How many patents does it take to signal innovation quality? *

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Abstract

In this paper, we offer a novel explanation to the surge in patenting observed during the last years. With low patentability standards at PTOs (Patent and Trademark Offices awarding so-called bad patents), not only "false" innovators have the chance of being granted patents but also, and more interestingly, "true" innovators are forced to patent more intensively trying to signal their type; however, if they are liquidity constrained, true innovators may fail to separate and this fact reduces the incentives to exert effort in R&D activities. In the last part of the paper, we investigate some of the proposals that have been put forward in order to mitigate the bad patents problem. We provide an intuitive condition under which a tightening of the patentability standards ("raising the bar") reduces the distortions caused by bad patents. Moreover, we show that introducing a two-tiered patent system is unlikely to improve market outcomes.

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1 Introduction

During the last years, a dramatic increase in patenting has been accompanied by a rise in the number of so-called "bad patents". As a matter of fact, PTOs (Patent and Trademark Offices) are increasingly granting patent protection to innovations that do not meet the novelty and/or the non-obviousness requirement and that would not get through a careful examination of the patentability standards.[1]

Starting with the seminal paper by Farrell and Shapiro (2008), the literature has investigated the economic consequences of bad patents.[2] According to several commentators, the vast majority of bad patents covers useless technologies or products that no one will ever use and, as such, is economically irrelevant.[3] Consistently with this view, PTO examiners should not pay more attention to every application being filed but they should rather concentrate on the few patents that may represent a too heavy burden to future innovators. This argument is clearly summarized by the following quote taken from Lemley et al. (2005, page 12): "The problem, then, is not that the Patent Office issues a large number of bad patents. Rather, it is that the Patent Office issues a small but worrisome number of economically significant bad patents...".

This view, however, overlooks an important role that patents play. When some relevant characteristic of the inventor is not observable, then patents might serve as a quality signal for third parties, such as potential investors or competitors (see Long, 2002). Several studies provide empirical evidence in favor of this hypothesis, particularly for start-ups with little or no track record and, more generally, for small firms.[4] Hsu and Ziedonis (2008), for instance, look at US semiconductor firms that received venture financing and show that having a large stock of patent applications increases both the likelihood of the company being financed by venture capitalists as well as the amount of financial aid received. By using their estimates, Gambardella (2013) calculates that the

[1]The issue is particulary relevant for the U.S. Patent and Trademark Office. Lemley and Sampat (2008) report that a share between 75 to 97% of patent applications filed in the U.S. is finally approved and, rather provocatively, they ask themselves whether the USPTO has become a rubber-stamp that grants patents to every application being filed.

[2]The role of licensing negotiations in mitigating the consequences of bad patents is investigated in Farrell and Shapiro (2008) and in Choi (2005). Caillaud and Duchene (2011), instead, focus on the overload problem at the PTO; they assume that the probability that a bad patent is granted increases with the overall number of applications (i.e. the PTO examination process worsens under congestion). In this setting, the authors show that a "low R&D equilibrium" may emerge: firms invest little in R&D, they file applications also based on bogus ideas, and the (congested) PTO grants bad patents with large probability.

[3]Lemley et al. (2005) report a series of curious patents awarded by the USPTO, such as patents "covering obvious inventions like the crustless peanut butter and jelly sandwich, ridiculous ideas like a method of exercising a cat with a laser pointer, and impossible concepts like travelling faster than the speed of light."

[4]The relevance of patents as signals for start-up firms has been confirmed by Mann (2005) and by the recent Berkeley Patent Survey (see Graham et al., 2010). Top-ranked motivations to patent are indeed related to the improved chances of securing outside investment and to the enhancement of the company's reputation. According to Czarnitzki et al. (2014), patents attenuates the financial constraints also for already established *small* companies.

value of patents as quality signal could be as high as 1.2 million US$, though he suggests that 93 thousand is a more reasonable estimate. Additional empirical evidence in favor of the signaling role of patents is provided by Greenberg (2013), Haeussler et al. (2009), Cockburn and MacGarview (2009) and by Conti et al. (2013a).[5]

The credibility of patents as quality signal can be substantially undermined by the adoption of low patentability standards. When "true innovators" as well as "false innovators" get through the examination process at the PTO, patents become only a noisy signal about the quality of the inventor/innovation.[6]

A couple of recent theoretical papers investigate the signaling role of patents in the presence of a PTO with low patentability standards. Koenen and Peitz (2012) present an infinite horizon game in which, at each period of time, the firm generates a patentable idea. The two authors determine the conditions under which reputational concerns induce the firm to apply for a patent only in the case it has generated a true innovation (and therefore refrain to file bad applications based on bogus ideas). Atal and Bar (2013) focus on one of the proposals suggested in the literature for mitigating the bad patents problem, namely the introduction of a two-tiered patent system where inventors are free to apply for a "gold-plated" (with larger fees, tighter PTO scrutiny but also offering stronger protection for the invention) or for a "regular" patent. The authors show that introducing a second patent-tier reduces the incidence of bad patents; however, they prove that economically more significant innovations do not necessarily turn out to apply for gold-plated patents.[7]

In this paper, we focus on a different mechanism true innovators may use in order to signal their type, namely the number of applications they file.[8] As a matter of fact, there is no one-to-one correspondence between innovations and patents and new products or processes may be covered by a series of patents,

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[5] Conti et al. (2013b) instead look at a multi-signal model where start-ups use patents to signal the quality of the invention and the money the founder invests in the venture as a signal of her commitment. The authors show that the signal which is most relevant to be sent depends on the typology/characteristics of the outside investors. In their empirical exercise, they find that venture capitalists care more about patents while business angels are more concerned with the money the founder has invested.

[6] The examination process at the PTO can be very long and one may wonder whether its approval/rejection decision actually conveys valuable information for third parties; this issue seems relevant especially for the European Patent Office (EPO). However, it is important to notice that before reaching the final decision the PTO publishes some interim reports which reveal information about the application. For instance, 18 months after the application has being filed, the EPO publishes the search report where references to existing prior art are listed and classified. Search reports are highly informative about the likelihood of a patent being granted/rejected. For instance, they list what references call the novelty or the inventive step of a claim into question (these are the so-called X and Y references); therefore, applications with many X and Y references are likely to be finally rejected. Greater details on the examination process at the EPO can be found in Harhoff and Wagner 2009.

[7] The signaling role of patents is investigated also by Anton and Yao (2003 and 2004) and Jansen (2011); these authors, however, do not consider the bad patent issue.

[8] The number of applications as a signal of quality is considered also by Conti et al. (2013a). Differently from our paper, these authors do not consider the issue of bad patents; moreover, they assume that when deciding how many patents to apply for the innovator is never liquidity constrained, which is also one of the key drivers of our analysis.

some of them applied possibly for some ancillary/secondary aspects of the innovation.[9] More generally, firms are involved in several different R&D projects and, therefore, they may decide for how many of them to ask for patent protection.

In the following sections, we consider a start-up company with limited financial resources involved in a multi-stage innovation game where patents have only a signaling role (they are used to signal whether the firm is a "true" or a "false" innovator). After exerting an R&D effort the firm observes the financial resources needed to complete the research project and chooses whether or not to invest them, i.e. it chooses whether to become a "true" or a "false" innovator. In the subsequent stage, the firm decides how many patent applications to file; since the PTO does not screen applications perfectly, also the false innovator can apply for patent protection. The maximum number of patents the firm can apply for is determined by the financial resources left after the investment decision. Hence, the assumption of limited financial resources implies that, even though the true innovator benefits the most from signaling, the false innovator is endowed with a greater budget for patenting, as it did not invest anything during the previous stage.[10]

We show that when the PTO has low patentability standards, true innovators increase the number of applications filed in an attempt to signal their type. However, if they are liquidity constrained at the patenting stage they are unable to separate from the false innovator. In addition to that, we prove that low standards at the PTO reduces the incentives to exert R&D efforts; as a matter of fact, with low standards, the signaling value of patents reduces which, in turn, lowers revenues true innovator can appropriate.

Our paper provides a novel explanation to the recent surge in patenting. While the literature suggests that, especially in high-tech sectors, companies amass large patent portfolios in order to use them strategically during negotiations or even to preempt competitors,[11] we argue that the observed rise in patenting can be explained by low patentability standards coupled with the signaling role of patents.[12] This results not only from the rather obvious fact

[9]In a study on the pharmaceutical industry, Ouellette (2010) reports that, on average, 3.5 patents cover one single drug with this number increasing up to about 5 in the case of blockbuster drugs. For the use of patents covering ancillary aspects of the innovation, see Hemphill (2012).

[10]The existence of a trade-off between the resources spent on patenting and investment in R&D activities is reported by Mann (2005), in a study on the software industry. The author reports that investors and developers of software firms emphasize that "attention to patents can be damaging to a startup because it has the potential to divert limited time and resources (p. 982)" and that "Every dollar we spend on [patenting] is a dollar we can't spend on a software engineer (pp. 982-3)". The trade-off between patenting and R&D is also evident when considering the expenses related to the patent application process. According to Graham et al. (2010), including attorney fees, the estimated cost of obtaining a patent in the U.S. is $35,000, a substantial amount of money especially for start-up companies.

[11]See Hall and Ziedonis (2001) among others.

[12]Implicitly, this fact is suggested also in Long (2002) . When patents stocks convey information, then there are incentives "to patent the smallest publishable unit, and divide what would normally be a single patent on an invention into multiple smaller patents on different facets of the same invention."

that, with low standards, the false innovator files applications since there is the chance of being granted patents. More interestingly, the true innovator is induced to increase the number of applications in the attempt to signal its type.

Note that our paper is consistent with a puzzling result that emerges in the empirical literature looking at the signaling role of patents. While the stock of patent applications significantly improves the chances of being financed, awarded patents have an ambiguous effect: they do not add anything in some studies (e.g. Haeussler et al, 2009) while they further increase the likelihood of receiving financial aid in others (e.g. Greenberg, 2013). This mixed evidence cannot be reconciled with previous theoretical contributions. However, it is consistent with the main result of our model in which the true innovator is able to separate through the number of applications it files in some cases while in others it is prevented from separating by financial constraint. As a result, the decision of the PTO is redundant in the former cases but it is informative in the latter ones.

In Section 4 of the paper, we discuss some of the proposals that have been put forward in order to mitigate the bad patents problem. We provide an intuitive condition under which a tightening of the patentability standards coupled with an increase in the patenting fees ("raising the bar") is likely to reduce the distortions caused by the presence of bad patents. Moreover, we show that a two-tiered patent system is likely to be ineffective once we depart from the standard assumption that the firm chooses either zero or one patent; when choosing also the number of patents to apply for, the firm can endogenously "gold-plate" its innovation; hence, introducing a two-tiered patent system is unlikely to increase market efficiency.

The paper is organized as follows. Section 2 presents the model while in Sections 3 we derive the equilibrium of the game and the main results. The policy implications of our findings are presented in Section 4 while Section 5 concludes. All proofs are collected in the Appendix which also presents some microfoundations of the reduced form of payoff we consider in the paper.

2 The Model

A start-up company endowed with an idea for an R&D project and with amount K of financial resources is involved in the following four-stage innovation game. In the first stage, the firm starts working on the idea while there is still uncertainty about whether the project is financially viable. Formally, we assume that the firm exerts a research effort e while ignoring the investment i necessary to complete the project; we assume that e increases the firm's revenues provided that the innovation is actually developed. In the second stage, once the effort has been exerted, the firm observes the investment i necessary to complete the R&D project; hence, uncertainty is resolved at this stage of the game. In the case the investment is undertaken, the innovation is developed and the firm "becomes a true innovator"; if the investment is not undertaken, the innovation does not materialize, effort e is lost, and the firm "becomes a false innovator".

During the third stage of the game, the firm chooses how many patent applications, $n \geq 0$, to file to the Patent and Trademark Office (PTO). As we clarify below, the PTO's screening of the patentability requirements is not perfect and this fact may encourage also the false innovator to file patent applications. In the last stage of the game, payoffs realize. More specifically, the four stages of the game are as follows.

t=1: effort stage

The firm chooses the effort level $e \geq 0$; this decision is taken before knowing the investment needed to complete the project and develop the innovation. Effort has a non-monetary cost $c(e) \geq 0$, with $c'(e) > 0$ and $c''(e) \geq 0$. A larger effort increases revenues but only if the innovation is developed; effort is useless in the case the firm does not invest i in order to develop the innovation.

t=2: investment stage

Once e has been chosen, the firm privately observes the amount of the investment required to complete the project: $i \in \{I_1, I_2, I_3\}$, with $I_1 < I_2 < K < I_3$. Clearly, when $i = I_3$ is observed, the firm has not enough financial resources to proceed, hence it terminates the project. By contrast, we assume that $I_1 = 0$ so that when $i = I_1$ the investment is certainly undertaken. Finally, when $i = I_2$ the investment is feasible though not necessarily profitable. We assume that the innovation technology is deterministic: whenever the investment is undertaken the innovation is realized and we say the firm becomes a true innovator; in the case the investment is not undertaken, the innovation does not materialize and the firm becomes a false innovator. The probabilities of I_1, I_2 and I_3 are p_1, p_2 and p_3, respectively, with $p_1+p_2+p_3 = 1$; these probabilities are publicly known and do not depend on the effort e.

t=3: patenting stage

In the third stage, the firm chooses the number of applications, $n \geq 0$, to file to the Patent and Trademark Office (PTO). For simplicity, in what follows we consider n as a continuous variable; moreover, we assume that patents are only a device to signal to the market the firm's own type (true or false innovator) and we abstract from any other motivation, such as protective reasons, that may induce the firm to patent.

The PTO has not enough time and resources to conduct an accurate screening of applications; as a result of this, its examination of the patentability requirements is not perfect and this fact may induce also the false innovator to file patent applications. In what follows, we let θ to parameterize the accuracy of the PTO's decisions.

More specifically, we assume that the firm is free to file any number of applications and that in this choice it is constrained only by the financial resources left after the investment stage. Formally, let P denote the patenting fees; then, the firm can apply at most for $n \leq (K - I)/P \equiv \bar{n}(I)$, where I is the amount invested at $t = 2$, with $I = i$ if the firm has made the investment and $I = 0$ in case it did not invest.

The following table formalizes the PTO's behavior:

	PTO "says true"	PTO "uninformative"	PTO "says false"
True innovator	$g(\theta, n)$	$(1 - g(\theta, n))$	0
False innovator	0	$(1 - g(\theta, n))$	$g(\theta, n)$

Table 1: PTO's behavior

With probability $g(\theta, n) \in [0, 1]$ the PTO's decision correctly certifies the firm's type: the PTO "says true" when the firm is actually a true innovator and it "says false" when the firm is actually a false innovator. With complementary probability, $1 - g(\theta, n)$, the PTO's decision is uninformative.[13]

We assume that the probability function $g(\theta, n)$ is differentiable in its two arguments and such that:

Assumption 1 *: i)* $\frac{\partial g(\theta,n)}{\partial \theta} > 0$, *ii)* $\frac{\partial g(\theta,n)}{\partial n} > 0$ *and iii)* $\frac{\partial^2 g(\theta,n)}{\partial n \partial \theta} \geq 0$.

Assumption 1-i) implies that the more accurate the PTO's screening process (the larger θ) the more informative its decision. Similarly, assumption 1-ii) requires that the likelihood of the PTO's decision being informative increases with the number of applications the firm files.[14] Finally, assumption 1-iii) implies that there is weak complementarity between the number of applications and the PTO accuracy.

Clearly, when the firm chooses not to file any application ($n = 0$) the PTO takes no decision, hence it does not reveal any information.

t=4: payoff stage

In the last stage of the game, the firm earns a payoff which depends on its type (true or false innovator), on the effort level e (if the firm is a true innovator) and on the beliefs that the market holds; in particular, the market forms its beliefs about the firm's type based on the number of applications that have been filed and on the PTO's decision. Formally, let ξ $(1 - \xi)$ denote the belief associated with the event "the firm is a true innovator" ("the firm is a false innovator" respectively). Clearly, when the PTO "says true", then $\xi = 1$, when the PTO "says false", then $\xi = 0$. Finally, we let $\xi = \xi_n$ denote the belief when the PTO's decision is uninformative.

[13] We are assuming that the probability of the PTO sending an uninformative signal is the same when the innovator is either true or false. In a previous version of the paper, we considered the asymmetric case where the PTO commits type I errors only (sometimes it fails to say false when the innovator is actually false). The main results *of the two models are qualitatively the same.*

[14] There are two alternative explanations to this assumption. As n rises, the information revealed by the applicant to the PTO also increases and this fact makes it more likely for the PTO to figure out the firm's type. An alternative interpretation relies on the fact that when filing an application the firm is actually purchasing a signal. Then, the more signals the firm purchases the more informative the decisions of the PTO.

We model the payoff stage in a reduced-form manner (in the Appendix, we provide three possible microfoundations of the reduced-form payoff we employ). More concretely, the revenues the firm collects are as follows:

- with probability $f(\xi)$ the firm earns a large profit: the false innovator earns R, the true innovator earns $Rl(e)$, with $l(0) = 1$, $l'(e) > 0$ and $l''(e) < 0$. The function $l(e)$ accounts for the fact that effort increases the payoff only in the case the innovation is actually developed;
- with probability $1 - f(\xi)$ the firm earns a small profit: the false innovator earns r, the true innovator earns $rl(e)$, with $r < R$.

Therefore, when the market holds the belief ξ, the payoff that the false and the true innovator expect at stage $t = 4$ is:[15]

$$\begin{aligned} f(\xi)R + (1 - f(\xi))r &= f(\xi)\Delta + r, \\ f(\xi)Rl(e) + (1 - f(\xi))rl(e) &= f(\xi)\Delta l(e) + rl(e), \end{aligned} \tag{1}$$

respectively and where $\Delta \equiv R - r$.

By resorting to one of the microfoundations provided in the Appendix, we can interpret the above structure of payoffs in the following manner. At $t = 4$ the firm needs to be financed by a venture capitalist in order to bring the innovation to the market. The probability of being financed, $f(\xi)$, depends on the beliefs that the market (the venture capitalist) holds. The payoff the firm collects depends on its type and on the financing decision of the venture capitalist.

Throughout the paper we assume that $f(\xi)$ is an increasing function of ξ. Using the previous interpretation of the model, this means that the venture capitalist is more likely to finance the firm as the beliefs improve (as the probability that the venture capitalist attaches to the event "the firm is a true innovator" increases).

We look for the Perfect Bayesian Equilibrium (PBE) of the game and we require the out-of-equilibrium beliefs to be "reasonable"; in particular, we require the out-of-equilibrium beliefs to satisfy the D1 criterion.

3 Equilibrium and results

As usual, we solve the model by backward induction starting from the patenting choice at $t = 3$.

3.1 Patenting stage ($t = 3$)

At the patenting stage, what matters is whether or not the firm has developed the innovation (i.e. whether it is a true or a false innovator) and the amount of financial resources that it can use to file patent applications. In what follows, we assume that I_2 is low enough and such that the firm undertakes the investment

[15] Expected payoffs are computed net of K, $c(e)$, i and nP.

when $i = I_2$. This implies that at the patenting stage the firm can actually be of three different types.[16] If it did not complete the project it is a false innovator that can apply for at most $\bar{n}(0) = K/P$ patents. If instead it developed the innovation it can be of two different types according to the amount of resources invested at $t = 2$. The firm can be a true innovator that has spent I_2 and that therefore can apply for at most $\bar{n}(I_2) = (K - I_2)/P$ patents; for reasons that we clarify below, we refer to this type of the firm as the "true liquidity constrained"; or, it can be a true innovator that has spent $I_1 = 0$ and that therefore can file at most $\bar{n}(0) = K/P$ applications; we refer to this type of the firm as the "true with deep pockets". For the sake of simplicity, when we refer to both the true liquidity constrained and to the true with deep pockets we just say "true types".

The next lemma proves a very important preliminary/benchmark result concerning the patenting subgame. Suppose, for the moment being, that the firm can apply for whatever number of patents n it likes and that it is not constrained to choose $n \leq \bar{n}(I_2)$ or $n \leq \bar{n}(0)$; according to Lemma 1, the true types are the ones who have more incentives to increase the number of applications.

Lemma 1 (Benchmark: no constraint on n) *Suppose n is chosen with some positive probability by the false innovator and by at least one of the true types; then in any PBE satisfying the D1 criterion, the true types prefer $n + \varepsilon$ to n, where ε is some positive, negligible number.*

If there were no financial constraints to the number of applications the firm can file, then the true types certainly would prefer to increase n in order to improve market beliefs, and possibly separate from the false innovator. The reason why this occurs is related to the fact that by increasing n the true types increase the probability $g(\theta, n)$ that the PTO "says true", thus rising the beliefs up to $\xi = 1$ (while the false innovator increases the likelihood that the PTO says false); moreover, as shown in (1), the true types benefit more than the false innovator from an improvement in market beliefs: an increase in ξ implies a benefit proportional to $\Delta l(e)$ for the true types and proportional to Δ for the false innovator, with $\Delta l(e) > \Delta$.

Consider now the behavior of the firm when the financial constraints $n \leq \bar{n}(I_2)$ or $n \leq \bar{n}(0)$ are in place. In particular, in what follows, we focus on the most interesting case in which $\bar{n}(0)$ is so large that the true with deep pockets can separate, while $\bar{n}(I_2)$ is low enough that the true liquidity constrained cannot separate from the false innovator. Next assumption guarantees that this is indeed the case.[17]

[16] In case the investment I_2 is not profitable, there are two firm's types at the patenting stage: a false innovator (which has observed either I_2 or I_3) and a true innovator with deep pockets. It would be easy to check that the equilibrium of the patenting subgame would be separating: the false innovator would not apply for any patent while the true innovator would apply for the minimum number of patents the false is not willing to file.

[17] It can be proved that if assumption 2-ii) is not satisfied, the unique equilibrium is such that the true types separate from the false innovator by applying a sufficiently large number of patents that the latter is not willing to match. By contrast, if condition 2-i) is not satisfied,

Assumption 2 *i)* $\bar{n}(0)P > (1 - g(\theta, \bar{n}(0)))(f(1) - f(0))\Delta)$ *and ii)* $\bar{n}(I_2)P < (1 - g(\theta, \bar{n}(I_2)))(f(1) - f(0))\Delta)$.

Assumption 2-i) ensures that by applying for $\bar{n}(0)$ patents the true with deep pockets separates from the false innovator. More specifically, Assumption 2-i) implies that the false innovator does not benefit from mimicking the true with deep pockets when this latter applies for the maximum affordable number of patents.[18]

Putting together Lemma 1 (the true with deep pockets prefers to separate from the false innovator) and Assumption 2-i) (separation is financially viable for the true with deep pockets) it follows that:

Lemma 2 *In any PBE satisfying the D1 criterion, the true with deep pockets separates from the false innovator.*

By contrast, Assumption 2-ii) implies that $\bar{n}(I_2)$ is low enough so that the false innovator benefits from imitating the true liquidity constrained, even in the case this latter applies for the maximum affordable number of patents. As a consequence, an implication of Assumption 2-ii) is the following:

Lemma 3 *There is no PBE in which the false innovator separates with probability* 1.

By combining Assumption 2-ii) (the false innovator benefits from imitating the true liquidity constrained) and Lemma 1 (the true with liquidity constrained benefits from choosing $n + \varepsilon$ when n is chosen by the false innovator) we derive the optimal strategy chosen by the true liquidity constrained in any equilibrium of the patenting subgame.

Lemma 4 *In any PBE satisfying the D1 criterion, the true liquidity constrained applies for the maximum affordable number of patents,* $\bar{n}(I_2)$, *with probability* 1.

We are now in the position to determine the equilibrium of the patenting subgame. We distinguish two cases depending on whether $\bar{n}(I_2)$ is small or not. The following Proposition considers the former case.

Proposition 1 (Equilibrium 1) *For small values of* $\bar{n}(I_2)$, *the unique PBE satisfying the D1 criterion is the following:*

- the false innovator and the true liquidity constrained apply for $\bar{n}(I_2)$ *patents;*

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then the unique equilibrium is such that, the true types apply for the maximum number of patents they can afford, $\bar{n}(I_2)$ and $\bar{n}(0)$, while the false innovator randomizes between $\bar{n}(I_2)$ and $\bar{n}(0)$ (and possibly $n = 0$).

[18] Formally, the condition ensures that the false innovator prefers not to file any application ($n = 0$), thus revealing its type, rather than imitate the true with deep pockets, by filing $\bar{n}(0)$ patents, and being detected with probability $g(\theta, \bar{n}(0))$.In the first case it obtains $f(0)\Delta + r + K$; while in the second it obtains $g(\theta, \bar{n}(0))f(0)\Delta + (1 - g(\theta, \bar{n}(0)))f(1)\Delta + r + K - \bar{n}(0)P$. Notice that the false innovator prefers $n = 0$ to $\bar{n}(0)$ also when the best possible beliefs are associated to the latter choice, namely $\xi_{\bar{n}(0)} = 1$ provided that the PTO does not detect the false innovator.

- the true with deep pockets applies for $n^* > \bar{n}(I_2)$ patents (n^* is defined in the Appendix).

The beliefs are: $\xi = 1$ if the PTO says true, $\xi = 0$ if the PTO says false; when the PTO sends an uninformative signal, the beliefs are: $\xi = 1$ if $n \geq n^*$, $\xi = \frac{p_2}{p_2+p_3}$ if $n = \bar{n}(I_2)$ and $\xi = 0$ in all other cases.

When $\bar{n}(I_2)$ is very low, the true liquidity constrained applies for the maximum affordable number of patents, $\bar{n}(I_2)$, and the false innovator imitates it with probability 1. Hence, despite applying for the largest number of patents it can file, the true liquidity constrained is unable to separate from the false innovator. Observe that in this case the equilibrium strategies do not lead to separation and market beliefs depend crucially on the PTO decision. If the PTO "says true" the market holds the belief $\xi = 1$, if the PTO "says false" $\xi = 0$, while if the PTO sends an uninformative signal the market updates its belief according to Bayes' rule and equilibrium strategies and $\xi = p_2/(p_2 + p_3)$.

Contrary to the liquidity constrained, the true with deep pockets is able to separate from the false innovator by increasing the number of applications above $\bar{n}(I_2)$. In this case, the decision taken by the PTO (either true or uninformative signal) is irrelevant since the market, by simply observing the number of applications n^*, can infer the type of innovator and $\xi = 1$. As shown in the Appendix, in equilibrium, n^* is the minimum number of applications that the false is not willing to file. In particular, n^* is such that the false innovator is just indifferent between mimicking the true with deep pockets and playing the equilibrium strategy, $\bar{n}(I_2)$.

As $\bar{n}(I_2)$ gets larger, mimicking the true liquidity constrained becomes costlier and hence less appealing for the false innovator; the equilibrium of the patenting subgame is as follows:

Proposition 2 (Equilibrium 2) *For intermediate values of $\bar{n}(I_2)$, the unique PBE satisfying the D1 criterion is:*

- the false innovator plays mixed strategies: it applies for $\bar{n}(I_2)$ with probability $0 < h(\bar{n}(I_2)) < 1$ and $n = 0$ with complementary probability;

- the true liquidity constrained applies for $\bar{n}(I_2)$ patents;

- the true with deep pockets applies for $n^{**} > \bar{n}(I_2)$ patents (n^{**} is defined in the Appendix).

The beliefs are: $\xi = 1$ if the PTO says true, $\xi = 0$ if the PTO says false; when the PTO sends an uninformative signal, the beliefs are: $\xi = 1$ if $n \geq n^{**}$, $\xi = \frac{p_2}{p_2+p_3 h(\bar{n}(I_2))}$ if $n = \bar{n}(I_2)$ and $\xi = 0$ in all other cases.

For intermediate values of $\bar{n}(I_2)$, the true liquidity constrained still applies for the maximum affordable number of patents, $\bar{n}(I_2)$, while the false innovator plays mixed strategies and imitates the true liquidity constrained with probability $h(\bar{n}(I_2))$ smaller than 1. Hence, also in this case the true liquidity constrained is unable to separate from the false innovator and the PTO's decision determines the market beliefs, in a way similar to what happens in Proposition 1. Still, the true with deep pockets separates by applying the minimum number of patents that the false innovator is not willing to file.

Summarizing, Propositions 1 and 2 highlight some interesting consequences stemming from the imperfect screening of the PTO. Firstly, the false innovator gets the chance of disguising its type by filing applications. Secondly, the true types increase the number of patents they file in the attempt to credibly signal that they have developed the innovation; the true liquidity constrained devotes its entire budget *to filing* applications while the true with deep pockets raises n up to the point where the false innovator does not profit from imitating its behavior. Finally, the PTO decision determines the market beliefs but only in the case the firm is unable to separate through the number of applications; by contrast, when n is a separating strategy, the PTO decision is irrelevant.

Comparative Statics

The previous analysis has shown that a non-perfect screening by the PTO induces the false innovator to file patent applications and this, in turn, forces the true innovators to increase n in the attempt to signal their type. Hence, "over-patenting" is the optimal reaction of firms to the low patentability standards applied by the PTO. This finding is reinforced by the simple comparative statics analysis that we present in this section.

We start our analysis by determining the effect of an increase in θ on the number of patents filed by the true with deep pockets (n^* in equilibrium 1 and n^{**} in equilibrium 2).

Corollary 1 *In equilibrium the number of patents applied for by the true with deep pockets decreases as θ increases.*

For the sake of brevity, in what follows we provide the intuition of the result focussing on the case of small $\bar{n}(I_2)$ described in equilibrium 1 (the intuition for the other case is analogous). In such an equilibrium, the true with deep pockets applies for n^* patents defined as the number of applications that makes the false innovator indifferent between the equilibrium strategy, $\bar{n}(I_2)$, and n^* itself. An increase in θ enlarges the probability of the PTO detecting the false innovator, thus reducing the expected payoff of the latter; since $\partial^2 g(\theta, n)/\partial n \partial \theta \geq 0$, this (negative) effect is stronger when n^* rather than $\bar{n}(I_2)$ is chosen (recall that $n^* > \bar{n}(I_2)$). Hence, as the accuracy of the PTO improves, applying for $\bar{n}(I_2)$ becomes comparatively more profitable for the false innovator and this fact implies that n^* must decrease to maintain indifference between $\bar{n}(I_2)$ and n^*.

The next corollary investigates the effect of a larger θ on the overall number of applications.

Corollary 2 *The overall number of patent applications filed in equilibrium decreases as θ increases.*

The comparative statics analysis of equilibrium 1 is immediate. Corollary 1 shows that n^* reduces as θ rises; since $\bar{n}(I_2)$ does not depend on θ, this implies that the overall number of patents also decreases as the PTO screening becomes more accurate. The effect of a rise in θ is less clear-cut when equilibrium 2 prevails. As a matter of fact, the false innovator plays mixed strategies selecting either $\bar{n}(I_2)$ or $n = 0$ randomly. However, in the Appendix we show that as θ increases the false innovator chooses $\bar{n}(I_2)$ with smaller probability, thus contributing to the reduction of the overall number of applications.

3.2 Effort choice $t = 1$

At stage $t = 1$, the firm chooses the level of effort; this decision is taken before observing the investment $i \in \{I_1, I_2, I_3\}$ required to complete the project and it is based on the expected return. Recall, that effort e increases revenues only in the case the innovation is developed i.e. only when the firm becomes a true innovator.

Assuming that equilibrium 1 of the patenting subgame is going to be played, at $t = 1$ the firm anticipates the following (a similar reasoning applies in the case of equilibrium 2):

- with probability p_1 it will become a true with deep pockets that applies for n^* patents, separates and obtains:

$$f(1)\Delta l(e) + rl(e) + K - n^* P \equiv \pi_1;$$

- with probability p_2 it will become a true liquidity constrained that spends its entire budget filing $\bar{n}(I_2)$ applications, is unable to separate from the false innovator, and obtains:

$$g(\theta, \bar{n}(I_2))(f(1)\Delta l(e)+rl(e))+(1-g(\theta, \bar{n}(I_2)))(f(\xi_{\bar{n}(I_2)})\Delta l(e)+rl(e)) \equiv \pi_2;$$

- with probability p_3 it will become a false innovator that applies for $\bar{n}(I_2)$ patents and obtains

$$g(\theta, \bar{n}(I_2))\left(f(0)\Delta + r\right)+(1-g(\theta, \bar{n}(I_2)))\left(f(\xi_{\bar{n}(I_2)})\Delta + r\right)+K-\bar{n}(I_2)P \equiv \pi_3.$$

Hence, at $t = 1$ the firm chooses e in order to maximize:

$$p_1\pi_1 + p_2\pi_2 + p_3\pi_3 - c(e).$$

By implicitly differentiating the first order condition of the maximization problem above (and analogously when equilibrium 2 is played), it is possible to show that:

Proposition 3 *The optimal effort level increases with θ.*

An increase in θ makes the PTO's decision more informative, as $g(\theta, n)$ rises. This affects the payoff of the true liquidity constrained: as discussed above, this type of the firm is unable to separate from the false innovator, hence it benefits from a more precise signal sent by the PTO. Clearly, an increase in π_2 provides a stronger incentive to exert effort at $t = 1$.

4 Policy proposals

Different proposals for fixing the bad patents problem and reducing the number of applications based on bogus ideas have been put forward by economists, lawyers and practitioners. Below, on the basis of our model, we discuss the possible effects of some of these proposals.

4.1 Rasing the bar

The most obvious fix to the bad patents problem is to make the PTO screening more stringent. This would reduce the rate of patents awarded by the Patent Office, with a more marked reduction in the rate of bad patents. Hence, PTO's decisions would become more informative.

In terms of the previous model, "raising the bar" can be interpreted as an increase in θ, the accuracy of the screening activity. We have already seen how a rise in θ affects the number of patent applications in the previuos section. However the findings presented in Corollaries 1 and 2 are derived under the assumption that patenting fees P are constant. Clearly this is not realistic if the PTO is, at least partially, self-financed. In this case a rise in θ, by requiring to devote more resources to screening patent applications, might result in higher patenting fees to offset the increase in the costs borne by the PTO. To capture this, in what follows, we assume that the patenting fees are a (weakly) increasing function of $\theta : dP/d\theta \geq 0$. Hence, the analysis below where we allow the patenting fees to increase with the accuracy of the examination process is a generalization of what shown in the previous Corollaries 1 and 2 and in Proposition 3.

As we show below, the effect of raising the bar crucially depends on how a larger θ affects the probability of the PTO correctly identifying the type of the firm when $\bar{n}(I_2)$ is chosen, $g(\theta, \bar{n}(I_2))$. We call $g(\theta, \bar{n}(I_2))$ the "probability of correct screening" since, as argued previously, the PTO's decision is relevant only when $\bar{n}(I_2)$ is chosen. The change in $g(\theta, \bar{n}(I_2))$ due to a rise in θ is:

$$\left[\underbrace{\frac{\partial g(\theta, \bar{n}(I_2))}{\partial \theta}}_{(+)} + \underbrace{\frac{\partial g(\theta, \bar{n}(I_2))}{\partial n}}_{(+)} \underbrace{\frac{\partial \bar{n}(I_2)}{\partial P}}_{(-)} \underbrace{\frac{dP}{d\theta}}_{(+)} \right].$$

The first term is positive by Assumption 1. By constrast the second term is negative: a more accurate screening by the PTO imposes larger patenting fees which, in turn, reduce $\bar{n}(I_2)$. Thus we have two contrasting effects stemming from a rise in θ. The following propositions show that, overall, an increase in θ is desirable when the positive effect on $g(\theta, \bar{n}(I_2))$ dominates the negative one; in this case in fact the informativeness of the PTO's decisions improve.

As before, we consider first the effect of a larger θ on the number of application filed by the true with deep pockets.

Proposition 4 *When $dP/d\theta \geq 0$, the number of patents that the true with deep pockets applies for in equilibrium decreases as θ rises.*

An increase in θ makes $\bar{n}(I_2)$ comparatively more appealing than n^* for the false innovator; hence n^* must decrease to restore indifference between the two strategies.[19] The comparative increase in the payoff associated with $\bar{n}(I_2)$ is due to a threefold effect. First, as in Corollary 1, a higher θ increases the probability of detecting the false innovator, a (negative) effect which is stronger when the firm applies for n^* patents. The other two effects, which reinfornce the first one, stem from the fact that a larger θ imposes an increase in the patenting fees. A larger $P(\theta)$ lowers $\bar{n}(I_2)$ and hence it reduces the probability that the false innovator is detected when choosing such strategy. Finally, larger patenting fees also imply larger expenses when the false innovator applies for n^* patents.

Proposition 5 focuses on the effect of a more accurate examination process on the overall number of applications.

Proposition 5 *Let $dP/d\theta \geq 0$; as θ rises, the overall number of patents filed:*

- always decreases in equilibrium 1;

- decreases in equilibrium 2, provided that the "probability of correct screening", $g(\theta, \bar{n}(I_2))$, increases with θ.

The overall number of applications filed in the case of equilibrium 1 certainly decreases: Proposition 4 proves that n^* diminishes while $\bar{n}(I_2)$ shrinks since the patenting fees increase with θ; hence, each type of the firm reduces the number of application it files. As in Corollary 2, the case of equilibrium 2 is more complex given that the false innovator selects either $\bar{n}(I_2)$ or $n = 0$ in mixed strategies. The condition on the "probability of correct screening" implies that the probability of the false innovator selecting $\bar{n}(I_2)$ reduces; hence each type of the firm decreases the number of applications also when equilibrium 2 is played.

Finally, Proposition 6 completes the analysis showing the effect of a larger θ on the effort level.

Proposition 6 *When $dP/d\theta \geq 0$, the optimal effort level increases with θ provided that the "probability of correct screening", $g(\theta, \bar{n}(I_2))$, increases as the accuracy of the PTO augments.*

The intuition in Proposition 6 is the same as in Proposition 3. When the "probability of correct screening" increases the true liquidity constrained obtains a larger payoff and this fact provides stronger effort incentives at $t = 1$.

4.2 Two-tiered patent system

Another proposal for mitigating the proliferation of bad patents is to introduce a two-tiered system based on two patent types: a "gold-plate" patent entailing larger patenting fees, stricter scrutiny by the PTO and providing stronger

[19] Also in this case we focus our comments to the case of equilibrium 1 only.

protection when awarded, and a "regular" patent requiring lower fees and less scrutiny but granting weaker rights to the patentee (see Atal and Bar, 2013).

What we argue below is that a two-tiered patent system is likely to be of little use once we depart from the standard assumption that the firm can only choose whether to apply for one or none patents. When the firm can also select the number of applications to file it can endogenously gold-plate its innovation by increasing n; hence having two types of patents is redundant.

Consider the following setting. There are two types of patents: a regular patent with fees P_r and accuracy θ_r and a gold-plate patent with fees $P_g (> P_r)$ and accuracy $\theta_g (> \theta_r)$. For the sake of simplicity, we assume that the firm can choose one type of patents only, i.e. all applications filed by the firm must belong to the same tier. Moreover, we suppose that Assumption 2 applies both for gold as well as for regular patents.

Let x be the amount of resources the firm spends in patenting i.e. $x = nP$; with a slight change in notation we let $g(\theta_i, x)$ be the probability of correct screening by the PTO when the firm spends $x > 0$ in applying for type $i = r, g$ patents. Without loss of generality, we assume that:

Assumption 3 $g(\theta_g, x) = g(\theta_r, x)$, *for all* $x > 0$.

Assumption 3 implies that if the firm spends the same amount of money in patenting the probability of correct screening by PTO is the same irrespectively of whether the firm applies for regular or gold-plate patents. In other words, the two tiers are "equally efficient" in revealing information on firm's type.

Proposition 7 compares a single-tier patent system (composed of, for instance, regular patents only) and a two-tiered patent system.

Proposition 7 *The equilibrium of the patenting subgame with a two-tiered patent system is equivalent to that with a single tier.*

In the Appendix we show that the equilibrium of the patenting subgame that one obtains with a single or with a two-tiered patent system is payoff equivalent. Hence, adding a second tier do not alter the market outcome whatsoever.

Proposition 7 is based on the assumption that the two tiers are equally efficient (Assumption 3). However, the result can be easily generalized; it would be immediate to show that if one tier is more efficient than the other (e.g. the gold-plate patent is more efficient: $g(\theta_g, x) > g(\theta_r, x)$ for any $x > 0$), then in equilibrium none would select the less efficient tier which therefore plays no role at all.

4.3 Penalties/patent bounties

One possible way to reduce the number of bad patents is by fining the applicant in the case the PTO rejects the application. Fines/penalties may take the form of, for instance, patent bounties (see Thomas, 2001).

In our paper, the PTO is a "black box" whose behavior is summarized by Table 1; as such, the probabilities $g(\theta, n)$ and $1 - g(\theta, n)$ are not directly linked to the probabilities of rejecting patent applications. Hence our setting is not suited to appropriately discuss the role of penalties. However, some considerations are worthwhile mentioning.

Clearly, if the PTO commits type I errors only (it awards patents to the false innovator), penalties may alleviate the bad patents problem by reducing the incentives to patent of the false innovator. However, in the most realistic case in which the Patent Office commits also type II errors (reject an application filed by a true innovator) penalites could also harm true innovators. As a matter of fact, penalties may reduce the ability of the true liquidity constrained to signal its types. If the PTO imposes penalties on top of the patenting fees P, then the budget for patenting shrinks thus reducing the informativeness of the signal sent through the PTO. Substituting the patenting fees P with the penalties (i.e. having a PTO that finances itself only through the fees it collects when rejecting patent applications) may not work either. Besides distorting the behavior of the PTO that would become more prone to rejecting applications, it might reduce substantially the number of patens filed by true innovators that are liquidity constrained. When choosing how many patent applications to file firms need to consider the "worst case scenario" in which many of their applications are rejected. Hence in order to be able to cope with the (potentially) very large penalties, firms may need to reduce the number of applications they files substantially, thus decreasing, once again, the informativeness of teh signal sent by the PTO.

5 Conclusions

In this paper, we develop a simple model which focuses on an often overlooked cost of bad patents. Empirical evidence shows, indeed, that patents play a crucial role in mitigating the asymmetry of information between start-up companies looking for financial aid and external investors; hence, when PTOs award patents based on a cursory screening of the novelty and non-obviousness requirements, patents become noisy signals with reduced informational content.

In the paper, we show that when the screening process by the PTO is imperfect, true innovators are forced to increase the number of applications they file in the attempt to signal their type. However, in the case they are liquidity constrained at the patenting stage, they fail to separate and they are mixed-up with false innovators; this inability to signal their type reduces expected revenues of true innovators, thus diminishing R&D incentives.

Our analysis provides an alternative explanation to the surge in patenting observed during the last years. With low patentability standards, not only false innovators have the chance of being granted patents but also, and more interestingly, true innovators patent more intensively trying to reveal their type.

A very preliminary and indirect evidence of our argument can be found in Table 1 of Haessler et al. (2009, p.25). The table shows that there is a positive correlation between the size of the patent stock a firm holds and the average quality of each patent (measured in terms of forward citations). This evidence is consistent with our finding according to which true innovators are the ones which patent more intensively.

An interesting finding of our paper is that the firm's type is revealed either by the number of applications that are filed or by the number of patents granted (i.e. by the PTO's decision), depending on whether the true innovator is able to separate or not. This result reconciles some apparently contradictory findings in the empirical literature. As a matter of fact, while some studies show that patent grants do not increase the chances of receiving the financial aid (e.g. Haessler et al., 2009), others report that the PTO decision to award a patent further enhances the likelihood of the company being financed (e.g. Greenberg, 2013).

In Section 4, we consider several policy interventions that have been proposed to reform the patent system and cope with the bad patent issue. Based on our model, we provide an intuitive condition under a tightening of the patentability standards ("raising the bar"), possibly in conjunction with an increase in the patenting fees, is likely to mitigate the negative consequences of bad patents. Finally, we show that the proposal to introduce a two-tiered patent system is unlikely to be effective, once we depart from the standard assumption that the firms can apply for either zero or one patent. As a matter of fact, firms can "gold-plate" their innovations simply by increasing the number of applications they file so that the gold-plate tier of the system becomes redundant.

More generally, from a policy perspective, our model suggests that patent reform proposals should carefully consider the effects of the screening process by the Patent Offices on the signaling role played by patents. This is particularly important for startups and young companies that rely more on patents as signaling device since they lack alternative means of conveying relevant information to the market.

6 Appendix (1): proofs

Proof of Lemma 1

We need to consider three cases. Case i): $n+\varepsilon$ is an equilibrium choice of the true types only; in this case the equilibrium beliefs associated to $n+\varepsilon$ is $\xi = 1$ and therefore the true innovator certainly prefers $n+\varepsilon$ to n.

Case ii): $n+\varepsilon$ is an equilibrium choice also of the false innovator. In this case, the false innovator is indifferent between $n+\varepsilon$ and n; hence it expects the same payoff when choosing $n+\varepsilon$ $(g(\theta,n+\varepsilon)f(0)\Delta+(1-g(\theta,n+\varepsilon))f(\xi_{n+\varepsilon})\Delta+r+K-(n+\varepsilon)P)$ and when choosing n $(g(\theta,n)f(0)\Delta+(1-g(\theta,n))f(\xi_n)\Delta+r+K-nP)$; hence, it follows that

$$\varepsilon P = [(g(\theta,n+\varepsilon)-g(\theta,n))f(0)+(1-g(\theta,n+\varepsilon))f(\xi_{n+\varepsilon})-(1-g(\theta,n))f(\xi_n)]\Delta \text{ iff}$$
$$\varepsilon P = \Gamma\Delta, \text{ with } \Gamma \equiv (g(\theta,n+\varepsilon)-g(\theta,n))f(0)+(1-g(\theta,n+\varepsilon))f(\xi_{n+\varepsilon})-(1-g(\theta,n))f(\xi_n)$$

Next we show that when $\varepsilon P = \Gamma\Delta$, then the true types strictly prefer $n+\varepsilon$ (with associated payoff $g(\theta,n+\varepsilon)f(1)\Delta l(e)+(1-g(\theta,n+\varepsilon))f(\xi_{n+\varepsilon})\Delta l(e)+$

$rl(e) + K - (n+\varepsilon)P)$ to n (with associated payoff $g(\theta, n)f(1)\Delta l(e) + (1 - g(\theta, n))f(\xi_n)\Delta l(e) + rl(e) + K - nP)$; this occurs when

$$\varepsilon P < [(g(\theta, n+\varepsilon) - g(\theta, n))f(1) + (1 - g(\theta, n+\varepsilon))f(\xi_{n+\varepsilon}) - (1 - g(\theta, n))f(\xi_n)]\Delta l(e) \text{ iff}$$
$$\varepsilon P < \chi \Delta l(e), \text{ with } \chi \equiv (g(\theta, n+\varepsilon) - g(\theta, n))f(1) + (1 - g(\theta, n+\varepsilon))f(\xi_{n+\varepsilon}) - (1 - g(\theta, n))f(\xi_n)$$

Notice that $\Delta l(e) \geq \Delta$ (iff $l(e) \geq 1$), $\chi > \Gamma$ (iff $g(\theta, n+\varepsilon) > g(\theta, n)$) and therefore $\varepsilon P < \chi \Delta l(e)$ when $\varepsilon P = \Gamma \Delta$. Hence, when $n+\varepsilon$ and n are indifferent for the false innovator, then the true types strictly prefer $n+\varepsilon$ to n.

Case iii): $n+\varepsilon$ is an out of equilibrium choice. By using the previous computations, the true types prefer $n+\varepsilon$ to n provided that the out-of-equilibrium belief $\xi_{n+\varepsilon}$ is such that $\varepsilon P < \chi \Delta l(e)$; in turn, the false innovator prefers $n+\varepsilon$ to n iff $\varepsilon P < \Gamma \Delta$. Since, $\Delta l(e) \geq \Delta$ and $\chi > \Gamma$ as argued above, then condition $\varepsilon P < \Gamma \Delta$ implies condition $\varepsilon P < \chi \Delta l(e)$ while the reverse implication does not hold. Therefore, the D1 criterion implies the out of equilibrium belief $\xi_{n+\varepsilon} = 1$; in turn this fact implies that the true types prefer $n+\varepsilon$ to n.⊡

Proof of Lemma 2

Follows directly from Lemma 1 and Assumption 2-i).⊡

Proof of Lemma 3

We prove the lemma by contradiction. Suppose that the false innovator separates with probability 1; then it certainly chooses $n = 0$, thus obtaining $f(0))\Delta + r + K$. In turn, the true liquidity constrained chooses some $0 < n' \leq \bar{n}(I_2)$. By deviating and choosing n', in a separating equilibrium, the false innovator obtains $g(\theta, n')f(0))\Delta + (1 - g(\theta, n'))f(1)\Delta + r + K - n'P$. One can easily check that under assumption 2-ii) the false innovator benefits from deviating and choosing n' for any $0 < n' \leq \bar{n}(I_2)$.⊡

Proof of Lemma 4

We prove the lemma by contradiction. Suppose that the liquidity constrained type chooses $n < \bar{n}(I_2)$ with some positive probability. Two cases are possible. Case i): also the false innovator chooses n with some positive probability. In this case, from Lemma 1 we know that the true liquidity constrained prefers to deviate and choose $n+\varepsilon$; therefore, we have a contradiction.

Case ii): the false innovator does not choose n. Clearly, the false innovator does not select an $n' > 0$ not chosen by any of the true types (by doing so it would separate and therefore $n = 0$ would be preferred); moreover, from Lemma 2 it follows that the false innovator cannot choose the same number of applications as the true with deep pockets. Therefore, the only possibility is that the false innovator chooses $n = 0$ with probability 1 but this violates Lemma 3 and we are back to case i).⊡

Proof of Proposition 1

Consider the false innovator. Given the equilibrium beliefs, it prefers $n = 0$ to any $n \in (0, \bar{n}(I_2))$ and to any $n \in (\bar{n}(I_2), n^*)$. Moreover it prefers $\bar{n}(I_2)$ to $n = 0$ provided that

$$g(\theta, \bar{n}(I_2))f(0)\Delta + (1 - g(\theta, \bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta + r + K - \bar{n}(I_2)P \geq f(0)\Delta + r + K$$
$$\Leftrightarrow \; \bar{n}(I_2)P \leq (1 - g(\theta, \bar{n}(I_2)))(f(\xi_{\bar{n}(I_2)}) - f(0))\Delta.$$

The minimum number of applications that the false innovator is not willing to file is n^* defined as the value of n such that

$$g(\theta, \bar{n}(I_2))f(0)\Delta + (1 - g(\theta, \bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta + r + K - \bar{n}(I_2)P =$$
$$g(\theta, n)f(0)\Delta + (1 - g(\theta, n))f(1)\Delta + r + K - nP.$$

Consider the true with deep pockets; given the equilibrium beliefs, it prefers n^* to any $n > n^*$. Moreover, Lemma 1 implies that the true innovator prefers n^* to any $n < n^*$ since the false does so. Similarly, the true with deep pockets prefers n^* to $\bar{n}(I_2)$, given that the false is indifferent between the two choices.

Consider now the beliefs. Since $n = n^*$ is chosen by the true only, then it follows that $\xi = 1$. From the definition of n^*, the false innovator never benefits from choosing $n > n^*$, therefore $\xi = 1$ is associated with any $n > n^*$. When $n = \bar{n}(I_2)$ and the PTO sends an uninformative signal, the belief $\xi = p_2/(p_2+p_3)$ follows from Bayes rule and equilibrium strategies. Consider now an out-of-equilibrium choice $n \in (\bar{n}(I_2), n^*)$. Clearly, the true liquidity constraint cannot apply for $n \in (\bar{n}(I_2), n^*)$ patents. The true with deep pockets prefers $n \in (\bar{n}(I_2), n^*)$ to n^* iff

$$g(\theta, n)f(1)\Delta l(e) + (1 - g(\theta, n))f(\xi_n)\Delta l(e) + rl(e) + K - nP \geq f(1)\Delta l(e) + rl(e) + K - n^*P$$
$$(n^* - n)P \geq (1 - g(\theta, n))[f(1) - f(\xi_n)]\bar{\Delta}$$

From the definition of n^*, we know that the false innovator is indifferent between n^* and $\bar{n}(I_2)$. Therefore we check under which conditions the false prefers $n \in (\bar{n}(I_2), n^*)$ to n^*. This occurs whenever:

$$g(\theta, n^*)f(0)\Delta + (1 - g(\theta, n^*))f(1)\Delta + r + K - n^*P \leq g(\theta, n)f(0)\Delta + (1 - g(\theta, n))f(\xi_n)\Delta + r + K - nP$$
$$(n^* - n)P \geq [g(\theta, n^*) - g(\theta, n)][f(0) - f(\xi_n]\Delta + (1 - g(\theta, n^*))[f(1) - f(\xi_n)]$$

A simple comparison of the above inequalities implies that whenever the condition is satisfied for the true with deep pockets it is satisfied for the false while the reverse is not true. Therefore, the D1 criterion implies that the out of equilibrium belief $\xi = 0$ is associated with $n \in (\bar{n}(I_2), n^*)$.

Consider now an out-of-equilibrium choice $n \in (0, \bar{n}(I_2))$. The true liquidity constrained prefers n to the equilibrium strategy $\bar{n}(I_2)$ iff

$$g(\theta, \bar{n}(I_2))f(1)\Delta l(e) + (1 - g(\theta, \bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta l(e) + rl(e) + K - \bar{n}(I_2)P \leq$$
$$g(\theta, n)f(1)\Delta l(e) + (1 - g(\theta, n))f(\xi_n)\Delta l(e) + rl(e) + K - nP$$

or

$$(\bar{n}(I_2)-n)P \geq [(g(\theta,\bar{n}(I_2))-g(\theta,n))f(1)+(1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})-(1-g(\theta,n))f(\xi_n)]\Delta l(e) \tag{A1}$$

The false innovator prefers n to the equilibrium strategy $\bar{n}(I_2)$ iff

$$g(\theta,\bar{n}(I_2))f(0)\Delta+(1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta+r+K-\bar{n}(I_2)P \leq$$
$$g(\theta,n)f(0)\Delta+(1-g(\theta,n))f(\xi_n)\Delta+r+K-nP$$

or

$$(\bar{n}(I_2)-n)P \geq [(g(\theta,\bar{n}(I_2))-g(\theta,n))f(0)+(1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})-(1-g(\theta,n))f(\xi_n)]\Delta \tag{A2}$$

A simple comparison of inequality A1and A2 implies that whenever the condition is satisfied for the true liquidity constrained innovator (A1) it is also satisfied for the false (A2) while the reverse is not true. Moreover, note that since the true innovator obtains a higher pay-off from playing n^* than playing $\bar{n}(I_2)$, we can conclude that the D1 criterion implies that the out of equilibrium belief $\xi=0$ is associated to $n \in (0,\bar{n}(I_2))$.

Concluding, this equilibrium exists for $\bar{n}(I_2)P \leq (1-g(\theta,\bar{n}(I_2)))(f(\xi_{\bar{n}(I_2)})-f(0))\Delta$.⊡

Proof of Proposition 2

Consider the false innovator. Given the equilibrium beliefs, it prefers $n=0$ to any $n \in (0,\bar{n}(I_2))$ and to any $n \in (\bar{n}(I_2),n^{**})$. In order to show, that there exists a $h \in (0,1)$ such that the false innovator is indifferent between $n=0$ and $\bar{n}(I_2)$ we need to show that:

- $g(\theta,\bar{n}(I_2))f(0)\Delta+(1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta+r+K-\bar{n}(I_2)P$ is decreasing in h. This is true given that $\xi_{\bar{n}(I_2)}=\frac{p_2}{p_2+p_3h}$;

- when $h=0$, it has to be $g(\theta,\bar{n}(I_2))f(0)\Delta+(1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta+r+K-\bar{n}(I_2)P > f(0)\Delta+r+K$. Notice that when $h=0$ then $\xi_{\bar{n}(I_2)}=1$ and therefore the previous condition reduces to $\bar{n}(I_2)P < (1-g(\theta,\bar{n}(I_2)))(f(1)-f(0))\Delta$

- when $h=1$ it has to be $g(\theta,\bar{n}(I_2))f(0)\Delta+(1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta+r+K-\bar{n}(I_2)P < f(0)\Delta+r+K$; this inequality is verified provided that $\bar{n}(I_2)P > (1-g(\theta,\bar{n}(I_2)))(f(\xi_{\bar{n}(I_2)})-f(0))\Delta$ where $\xi_{\bar{n}(I_2)}=\frac{p_2}{p_2+p_3}$ since $h=1$.

Therefore, when $\bar{n}(I_2)P \in ((1-g(\theta,\bar{n}(I_2)))(f(\xi_{\bar{n}(I_2)})-f(0))\Delta,(1-g(\theta,\bar{n}(I_2)))(f(1)-f(0))\Delta)$ there exists a $h \in (0,1)$ such that the false innovator is indifferent between $n=0$ and $\bar{n}(I_2)$; we call $h(\bar{n}(I_2)$ such value, that is the value of h such that $g(\theta,\bar{n}(I_2))f(0)\Delta+(1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta+r+K-\bar{n}(I_2)P = f(0)\Delta+r+K$, where

The minimum number of applications that the false innovator is not willing to file is n^{**} defined as the value of n such that

$$f(0)\Delta+r+K = g(\theta,n)f(0)\Delta+(1-g(\theta,n))f(1)\Delta+r+K-nP.$$

Consider the true with deep pockets. Given the equilibrium beliefs, the it prefers n^{**} to any $n > n^{**}$. Lemma 1 implies that the true innovator prefers n^{**} to any $n < n^{**}$ since the false does so. Similarly, the true prefers n^{**} to $\bar{n}(I_2)$, given that the false is indifferent between the two choices; moreover, the true prefers n^{**} to $n = 0$, given that the false is indifferent between the two choices.

By applying the same arguments as in Proposition 1, it easy to demonstrate that the beliefs specified in the proposition satisfy the D1 criterion.

Concluding, equilibrium 2 exists for $\bar{n}(I_2)P \in ((1 - g(\theta, \bar{n}(I_2)))(f(\xi_{\bar{n}(I_2)}) - f(0))\Delta, (1 - g(\theta, \bar{n}(I_2)))(f(1) - f(0))\Delta)$.⊡

Proof of Corollary 1

This is a particular case of Proposition 4.⊡

Proof of Corollary 2

This is a particular case of Proposition 5.⊡

Proof of Proposition 3

This is a particular case of Proposition 6.⊡

Proof of Proposition 4

Consider equilibrium 1 first. The true innovator with deep pockets applies for n^* patents; n^* is defined as the number of applications that make the false innovator indifferent between the equilibrium strategy $\bar{n}(I_2)$ and n^*; formally, n^* is the value of n such that:

$$g(\theta, \bar{n}(I_2))f(0)\Delta + (1 - g(\theta, \bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta + r + K - \bar{n}(I_2)P = g(\theta, n)f(0)\Delta + (1 - g(\theta, n))f(1)\Delta + r + K - nP.$$

$$\iff (g(\theta, \bar{n}(I_2)) - g(\theta, n))f(0)\Delta + (1 - g(\theta, \bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta - (1 - g(\theta, n))f(1)\Delta + I_2 - K + nP = 0$$

(here above we exploit the fact that $K - \bar{n}(I_2)P = I_2$.)

By implicitly differentiating the last equality, it follows that

$$\frac{\partial n^*}{\partial \theta} = -\frac{\partial(\cdot)/\partial\theta}{\partial(\cdot)/\partial n}$$

where

$$\frac{\partial(\cdot)}{\partial\theta} = \frac{\partial g(\theta, \bar{n}(I_2))}{\partial\theta}\left(f(0) - f(\xi_{\bar{n}(I_2)})\right)\Delta - \frac{\partial g(\theta, n^*)}{\partial\theta}\left(f(0) - f(1)\right)\Delta + \frac{\partial g(\theta, \bar{n}(I_2)}{\partial n}\frac{\partial\bar{n}(I_2)}{\partial P}\frac{dP}{d\theta}\left(f(0) - f(\xi_{\bar{n}(I_2)})\right)\Delta + n\frac{dP}{d\theta}$$

The sum of the first two terms is positive since $\frac{\partial^2 g(\theta,n)}{\partial n \partial\theta} \geq 0$ and $n^* > \bar{n}(I_2)$; the third and the fourth terms are positive. Moreover,

$$\frac{\partial(\cdot)}{\partial n} = -\frac{\partial g(\theta,n)}{\partial n}\left(f(0)-f(1)\right)\Delta + P > 0.$$

Thus it follows that $\partial n^*/\partial\theta < 0$. In other terms the number of applications filed by the true innovator decreases with θ.

Consider now equilibrium 2. The true innovator with deep pockets applies for n^{**} patents; n^* is defined as the number of applications that make the false innovator indifferent between the equilibrium strategy $n = 0$ and n^{**}; formally, n^{**} is the value of n such that:

$$f(0)\Delta + r + K = g(\theta,n)f(0)\Delta + (1-g(\theta,n))f(1)\Delta + r + K - nP.$$
$$\iff (1-g(\theta,n))(f(1)-f(0))\Delta - nP = 0.$$

By applying the implicit function theorem and following the same steps as for equilibrium 1, also in this case it follows that $\partial n^*/\partial\theta < 0.\boxdot$

Proof of Proposition 5

The case of equilibrium 1 follows immediately from Proposition 4 and from the fact that $\bar{n}(I_2)$ decreases with θ through the increase in the patenting fees. Consider now equilibrium 2. The false innovator applies for $\bar{n}(I_2)$ with probability $h(\bar{n}(I_2)$ defined as the value of h such that:

$$g(\theta,\bar{n}(I_2))f(0)\Delta + (1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta + r + K - \bar{n}(I_2)P = f(0)\Delta + r + K$$
$$\Leftrightarrow (1-g(\theta,\bar{n}(I_2)))(f(\xi_{\bar{n}(I_2)}) - f(0))\Delta + I_2 - K = 0$$

where $\xi_{\bar{n}(I_2)} = \frac{p_2}{p_2+p_3h}$ and where we have used the fact that $K - \bar{n}(I_2)P = I_2$. By implicitly differentiating the last equality, it follows that

$$\frac{\partial h(\bar{n}(I_2)}{\partial\theta} = -\frac{\partial(\cdot)/\partial\theta}{\partial(\cdot)/\partial h}$$

where

$$\frac{\partial(\cdot)}{\partial\theta} = \left[\frac{\partial g(\theta,\bar{n}(I_2))}{\partial\theta} + \frac{\partial g(\theta,\bar{n}(I_2)}{\partial n}\frac{\partial\bar{n}(I_2)}{\partial P}\frac{dP}{d\theta}\right]\left(f(0) - f(\xi_{\bar{n}(I_2)})\right)\Delta,$$

this derivative is negative provided that the term into the square brackets is positive. Moreover,

$$\frac{\partial(\cdot)}{\partial h} = (1-g(\theta,\bar{n}(I_2)))\,\frac{df(\xi_{\bar{n}(I_2)})}{d\xi}\frac{d\xi_{\bar{n}(I_2)}}{dh},$$

is negative since $\xi_{\bar{n}(I_2)}$ decreases with h. Therefore, if $\left[\frac{\partial g(\theta,\bar{n}(I_2))}{\partial\theta} + \frac{\partial g(\theta,\bar{n}(I_2)}{\partial n}\frac{\partial\bar{n}(I_2)}{\partial P}\frac{dP}{d\theta}\right] > 0$, then $\frac{\partial h(\bar{n}(I_2)}{\partial\theta} < 0$. This latter is a sufficient condition for the overall number of patent applications to decrease with θ in the case of equilibrium 2.⊡

Proof of Proposition 6

Consider equilibrium 1. The optimal effort level e^* is the value of effort e that satisfies the first order condition of the maximization problem stated in the text; namely, the value of e such that:

$$p_1 l'(e)\left(f(1)\Delta + r\right) + p_2 l'(e)[g(\theta,\bar{n}(I_2))f(1)\Delta + (1-g(\theta,\bar{n}(I_2)))f(\xi_{\bar{n}(I_2)})\Delta + r] - c'(e) = 0,$$

where $\xi_{\bar{n}(I_2)} = p_2/(p_2+p_3)$. By implicitly differentiating the first order condition it follow that:

$$\frac{de^*}{d\theta} = -\frac{d(\cdot)/d\theta}{d(\cdot)/de}.$$

where:

$$\frac{d(\cdot)}{d\theta} = p_2 l'(e)\left[\frac{\partial g(\theta,\bar{n}(I_2))}{\partial\theta} + \frac{\partial g(\theta,\bar{n}(I_2))}{\partial n}\frac{\partial\bar{n}(I_2)}{\partial P}\frac{dP}{d\theta}\right](f(1) - f(\xi_{\bar{n}(I_2)}))\Delta.$$

This expression is positive when the term in the square brackets is positive. The derivative $\frac{d(\cdot)}{de}$ is negative by the concavity of the profit function. Therefore, when $\left[\frac{\partial g(\theta,\bar{n}(I_2))}{\partial\theta} + \frac{\partial g(\theta,\bar{n}(I_2))}{\partial n}\frac{\partial\bar{n}(I_2)}{\partial P}\frac{dP}{d\theta}\right]$ then $\frac{de^*}{d\theta} > 0$.

Consider now equilibrium 2. The first order condition is the same as the one stated above for the equilibrium 1 case, the only difference being $\xi_{\bar{n}(I_2)} = p_2/(p_2 + p_3 h(\bar{n}(I_2))$. By implicitly differentiating the first order condition it follows that:

$$\frac{d(\cdot)}{d\theta} = p_2 l'(e)\left[\frac{\partial g(\theta,\bar{n}(I_2))}{\partial\theta} + \frac{\partial g(\theta,\bar{n}(I_2))}{\partial n}\frac{\partial\bar{n}(I_2)}{\partial P}\frac{dP}{d\theta}\right](f(1) - f(\xi_{\bar{n}(I_2)}))\Delta +$$
$$p_2 l'(e)\left(1 - g(\theta,\bar{n}(I_2))\right)\frac{df(\xi_{\bar{n}(I_2)})}{d\xi}\frac{d\xi_{\bar{n}(I_2)}}{dh}\frac{dh(\bar{n}(I_2)}{d\theta}.$$

Both terms here above are positive provided that $\left[\frac{\partial g(\theta,\bar{n}(I_2))}{\partial\theta} + \frac{\partial g(\theta,\bar{n}(I_2))}{\partial n}\frac{\partial\bar{n}(I_2)}{\partial P}\frac{dP}{d\theta}\right] > 0$; in particular, the second term is positive since $dh(\bar{n}(I_2)/d\theta < 0$, as shown in the proof of Proposition 5. The derivative $d(\cdot)/de$ is negative by the concavity of the profit function. Therefore, when $\left[\frac{\partial g(\theta,\bar{n}(I_2))}{\partial\theta} + \frac{\partial g(\theta,\bar{n}(I_2))}{\partial n}\frac{\partial\bar{n}(I_2)}{\partial P}\frac{dP}{d\theta}\right]$ then $\frac{de^*}{d\theta} > 0$ also in the case of equilibrium 2.⊡

Proof of Proposition 7

We prove Proposition 7 by means of three claims. We focus on the case in which $\bar{n}_g(I_2) \equiv (K - I_2)/P_g$ and $\bar{n}_r(I_2) \equiv (K - I_2)/P_r$ are small enough so that the false innovator imitates the true liquidity constrained with probability 1 (i.e. we focus on the case analogous to equilibrium 1). A similar reasoning applies for the case in which the false innovator plays mixed strategies (the case analogous to equilibrium 2).[20]

Claim 1: Lemmas 1, 2 and 3 still apply.

Proof. One can easily check that Lemma 1 applies "within the tier", that is if n_i is chosen with some positive probability by the false innovator and by at least one of the true types, then the true types prefer to apply for $n_i + \varepsilon$ patents rather than for n_i, with $i = r, g$. Lemmas 2 and 3 continue to hold since Assumption 2 is assumed to be true for both gold-plate and regular patents. ■

Claim 2: the true liquidity constrained applies for either $\bar{n}_r(I_2)$ regular patents or for $\bar{n}_g(I_2)$ gold plate patents and the false innovator imitates it with probability 1.

Proof. Lemma 4 applies "within the tier" i.e. it shows that it cannot be that the true liquidity constrained applies for $n_i < \bar{n}_i(I_2)$ patents of type $i = r, g$. Notice that the total expenditure in patenting is the same, $\bar{n}_g(I_2)P_g = \bar{n}_r(I_2)P_r = K - I_2$, hence, by Assumption 3, the two choices generate the same probabilities of correct screening $g(\theta_g, \bar{n}_g(I_2)P_g) = g(\theta_r, \bar{n}_r(I_2)P_r)$. This fact implies that there are two possible equilibria that are payoff equivalent: the true liquidity constrained and the false innovator apply $\bar{n}_r(I_2)$ regular patents or they both apply for $\bar{n}_g(I_2)$ gold plate patents. Consider the first equilibrium for instance; both types choosing $\bar{n}_r(I_2)$ regular patents is supported by the following out of equilibrium belief: if $\bar{n}_g(I_2)$ is observed then, in case the PTO sends an uninformative signal, $\xi = 0$. Since $\bar{n}_g(I_2)$ and $\bar{n}_r(I_2)$ are payoff equivalent for both types, then the D1 criterion is silent and, therefore, $\xi = 0$ trivially satisfies it. ■

Claim 3: the true with deep pockets applies for n_g^* gold plate patents or n_r^* for regular patents with $n_g^* P_g = n_r^* P_r$.

Proof. Call n_r^* the number of regular patents that makes the false indifferent between the equilibrium strategy $\bar{n}_r(I_2)$ and imitating the true with deep pockets by filing n_r^* patents; formally, n_r^* is the value of n_r such that:

$$g(\theta_r, \bar{n}_r(I_2)P_r)f(0)\Delta + (1 - g(\theta_r, \bar{n}_r(I_2)P_r))f(\xi_{\bar{n}_r(I_2)})\Delta + r + K - \bar{n}_r(I_2)P_r =$$
$$g(\theta_r, n_r P_r)f(0)\Delta + (1 - g(\theta_r, n_r P_r))f(1)\Delta + r + K - n_r P_r.$$

Similarly, call n_g^* the number of gold-plate patents that makes the false indifferent between the equilibrium strategy $\bar{n}_r(I_2)$ and imitating the true with deep pockets by filing n_g^* patents; formally, n_g^* is the value of n_g such that:

$$g(\theta_r, \bar{n}_r(I_2)P_r)f(0)\Delta + (1 - g(\theta_r, \bar{n}_r(I_2)P_r))f(\xi_{\bar{n}_r(I_2)})\Delta + r + K - \bar{n}_r(I_2)P_r =$$
$$g(\theta_g, n_g P_g)f(0)\Delta + (1 - g(\theta_g, n_g P_g))f(1)\Delta + r + K - n_g P_g.$$

[20] XXXXX In realtà questo non lo ho controllato, anche se non vedo perché non dovrebbe funzionare XXXXXX

Since by Assumption 3 $g(\theta_g, x) = g(\theta_r, x)$ for all $x > 0$, it follows that $n_g^* P_g = n_r^* P_r$; hence, n_g^* and n_r^* are payoff equivalent and the true with deep pockets is indifferent between the two strategies. ■

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7 Appendix (2): microfoundations

In this section, we present three possible microfoundations of the probability $f(\xi)$ according to which the firm obtains the payoff $Rl(e)$ (in the case of true innovator) or R (in the case of false innovator).

7.1 Entrant case

Consider the following setting. The firm is the incumbent that, at $t = 1$ and at $t = 2$ exerts effort and then makes the investment i to develop an innovation which can reduce its production cost; when investing $i = \{I_1, I_2\}$, the firm operates with a constant marginal production cost c_L.Otherwise, its marginal production cost is c_H, with $c_L < c_H$.

At time $t = 3$, there is a potential entrant which decides whether or not to enter the market. Before taking its decision, the potential entrant observes the number of patents applied for by the incumbent, and the PTO's decisions. Then, it forms its beliefs ξ as specified in the text. Moreover, the entrant observes the costs it needs to sunk in order to enter the market, $\rho\chi$. The value of χ is common knowledge while the realization of ρ is private information of the entrant. Its cumulative distribution function $\Gamma(p)$ is common knowledge.

At $t = 4$, there is the production stage. In the case the entrant has not entered the market, then: the incumbent is the monopolist and obtains profits $Rl(e)$ (in the case of true innovator) or R (in the case of false innovator) while the entrant obtains 0. In the case the entrant has entered the market: we assume that the marginal production cost of the incumbent (c_L or c_H) becomes common knowledge and that firms compete à la Cournot; the profits of the incumbent and the entrant are respectively $rl(e)$ and $\pi_E(c_L)$ when the incumbent operates with the marginal cost c_L and r and $\pi_E(c_H)$ when the incumbent operates with the marginal cost c_H. We assume that $\pi_E(c_H) > \pi_E(c_L)$ i.e. the profits of the entrant are larger when the incumbent operates with the marginal cost c_H.

Consider the decision at $t = 3$. The entrant enters the market provided that:

$$\xi\pi_E(c_L) + (1-\xi)\pi_E(c_H) - \rho\chi \geq 0 \text{ iff } \rho \leq \frac{\xi\pi_E(c_L) + (1-\xi)\pi_E(c_H)}{\chi} \equiv \bar{\rho}.$$

Notice that, since $\pi_E(c_H) > \pi_E(c_L)$, then $\bar{\rho}$ decreases with ξ.

Consider now, the incumbent that takes its decisions at $t = 1$ and at $t = 2$ and that knows that ρ is distributed according to $\Gamma(\rho)$. The incumbent anticipates that at $t = 3$ there is entry with probability $\Gamma(\bar{\rho})$ and that this probability decreases with ξ, given that $\bar{\rho}$ is a decreasing function of the beliefs. The probability $\Gamma(\bar{\rho})$ corresponds to the probability $1 - f(\xi)$ in the model: it is the probability that the incumbent obtains profits $rl(e)$ (in the case of true innovator) or r (in the case of false innovator).

7.2 Licensing

Consider the following setting. The firm is unable to exploit its innovation commercially and therefore, at time $t = 3$, once the investment and the patenting decisions have been taken, it sells/licenses the innovation to a licensee. More specifically, we assume the firm is matched with a licensee, and then it proposes the following royalty contract: the licensee purchases the innovation and pays the firm a share $(1 - s)$ of the profits that the innovation generates (and keeps the remaining share s). We assume that licensees differ in terms of the ability to exploit the innovation commercially: the overall profits generated by the innovation equal $\rho\bar{\pi}$ (in case of true innovator) and $\rho\underline{\pi}$ in case of false innovator, with $\bar{\pi} > \underline{\pi}$. The exact realization of ρ is private information of the licensee while the cumulative distribution $\Gamma(\rho)$ is common knowledge. Notice that $\bar{\pi}$ and $\underline{\pi}$ could be to be interpreted as expected values; the realization of π occurs according to some probability distribution function $\bar{G}(\pi)$ and $\underline{G}(\pi)$ defined over the same support.

The licensee decides whether or not to accept the royalty contract s after observing the following information: its type ρ, the number of patents applied for by the firm, the PTO's decision, the contract s proposed by the firm. Denoting by $\xi \in [0, 1]$ the belief that it holds when taking its decision, then the licensee accepts the proposal of the firm provided that:

$$\xi s\rho\bar{\pi} + (1 - \xi)s\rho\underline{\pi} \geq \underline{U} \text{ iff } \rho \geq \frac{\underline{U}}{\xi s\bar{\pi} + (1 - \xi)s\underline{\pi}} \equiv \bar{\rho},$$

where $\bar{\rho}$ is decreasing in ξ and in s.

When offering the contract s the firm anticipates that its proposal is going to be accepted with probability $1 - \Gamma(\bar{\rho})$; notice that the probability $1 - \Gamma(\bar{\rho})$ is increasing ξ and in s, given that $\bar{\rho}$ increases with these two variables. To make explicit the fact that the probability that the licensing contract is signed depends on the beliefs ξ the licensee holds as well as on the share of profits it obtains, s, we use the notation $q(\xi, s)$ with $q(\xi, s) \equiv 1 - \Gamma(\bar{\rho})$.

Finally, notice that in terms of the notation we use in the model: $rl(e) = \bar{\pi}$, $r = \underline{\pi}$ and $rl(e) = r = 0$; moreover, the function $f(\xi)$ corresponds to $q(\xi, s)(1 - s)$ which is increasing in the beliefs held by the licensee

7.2.1 Equilibrium of the sub-game at $t = 3$

In order to determine the equilibrium at the financing stage $t = 3$, we need to distinguish two cases depending on whether at the patenting stage played at $t = 2$ the true and the false innovators separate or not (either through the number of applications they file or through the PTO's decision). Consider the case of separation first. At $t = 3$, the VC knows the type of the firm with certainty, and therefore by offering a royalty contract s the true innovator obtains $q(1, s)(1 - s)\overline{\pi}$: the licensing contract is signed with probability $q(1, s)$ and the firm obtains a share $(1 - s)$ of the expected profits; therefore, in this case, the optimal proposal of the true innovator is:

$$s^T \in \arg\max_s q(1, s)(1 - s).$$

Similarly, the optimal proposal for the false innovator is:

$$s^F \in \arg\max_s q(0, s)(1 - s).$$

Consider now the case in which there is no separation at the patenting stage so that at $t = 3$, before receiving the proposal of the firm, the licensee holds the belief $\xi_n(\theta) \in (0, 1)$. In this case, the following is the equilibrium of the licensing sub-game:

i) the two types of the firm propose s^* such that $q(\xi_n(\theta), s^*)(1 - s^*) \geq q\left(0, s^F\right)\left(1 - s^F\right)$, and the licensee holds the belief $\xi = \xi_n(\theta)$;

ii) the licensee holds the belief $\xi = 0$ when receiving a proposal different from s^*.

Proof. Recalling that $s^F \in \arg\max_s q(0, s)(1 - s)$ and, given the out-of-equilibrium beliefs of the licensee, the best possible deviation for the firm (both for the true and the false innovator) is s^F. However condition i) implies that the equilibrium profits ($q(\xi_n(\theta), s^*)\overline{\pi}(1 - s^*)$ for the true innovator and $q(\xi_n(\theta), s^*)\underline{\pi}(1 - s^*)$ for the false innovator) are larger than or equal to the profits it obtains by deviating. Therefore, both types find it optimal to propose s^*. Notice that s^* satisfying condition i) does exist since $q(\xi, s)$ is an increasing function of the beliefs. Finally, notice that the D1 criterion is silent about the out-of-equilibrium beliefs, and therefore the beliefs specified in part ii) trivially satisfy the criterion; one can easily check this last point since both types benefit from proposing $s \neq s^*$ if and only if the licensee holds a belief ξ such that $q(\xi, s)(1 - s) > q(\xi, s^*)(1 - s^*)$.■

Summarizing what we have found above, then given any belief $\xi \in [0, 1]$ formed at the patenting stage, the probability that the true innovator obtains $rl(e)$ (and that the false innovator obtains r) equals $q(\xi, s)(1 - s)$ which is an increasing function of ξ.

7.3 Venture capitalist

Consider the following setting consistent with our model. At $t = 4$, in order to bring the innovation to the market, the firm needs some financial aid from a venture capitalist (VC). Conditional on being financed by the VC, the true innovator receives a payoff $\bar{\pi}$ while the false innovator obtains ($\underline{\pi}$), with $\bar{\pi} > \underline{\pi}$. Notice that $\bar{\pi}$ and $\underline{\pi}$ should be interpreted as expected values; the realization of π occurs according to some probability distribution function $\bar{G}(\pi)$ and $\underline{G}(\pi)$ defined over the same support. If not financed by the VC, the firm (true or false innovator) obtains 0.

More specifically, we assume that at $t = 4$, the firm is matched with a venture capitalist, and then it makes a take-it-or-leave-it proposal s: the VC obtains a share $s \in [0, 1]$ of the profits (the firm obtains the remaining $1 - s$ share). The cost borne by the VC in order to finance the firm is ρ. The exact realization of ρ is privately observed by the venture capitalist before deciding whether or not to accept the proposal of the firm; in turn, the firm knows that ρ is distributed according to a cumulative distribution function $\Gamma(\rho)$.

Consider first the decision of the venture capitalist. The VC decides whether or not to accept the proposal after observing the following information: its type ρ, the number of patents applied for by the firm, the PTO's decision, the contract s proposed by the firm. Denoting by $\xi \in [0, 1]$ the belief that it holds when taking its decision, then the venture capitalist accepts the proposal of the firm provided that:

$$\xi s\bar{\pi} + (1 - \xi)s\underline{\pi} - \rho \geq 0 \text{ iff } \rho \leq \xi s\bar{\pi} + (1 - \xi)s\underline{\pi} \equiv \bar{\rho},$$

where $\bar{\rho}$ is increasing in ξ and in s.

When offering contract s the firm anticipates that its proposal is going to be accepted with probability $\Gamma(\bar{\rho})$. Notice that since $\bar{\rho}$ is increasing in ξ and s so does $\Gamma(\bar{\rho})$. To make explicit the fact that the probability that the VC finances the firm depends on the beliefs ξ as well as on its share of profits, s, we use the notation $q(\xi, s)$ with $q(\xi, s) \equiv \Gamma(\bar{\rho})$.

Consider the firm that makes the proposal s knowing that the VC holds the belief ξ. The firm knows that the proposal is going to be accepted with probability $q(\xi, s)$ and its expected profits are $q(\xi, s)\,\bar{\pi}\,(1 - s)$ (in the case of true innovator) and $q(\xi, s)\,\underline{\pi}\,(1 - s)$ (in the case of false innovator). Notice that in terms of the notation we use in the model: $rl(e) = \bar{\pi}$, $r = \underline{\pi}$ and $rl(e) = r = 0$; moreover, the function $f(\xi)$ corresponds to $q(\xi, s)\,(1 - s)$ which is increasing in the beliefs held by the VC.

7.3.1 Equilibrium of the sub-game at $t = 4$

In order to determine the equilibrium at the financing stage $t = 4$, we need to distinguish two cases depending on whether at the patenting stage played at $t = 3$ the true and the false innovators separate or not (either through the number of applications they file or through the PTO's decision).

Consider the case of separation first. At $t = 4$, the VC knows the type of the firm with certainty, and therefore by offering contract s the true innovator obtains $q(1,s)(1-s)\bar{\pi}$: it receives the financial aid with probability $q(1,s)$ and it obtains a share $(1-s)$ of the expected profits $\bar{\pi}$; therefore, in this case, it is optimal for the true innovator to propose the contract that maximizes $q(1,s)(1-s)$, namely:

$$s^T \in \arg\max_s q(1,s)(1-s).$$

Similarly, the false innovator finds it optimal to propose the contract that maximizes $q(0,s)(1-s)$, namely:

$$s^F \in \arg\max_s q(0,s)(1-s).$$

Consider now the case in which there is no separation at the patenting stage. Since the true with deep pockets always separates, the VC knows that At $t = 4$, before receiving the proposal of the firm, the VC the knows the type of the true with deep pockets, but if he observes $\overline{n}(I)$ does not know whether it is the true liquidity constraint or the false innovator and holds the belief $\xi_n(\theta) \in (0,1)$. In this case, the following is the equilibrium of the financing sub-game:

i) The true with deep pockets offers s^T while the other two types of the firm propose s^* such that $q(\xi_n(\theta), s^*)(1-s^*) \geq q\left(0, s^F\right)\left(1-s^F\right)$, and the VC holds the belief $\xi_n(\theta)$;

ii) the VC holds the belief $\xi = 0$ when receiving a proposal $s \neq s^*$.

Proof. Recalling that $s^F \in \arg\max_s q(0,s)(1-s)$ and, given the out-of-equilibrium beliefs of the VC, the best possible deviation for the firm (both for the true and the false innovator) is the proposal s^F; however condition i) implies that the equilibrium profits ($q(\xi_n(\theta), s^*)\bar{\pi}(1-s^*)$ for the true innovator and $q(\xi_n(\theta), s^*)\underline{\pi}(1-s^*)$ for the false innovator) are larger than or equal to the profits it obtains by deviating. Therefore, both types find it optimal to propose s^*. Notice that s^* satisfying condition i) does exist since $q(\xi, s)$ is an increasing function of the beliefs. Finally, notice that the D1 criterion is silent about the out-of-equilibrium beliefs, and therefore the beliefs specified in part ii) trivially satisfy it; one can easily check this last point since both types benefit from proposing $s \neq s^*$ if and only if the VC holds a belief ξ such that $q(\xi, s)(1-s) > q(\xi_n(\theta), s^*)(1-s^*)$.■

Summarizing what we have found above, then given any belief $\xi \in [0,1]$ formed at the patenting stage, the probability that the true innovator obtains $rl(e)$ (and that the false innovator obtains r) equals $q(\xi, s)(1-s)$ which is an increasing function of ξ.

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