Natural disasters, mitigation investment and financial aid

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Abstract

We consider firms facing the risk of natural disasters and study their problem of investing in mitigation if financial insurance is not available. The firms' problem is to choose the optimal timing and size of the investment. The timing problem leads to a critical productivity (firm) size where firms above it invest in mitigation while firms below the threshold decide to not invest. We investigate how cash aid, such as emergency response, and targeted financial aid, such as in-kind aid, reconstruction, rehabilitation or disaster risk reduction investments, affect the critical productivity threshold and the optimal investment size and characterize the international donor's optimal charity strategy.

Keywords: Natural Disasters; Mitigation Investment; Disaster Aid; Real Options.

JEL: D81; G1; O16; O22; Q54.

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1 Introduction

Natural disasters like Typhoon Haiyan laying waste to much of the central Philippines in 2013, Hurricane Katrina devastating the Gulf Coast of the United States in 2005, L'Aquila Earthquake striking central Italy in 2009, damaging more than 3,000 buildings and killing more than 300 persons, and the Indian Ocean Tsunami of 2004 that was generated by an undersea mega-thrust earthquake with epicenter off the west coast of Sumatra and that killed over 230,000 people in fourteen countries, are just few examples of catastrophe hazards which remind us that we live in an unpredictable and increasingly unstable world. All these catastrophe events have the power to disrupt, or cause uncertainty to economic activities, the environment and living species. Moreover, there is strong evidence that the number of natural disasters occurring worldwide has been rapidly increasing and that there has been an increase both in their severity and potential for disruptions. Munich Re for example stated in the 2012 annual report on natural hazards that since 1980, there has been a long-term upward trend in the number of events and the amount of economic and insured losses. Losses from natural catastrophes alone increased from \$528 billion of period 1981-1990, to \$1.2 trillion of period 1991-2000 and to \$1.6 trillion of the period 2001-2011.[1]

Financial insurance against catastrophes is often not available. This is the case of many developing countries where lack of formal titles of property, the shortage of adequate loss and frequency data necessary for the calculation of the risk premium and limited opportunities for diversification hinder the development of catastrophic risk insurance (Litan, 2000). As a consequence, mitigation investments that are directed at either reducing exposure to catastrophe events or at increasing the ability of structures to withstand the impact of catastrophes become an important strategy to prevent major business disruptions. But the underestimation of the probability of catastrophic events, short-term horizons (that is, the search for a quick return on investment), aversion to large upfront costs and expectation of disaster assistance are important factors that discourage such investments (Kunreuther, 2000, 2001).

In this paper we analyze incentives of firms to engage in mitigation investments that reduce the damage caused by natural disasters if financial insurance is not available and investigate how these incentives are affected by financial aid programs. Kallett and Caravani (2013) report that despite \$862 billion of losses due to natural disasters in developing nations, the total funding for natural disasters over the period 1991-2010 was only 106,7 billion. Moreover, only 12.7% was dedicated to disaster risk

[1] See http://www.munichre.com/en/reinsurance/business/non-life/georisks/natcatservice/annual_statistics.aspx.

reduction, 21.8% for reconstruction and rehabilitation, while 65.5% was for emergency response. A recent joint report by the World Bank and United Nations shows that in low-income countries bilateral and multilateral donors currently allocate 99% of their disaster management funds for relief and reconstruction and only 1% to reduce future loss exposure and vulnerability.[2] Financial aid may create dependence and discourage preventive actions such as mitigation investments: the Samaritan's dilemma (Buchanan 1975, Coate 1995). Raschky and Schwindt (2009) investigates how charity from foreign governments affects the degree of protection against large-scale disasters and reports ambiguous effects on the decision to invest by recipient's country. The authors' policy implication is to rethink strategies for international aid and to redesign existing aid programs. However, to our knowledge, the connection between alternative financial aid programs and their influence on investment decisions has, so far, not been theoretically analyzed. Lewis and Nickerson (1989) examine a model where individuals face a decision to self-insure against natural catastrophes when market insurance is unavailable and when financial aid expectations are taken into account and find that individuals underinvest in mitigation. Finally, they analyze the design of government policies to induce less costly levels of private expenditures on self-insurance. We investigate how different financial aid strategies affect the decision to invest in mitigation, the critical firm size and the size of the investment and characterize the international donor's optimal charity strategy.

We consider mitigation investments as irreversible and assume that firm productivity can be described by a jump-diffusion process where downward jumps represent natural disasters whose amplitude (damage size) is assumed to follow a Pareto law. The firm problem is to maximize its value and to choose the optimal timing and size of the investment. The solution to the timing problem leads to an investment trigger where firms with a sufficiently large productivity level, that is, sufficiently large firms, invest in mitigation while firms below the threshold decide to not invest. We then analyze the donor's optimal aid strategy. In particular, under the assumption that there is complete information about damages caused by natural hazards, we consider how cash aid, such as emergency response, and targeted financial aid, such as in-kind aid, reconstruction, rehabilitation or disaster risk reduction investments, if perfectly anticipated by firms, affect their critical investment threshold and the optimal investment size. Depending on the donor's objective function, we characterize the optimal aid strategy. In the case of cash aid we assume that the firm in the event of a natural disaster is given a cash flow proportional to the profit-loss, while in the case of targeted aid we assume that part of the

[2]See http://www.unisdr.org/2005/wcdr/intergover/official-doc/L-docs/Hyogo-framework-for-action-english.pdf

firm's productivity is reintegrated. For simplicity's sake we take the extreme case where firms do not use the additional cash flow to reintegrate it's productivity, but qualitative results below hold as long as the cash flow is not entirely used to reintegrate productivity. In the case of cash aid firms may not use all the cash flow to reintegrate its productivity because they may not have readily access to such capital, or because, for emergency reasons, it is used for other purposes, while in the case of targeted aid the international donor directly reintegrates destroyed productivity. We find that both financial aid strategies increase the critical firm size above which investment in mitigation is optimal thereby delaying them. This result reminds the charity hazard problem discussed in the financial insurance literature, where it has been pointed out that the expectation of financial assistance may lead individuals to not insure against natural hazards (see Browne and Hoyt, 2000; Raschky and Weck-Hannemann, 2007; Raschky et al., 2013). Moreover, we show that given that the two aid strategies have on average the same economic value, targeted financial aid increases the critical firm size more than cash aid does. We also find that cash aid does not alter the size of the mitigation investment, while targeted financial aid increases the investment size. As a policy conclusion, if the international donor's aim is to speed up mitigation investments, or put differently, if their aim is to reduce the firm size above which mitigation investments are optimal, then cash aid is preferable to targeted financial aid. On the contrary, if the international donor's aim is to increase the size of mitigation investments, thereby reducing the average damage caused by natural disasters, then targeted financial aid is preferable.

We propose an empirical application of our model using data on St. Lucia that aims at investigating the critical firm size above which mitigation investment is optimal and how this threshold may be affected by aid strategies. St. Lucia as an island in the southeast Caribbean basin lies within the hurricane belt. As such, the agricultural sector, among others, is highly affected by the impact of windstorms and hurricanes. In addition, the market for catastrophic risk insurance for small-scale farmers is still in its infancy. This makes small farmers extremely vulnerable to hurricane risk especially in the livestock sector. In this paper we consider the construction of a hurricane-resistant small ruminant shelter as a risk reduction measure against natural disasters. To do so we calibrate our model using data taken from FAO and the Emergency Disasters Database (EM-DAT) among others.

The paper is organized as follows. Section 2 provides a literature review. Section 3 describes the setup of the model and characterizes optimal investment decisions without financial aid. Section 4 studies how financial aid affects investment in mitigation. In particular, Section 4.1 discusses the optimal investment problem with cash aid and studies how it affects the timing and size of investment in mitigation. Section 4.2 presents the optimal investment problem with targeted financial aid consisting of a (partial) productivity restoration and studies how it affects the timing and size of investment in mitigation. Section 4.3 compares the two aid strategies and discusses the donor's optimal policy. Numerical results are presented in Section 5. Section 6 concludes the paper. All proofs are in the Appendix.

2 Literature review

Our paper is related to Raschky and Schwindt (2012) where the channel of international aid, that is, cash or in-kind as in our model, is investigated. The paper examines aid after 228 disasters over years 2000–2007 and finds that the choice of the channel and type of disaster assistance is mainly determined by the quality of institutions in the recipient country and strategic trade and natural resource interests, while humanitarian aspects appear to play only a minor role. In particular their results show that higher levels of rule of law and corruption control significantly increase the likelihood of receiving cash rather than in-kind transfers. Trading partners are more likely to receive cash aid, while oil exporting countries have a higher probability of receiving in-kind transfers. Finally, they find differences in the aid allocation behavior of OECD and non OECD countries showing that OECD countries are less likely to donate cash to large-scale disasters.

De Mel et al. (2012) study how relief from international aid flows affect the recovery of enterprises in developing countries, i.e., if these speed up the ex-post recovery process. The authors provide a microeconomic study on the process of recovery of Sri Lankan microenterprises affected by the tsunami of December 2004. They investigate whether better targeting of aid would speed up recovery of enterprises and find unclear results since damages due to the tsunami extend beyond real assets, destroying also trading relationships and supply chains. Then they implement a field experiment providing grants to randomly selected enterprises and examine whether the grant speeds up the recovery process. The authors find that firms that received grants recover profit levels in the 24 months following the tsunami. Moreover, they document that grants have a significantly larger impact on firms in the retail sectors and very little effects on firms in manufacturing and services.

A large body of economic literature studies ex ante adoption of disaster risk mitigation measures and shows how negative impacts of natural disasters can be blunted by investment in mitigation.[3] While

[3] A survey of the economic literature examining the aggregate impact of disasters and of the main disaster data sources available can be found in Cavallo and Noy (2009) and Kellenberg and Mobarak (2011).

it is clear that these investments are beneficial in mitigating specific risks, one of the main challenges is the assessment of costs associated with these investments versus the additional benefits that they generate. Theoretically speaking, the costs of implementing mitigating strategies can be considered as investment costs, but it is difficult to evaluate the potential return on such an investment for which benefits are mostly avoided or reduced damages and losses, and for which statistical data often are limited. OECD countries such as the United Kingdom and the United States, as well as international financial institutions such as the World Bank, have often used the cost-benefit analysis to evaluate disaster risk reduction measures in developing countries (Kull et al. 2013). Michel-Kerjan et al. (2013) employed a probabilistic cost-benefit analysis to examine disaster risk reduction investments providing an application to the hurricane risk management in St. Lucia. Other studies analyze, in the context of climate change, the effects of cost and benefit uncertainty of mitigation policies. [4] Tsur and Zemel (2009) study optimal mitigation for a growing economy under threats of catastrophic climate change and find that the optimal policy should eliminate total emissions at a finite time so that the catastrophic risk will vanish in the long run. Zeeuw and Zemel (2012) develop an optimal control model for managing a dynamic system subject to pollution damage and under the risk of an abrupt regime shift.

Our paper also contributes to the real-option literature that studies investment decisions in the presence of rare events such as innovations or natural disasters. Technically these are modeled as positive and negative jumps in the underlying continuous-time process. This argument is briefly introduced by Dixit and Pindyck (1994) and thoroughly examined by Boyarchenko (2004), Boyarchenko and Levendorskiĭ (2002) and Mordecki (2002). However, none of them deal with the optimal investment in mitigation in the presence of alternative disaster relief programs as we do. Baranzini et al. (2003) examine the optimal timing of implementing an abatement policy under uncertainty and climate catastrophes. In their model the underlying ratio of the sum of expected discounted benefits and costs of reducing global warming is stochastic and jumps of deterministic size account for the impact of catastrophe events. The decision maker is a governmental agency who looks for the optimal timing to reduce the effect of the accumulation of greenhouse emissions on climate change. Our assessment of catastrophe risk and that by Baranzini et al. (2003) are similar. However, we model natural disaster by reducing the general productivity of the firm, while mitigation reduces the expected loss at a cost. In this framework, we analytically compute the critical level of the firm's productivity (which

[4] The natural hazard literature, and this paper, refers to these actions as mitigation investments, whereas in the climate literature, mitigation refers to reductions of greenhouses emissions.

corresponds to a critical firm size) at which the investment in mitigation is undertaken. Truong and Truck (2010) provide a framework for the analysis of investment in mitigation of catastrophe losses under the impacts of climate change and discuss an application to the case of bushfire management in Australia. Woodward et al. (2013) use real options approach to evaluate investments in mitigation of flood risk in an estuarine area under climate change uncertainty.

Recent studies used historical data in order to estimate the likelihood and expected impact of catastrophe events on capital stock, GDP and wealth and provide a list of risk management strategies both in macro and economics of insurance literature. In two recent papers Barro (2006, 2009) tries to explain the equity premium and related asset-pricing puzzles using historical data on three biggest disaster events of the last century: the two World Wars and the Great Depression. Pindyck and Wang (2013) study a general equilibrium model and analyze the impact of possible catastrophic events on consumption, calculating their implications for catastrophic risk insurance, and evaluating tax policies to reduce their severity.

3 The mitigation investment decision without financial aid

We study a simple partial-equilibrium model of a small open economy where firms in the industry are price takers and produce a homogeneous product. Let us consider a representative firm producing q units of output at each moment of time according to the simple production function

$$q\left(K_t,\theta_t\right)=\theta_t K_t^a,$$

where K is the capital used as an input in the production process, a is the constant output elasticity $(0<a<1)$, and θ is the firm's productivity. Let p be the fixed international price of output and ρ the fixed unit cost of capital. The instantaneous operating profit is

$$\pi\left(K_t,\theta_t\right)=p\theta_t K_t^a-\rho K_t.$$

We assume that capital input K_t can be instantaneously adjusted, and thus the firm chooses K_t to maximize the instantaneous operating profit $\pi\left(K_t,\theta_t\right)$. It is easily shown that

$$\pi\left(\theta_t\right)\equiv\max_{K_t}\pi\left(K_t,\theta_t\right)=\Psi\theta_t^{\delta}$$

where $\Psi \equiv (1-a)\left(\frac{a}{\rho}\right)^{\frac{a}{1-a}} p^{\frac{1}{1-a}}$ and $\delta \equiv \frac{1}{1-a} > 1$. The value of K that maximizes the firm's profit is $\left(\frac{ap\theta}{\rho}\right)^{\delta}$. π is a measure of firm size and thus the larger is θ, the larger the firm size.

We model natural hazard by decreasing θ_t, i.e., a natural calamity can cause damages which reduce the general productivity of the technology in place, etc. More formally, productivity evolves according to a geometric jump-diffusion process

$$d\theta_t = \mu\theta_t dt + \sigma\theta_t dW_t + (Y-1)\,\theta_t dQ_t. \tag{1}$$

$\mu\,(>0)$ is the instantaneous drift, $\sigma\,(>0)$ is the volatility of the Brownian part of the process, dW_t is a standard Gauss-Wiener process and dQ_t is a Poisson process where $dQ_t = 1$ with probability λdt and $dQ_t = 0$ with probability $(1-\lambda dt)$. The productivity parameter θ as given by (1) has two sources of uncertainty. The term σdW_t corresponds to "business-as-usual" uncertainty, while the term dQ_t is the jump uncertainty which describes natural hazards. Within an infinitesimal small time interval dt, a jump occurs with probability λdt, that is, λ denotes the arrival rate of a natural disaster. In addition to uncertainty associated with the timing of natural calamities, we also assume that the magnitude of the jump is uncertain (throughout the processes W, Q and Y are supposed to be independent). Upon the occurrence of such an event the general productivity of the technology instantaneously drops from θ to $Y\theta$, where $0 \leq Y \leq 1$ is the fraction of productivity in place after a natural disaster has occurred. If $Y = 0$, then the calamity destroys all the productivity, while if $Y = 1$, then it has no effect on the productivity. We assume that Y is drawn from a power distribution over the interval $[0,1]$ with parameter $\alpha\,(>0)$; that is, the distribution function is defined as

$$F(Y) = Y^{\alpha}, \text{ with } 0 \leq Y \leq 1 \tag{2}$$

and the expected fraction of productivity lost is $\mathbb{E}(1-Y) = \int_0^1 (1-Y)\,dF(Y) = \frac{1}{\alpha+1}$. Hence, an increase in α decreases the expected loss or, otherwise, a larger value of α increases $\mathbb{E}(Y)$, the expected fraction of productivity still in place after the calamity. Equation (2) implies that $\mathbb{E}(Y^n) = \frac{\alpha}{\alpha+n}$, provided that $\alpha + n > 0$.

We define the following function $Dis(x) \equiv r - \mu\delta - \frac{1}{2}\sigma^2\delta(\delta-1) + \frac{\lambda\delta}{x+\delta}$, where r is the market interest rate. Note that $Dis(x)$ is decreasing in x and let us define $Dis(\infty) \equiv lim_{x\to\infty} Dis(x)$. Throughout the paper we use the following assumption.

Assumption 1 *Parameters are such that* $Dis(x) \in (0,1)$ *for each* $x \geq 0$ *and that* $r \in (0,1)$.

Assumption $Dis(x) > 0$ guarantees that firm values below are well defined, while the assumption $Dis(x) < 1$ requires that λ is sufficiently small, that is, natural disasters are sufficiently rare.

In order to simplify notation we omit the time subscript wherever this does not lead to confusion. We indicate with subscript *be* investment trigger and value functions in the (benchmark) case with no financial aid.

We assume that the firm can reduce the damage caused by a natural disaster through a mitigation investment. In particular, we assume that if the firm spends I, then the probability distribution of the fraction of productivity that is in place after a natural calamity is

$$F_I(Y;I) = Y^{\alpha(I)}, \text{ with } 0 \leq Y \leq 1, \tag{3}$$

where $\alpha(I)$ is a linear function of I, i.e. $\alpha(I) = \alpha + \varepsilon I$, for $\varepsilon > 0$. Hence, if the firm does not invest then $\alpha(0) = \alpha$ and thus (3) and (2) are the same. Given two investment levels I' and I'', where $I' < I''$, we have that $F_I(Y;I'') < F_I(Y;I')$ and thus $F_I(Y;I'')$ first order stochastically dominates $F_I(Y;I')$. As a consequence, the greater is I, the lower the average damage of a natural calamity, i.e. $\int_0^1 (1-Y)\, dF_I(Y;I'') < \int_0^1 (1-Y)\, dF(Y;I')$. ε measures how strongly α is increased for a given investment level and thus captures the efficiency of the mitigation investment. The entrepreneur chooses I with the aim to maximize the mitigation option value and hence the firm value.

The firm's problem is to choose the optimal investment strategy such that the expected present value of its cash flow is maximized. Formally, the following optimal investment problem has to be solved

$$V(\theta_0) = \sup_{I,\tau \in \mathcal{T}} \mathbb{E}_0 \left\{ \int_0^\infty \pi(\theta_t)\, e^{-rt} dt - I e^{-r\tau} \right\}, \tag{4}$$

subject to Equation (1), where Y is distributed according to $F(Y)$ for $0 < t \leq \tau$, and according to $F_I(Y;I)$ for $t > \tau$. Assumption 1 guarantees that the integral in (4) is well defined. $\mathcal{T}$ is the class of admissible implementation times conditional on the filtration generated by the stochastic process θ_t.

The firm value satisfies the following Hamilton-Jacobi-Bellman (H-J-B) equation[5]

$$rV(\theta)\, dt = \Psi \theta^\delta dt + \mathbb{E}(dV). \tag{5}$$

[5] We suppress time subscripts unless they are needed for clarity.

Equation (5) has a straightforward economic interpretation. The rate of return consists of the cash flow $\Psi\theta^{\delta}$ plus the expected capital gain $\mathbb{E}(dV)$. Optimality requires that the total expected return of the investment equals the market return r. To calculate the expected change in the firm value, $\mathbb{E}d(V)$, we apply Ito's Lemma for jump-diffusion processes to obtain

$$\frac{1}{dt}\mathbb{E}(dV) = \begin{cases} \mu\theta\frac{\partial V}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 V}{\partial\theta^2} + \lambda\int_0^1 [V(\theta Y) - V(\theta)]\, dF(Y) & \text{for } 0 < t \leq \tau \\ \mu\theta\frac{\partial V}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 V}{\partial\theta^2} + \lambda\int_0^1 [V(\theta Y) - V(\theta)]\, dF_I(Y; I) & \text{for } t > \tau \end{cases}.$$

We can rewrite the H-J-B equation as follows

$$rV(\theta) = \begin{cases} \Psi\theta^{\delta} + \mu\theta\frac{\partial V}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 V}{\partial\theta^2} + \lambda\int_0^1 [V(\theta Y) - V(\theta)]\, dF(Y) & \text{for } 0 < t \leq \tau \\ \Psi\theta^{\delta} + \mu\theta\frac{\partial V}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 V}{\partial\theta^2} + \lambda\int_0^1 [V(\theta Y) - V(\theta)]\, dF_I(Y; I) & \text{for } t > \tau \end{cases} \tag{6}$$

We call $V_{d,be}$ and $V_{m,be}$ the value functions in the continuation region ($0 < t \leq \tau$), where investment is delayed, and termination region ($t > \tau$), where the investment in mitigation has already been undertaken, respectively; as mentioned above, the index *be* indicates that this is the benchmark case, while d and m indicates "delay" and "mitigate", respectively.

Since investment in mitigation is costly, it is undertaken only by firms whose productivity is sufficiently large. In other words, firms have to be sufficiently large for the investment in mitigation to be profitable. The intuition for this result is that if productivity is too low then the costs of a mitigation investment are large compared with its gains and the firm prefers to take its chances and to wait and to weather out the storm. Consequently, the solution to the optimal stopping problem (4) consists in finding a critical productivity level θ_{be}, and a corresponding critical firm size $\pi(\theta_{be})$, above which it is optimal to invest in mitigation. Hence, investment in mitigation should be undertaken the first time the process θ_t crosses the threshold θ_{be} from below.

Consider first the firm value in the termination region $V_{m,be}$. We consider the simple case where only a single investment is allowed, which means that in the termination region there is no further investment option. The general solution to (6) is of the form $\Psi_m\theta^{\delta}$, where Ψ_m is a constant to be determined. Substituting this expression into (6) and solving for $V_{m,be}$ we find that[6]

$$V_{m,be}(\theta) = \frac{\Psi\theta^{\delta}}{Dis(\alpha(I))}. \tag{7}$$

[6] This result is formally derived in the Appendix.

Assumption 1 assures that (7) is well defined. Equation (7) has a straightforward economic interpretation. $V_{m,be}(\theta)$ is the expected present value of the profit flow if the productivity level is θ and where the denominator is the discount factor that takes the mitigated expected losses due to natural disasters into account.

When the firm decides to invest in mitigation, the expected payoff is the expected present value of the future profit stream minus the investment cost, that is, the termination payoff is $\Omega_{m,be}(\theta; I) = \frac{\Psi\theta^{\delta}}{Dis(\alpha(I))} - I$. The entrepreneur chooses the investment level I that maximizes the value of termination, where

$$\Omega_{m,be}(\theta) = \max_{I}\left[\frac{\Psi\theta^{\delta}}{Dis(\alpha(I))} - I\right], \tag{8}$$

and where $I_{be} = \arg\max_I \left[\frac{\Psi\theta^{\delta}}{Dis(\alpha(I))} - I\right]$.

Consider next the firm value in the continuation region $V_{d,be}$. It can be verified by direct substitution that a general solution to the differential equation (6) is

$$V_{d,be}(\theta) = \Psi_d\theta^{\delta} + \Phi_{1,be}\theta^{\phi_1} + \Phi_{2,be}\theta^{\phi_2} + \Phi_{3,be}\theta^{\phi_3}, \tag{9}$$

where $\Phi_{1,be}$, $\Phi_{2,be}$ and $\Phi_{3,be}$ are constants, ϕ_1, ϕ_2 and ϕ_3 are the roots of the characteristic equation

$$\frac{1}{2}\sigma^2\phi(\phi-1) + \mu\phi - \frac{\lambda\phi}{\alpha+\phi} = r \tag{10}$$

and $\Psi_d = \frac{\Psi}{Dis(\alpha)}$. Assumption 1 assures that $\Psi_d > 0$. The last three terms in (9) can be interpreted as the investment option value, while the first term is the expected present value of the profit flow $\Psi\theta_t^{\delta}$. Since $\alpha(I) > \alpha$, $Dis(\alpha(I)) < Dis(\alpha)$ and thus $\theta^{\delta}\Psi_d$ is lower than $\frac{\Psi\theta^{\delta}}{Dis(\alpha(I))}$. The following lemma characterizes the roots of the cubic equation (10).

Lemma 1 *The three roots of the characteristic equation (10) are* $\phi_2 < -\alpha < \phi_1 < 0 < \delta < \phi_3$.

The solution for $V_{d,be}$ must satisfy the following set of boundary conditions

$$V(0) = 0, \tag{11}$$

$$V(\theta) = \Omega(\theta), \tag{12}$$

$$\frac{\partial V(\theta)}{\partial\theta} = \frac{\partial\Omega(\theta)}{\partial\theta}, \tag{13}$$

computed at $\theta = \theta_{be}$ and where $V = V_{d,be}$ and $\Omega = \Omega_{m,be}$; conditions (11) (12) and (13) determine the constants $\Phi_{1,be}$, $\Phi_{2,be}$, $\Phi_{3,be}$ and θ_{be}. Condition (11) states that the firm value is zero if productivity is zero which, since the sign of ϕ_1 and ϕ_2 is negative (see Lemma 1) condition (11), implies that $\Phi_{1,be} = \Phi_{2,be} = 0$. (12) and (13) represent the value matching and smooth pasting condition, respectively, and guarantee that at the investment threshold θ_{be} the firm value as well as its derivative are continuous and jointly determine the value of θ_{be} and $\Phi_{3,be}$, ruling out arbitrage possibilities. θ_{be} is a free boundary which separates the investment from the no-investment regions. It is also the solution to the stopping problem (4) $\tau = \inf\{t > 0,\ \theta \geq \theta_{be}\}$.

Proposition 1 *The value function in the continuation and termination region is, respectively,*

$$V_{d,be}(\theta) = \frac{\Psi\theta^{\delta}}{Dis(\alpha)} + \left[\frac{1}{Dis(\infty)}\left(\sqrt{\Psi\theta_{be}^{\delta}} - \sqrt{\frac{\lambda\delta}{\varepsilon}}\right)^2 + \frac{1}{\varepsilon}(\alpha+\delta) - \frac{\Psi\theta_{be}^{\delta}}{Dis(\alpha)}\right]\left(\frac{\theta}{\theta_{be}}\right)^{\phi_3}, \tag{14}$$

$$\Omega_{m,be}(\theta) = \frac{1}{Dis(\infty)}\left(\sqrt{\Psi\theta^{\delta}} - \sqrt{\frac{\lambda\delta}{\varepsilon}}\right)^2 + \frac{1}{\varepsilon}(\alpha+\delta). \tag{15}$$

The investment threshold is

$$\theta_{be} = \left[\frac{(\alpha+\delta)\,Dis(\alpha)}{\sqrt{\varepsilon\delta\lambda\Psi}}\frac{\phi_3}{\phi_3-\delta}\right]^{\frac{2}{\delta}} \tag{16}$$

and the size of the investment is

$$I_{be} = \frac{\delta(\alpha+\delta)}{\varepsilon(\phi_3-\delta)}\frac{Dis(\alpha)}{Dis(\infty)}.$$

Corollary 1 *The value function $V_{d,be}(\theta)$ and the value of termination $\Omega_{m,be}(\theta)$ are increasing in ε.*

(14) is the firm value as long as $\theta < \theta_{be}$, while once $\theta \geq \theta_{be}$, the firm invests in mitigation and the firm value becomes (15). The second term in (14) represents the investment option value, which is always positive (this can be seen by substituting (16) into (14)). For a given investment level, the larger is ε, the more efficient are mitigation investments and the lower the average damage of a catastrophic event. From Proposition 1 we can observe that θ_{be} and I_{be} are decreasing in ε and thus, taking into account the results stated in Corollary 1, we can conclude that, the larger is ε, the lower the optimal investment in mitigation, the lower the critical firm size that triggers investment, that is, the earlier the investment is undertaken, and the larger the firm value.

4 The mitigation investment decision with financial aid

In this section we study how financial aid, if perfectly anticipated by firms, affects their mitigation investment strategy. Financial assistance through donations is provided to help individuals in the recovery process and to alleviate the financial burden they may experience after a natural disaster. In particular we study how charity affects the invest threshold and the size of the investment. We first consider cash aid in the form of an instantaneous cash flow and afterward we consider a targeted financial aid program aimed at restoring firm's productivity. We assume that the instantaneous cash flow cannot be used to restore the firm's productivity. The analysis below could be extended to the case where the cash flow is partially used to restore productivity without altering our qualitative results. We indicate with index *ca* investment trigger and value functions if financial aid is in the form of cash, while with *re* if aid is targeted in the form of a productivity restoration program.

4.1 Cash aid

Throughout this section we assume that upon occurrence of a natural calamity the firm receives an instantaneous cash flow. More formally, we assume that a natural disaster triggers a cash flow proportional to firm profits prior to the event[7]

$$Aid_t dt = \kappa(1 - Y^\delta)\Psi\theta_t^\delta dQ_t,$$

where $0 < \kappa \leq 1$ is an exogenous donation rate, and where $dQ_t = 1$ with probability λdt and $dQ_t = 0$ with probability $1 - \lambda dt$. The expected instantaneous cash flow due to donations without mitigation investment, where Y is distributed according to $F(Y)$ in (2), is

$$\mathbb{E}_t\left(Aid\right) = \frac{\lambda\kappa\delta}{\alpha + \delta}\Psi\theta_t^\delta,$$

which is increasing in the donations rate κ, the arrival rate of natural disasters λ and also in the severity of the disaster as measured by the damages caused (the lower α, the more severe the damages). The reason for this latter assumption is that the more adverse the effects of a calamity, the greater the

[7]Let us define $\pi(\theta_t) = \Psi\theta_t^\delta$, the application of Ito's Lemma leads to:

$$d\pi(\theta_t) = \left[\mu + \frac{\sigma^2}{2}(\delta - 1)\right]\delta\pi(\theta_t)dt + \sigma\delta\pi(\theta_t)dW_t + \left(Y^\delta - 1\right)\pi(\theta_t)dQ_t;$$

if a natural disaster occurs the firm's profit flow drops from $\Psi\theta^\delta$ to $\Psi\left(Y\theta\right)^\delta$, where $0 \leq Y^\delta \leq 1$ is the fraction of profit flow obtained after a natural disaster has occurred.

worldwide news coverage, and therefore the more funds will be raised.[8]

With financial charity the firm's investment problem is

$$V(\theta_0) = \sup_{I,\tau_{ca}\in\mathcal{T}} \mathbb{E}_0 \left\{ \int_0^\infty e^{-rt}\Psi\theta_t^\delta dt + \int_0^\infty e^{-rt}\kappa(1-Y^\delta)\Psi\theta_t^\delta dQ_t - Ie^{-r\tau_{ca}} \right\},$$

subject to equation (1), where for $0 < t \leq \tau_{ca}$ the distribution of Y is $F(Y)$, while for $t > \tau_{ca}$ it is $F_I(Y; I)$.

Following the argument in the previous section we must calculate the expected discounted present value of the termination value with cash aid

$$\mathbb{E}_0 \left[\int_0^\infty e^{-rt}\Psi\theta_t^\delta dt + \int_0^\infty e^{-rt}\kappa(1-Y^\delta)\Psi\theta_t^\delta dQ_t \right],$$

where the dynamics of θ_t are given by (1) with Y distributed as explained above. Following the calculations presented in the Appendix we can show that the expected present value of the termination cash flow is $\frac{(\alpha(I)+\delta+\lambda\kappa\delta)}{(\alpha(I)+\delta)Dis(\alpha(I))}\Psi\theta^\delta$. Hence, the termination payoff is $\Omega_{m,ca}(\theta; I) = \frac{(\alpha(I)+\delta+\lambda\kappa\delta)}{(\alpha(I)+\delta)Dis(\alpha(I))}\Psi\theta^\delta - I$ and the entrepreneur chooses the investment level I that maximizes the value of termination, where

$$\Omega_{m,ca}(\theta) = \max_I \left\{ \frac{(\alpha(I)+\delta+\lambda\kappa\delta)}{(\alpha(I)+\delta)\, Dis(\alpha(I))}\Psi\theta^\delta - I \right\}$$

and where $I_{ca} = \arg\max_I \left\{ \frac{(\alpha(I)+\delta+\lambda\kappa\delta)}{(\alpha(I)+\delta)Dis(\alpha(I))}\Psi\theta^\delta - I \right\}$.

In the continuation region d the firm does not invest in mitigation and the firm value is implicitly defined by the fundamental equation of optimality

$$rV_{d,ca}(\theta)\, dt = \Psi\theta^\delta dt + \kappa\mathbb{E}\left[(1-Y^\delta)dQ_t\right]\Psi\theta_t^\delta + \mathbb{E}(dV_{d,ca}),$$

where the second term on the right-hand-side is the expected cash flow due to financial aid. We apply Ito's Lemma for jump-diffusion processes to obtain

$$rV_{d,ca}(\theta) = \Psi\theta^\delta + \kappa\lambda\Psi\theta^\delta \int_0^1 \left(1-Y^\delta\right) dF(Y) + \mu\theta\frac{\partial V_{d,ca}}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 V_{d,ca}}{\partial\theta^2} + \lambda\int_0^1 \left[V_{d,ca}(\theta Y) - V_{d,ca}(\theta)\right] dF(Y),$$

[8] Strömberg (2007) shows that international aid related to natural disasters increases with the severity of the disaster, as measured by the number of individuals killed and affected, and rises with news coverage. Other factors that drive international relief are geographic proximity, cultural and colonial connections. Eisensee and Strömberg (2007), examine the determinants of the size of international aid flows and find that news effects bias relief towards disasters that are more newsworthy or stuck in more developed (hazard-prone) countries. We build up on these findings and assume that financial aid is determined by the severity of catastrophes as measured by the damages to the firm's productivity.

It can be easily verified by direct substitution that a general solution to this differential equation is $V_{d,ca}(\theta) = \frac{(\alpha+\delta+\kappa\lambda\delta)\Psi\theta^{\delta}}{(\alpha+\delta)Dis(\alpha)} + \Phi_{3,ca}\theta^{\phi_3}$, where $\Phi_{3,ca}$ is a constant to be determined and ϕ_3 is the positive root of the characteristic equation (10).

The solution for $V_{d,ca}$ must satisfy the absorbing barrier (11), value matching (12) and smooth pasting (13) conditions at the critical threshold θ_{ca}, with $\Omega = \Omega_{m,ca}$.

The following proposition describes the firm value.

Proposition 2 *The value function in the continuation and termination region is, respectively,*

$$V_{d,ca}(\theta) = \frac{\Psi\theta^{\delta}(\alpha+\delta+\kappa\lambda\delta)}{Dis(\alpha)(\alpha+\delta)} +$$

$$\left[\frac{\Psi\theta_{ca}^{\delta} - 2\sqrt{\frac{1}{\varepsilon}\Psi\theta_{ca}^{\delta}\delta\lambda(1-\kappa Dis(\infty)) + \frac{1}{\varepsilon}Dis(\alpha)(\alpha+\delta)}}{Dis(\infty)} - \frac{\Psi\theta_{ca}^{\delta}(\alpha+\delta+\kappa\lambda\delta)}{Dis(\alpha)(\alpha+\delta)}\right]\left(\frac{\theta}{\theta_{ca}}\right)^{\phi_3},$$

$$\Omega_{m,ca}(\theta) = \frac{1}{Dis(\infty)}\left[\Psi\theta^{\delta} - 2\sqrt{\frac{1}{\varepsilon}\Psi\theta^{\delta}\delta\lambda(1-\kappa Dis(\infty)) + \frac{1}{\varepsilon}Dis(\alpha)(\alpha+\delta)}\right].$$

The investment threshold is

$$\theta_{ca} = \left[\frac{(\alpha+\delta)Dis(\alpha)}{\sqrt{\varepsilon\delta\lambda\Psi[1-\kappa Dis(\infty)]}}\frac{\phi_3}{\phi_3-\delta}\right]^{\frac{2}{\delta}} > 0,$$

and the size of the investment is

$$I_{ca} = \frac{\delta(\alpha+\delta)Dis(\alpha)}{\varepsilon(\phi_3-\delta)Dis(\infty)}.$$

In the following Proposition we compare investment thresholds θ_{ca} and θ_{be} and investment sizes I_{ca} and I_{be}.

Proposition 3 *The optimal investment threshold with cash aid, θ_{ca}, is increasing in the donation rate κ and it is always larger than the optimal investment threshold without financial aid, θ_{be}. The size of the investment with and without cash aid is the same.*

Financial charity has the effect of increasing the critical firm size above which investment in mitigation is optimal, thereby delaying the investment. The greater the donation rate κ, the larger the critical firm size and thus the later the firm invests in mitigation. Note that even for $\kappa = 1$, that is, the case where the cash flow from the international donor is equal to the size of the damage, investing in mitigation is still optimal for sufficiently large firms. The reason for this result is that the donor's cash flow is not (or cannot) be used for restoring the firm's productivity. Since what matters for the firm value is the productivity level, investing in mitigation in order to reduce the damage caused by natural hazards is still optimal if the firm is sufficiently large.

Cash aid produces two effects on the investment size. Firstly, for a given productivity level θ, it has a direct negative effect because the expectation of financial assistance crowds out private investments. Secondly, since it increases the critical productivity threshold, it has an indirect positive effect on the investment size. The two exactly offset each other and thus the overall effect of cash aid on the investment size is nil.

4.2 Targeted financial aid

In this section we consider targeted financial aid programs that aim at restoring (partially) productivity (i.e, plants, factories, machinery) destroyed by natural disasters to a pre-disaster condition. Let the probability distribution of the fraction of productivity in place after the occurrence of a natural disaster with a targeted financial aid program be

$$F_{\xi}\left(Y;\xi\right) = Y^{\alpha(\xi)}, \text{ with } 0 \leq Y \leq 1, \tag{17}$$

where ξ captures the size of financial relief and $\alpha\left(\xi\right)$ is a linear function of ξ, i.e. $\alpha\left(\xi\right) = \alpha + \xi$. Consider ξ' and ξ'', where $\xi' > \xi''$, then $F_{\xi}\left(Y;\xi'\right)$ first order stochastically dominates $F_{\xi}\left(Y;\xi''\right)$. As a consequence the larger is ξ, the lower the expected productivity loss. For $\xi = 0$ financial aid is zero and thus (2) and (17) are the same, while for $\xi \to \infty$ productivity restoration is complete.

Akin to the previous sections we assume that if the firm spends I in mitigation, then the probability distribution of the fraction of productivity that survives a catastrophic event is

$$F_{\xi I}\left(Y;I,\xi\right) = Y^{\alpha(I,\xi)}, \text{ with } 0 \leq Y \leq 1,$$

where $\alpha\left(I,\xi\right) = \alpha + \varepsilon I + \xi$.

The firm's problem is to choose the optimal investment strategy such that the expected present value of its cash flow is maximized. Formally, the optimal investment problem (4) has to be solved, subject to Equation (1), where Y is distributed according to $F_{\xi}\left(Y;\xi\right)$ for $0 < t \leq \tau_{re}$, and according to $F_{\xi I}\left(Y;I,\xi\right)$ for $t > \tau_{re}$ and where the subscript re indicates that a restoration program is in place.

If the firm decides to invest in mitigation, then the expected payoff is the expected present value of the future profit stream minus the investment cost. Thus, the value of termination can be written as

$$\Omega_{m,re}(\theta; I) = \frac{\Psi\theta^{\delta}}{Dis(\alpha(I,\xi))} - I, \tag{18}$$

The entrepreneur chooses the investment level I that maximizes the value of termination

$$\Omega_{m,re}(\theta) = \max_{I}\left\{\frac{\Psi\theta^{\delta}}{Dis(\alpha(I,\xi))} - I\right\},$$

where $I_{re} = \arg\max_I \left\{\frac{\Psi\theta^{\delta}}{Dis(\alpha(I,\xi))} - I\right\}$.

In the continuation region d, where investment in mitigation is delayed, the firm value is implicitly defined by the fundamental equation of optimality

$$rV_{d,re}(\theta) = \Psi\theta^{\delta} + \mu\theta\frac{\partial V_{d,re}}{\partial\theta} + \frac{1}{2}\sigma^2\theta^2\frac{\partial^2 V_{d,re}}{\partial\theta^2} + \lambda\int_0^1 \left[V_{d,re}(\theta Y) - V_{d,re}(\theta)\right] dF_{\xi}(Y;\xi)$$

It can be easily verified by direct substitution that a general solution to this differential equation is $V_{d,re}(\theta) = \frac{\Psi\theta^{\delta}}{Dis(\alpha(\xi))} + \Phi_{3,re}\theta^{\phi_3}$, where $\Phi_{3,re}$ is a constant to be determined and ϕ_3 is the positive root of the characteristic equation (10). Assumption 1 assures that $Dis(\alpha(\xi)) > 0$.

The solution for $V_{d,re}$ must therefore satisfy the absorbing barrier (11), value matching (12) and smooth pasting (13) conditions at the critical threshold θ_{re}, with $\Omega = \Omega_{m,re}$. The following proposition describes the firm value.

Proposition 4 *The value function in the continuation and termination region is, respectively,*

$$V_{d,re}(\theta) = \frac{\Psi\theta^{\delta}}{Dis(\alpha(\xi))} + \left[\frac{1}{Dis(\infty)}\left[\sqrt{\Psi\theta_{re}^{\delta}} - \sqrt{\frac{\delta\lambda}{\varepsilon}}\right]^2 + \frac{1}{\varepsilon}(\alpha+\delta+\xi) - \frac{\Psi\theta_{Re}^{\delta}}{Dis(\alpha(\xi))}\right]\left(\frac{\theta}{\theta_{re}}\right)^{\phi_3},$$

$$\Omega_{m,re}(\theta) = \frac{1}{Dis(\infty)}\left[\sqrt{\Psi\theta^{\delta}} - \sqrt{\frac{\delta\lambda}{\varepsilon}}\right]^2 + \frac{1}{\varepsilon}(\alpha+\delta+\xi).$$

The investment threshold is

$$\theta_{re} = \left[\frac{(\alpha+\delta+\xi)\,Dis(\alpha(\xi))}{\sqrt{\varepsilon\delta\lambda\Psi}}\frac{\phi_3}{\phi_3-\delta}\right]^{\frac{2}{\delta}},$$

and the size of the investment is

$$I_{re} = \frac{\delta\left(\alpha+\delta+\xi\right)Dis\left(\alpha\left(\xi\right)\right)}{\varepsilon\left(\phi_3-\delta\right)Dis\left(\infty\right)}.$$

From a comparison between the investment thresholds with restoration program θ_{re} and without restoration θ_{be} we can get the following result.

Proposition 5 *The optimal investment threshold with restoration program, θ_{re}, is increasing in ξ and it is always larger than the optimal investment threshold without financial aid, θ_{be}. The size of the investment with restoration program is larger than the one without it.*

These results show that targeted financial aid delays mitigation investments and that the critical threshold level above which firms invest in mitigation is increasing in the size of financial relief. Note that as ξ becomes infinitely large the productivity threshold, and hence the critical firm size, becomes infinitely large and thus the firm never invests in mitigation. This result is very intuitive: if financial restoration programs cover all losses by restoring productivity to its previous level, then the firm has never an incentive to engage in costly investments that reduce damages to productivity due to natural calamities. Akin to cash aid, targeted financial aid produces two effects on the investment size. Firstly, for a given productivity level θ, it affects the investment size negatively because of a crowding out effect. Secondly, since it increases the critical productivity threshold, it has an indirect positive effect on the investment size. This latter effect is stronger than the former one and thus the overall effect of targeted financial aid on the investment size is positive. Note that θ_{re} and I_{re} are both increasing in ξ, and thus the greater is financial aid, the later the firm invests in mitigation, i.e. the larger the firm size, and the larger is the investment size, that is, the lower is the average damage of natural hazards.

4.3 International donor: Cash aid vs. targeted financial aid

In this section we consider the international donor's problem who has to choose between cash aid and targeted aid. For this purpose we assume that the donor's objective function depends negatively on the time of the investment, that is, the critical firm size above which firms decide to invest in mitigation, and positively on the investment size. We restrict ourself to the case where all aid is either in cash or targeted, even though the analysis could be straightforwardly extended to the case of mixed strategies where the optimal policy mix could be identified. We assume that the international donor maximizes her objective function under the constraint that the economic value for the recipient is on average the same under both strategies.

Cash aid consists of a cash flow in the case of a natural disaster while targeted financial aid entails the (partial) restoration of the firm's productivity to pre-disaster levels. We require the effects of the two policies in terms of firm profits to be on average the same. Since we are interested in comparing investment triggers we require the value of the two aid strategies to be equal before the investment has been undertaken. Formally, we require that

$$\kappa \int_0^1 \left[(1-Y^\delta)\Psi\theta_t^\delta\right] dF\left(Y\right) = \int_0^1 \Psi\left(Y\theta_t\right)^\delta dF_\xi\left(Y;\xi\right) - \int_0^1 \Psi\left(Y\theta_t\right)^\delta dF\left(Y\right)$$

The left-hand-side is the expected cash flow of the cash aid program, while the right-hand-side is the expected value in terms of firm profits of the targeted financial aid program.

Rearranging this equality we obtain for a given κ the critical value $\tilde{\xi}\left(\kappa\right)$ such that the economic value of the two programs is on average the same

$$\tilde{\xi}\left(\kappa\right) = \frac{\kappa}{1-\kappa}\left(\alpha+\delta\right). \tag{19}$$

Proposition 6 *For any $\xi > 0$ and $\kappa > 0$, the optimal investment size with targeted financial aid, I_{re}, is larger than the optimal investment size with cash aid I_{ca} and without financial aid, I_{be}. For any κ and $\xi = \tilde{\xi}\left(\kappa\right)$, $\theta_{re} > \theta_{ca} > \theta_{be}$.*

The first part of Proposition 6 follows straightforwardly from Proposition 3 and Proposition 5: the investment size with targeted financial aid is larger than the one with cash aid and without aid. Moreover, since I_{re} is increasing in ξ, the greater is the economic value of targeted financial aid, the greater is the difference between the investment sizes. The second part of Proposition 6 states that the investment threshold with restoration program is always larger than the investment threshold with cash aid which is always larger than the investment threshold without financial aid. The intuition for this latter result is that cash aid, by assumption, cannot be used to restore the firm's productivity and thus the effect of an increase in the donation rate on the critical investment threshold is small. On the other side, in the case of targeted financial aid, productivity is directly restored and thus the firm has a much weaker incentive to engage in mitigation investments.

Many residents living in hazard-prone areas rarely undertake loss prevention measures to protect their property. Proposition 6 gives us a policy indication to cope with this reluctance by firms to invest in mitigation. If the goal of the policy-maker is to provide incentives to speed-up investments in loss reduction measures, or put differently, if its aim is to reduce the critical firm size above which mitigation investments are optimal, then cash aid is preferable to targeted financial aid. On the contrary, if the policy-maker's aim is to increase the size of the investment and thus to reduce the average damage caused by natural disasters then targeted financial aid is preferable. If we allow also for mixed policies and if the donor places more importance on the timing, then the optimal donation strategy consists of relatively more cash, while if greater weight is placed on the investment size, then the optimal donation strategy consists of relatively more targeted aid.

5 Numerical application

In this section we provide a numerical application using data on St. Lucia taken from the FAO's reports on hurricane hazard mitigation investments in Latin America and the Caribbean and data for natural disasters taken from EM-DAT: The OFDA/CRED Emergency Disasters Database.

St. Lucia, is a small developing country located in the tropical hurricane belt, south of Martinique and north of St. Vincent in the Caribbean Sea. Due to its small size and relative lack of geological resources, its economy relies primarily on the sale of banana crops, and income generated from tourism, with additional inputs from small-scale enterprises (especially livestock and manufacturing) and fishery. Due to its geographic location in the hurricane belt as well as in a tectonically-active area, St. Lucia is regularly exposed to natural hazards including tropical storms, hurricanes, floods, localized floods, drought spells, landslides and earthquakes, which regularly affect the agriculture, tourism and livestock sectors. Small ruminants are an important livelihood asset of farmers. According to the Food and Agriculture Organization of the United Nations (FAO)[9] there is at present no meaningful or effective catastrophe coverage for vulnerable farmers with very few exceptions for banana/crop producers. Livestock farmers in St. Lucia have no means to transfer risks hence they are extremely vulnerable to natural disasters. To cope with this lack of insurance coverage for small scale farmers we consider the construction of a hurricane-resistant small ruminant shelter. The construction of a hurricane-resistant small ruminant housing unit incorporates building design features to securely bolt down the roof and reinforce the foundation of the structures. The estimated investment cost (I) is US$ 3000 for materials plus labor which is 40 percent of material cost.[10]

[9] See the FAO report 2011 on rural finance: http://www.fao.org/climatechange/32723-0afc542f1c02f5c5badb5fe1d7349152d.pdf.

[10] See more at: http://teca.fao.org/technology/construction-hurricane-resistant-small-ruminant-shelter-st-lucia.

Hurricane classification is based on the intensity of the storm, which reflects damage potential. The most commonly used categorization method is the one developed by H. Saffir and R.H. Simpson. The Saffir-Simpson hurricane wind scale is a 1 to 5 rating based on a hurricane's sustained wind speed.[11] Levels of storm surge fluctuate greatly due to atmospheric and bathymetric conditions. Thus, the expected storm surge levels are general estimates of a typical hurricane occurrence. According to data published by the Caribbean Hurricane Network,[12] only 14 hurricanes have moved closer than 60 miles to St. Lucia since 1850. Of those, none has reached Category 5 on the Saffir-Simpson scale, only one has been Category 4 and one Category 3. The islands easterly location also insures that most hurricanes don't spend enough time over open water to build strength in their destructive wind forces. This is why almost every hurricane to hit the island is category 1 or 2. In the last 10 years five major hurricanes affected the country, including hurricanes Lili (2002), Ivan (2004), Emily (2005), Dean (2007) and Tomas (2010) where the last two were Category 2 storms. Banana, root crops and livestock of small scale farmers and fisherfolk were all severely affected.

The information on economic damages presented here is taken from the EM-DAT: Emergency Disasters Database.[13] Looking at the EM-DAT data on top natural disasters in St. Lucia for the period 1900 to 2014 sorted by economic damage costs, we see that 14 major tropical storms hit St. Lucia with total damages of US$ 1.137 billion and an average damage per event of about US$ 142 million. There are several methodologies to quantify the cost of disasters, but there is no standard measure to determine a global figure for economic impact. Here, total estimated damages include damages and economic losses directly or indirectly related to the tropical storm. Moreover, these are calculated as money damage in relation to the GDP of St. Lucia. Nonsignificant disasters were excluded, a significant disaster being defined as one that caused economic losses greater than 500000 US$.

Table 2 summarizes top 8 tropical storms in St. Lucia for the period 1900 to 2014 sorted by economic damage costs [14]:

[11] Source: http://www.nhc.noaa.gov/aboutsshws.php.

[12] See more at: stormcarib.com

[13] In order for a disaster to be entered into this database at least one of the following criteria has to be fulfilled: 10 or more people reported killed, 100 people reported affected, declaration of a state of emergency, call for international assistance.

[14] We are indebted with Paul Cashin for providing us with data for GDP. See also Cashin (2006). *St. Lucia real GDP in 1963 and 1967 are 193 and 228 Million EC$, respectively. Nominal GDP is computed using exchange rates 1.7 (1963) and 1.8 (1967) and using the consumer price index to approximate the GDP deflator. Source: World Bank and Federal Reserve Bank of St. Louis; for the year 1963 we considered a Consumer Price Index of 9.

Table 1: Top 8 storms in St. Lucia sorted by economic damage costs

Date	Damage (current prices US$ $\times 10^6$)	GDP (current prices US$ $\times 10^6$)
31/07/1980	88	136
08/09/2004	0.5	831
30/10/2010	0.5	1203
17/08/2007	40	1063
25/09/1963	3.5	15*
07/09/1967	3	17*
01/09/1983	1.3	157
11/09/1988	1000	344
Total	1137	3768

More information and data on: www.emdat.be/; http://www.econstats.com/weo/CLCA.htm.
Source of data: (1) EM-DAT: The OFDA/CRED International Disaster Database, University catholique de Louvain, Brussels; (2) IMF World Economic Outlook.

Hence, average damage per event is estimated at about 30 percent of GDP. Throughout we assume that the damage is uniformly distributed over all firms and hence, if a hurricane occurs, the expected loss for each is $\mathbb{E}\left(1 - Y^{\delta}\right) = \frac{\delta}{\alpha+\delta} = 0.3$.

Next we calculate how often St. Lucia gets affected by tropical storms. We consider the period 1963-2014 when major hurricanes occurred. In 51 years 8 major hurricanes hit the island and thus the mean waiting time is 6.4 years. Hence, since $\frac{1}{\lambda} = 6.4$, the annual frequency (λ) of hurricanes is $\lambda = 0.16$.

Livestock productivity can be measured by the amount of meat or milk (wool, eggs etc.) produced per animal per year.[15] Higher productivity is a compound of higher off-take rates (shorter production cycles by, for example, faster fattening), and higher dressed weight or milk or wool yields. We assume that the small ruminant's productivity in St. Lucia, proxied by the average sheep and goat dressed weight, is identical to the one in Latin America and the Caribbean, which is $\theta_t = 16$ kg for each animal.[16] Market prices for sheep and goat meat in various Caribbean islands are given in Singh et al. (2006). The average market price for small ruminants meat in St. Lucia is US$ 2.88 for each kg of living animal[17] and we assume that the price for each kg of dressed weight (p) is US$ 2.

The elasticity of small ruminant meat with respect to capital stock (livestock and other equipments) is the percentage increase in livestock output resulting from a 1 percent increase in the capital stock. The estimation of the Cobb-Douglas production function for small ruminants meat industry suggests

[15]Source: World Agriculture: Towards 2015/2030. An FAO perspective. Ch. 5.
[16]Source: http://www.fao.org/docrep/005/y4252e/y4252e07.htm.
[17]See more at: http://www.caricom.org/jsp/community/agribusiness_forum/small_ruminant_competitiveness_development.pdf, p. 62. Values based on 2006/07 data.

that livestock products are rather sensitive to change in capital stock. For the following we assume $a = 0.5$ which gives $\delta = 2$. Using this latter figure and the expression for the average loss we get an estimation of $\alpha = 4.7$.

Taking a unit cost of capital $\rho = 0.2$ we get $\pi_t = (1-a)\left(\frac{a}{\rho}\right)^{\frac{a}{1-a}}(p\theta_t)^{\frac{1}{1-a}} = 1280\$$ which is in keeping with the empirical evidence provided by Singh et al. (2006) who report a projected annual gross farm income for St. Lucia island of about 1300 US$/year.

We calibrate ε assuming that the optimal investment size if there is no financial aid (which is identical to the investment size in the case of cash aid) is $I_{ca} = I_{be} = 4200\$$. In this way we obtain $\varepsilon = 0.00229336$ and hence that the expected damage after the investment without financial aid and with cash aid is 12.2% of income. We further assume that after the occurrence of a major hurricane the firm expects financial aid in cash or through a productivity restoration program equivalent to 10%, 20%, 30%, 40% and 50% of its profit losses, that is $\kappa = 0.1$, 0.2, 0.3, 0.4 and 0.5. Aid strategies are compared using (19) which implies that their benefits in terms of firm profits are the same.

Other parameter values are assumed as follows: $r = 0.07$ (long run estimate from World Bank database), $\mu = 0.01$ (drift-rate of the productivity shock) and $\sigma = 0.1$ (volatility of the productivity shock).

Table 1 summarizes the base-case parameter values.

Table 2: Summary of base-case parameter values

Input	Symbol	Value	Input	Symbol	Value
Investment cost	I	4200	Risk-free interest rate	r	0.07
Frequency of Hurricanes	λ	0.16	Drift-rate of the productivity shock	μ	0.01
Distribution parameter	α	4.7	Volatility of the productivity shock	σ	0.1
Price of livestock products	p	2	Distribution parameter with mitigation	ε	0.00196882
Elasticity of output to capital	a	0.5	Donation rate	κ	$0.1, 0.2, 0.3, 0.4, 0.5$
Livestock productivity	θ	16	Unit cost of capital	ρ	0.2

We carry out simulations with the intent to investigate investment thresholds and investment sizes and how they are affected by financial aid.

Table 3: Critical thresholds for $r = 0.07$; donation rates $\kappa = 0, 0.1, 0.2, 0.3, 0.4$ and 0.5; $I = 4200$ and $\varepsilon = 0.00201499$.

	$\kappa = 0$	$\kappa = 0.1$	$\kappa = 0.2$	$\kappa = 0.3$	$\kappa = 0.4$	$\kappa = 0.5$
θ_{ca}	16.1	16.1	16.1	16.2	16.2	16.2
$\pi(\theta_{ca})$	1290.8	1296	1301.2	1306.5	1311.8	1317.1
θ_{re}	16.1	16.9	18	19.2	20.9	23.4
$\pi(\theta_{re})$	1290.8	1424.9	1601.7	1844.3	2194.4	2735.6
I_{re}	4200	4412.7	4678.6	5020.4	5476.2	6114.3
average damage	12.2%	11.9%	11.5%	11%	10.4%	9.7%

Firstly, as predicted by theory, simulation results show that a restoration program increases the critical firm size much more than cash aid. Thus, while cash aid has only a weak impact on the critical firm size, from Table 3 we observe that a restoration program has a much stronger impact on the investment decision. Secondly, we observe that, in the case of a restoration program, the size of the investment is increasing in the donation rate, which also implies that the expected damage after the investment has been undertaken is decreasing in the donation rate. Table 3 evidences the trade-off the international donor is facing: cash aid leads to a lower critical firm size but also to a lower investment level (which implies a higher average damage), while a productivity restoration program leads to a larger critical firm size but also a larger investment level (which implies a lower average damage). Thus, if firms are heterogeneous in their income, then increasing the donation rate, by increasing the critical firm size, induces fewer firms to invest in mitigation. Moreover, if aid is in the form of a restoration program then firms investing will invest more than if aid is in cash, even though fewer firms will invest.

We carry out some sensitivity analysis by increasing the calibrated cost of the investment I from $I_{ca} = I_{be} = 4200\$$ to $I_{ca} = I_{be} = 4300\$$. Note that this is equivalent to assuming a lower efficiency of the investment ε. In particular, by calibrating the model assuming $I_{ca} = I_{be} = 4300\$$ we obtain $\varepsilon = 0.00224002$, which is lower than the one obtained in the previous case, while the expected damage after investment remains unchanged (12.2%).[18] From Table 4 we observe that an increase in I by approximately 2.38% increases the investment thresholds by the same percentage. Hence innovations that increase the effectiveness of mitigation investments lead to earlier adoptions and to lower investment sizes, without affecting the expected damage after the investment.

[18]The reason for the unchanged expected damage after investment is that a lower efficiency of investment is compensated by a larger investment size and thus, because of the envelope theorem, the expected damage after investment is unchanged.

Table 4: Critical thresholds for, $r = 0.07$; donation rates $\kappa = 0, 0.1, 0.2, 0.3, 0.4$ and 0.5; $I = 4300$ and $\varepsilon = 0.00224002$.

	$\kappa = 0$	$\kappa = 0.1$	$\kappa = 0.2$	$\kappa = 0.3$	$\kappa = 0.4$	$\kappa = 0.5$
θ_{ca}	16.3	16.3	16.3	16.4	16.4	16.4
$\pi(\theta_{ca})$	1321.5	1326.8	1332.2	1337.6	1343	1348.5
θ_{re}	16.3	17.1	18.1	19.4	21.2	23.7
$\pi(\theta_{re})$	1321.5	1458.8	1639.9	1888.2	2246.7	2800.7
I_{re}	4300	1458.8	4790	5139.9	5606.6	6259.9
average damage	12.2%	11.9%	11.5%	11%	10.4%	9.7%

In Table 5 we show simulation results for investment costs $I_{ca} = I_{be} = 4200\$$ if the interest rate raises from 0.07 to 0.075. Compared with the base line simulation results in Table 3 we observe that a small increase in the interest rate strongly increases the investment triggers. Hence policies that reduce the real interest rate have the effect of reducing investment thresholds, thereby speeding up mitigation investments.

Table 5: Critical thresholds for $r = 0.075$; donation rates $\kappa = 0, 0.1, 0.2, 0.3, 0.4$ and 0.5; $I = 4200$ and $\varepsilon = 0.00208388$.

	$\kappa = 0$	$\kappa = 0.1$	$\kappa = 0.2$	$\kappa = 0.3$	$\kappa = 0.4$	$\kappa = 0.5$
θ_{ca}	17.6	17.6	17.7	17.7	17.7	17.8
$\pi(\theta_{ca})$	1546	1553	1560	1567.2	1574.4	1581.6
θ_{re}	17.6	18.5	19.7	21.2	23.3	26.1
$\pi(\theta_{re})$	1546	1717.1	1943.7	2255.7	2707.7	3409.8
I_{re}	4200	4426.4	4709.4	5073.2	5558.3	6237.5
average damage	12.9%	12.6%	12.1%	11.6%	10.9%	10.2%

6 Conclusion

Disaster involving natural hazards can have devastating short and long-term impacts on the society and the economy of any country, adversely affecting progress towards sustainable development. They cause loss of life, social disruption and affect economic activities. The general increase in vulnerability of societies worldwide has motivated the 2005 United Nations World Conference on Disaster Reduction and its resulting report, the Hyogo "Framework for Action", which emphasizes the need for pro-active disaster management including cost-effective risk reduction investments and where this is not possible, risk transfer through insurance and other catastrophe linked-securities (i.e., Cat Bonds). In this paper we analyze how international aid programs in the form of cash transfer and targeted financial aid affect mitigation investments such as the inclusion of specific safety or vulnerability reduction measures in the design and construction of new facilities, the retrofitting of existing facilities against seismic risk, or the building of structural flood defense measures, and do not consider risk transfer through insurance and other financial markets instruments. We find that: 1) both financial aid strategies increase the critical firm size thereby delaying investment in mitigation; 2) targeted financial aid increases the critical firm size more than cash aid does and thus delays investment in mitigation more than under a cash aid program; 3) cash aid does not alter the size of the mitigation investment, while targeted financial aid increases it. These results have important implications for international donors. Donors may want to accelerate the adoption of measures reducing vulnerability of small firms. In this case they should provide relief in the form of cash aid which outperforms targeted financial aid and speeds up investment in mitigation. On the contrary, if their aim is to reduce the average damage caused by natural hazards, then targeted financial aid is preferable.

Some extensions and further directions of research might be fruitful. One is to incorporate environmental risk, accounting for the uncertainty in the arrival rate of catastrophe events or in the future costs of environmental damage. Another extension would be taking informational asymmetries into account, by assuming that the international donor cannot observe the damage caused by natural hazards, and to solve the corresponding principle-agent problem.

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Appendix

The profit flow $\pi(\theta) = \Psi\theta^{\delta}$, where θ follows the geometric jump-diffusion process (1). We want to calculate the expected discounted present value

$$V(\theta_0) = \mathbb{E}_0\left[\int_0^{\infty} e^{-rt}\Psi\theta^{\delta}dt\right].$$

Let us define $f(\theta) = ln\left(\theta^{\delta}\right)$. Ito's Lemma in the case of jump-diffusion processes is given in Cont and Tankov (2004)

$$df(\theta_t, t) = \frac{\partial f(\theta_t, t)}{\partial t}dt + \mu\theta_t\frac{\partial f(\theta_t, t)}{\partial \theta}dt + \frac{\sigma^2}{2}\theta_t^2\frac{\partial^2 f(\theta_t, t)}{\partial \theta^2}dt + \sigma\theta_t\frac{\partial f(\theta_t, t)}{\partial \theta}dW_t +$$

$$+\left[f(\theta_{t-} + \triangle\theta_t, t) - f(\theta_{t-}, t)\right],$$

where the derivatives are $\frac{\partial f(\theta)}{\partial \theta} = \delta\frac{1}{\theta}$ and $\frac{\partial^2 f(\theta)}{\partial \theta^2} = -\delta\frac{1}{\theta^2}$.

By applying Ito's formula to $ln\left(\theta^{\delta}\right)$

$$ln\left(\theta_t^{\delta}\right) = ln\left(\theta_0^{\delta}\right) + \left(\mu - \frac{\sigma^2}{2}\right)\delta t + \sigma\delta W_t + \sum_{i=1}^{Q_t} ln\left(Y_i\right)^{\delta},$$

which can be written as

$$\theta_t^\delta = \theta_0^\delta e^{\left(\mu - \frac{1}{2}\sigma^2\right)\delta t + \sigma\delta W_t + \sum_{i=1}^{Q_t} ln(Y_i)^\delta},$$

and whose expectation is

$$\mathbb{E}_0\left[\theta_t^\delta\right] = \theta_0 e^{\left(\mu + \frac{1}{2}\sigma^2(\delta-1)\right)\delta t}\mathbb{E}_0 e^{\sum_{i=1}^{Q_t} \delta ln(Y_i)}.$$

Using this result we can now compute the expected discounted value of the profit flow $\Psi\theta_t^\delta$

$$V(\theta_0) = \left\{\int_0^\infty e^{-rt}\Psi\theta_0^\delta e^{\left(\mu + \frac{1}{2}\sigma^2(\delta-1)\right)\delta t}\mathbb{E}_0 \exp\left[\delta\sum_{i=1}^{Q_t}\ln(Y_i)\right]dt\right\}$$

$$= \Psi\theta_0^\delta\left\{\int_0^\infty e^{-rt}e^{\left(\mu+\frac{1}{2}\sigma^2(\delta-1)\right)\delta t}e^{\left(\lambda t\left(\mathbb{E}\left(Y^\delta\right)-1\right)\right)}dt\right\}$$

$$= \frac{\Psi\theta_0^\delta}{r - \mu\delta - \frac{1}{2}\sigma^2\delta(\delta-1) + \frac{\lambda\delta}{\alpha+\delta}}.$$

Proofs

Proof of Lemma 1. The characteristic equation (10) can be rewritten as $G(\phi) = 0$, where

$$G(\phi) = \left[\frac{1}{2}\sigma^2\phi(\phi-1) + \mu\phi - r\right](\alpha+\phi) - \lambda\phi$$

It is easy to see that $G(0) = -r\alpha - \lambda\phi < 0$. Therefore, since $G(-\alpha) > 0$ and $lim_{\phi\to-\infty}G(\phi) = -\infty$ a first negative root ϕ_1 between $-\alpha$ and 0 and a second negative root $\phi_2 < -\alpha$ exists. Since $lim_{\phi\to\infty}G(\phi) = \infty$, to prove that $\phi_3 > \delta$ it is sufficient to show that $G(\delta) < 0$. Because of Assumption 1, $r > \mu\delta + \frac{1}{2}\sigma^2\delta(\delta-1)$, and thus

$$G(\delta) = \left[\frac{1}{2}\sigma^2\delta(\delta-1) + \mu\delta - r\right](\alpha+\delta) - \lambda\delta < -\lambda\delta < 0$$

which proves the result.

Using the implicit function theorem $\frac{d\phi_3}{d\lambda} = -\frac{G_\lambda}{G_\phi}$. Since $G_\lambda(\phi_3) < 0$ and $G_\phi(\phi_3) > 0$, $\frac{d\phi_3}{d\lambda} > 0$. Moreover, for $\lambda\to\infty$, $\phi_3\to\infty$. ■

Proof of Proposition 1. Consider first the firm's optimal investment decision. The first order condition for the maximization problem (8) is

$$\frac{\varepsilon\delta\lambda\Psi\theta^{\delta}}{Dis\left(\alpha\left(I\right)\right)^{2}\left(\alpha+\delta+\varepsilon I\right)^{2}}=1,$$

which yields

$$I_{be}=\frac{1}{\varepsilon}\left[\frac{\sqrt{\varepsilon\delta\lambda\Psi\theta^{\delta}}-\lambda\delta}{Dis\left(\infty\right)}-\left(\alpha+\delta\right)\right].$$

Substituting I_{be} into $\Omega_{m,be}\left(\theta;I\right)$ we obtain

$$\frac{\Psi\theta^{\delta}}{Dis\left(\infty\right)}-2\frac{\sqrt{\lambda\delta\Psi\theta^{\delta}}}{Dis\left(\infty\right)\sqrt{\varepsilon}}+\frac{\lambda\delta}{\varepsilon Dis\left(\infty\right)}+\frac{1}{\varepsilon}\left(\alpha+\delta\right).$$

We have to solve (6) with termination value (15) under conditions (11) - (13) at the critical threshold θ_{be}. The general solution to (6) is $V_{d,be}\left(\theta\right)=\frac{\Psi}{Dis(\alpha)}\theta^{\delta}+\Phi_{1,be}\theta^{\phi_1}+\Phi_{2,be}\theta^{\phi_2}+\Phi_{3,be}\theta^{\phi_3}$. Since roots ϕ_1 and ϕ_2 are negative, boundary condition (11) requires that the coefficients $\Phi_{1,be}$ and $\Phi_{2,be}$ are zero. Consequently, we can rewrite the firm value before exercising the option to invest in mitigation as $V_{d,be}\left(\theta\right)=\frac{\Psi}{Dis(\alpha)}\theta^{\delta}+\Phi_{3,be}\theta^{\phi_3}$. In order to find the critical threshold of investing in mitigation θ_{be} and the constant $\Phi_{3,be}$ we use the value-matching condition (12)

$$\Phi_{3,be}\theta_{be}^{\phi_3}=\Psi\theta_{be}^{\delta}\frac{Dis\left(\alpha\right)-Dis\left(\infty\right)}{Dis\left(\alpha\right)Dis\left(\infty\right)}-\frac{1}{Dis\left(\infty\right)}\left(2\sqrt{\frac{\delta\lambda\Psi\theta_{be}^{\delta}}{\varepsilon}}-\frac{\lambda\delta}{\varepsilon}\right)+\frac{1}{\varepsilon}\left(\alpha+\delta\right)\tag{20}$$

and the smooth pasting condition (13)

$$\Phi_{3,be}\phi_3\theta_{be}^{\phi_3}=\frac{Dis\left(\alpha\right)-Dis\left(\infty\right)}{Dis\left(\infty\right)Dis\left(\alpha\right)}\Psi\delta\theta_{be}^{\delta}-\frac{\delta}{Dis\left(\infty\right)}\sqrt{\frac{\delta\lambda\Psi\theta_{be}^{\delta}}{\varepsilon}}\tag{21}$$

Hence, substituting (21) into (20) gives

$$\frac{\phi_3-\delta}{\phi_3}\frac{Dis\left(\alpha\right)-Dis\left(\infty\right)}{Dis\left(\alpha\right)}\Psi z^{2}-\frac{2\phi_3-\delta}{\phi_3}\sqrt{\frac{\delta\lambda\Psi}{\varepsilon}}z+\frac{\alpha+\delta}{\varepsilon}Dis\left(\alpha\right)=0.$$

where $z=\sqrt{\left(\theta_{be}\right)^{\delta}}$ and where $Dis\left(\alpha\right)-Dis\left(\infty\right)=\frac{\lambda\delta}{\alpha+\delta}$. This second order equation has two positive roots

$$z=\begin{cases}\frac{(\alpha+\delta)Dis(\alpha)}{\sqrt{\varepsilon\delta\lambda\Psi}}\frac{2\phi_3}{2\phi_3-2\delta}\\ \frac{(\alpha+\delta)Dis(\alpha)}{\sqrt{\varepsilon\delta\lambda\Psi}}\end{cases}$$

Since $I_{be} > 0$ we need that

$$z > \frac{\alpha+\delta}{\sqrt{\varepsilon\delta\lambda\Psi}} Dis\left(\alpha\right)$$

and thus results in the proposition can be obtained. ■

Proof of Corollary 1. Take the derivative of $\Omega_{m,be}\left(\theta\right)$ with respect to ε and evaluate the result at θ_{be} and we obtain

$$\left.\frac{\partial\Omega_{m,be}\left(\theta\right)}{\partial\varepsilon}\right|_{\theta=\theta_{be}} = \frac{1}{Dis\left(\infty\right)}\left(\frac{1}{\varepsilon^2}\left(\alpha+\delta\right)Dis\left(\alpha\right)\frac{\phi_3}{\phi_3-\delta} - \frac{\lambda\delta}{\varepsilon^2}\right) - \frac{1}{\varepsilon^2}\left(\alpha+\delta\right)$$

After rearranging terms it is easy to see that $\left.\frac{\partial\Omega_{m,be}(\theta)}{\partial\varepsilon}\right|_{\theta=\theta_{be}} > 0$ which proves the result. ■

Proof of Proposition 2. Consider the firm's optimal investment decision. The first order condition for the maximization problem $\Omega_{m,ca}\left(\theta; I\right)$ is

$$\frac{\varepsilon\delta\lambda\Psi\theta^\delta}{Dis\left(\alpha\left(I\right)\right)^2\left(\alpha+\delta+\varepsilon I\right)^2}\left[1-Dis\left(\infty\right)\kappa\right] = 1,$$

which yields

$$I_{ca} = \frac{1}{\varepsilon}\left[\frac{\sqrt{\varepsilon\delta\lambda\Psi\theta^\delta\left[1-Dis\left(\infty\right)\kappa\right]}-\delta\lambda}{Dis\left(\infty\right)} - \left(\alpha+\delta\right)\right]$$

Substituting I_{ca} into $\Omega_{m,ca}\left(\theta; I\right)$ we obtain

$$\frac{\Psi\theta^\delta}{Dis\left(\infty\right)} - 2\frac{\sqrt{\delta\lambda\Psi\theta^\delta\left(1-\kappa Dis\left(\infty\right)\right)}}{Dis\left(\infty\right)\sqrt{\varepsilon}} + \frac{Dis\left(\alpha\right)}{\varepsilon Dis\left(\infty\right)}\left(\alpha+\delta\right).$$

In order to find the critical threshold of investing in mitigation θ_{ca} and the constant $\Phi_{3,ca}$ we use the value-matching (12)

$$\Phi_{3,ca}\theta_{ca}^{\phi_3} = \Psi\theta_{ca}^\delta\frac{\delta\lambda\left[1-Dis\left(\infty\right)\kappa\right]}{\left(\alpha+\delta\right)Dis\left(\alpha\right)Dis\left(\infty\right)} - 2\frac{\sqrt{\delta\lambda\Psi\theta_{ca}^\delta\left(1-\kappa Dis\left(\infty\right)\right)}}{Dis\left(\infty\right)\sqrt{\varepsilon}} + \frac{Dis\left(\alpha\right)}{\varepsilon Dis\left(\infty\right)}\left(\alpha+\delta\right) \qquad (22)$$

and the smooth pasting condition (13)

$$\Phi_{3,ca} = \frac{\delta}{\phi_3}\Psi\theta_{ca}^{\delta-\phi_3}\frac{\delta\lambda\left[1-Dis\left(\infty\right)\kappa\right]}{\left(\alpha+\delta\right)Dis\left(\alpha\right)Dis\left(\infty\right)} - \delta\frac{\sqrt{\delta\lambda\Psi\theta_{ca}^\delta\left(1-\kappa Dis\left(\infty\right)\right)}}{\theta_{ca}^{\phi_3}\phi_3 Dis\left(\infty\right)\sqrt{\varepsilon}} \qquad (23)$$

Hence, substituting (23) into (22) gives

$$\frac{\phi_3-\delta}{\phi_3}\frac{\delta\lambda\left[1-\kappa Dis\left(\infty\right)\right]}{\left(\alpha+\delta\right)Dis\left(\alpha\right)}\Psi\left(z'\right)^2-\left(\frac{2\phi_3-\delta}{\phi_3}\right)\frac{\sqrt{\delta\lambda\Psi\left(1-\kappa Dis\left(\infty\right)\right)}}{\sqrt{\varepsilon}}z'+\frac{Dis\left(\alpha\right)}{\varepsilon}\left(\alpha+\delta\right)=0.$$

where $z'=\sqrt{\theta_{ca}^{\delta}}$. This second order equation has two positive roots

$$z'=\begin{cases}\frac{\phi_3}{\phi_3-\delta}\frac{(\alpha+\delta)Dis(\alpha)}{\sqrt{\varepsilon\delta\lambda\Psi[1-\kappa Dis(\infty)]}}\\ \frac{(\alpha+\delta)Dis(\alpha)}{\sqrt{\varepsilon\Psi\delta\lambda[1-\kappa Dis(\infty)]}}\end{cases}$$

Since $I_{ca}>0$ we need that

$$z'>\frac{\left(\alpha+\delta\right)Dis\left(\alpha\right)}{\sqrt{\varepsilon\delta\lambda\Psi\theta^{\delta}\left[1-Dis\left(\infty\right)\kappa\right]}},$$

and thus results in the proposition can be obtained. ■

Proof of Proposition 3. Comparing thresholds θ_{ca} and θ_{be} it is easy to see that $\theta_{ca}>\theta_{be}$ for any $\kappa>0$. Moreover, it is immediate to show that I_{ca} and I_{be} are identical. ■

Proof of Proposition 4. Consider the firm's optimal investment decision. The first order condition for the maximization problem $\Omega_{m,re}\left(\theta;I\right)$ is

$$\frac{\varepsilon\delta\lambda\Psi\theta^{\delta}}{Dis\left(\alpha\left(I,\xi\right)\right)^2\left(\alpha+\delta+\varepsilon I+\xi\right)^2}=1,$$

which yields

$$I_{re}=\left[\frac{\sqrt{\varepsilon\delta\lambda\Psi\theta^{\delta}}-\delta\lambda}{Dis\left(\infty\right)}-\left(\alpha+\delta+\xi\right)\right]\frac{1}{\varepsilon}\tag{24}$$

Substituting I_{re} into $\Omega_{m,re}\left(\theta;I\right)$ we obtain

$$\frac{\Psi\theta^{\delta}}{Dis\left(\infty\right)}-2\frac{\sqrt{\delta\lambda\Psi\theta^{\delta}}}{\sqrt{\varepsilon}Dis\left(\infty\right)}+\frac{Dis\left(\alpha\left(\xi\right)\right)}{\varepsilon Dis\left(\infty\right)}\left(\alpha+\delta+\xi\right).$$

In order to find the critical threshold of investing in mitigation θ_{re} and the constant $\Phi_{3,re}$ we use the value-matching condition (12)

$$\Phi_{3,re}\theta_{re}^{\phi_3}=\Psi\theta_{re}^{\delta}\frac{Dis\left(\alpha\left(\xi\right)\right)-Dis\left(\infty\right)}{Dis\left(\alpha\left(\xi\right)\right)Dis\left(\infty\right)}-2\frac{\sqrt{\delta\lambda\Psi\theta_{re}^{\delta}}}{\sqrt{\varepsilon}Dis\left(\infty\right)}+\frac{Dis\left(\alpha\left(\xi\right)\right)}{\varepsilon Dis\left(\infty\right)}\left(\alpha+\delta+\xi\right)\tag{25}$$

and the smooth pasting condition (13)

$$\Phi_{3,re} = \frac{\delta}{\phi_3}\Psi\theta_{re}^{\delta-\phi_3}\frac{Dis\left(\alpha\left(\xi\right)\right) - Dis\left(\infty\right)}{Dis\left(\alpha\left(\xi\right)\right)Dis\left(\infty\right)} - \frac{\delta}{\phi_3\theta_{re}^{\phi_3}}\frac{\sqrt{\delta\lambda\Psi\theta_{re}^{\delta}}}{\sqrt{\varepsilon}Dis\left(\infty\right)} \tag{26}$$

Hence, substituting (26) into (25) gives

$$\frac{\phi_3-\delta}{\phi_3}\Psi\frac{Dis\left(\alpha\left(\xi\right)\right) - Dis\left(\infty\right)}{Dis\left(\alpha\left(\xi\right)\right)}\left(z''\right)^2 - \frac{2\phi_3-\delta}{\phi_3}\sqrt{\frac{1}{\varepsilon}\delta\lambda\Psi}z'' + \frac{Dis\left(\alpha\left(\xi\right)\right)}{\varepsilon}\left(\alpha+\delta+\xi\right) = 0 \tag{27}$$

where $z'' = \sqrt{\left(\theta_{re}\right)^{\delta}}$. This second order equation has two positive roots

$$z'' = \begin{cases} \frac{\phi_3}{\phi_3-\delta}\frac{Dis(\alpha(\xi))(\alpha+\delta+\xi)}{\sqrt{\varepsilon\delta\lambda\Psi}} \\ \frac{Dis(\alpha(\xi))(\alpha+\delta+\xi)}{\sqrt{\varepsilon\delta\lambda\Psi}} \end{cases}$$

Since $I_{re} > 0$ we need that

$$z'' > \frac{Dis\left(\alpha\left(\xi\right)\right)\left(\alpha+\delta+\xi\right)}{\sqrt{\varepsilon\delta\lambda\Psi}},$$

and thus results in the proposition can be obtained. ■

Proof of Proposition 5. Straightforward calculations show that $\theta_{re} > \theta_{be}$ is true for any $\xi > 0$.

■

Proof of Proposition 6. For any given ξ, it follows that if $\kappa < \kappa^*$, then $\theta_{ca} < \theta_{re}$, where

$$\kappa^* = \frac{1 - \left(\frac{(\alpha+\delta)Dis(\alpha)}{(\alpha+\delta+\xi)Dis(\alpha(\xi))}\right)^2}{Dis\left(\infty\right)}$$

Therefore, it is sufficient to show that $\tilde{\kappa} < \kappa^*$, where $\tilde{\kappa} = \frac{1}{\alpha+\xi+\delta}\xi$ is the inverse function of $\tilde{\xi}$ in (19). Inequality $\tilde{\kappa} < \kappa^*$ can be written as

$$Dis\left(\alpha+\xi\right) < 1$$

which, in view of Assumption 1, is always true. ■