponding share in the S-sector, that is \(\alpha > \beta\).\(^{14}\)

For instance, in Figures 2 and 3, \(\alpha > \beta\). Hence given Proposition 6.1 starting from a small E-sector (\(\gamma\) close to 1), its expansion (driven by the dynamics of the model) induces a reduction of income inequality: that is a virtuous circle. In Figure 4 instead \(\alpha < \beta\) and hence to a reduction of \(\gamma\) is associated more inequality. Moreover, depending on which equilibrium arises, the model generates qualitatively different scenarios. For instance, in Figure 3, the increase in the E-sector is significantly higher than that in Figure 2 (since \(\gamma_{Z2}^* < \gamma_{Z3}^*\)) and hence also the associated reduction of income inequality is higher in case of Figure 3 than that of Figure 2. Through

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\(^{13}\)As is well known, the Gini Index is an increasing function of income inequality. In particular when \(G(\gamma) = 0\), the inequality is minimal (all consumers have the same income), while when \(G(\gamma) = 1\), the inequality is greatest.

\(^{14}\)It seems reasonable that in real economies the share of product going to profits is lower in the E-sector than in the standard one, since the respect of criteria, especially labor ones, can easily induce a reduction in the share of profits.
distributional and preference levers policy makers may shape the demand in the two sectors, shifting the equilibrium. In the next section we investigate the impact of such policies on the two goals: reduction of inequality and expansion of the ethical sector; that is on the building of a virtuous circle.

7 Policy Implications

We concentrate our analysis on two kinds of policies that affect preferences – through $\phi$ – and income distribution – through parameter $\sigma$. The model shows the following two properties:

a) Parameter $\phi$ does not influence $\Psi$, $\overline{\Psi}$ and $p_E$. Hence the values of $\gamma$ at which the excess demand function is discontinuous do not vary through changes in $\phi$. By contrast, $\phi$ influences $Z_2$ and $Z_3$ with $\frac{dZ_2}{d\phi} < \frac{dZ_3}{d\phi} = \frac{dZ_4}{d\phi} = 0$. Hence an increase in $\phi$ induces a lower value of $\gamma^*_{Z_2}$ and $\gamma^*_{Z_3}$.\footnote{There are other parameters which may affect income distribution (e.g. $\alpha$ and $\beta$) and the behavior of consumers. However, given our framework $\sigma$ and $\phi$ generate more interesting results and can be easily influenced by policy makers.}

b) Parameter $\sigma$ influences $\overline{\Psi}$ with $\frac{d(\overline{\Psi})}{d\sigma} < 0$. This implies that intervals of $\gamma$ in which $Z$ takes values of $Z_3$ and $Z_4$ can be influenced by $\sigma$. This happens when $\overline{\Psi}$ intersects $p_E$. Moreover, $\sigma$ influences $Z_3$ with $\frac{dZ_3}{d\sigma} < 0 = \frac{dZ_2}{d\sigma} = \frac{dZ_4}{d\sigma}$. Hence an increase in $\sigma$ induces a lower value of $\gamma^*_{Z_3}$.

Let us assume that the economy is at equilibrium $\gamma^*_{Z_2}$ or $\gamma^*_{Z_3}$ and policy makers induce an increase in $\phi$. This change always causes an expansion of the ethical sector. Indeed, the S-sector switches from an equilibrium position to an excess of supply. This in turn leads to a reduction in $\gamma^*$ and the extent of the E-sector increases (see Property “a” above). Finally if the economy is at a stable limit cycle, the effects of an increase in $\phi$ can produce different results whether the limit cycle is between $Z_4$ and $Z_3$ or between $Z_3$ and $Z_2$ (or between $Z_2$ and $Z_1$). Indeed, while in the first case policy makers cannot induce any change (since $Z_4$ is fixed), in the other two

\footnote{As we pointed out in Section 4, each $\gamma^*_{Z_j}$ ($j = 2, 3$) may not be an equilibrium. However, this result applies both when $\gamma^*_{Z_j}$ is and is not an equilibrium.}
cases the increase in $\phi$ may induce the S-sector to switch from an excess of demand to an excess of supply. Hence, the limit cycle disappears and the E-sector increases.

Differently from $\phi$, $\sigma$ does not affect preferences but may affect consumers’ behavior through changes in income distribution. For instance, an increase in $\sigma$ reduces the income of consumers receiving the share of profits, but increase their number. As we pointed out in Property “b”, this implies that both $\overline{\nu}$ and the excess demand function $Z_3$ shift downward. Hence, if the economy is at equilibrium $\gamma_{Z_2}^*$, a change in $\sigma$ has no consequences. If instead the economy is at equilibrium $\gamma_{Z_3}^*$, the increase in $\sigma$ implies an increase in the E-sector if, at equilibrium, the class of richest ethical consumer still purchase the ethical good. Otherwise, i.e. after the change in $\sigma$, equilibrium $\gamma_{Z_3}^*$ disappears, a stable limit cycle between $Z_3$ and $Z_4$ takes place and the size of the E-sector decreases since a lower number of consumers purchase the ethical good. The opposite applies when $\sigma$ decreases. Finally, if the economy lies in a limit cycle between $Z_3$ and $Z_2$, the increase in $\sigma$ has the same effect as an increase in $\phi$.

Changes in the relative sizes of the two sectors affect the level of inequality in the economy. We can characterize the effect of changes of $\phi$ and $\sigma$ on the Gini index derived in the previous section. Parameter $\phi$ does not directly affect $G(\gamma)$, see equation (34). However, as analyzed above, changes in $\phi$ can affect the extent of the E-sector, and hence through $\gamma$ the level of inequality. By Proposition 6.1, we prove that for $\alpha > \beta$, policies on preferences that increase the extent of the E-sector result in a reduction of inequality. Otherwise, policies on preferences that increase the extent of the E-sector result in an increase of inequality. In other words, when the share of product going to workers in the E-sector is greater than that in the S-sector, policies which induce an expansion of ethical sector also lead to a reduction of inequality, i.e. policies produce a virtuous circle.

Parameters $\sigma$ directly enter the Gini Index. Without considering the effect of $\sigma$ on $\gamma$, an increase in $\sigma$ induces a reduction in the Gini Index, see equation (34). However, as analyzed above, changes in $\sigma$ can also affect the extent of the E-sector. The effect of $\gamma$ on $G(\gamma)$ is given by Proposition 6.1. Hence, if $\alpha > \beta$ policies that increase the extent of the E-sector, through
an increase in $\sigma$, also reduce income inequality, i.e. they produce a virtuous circle. If instead $\alpha < \beta$, while the increase in $\sigma$ tends to reduce income inequality, the increase in the E-sector goes in the opposite direction. Hence, the dominant effect determines whether the inequality decreases, and hence whether redistributive policies result in an expansion of E-sector. We found that redistributive policies can generate a virtuous circle even if $\alpha < \beta$. As an example, Appendix A2 shows that this result holds for a wide range of parameters when the economy lies at the equilibrium $\gamma_{Z_3}^*$. 

Finally, the increase in the E-sector may be due to a reduction of $\sigma$. In this case, the effects of policies on $\sigma$ and on the expansion of the ethical sector work in the opposite directions to those illustrated above.\(^{17}\)

### 8 Concluding Remarks

We introduced CSR differentiation in a general equilibrium model. The main novelty was the analysis of the role of income distribution in CSR growth. We made two assumptions: i) socially concerned consumers do not purchase ethical goods below a threshold level of attainable utility, while, over this threshold, they switch from standard to ethical products (rather than consuming more of standard products) and they totally spends their income in the CSR sector; ii) there are only two classes of income, since profits are equally distributed among a fraction of the labor force. The model admits one single equilibrium. Depending on the values of parameters two alternative equilibrium configurations may arise, each characterized by a different size of the E-sector. Different hypotheses generate different scenarios but do not change the finding that income inequality is a deterrent to the diffusion of CSR. In our set-up, we found that when the share of product going to workers is higher in the ethical sector than in the standard one, there is a virtuous circle which ties CSR growth to inequality reduction. In this case, any policy which increases the demand for ethical commodities results in a reduction of inequality. Otherwise, only redistributive policies can generate the virtuous circle between those two policy targets. This result holds for a

\(^{17}\)That is, when $\alpha > \beta$ changes in $\sigma$ and $\gamma$ conflictingly affect the Gini Index while, when $\alpha < \beta$ they work in the same direction.
wide range of parameters.

The Lisbon Strategy identifies in CSR diffusion a valuable instrument for European development. Our contribution argued that income distribution and CSR cannot be independently analyzed.

Appendixes

A1. The Gini Index

The Gini Index is defined as the ratio of the area that lies between the line of equality and the Lorenz curve (marked $C$ in Figure 5) to the total area under the line of equality (the sum of areas $A$, $B$ and $C$ in Figure 5), i.e. the Gini Index, $G(\gamma)$ is given by the ratio $\frac{C}{A+B+C}$. Since in our model there are only two classes of income, the Lorenz curve drawn in Figure 5 is given by two segments of different shapes: in relative terms, $\frac{w+\theta}{y}$ for the share of poorest workers and $\frac{w+\theta}{y}$ for the share of richest ones, where $y$ is the average per capita income, i.e. $y = w + \frac{\Pi}{L}$. The share of workers which does not receive profits is $1 - \sigma$. Hence their cumulative income expressed in the vertical axis is $y_1 = \frac{w}{y}(1 - \sigma)$. By determining the areas $A$, $B$ and $C$, it holds that

$$G(\gamma) = \frac{1 - \sigma}{2} \left(1 - \frac{\alpha \beta}{L[\beta + \gamma(\alpha - \beta)]}\right).$$  \hspace{1cm} (36)

From equations (8), (19) and (36), we get equation (34) of Section 6.

A2. Policies and virtuous circle

Let us assume that the economy is located in $\gamma_{Z_3}^*$ and $L = 1$. From (34), it results that $\sigma$ influences directly both the Gini Index and $\gamma_{Z_3}^*$. Hence, to obtain the full effect of $\sigma$ on the Gini Index, we substitute $\gamma_{Z_3}^*$ in $G(\gamma)$ and we compute the derivative $\frac{\partial G(\gamma_{Z_3}^*)}{\partial \sigma}$:

$$\frac{\partial G(\gamma_{Z_3}^*)}{\partial \sigma} = \frac{A\sigma^2 + B\sigma + C}{[1 - \phi(\alpha - \beta)k(1 - \sigma)]^2},$$  \hspace{1cm} (37)

where

$$A = -\phi^2(\alpha - \beta)^2 < 0,$$  \hspace{1cm} (38)

18This assumption is made to simplify the analysis and do not modify results.
Cumulative share of workers

Cumulative share of income

Figure 5: Gini Index

\[ B = 2\phi(\alpha - \beta)[1 + \phi(\alpha - \beta)] \]  

(39)

and

\[ C = \beta - 1 - \phi(\alpha - \beta)[1 + \phi(\alpha - \beta)]. \]  

(40)

From (37), it holds that \( \frac{\partial G(\gamma Z_3)}{\partial \sigma} > 0 \) if and only if \( A\sigma^2 + B\sigma + C > 0 \) and \( \frac{\partial G(\gamma Z_3)}{\partial \sigma} < 0 \) otherwise. The numerator of (37) is a second-order polynomial which can be represented by a concave parabola – see (38) – whose roots are

\[ \sigma_1 = \frac{\phi(\alpha - \beta) + 1 + \sqrt{\Delta}}{\phi(\alpha - \beta)} \]  

(41)

and

\[ \sigma_2 = \frac{\phi(\alpha - \beta) + 1 - \sqrt{\Delta}}{\phi(\alpha - \beta)}, \]  

(42)

with \( \Delta \equiv B^2 - 4AC = \phi(\alpha - \beta) + \beta > 0 \) for any value of \( \alpha, \beta \) and \( \phi \).

When \( \alpha > \beta, \sigma_1 > \sigma_2 > 1 \) and hence \( A\sigma^2 + B\sigma + C < 0 \) for any \( \sigma \in [0, 1] \). Therefore, \( \frac{\partial G(\gamma Z_3)}{\partial \sigma} < 0 \). If instead \( \alpha < \beta, \sigma_1 < 0 \) and the sign of \( \sigma_2 \) depends on \( \alpha, \beta \) and \( \phi \). In particular:

- If \( \beta < \frac{3}{4} \) for any \( \alpha \in [0, 1], \sigma_1 < \sigma_2 < 0 \). Hence \( A\sigma^2 + B\sigma + C < 0 \) for any \( \sigma \in [0, 1] \) and \( \frac{\partial G(\gamma Z_3)}{\partial \sigma} < 0 \).
• If $\frac{3}{4} < \beta < 1 + \alpha - \sqrt{\alpha}$ and $\frac{1}{4} < \alpha < 1$, $\sigma_1 < \sigma_2 < 0$. Hence $A\sigma^2 + B\sigma + C < 0$ for any $\sigma \in [0, 1]$ and $\frac{\partial G(\gamma_Z)}{\partial \sigma} < 0$.

• If $\frac{3}{4} < \beta < 1 + \alpha - \sqrt{\alpha}$ and $\alpha < \frac{1}{4}$, $\sigma_1 < \sigma_2 < 0$ for $\phi < \frac{-1+\sqrt{-3+4\beta}}{2(\alpha-\beta)}$ or $\phi > \frac{-1-\sqrt{-3+4\beta}}{2(\alpha-\beta)}$, and $0 < \sigma_2 < 1$ for $\frac{-1+\sqrt{-3+4\beta}}{2(\alpha-\beta)} < \frac{-1-\sqrt{-3+4\beta}}{2(\alpha-\beta)}$. Hence, if $\phi < \frac{-1+\sqrt{-3+4\beta}}{2(\alpha-\beta)}$ or $\phi > \frac{-1-\sqrt{-3+4\beta}}{2(\alpha-\beta)}$, $\frac{\partial G(\gamma_Z)}{\partial \sigma} < 0$ for any $\sigma \in [0, 1]$, while, for $\frac{-1+\sqrt{-3+4\beta}}{2(\alpha-\beta)} < \phi < \frac{-1-\sqrt{-3+4\beta}}{2(\alpha-\beta)}$, $\frac{\partial G(\gamma_Z)}{\partial \sigma} < 0$ if and only if $\sigma_2 < \sigma < 1$, and $\frac{\partial G(\gamma_Z)}{\partial \sigma} > 0$ if and only if $0 < \sigma < \sigma_2$.

• If $1 + \alpha - \sqrt{\alpha} < \beta < 1$, $\alpha < \frac{1}{4}$ and $\alpha > \frac{1}{2}$, then $\sigma_1 < \sigma_2 < 0$ for $0 < \phi < \frac{-1+\sqrt{-3+4\beta}}{2(\alpha-\beta)}$, and $0 < \sigma_2 < 1$ for $\frac{-1+\sqrt{-3+4\beta}}{2(\alpha-\beta)} < \phi < 1$. Hence, if $0 < \phi < \frac{-1+\sqrt{-3+4\beta}}{2(\alpha-\beta)}$, $\frac{\partial G(\gamma_Z)}{\partial \sigma} < 0$ for any $\sigma \in [0, 1]$, while, for $\frac{-1+\sqrt{-3+4\beta}}{2(\alpha-\beta)} < \phi < 1$, $\frac{\partial G(\gamma_Z)}{\partial \sigma} < 0$ if and only if $\sigma_2 < \sigma < 1$, and $\frac{\partial G(\gamma_Z)}{\partial \sigma} > 0$ if and only if $0 < \sigma < \sigma_2$.

• Finally, if $1 + \alpha - \sqrt{\alpha} < \beta < 1$, $\frac{1}{4} < \alpha < \frac{1}{2}$, then results on Gini are identical to the case $\frac{3}{4} < \beta < 1 + \alpha - \sqrt{\alpha}$ and $\alpha < \frac{1}{4}$.

References


