

# Heterogeneous market beliefs, fundamentals and the euro-sovereign debt crisis

**Roberto Tamborini\***

Department of Economics and Management  
University of Trento  
Via Inama 5, 38100 Trento (Italy)  
+390461282228  
roberto.tamborini@unitn.it

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## Abstract

The unprecedented sovereign debt crisis across the European Monetary Union has prompted a new generation of models with "self-fulfilling" attacks to public debt. The key idea is that governments may be forced to default even though initial fundamental fiscal variables are sound. The model presented in this paper has two main features: (i) the government's default decision arises out of a cost-benefit analysis that sets the sustainable limit of the solvency primary balance; (ii) investors have no direct information about this variable, and are characterized by a frequency distribution of rational beliefs. As a consequence, a "good" and "bad" state of the debt market are possible; the latter is unstable and the model identifies an attraction domain of default within which the government is bound to default although initial solvency conditions are sustainable. The extent of this domain may be larger or smaller depending on the interplay between fiscal fundamentals and the *distribution* of investors' opinions. I then discuss several issues concerning the role of initial conditions, fiscal shocks, and the policy options to escape from the default domain. Under this new light, the institutional design of the European Monetary Union now appears seriously deficient and largely co-responsible for the gravity of the crisis.

**Keywords:** Models of public debt, speculative attacks, euro-sovereign debt crisis

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## 1. Introduction

(...) we are in a situation now where you have large parts of the euro area in what we call a "bad equilibrium", namely an equilibrium in which you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios. So, there is a case for intervening, in a sense, to "break" these expectations (...) But then, we should not forget why countries have found themselves in a bad equilibrium to start with (Draghi (2012, p. 4)).

This quotation from the presentation of the European Central Bank's (ECB) "Outright Market Transactions" new programme for purchases of government bonds certifies the official endorsement of a new "multiple equilibria" (ME) approach to sovereign debt analysis. This approach marks a substantial modification of theory and policy with respect to the orthodox view of "market discipline" and "credibility", based on the efficient market hypothesis with single rational-expectations equilibrium.

The gist of models with ME is that (at least) one possible equilibrium is the result of "self-fulfilling prophecies", that is market beliefs about future states of the economy that turn out to be true though they are unrelated to so-called "fundamentals". Self-fulfilling prophecies are a long-standing research field (e.g. Farmer (1993)). Financial and currency markets are natural fields where this class of models has proved able to yield valuable insights into complex phenomena such as bubbles, crashes, or speculative attacks. In fact, the closest antecedents to the ME approach to sovereign debt date back to the various "generations" of models of currency crisis and exchange-rate regime collapse of the 1980s and 1990s (e.g. Obstfeld (1995)). Early extensions to sovereign debt also appeared with special reference to emerging economies (e.g. Calvo (1988), Cole and Kehoe (2000)), but they are now being boosted by the dramatic euro-sovereign debt crisis that erupted in Greece in early 2009 and then propagated across the whole area.

ME models may have an intrinsic theoretical interest and motivation, and this paper is no exception. However, this new wave of studies seeks to address and explain in a consistent framework a set of phenomena that has rapidly grown to challenge the orthodox view traditionally endorsed by the European institutions:

- there is scant evidence of consistent "market discipline", that is, the correct "fundamental" pricing of bonds, *throughout* the life of the euro:

typically, (some) country risk spreads were too low until 2008; they have been too high since 2009 (Di Cesare et al. (2012))

- there is evidence that post-2009 spreads not only reflect country-specific fundamentals, but are also highly sensitive to "systemic risk" and other exogenous factors (Manganelli and Wolswijk (2009), Sgherri and Zoli (2009), Attinasi et al. (2009), Caceres et al. (2010)<sup>1</sup>, Favero and Missale (2011))
- there is evidence of "contagion", that is, the transmission of high spreads across countries via non-fundamental channels (Caceres et al. (2010), De Grauwe and Ji (2012a), Tola and Wäldi (2012))
- there is evidence of "self-fulfilling-prophecies" via the positive feedback mechanism among market beliefs of default, higher spread, higher fiscal effort, reinforcement of market beliefs (De Grauwe and Ji (2012b))

With regard to these phenomena, a key feature of ME models of sovereign debt crisis is that fundamental fiscal variables and market beliefs interact, so that one possible equilibrium is typically a self-fulfilling default prophecy due to the positive feedback mechanism described above<sup>2</sup>. Hence, a sovereign may be driven to default even though it is solvent in initial conditions. Along this perverse trajectory, contrary to fiscal orthodoxy, attempts at strong fiscal consolidation may be counterproductive. The possibility of this scenario has of course important policy implications.

Like ME models of currency crisis, also those of sovereign debt crisis now display different "generations". An earlier generation of models (e.g. Adrian and Gros (1999)) was concerned with the optimal choice of instruments whereby the government *can always remain solvent*, typically taxation or monetization (inflation). Following the seminal paper by Calvo (1988), the current generation of models is concerned with institutional set-ups where the government is constrained in the use of these instruments (for instance, euro-governments have no access to monetization) and therefore *it can in fact opt for default* (e.g. Cooper (2012), Corsetti and Dedola (2011), Gros

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<sup>1</sup> These authors find that systemic risk and other non-country-specific factors were more important in the build-up phase of the crisis (2008-09), whereas country-specific factors became more important subsequently, though these were not only related to fiscal fundamentals.

<sup>2</sup> This phenomenon is also called "reflexivity" by Soros after Popper (Soros (2012)), or "strategic uncertainty" by Cooper (2012).

(2012), De Grauwe (2011), Ghosh et al. (2013)).<sup>3</sup> This representation is supported by extensive historical evidence showing that default is almost always a government choice not necessarily forced by immediate inability to pay (Reinhart and Rogoff (2009), Tomz and Wright (2013)). This paper presents a ME model of sovereign debt crisis belonging to this latter generation, which fits the euro-sovereigns' institutional features quite easily. The model hinges on two key characterizations.

First, at each point in time the government faces a solvency condition in terms of a target primary-balance/GDP ratio  $b^*$ ; its decision of default *vs.* solvency is the result of a *cost-comparative analysis setting a threshold level of "fiscal effort"* (the primary balance to GDP ratio  $\bar{b}$ ) beyond which default is preferred.

Second, investors are risk neutral but operate under *uncertainty about the default event* in a way that characterizes the model with respect to the relevant literature. The earlier model by Calvo (1988) was deterministic, where all agents are perfectly informed about the government's choice model, there is no uncertainty and hence the no-default and default equilibria are rational-expectations equilibria (REE). However, uncertainty seems a necessary feature of the problem if the notion of default risk premium has to be introduced meaningfully.

Corsetti and Dedola (2011) and Cooper (2012) introduce uncertainty into the Calvo model in the form of an exogenous random shock to the government's ability to pay such that it opts for default<sup>4</sup>. This kind of "exogenous uncertainty" is also adopted by Ghosh et al. (2013). Therefore, this class of ME crucially depend on the shape of the probability distribution of shocks, while the rational expectations hypothesis still holds in that investors know the true probability distribution. As a consequence, however, the notion of non-fundamental, self-fulfilling default is ill posed. If the probability of an adverse shock is high, the risk premium will be high and the government's default will be more likely: there is the positive feedback mechanism underlying self-fulfilling default, but it remains virtual unless an exogenous shock hits. And the high probability of default conditional on

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<sup>3</sup> De Grauwe (2011) and De Grauwe and Ji (2012a) stress the different behaviour of risk premia *vis-à-vis* fiscal fundamentals for countries within and outside the EMU.

<sup>4</sup> An adverse shock may hit output, so that it may be too low to generate the no-default tax revenue.

an adverse shock *is* part of the fundamentals in the sense of the "true" information set exogenously given to investors.<sup>5</sup>

An interesting variation is proposed by Gros (2012). He introduces uncertainty by way of the political process leading to the government's default decision, such that a higher cost of solvency increases the probability that the pro-default party wins, though this result is not certain owing to other factors. Again, this probability information is known to investors who rationally use it in the calculation of the risk premium, so that all ME are REE.<sup>6</sup> This approach to uncertainty seems both more interesting and consistent with the focus on the government's decision process than assuming exogenous shocks to the government's ability to pay. Therefore, I depict investors who are uncertain because they understand the government's choice-theoretic setup, but they have no access to the full information necessary to know the true limit primary surplus  $\bar{b}$ . Hence, the critical variable that regulates the risk premium is the probability of default  $p$ , which is the probability that  $b^* \geq \bar{b}$ .

However, it does not seem sensible to impose that all investors share the same (objective?) probability assessment of  $\bar{b}$ , which leads to the other novel feature of the present model, namely that investors possess *heterogeneous (rational) beliefs about  $\bar{b}$* , which can be represented in a frequency distribution (think of those commonly used by surveys of professional forecasts). The technical complexities of sustainability assessment (e.g. Bohn (1995), Kanda (2011), IMF (2012)), and the political complexity of the default choice evoked by Gros, combine in a good case in support of Kurz's (2011) claim that "lack of knowledge of the truth is the foundation of belief diversity" (p. 191). Actually, default is a rare or unique event in each specific country for which "objective" inferences based on

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<sup>5</sup> In the model by De Grauwe (2011), it is the expectation of default itself that raises the risk premium and makes solvency too costly in the event of a shock. Hence, default is a REE but in the sense that expectations are self-fulfilling though unrelated to fundamentals or objective probability distributions. However, the model does not explain how default or non-default expectations are formed, or how investors coordinate on one expectation instead of the other. Also, the result is problematic since in reality governments may find it profitable to default when this is unexpected, and some investors do remain entrapped, rather than the other way round.

<sup>6</sup> One key finding stressed by Gros is that the region of multiple equilibria is smaller than in the case of certainty, which seems a counterintuitive result. As will be seen, this is not necessarily the case in the present model.

recurrent observations are not available (no default has occurred in Western Europe in the last sixty years). Being gauged on an individual, partial and conjectural basis, beliefs may also differ across investors.<sup>7</sup> Heterogeneous beliefs are an active research area. They involve highly complex theoretical issues<sup>8</sup>, but most of all they are matter of growing applications to financial markets because "heterogeneous beliefs are a fact of life" (Xiong (2013, p. 14)). Their economically relevant existence and persistence is by now documented and measured rigorously by empirical investigations of professional forecasts in various fields (e.g. Mankiw et al. (2004), Wieland and Wolters (2011), for macroeconomic variables) which now also cover fiscal forecasts (Poplawsky-Ribeiro and Rülke (2011)). Therefore, my aim here is to analyse the implications of the *existence* of heterogeneous beliefs regarding the emergence of ME and self-fulfilling default in the sovereign debt market.

My specific assumption is that investors possess "rational beliefs" (Kurz (1994, 1996, 2011)). Beliefs are rational as they are all consistent with the government's choice-theoretic model generating the default event except the exact dimension of  $\bar{b}$ . This also creates "*endogenous uncertainty*": the probability of default  $p$  is an "aggregate" (not individual) assessment of the market, defined as the cumulated frequency of the investors who believe that  $b^* \geq \bar{b}$ . As a result, I obtain an interest-rate function convex in  $b^*$  via the market's probability of default  $p$ . Thus, the model embeds the positive feedback mechanism between the market interest rate and the solvency constraint which can generate two non-default states, a "good equilibrium" (stable) with low "fiscal fatigue" and interest rate and a "bad equilibrium" (unstable) with high "fiscal fatigue" and interest rate, and a third default state. All equilibria are *a joint product* of fiscal fundamentals and market beliefs: the two components cannot be disentangled meaningfully. The extension of the domain of attraction of default crucially depends on the *first*

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<sup>7</sup> For a cognitive approach see Tamborini (1997). Another approach, particularly suited to professional agents and the market for forecasters and advisors, points out that acquisition of knowledge and information is a costly activity from which an individual competitive advantage is expected only if the results remain exclusive private goods (see the classical Grossman and Stiglitz (1980)). Hence diversification of knowledge and information is not just a "friction" imposed on agents from outside, but it is the result of conscious activity responding to economic incentives

<sup>8</sup> Thorough treatments are provided by Kurz (2011) and Xiong (2013).

two moments of the distribution of beliefs, a typical feature of heterogeneous beliefs models (Kurz (2011)).

Section 2 of the paper introduces the model. Section 3 expounds the model at work, covering the distinction between fiscal and market (i.e. investors' belief) shocks, large and small, puzzles in spread patterns across countries and over time, domestic *vs.* foreign debt, why "austerity" may not work. The aim of this section is not to provide detailed policy solutions or examine those under discussion in the European Monetary Union (EMU), but only to show how the model can be used to frame policy analysis to be further developed. Summary and conclusions follow in section 4.

## 2. The model

### 2.1. Basic notions

To begin with, let us examine the evolution of public debt<sup>9</sup> over time in a forward-looking perspective from the current year  $t$ . Hence, the nominal value of the level of debt in  $t+1$   $D_{t+1}$  will be

$$(1) \quad D_{t+1} = D_t + (i_{t+1}D_t - B_{t+1}) - M_{t+1} + S_{t+1}$$

i.e., the value of outstanding debt in  $t$   $D_t$  plus the government's net borrowing requirement (in brackets), minus central bank's loans ("monetization" for short,  $M_{t+1}$ ), plus extraordinary debt-management operations (e.g. asset sales,  $S_{t+1} < 0$ ) and other corrections (often called "stock-flow adjustments", see e.g. European Commission (2011)).<sup>10</sup> The government's net borrowing requirement in  $t+1$  will result from the difference between interest payments on the outstanding debt and the primary balance  $B_{t+1}$ . In this formulation, given  $D_t$ , the evolution of debt entirely depends on the set of future variables  $\Omega_{t+1} \equiv \{i_{t+1}, B_{t+1}, M_{t+1}, S_{t+1}\}$ . These may be matter of government's forecast and planning as well as of investors' assessment.

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<sup>9</sup> Debt held by the resident and non-resident private sector. Excluded is the debt held by other public institutions, namely the central bank. In fact, interests paid on the latter share of debt do not constitute net disbursements for the public sector as a whole.

<sup>10</sup> Monetization may take many different forms. Analytically,  $M_{t+1} > 0$  denotes direct purchases of bonds in the issuance market. Purchases in the secondary market are better represented as if  $S_{t+1} < 0$ , since they move a fraction of the outstanding debt out of the private sector (see fn. 9).

The key variable in budget planning is usually  $B_{t+1}$ . In the first place, let us reformulate equation (1) in terms of change in the debt level  $\Delta D_{t+1} \equiv D_{t+1} - D_t$ . Then, the principle is that the government should control  $\Delta D_{t+1}$  by choosing the target  $B^*_{t+1}$  such that

$$(2) \quad B^*_{t+1} = -\Delta D^*_{t+1} + i_{t+1}D_t - M_{t+1} + S_{t+1}$$

As in the Bohn (1998) approach to sustainability, this relationship can be read as a fiscal reaction function  $B^*_{t+1} = \phi(D_t, i_{t+1}, M_{t+1}, S_{t+1})$ . Identifying sustainability with  $\Delta D^*_{t+1} \leq 0$ , equation (1) implies  $\phi'(D_t) > 0$ , which is the key hypothesis in econometric tests of sustainability in this approach. A typical "Fiscal-Compact-style" plan in the EMU can be viewed as a normative version of this principle requiring each government to plan its primary balance so as to achieve a debt target such that the excess of the debt/GDP ratio above 60% is reduced by 1/20th per year. To keep the treatment manageable, I shall consider the solvency requirement of "no Ponzi game", or no "Minsky ultraspeculative position", that is, no new debt to pay interests on outstanding debt. This amounts to setting  $\Delta D^*_{t+1} = 0$ , or keeping the total budget in balance.

Now let us reformulate expression (2) in terms of GDP ratios by dividing both sides of the equation by  $Y_{t+1}$ . Note that  $Y_{t+1} = (1 + n_{t+1})Y_t$ , where  $n_{t+1}$  is the one-year nominal growth rate. This can be split into an invariant trend component  $n$ , and a time-variant shock component  $z_{t+1}$ . Hence, denoting GDP ratios with small-case letters, we obtain

$$(3) \quad b^*_{t+1} = \frac{i_{t+1}}{(1+n)(1+z_{t+1})}d_t - m_{t+1} + s_{t+1}$$

We are now in a position to appreciate the special status of a sovereign. Since  $b_{t+1}$  is the difference between the total fiscal revenue/GDP ratio  $\tau_{t+1}$  and total expenditure in goods and services/GDP ratio  $g_{t+1}$ , in expression (3) there are two variables that a sovereign, and only a sovereign, can control at will in order to achieve any debt target. One is the tax rate  $\tau_{t+1}$ , and the other is the monetization ratio  $m_{t+1}$ . In fact, by imposing taxation, a sovereign can raise its revenues, while by monetization it can expand its ability to pay, in ways that are precluded to any other ordinary debtor. To some extent also  $s_{t+1}$  can be manipulated by way of legislation, as is the case with forced debt redemption. Therefore, a sovereign *can always choose* the appropriate combination in the set of variables  $\Gamma_{t+1} \equiv \{\tau_{t+1}, g_{t+1}, m_{t+1}, s_{t+1}\}$

that satisfies the solvency constraint.<sup>11</sup> Where does a sovereign's solvency problem come from? As stated above, it may come from constraints imposed, or self-imposed, on its ability to manipulate  $\Gamma_{t+1}$  at will. Today the orthodox debt-management policies that democratic governments are expected to abide with prescribe that  $m$  and  $s$  should be small or zero. This is the case for euro-sovereigns, and  $\{m_{t+1}, s_{t+1}\} = 0$  are the *ex ante* constraints that will be embedded in the model.<sup>12</sup>

## 2.2. The default decision

The solvency constraint (3) with  $\{m_{t+1}, s_{t+1}\} = 0$  highlights that the government is constrained to choose a combination of  $\{\tau_{t+1}, g_{t+1}\}$ , all other variables being given. This is the key to models of the government's default choice (see Introduction). A larger  $b^*_{t+1}$  can only be obtained either by raising  $\tau_{t+1}$  or by cutting  $g_{t+1}$ . Both options involve welfare costs and/or political costs. There may also be fiscal effects on nominal growth  $n_{t+1}$  that feed back onto  $b^*_{t+1}$ . These are seldom considered, and I shall not model them explicitly either.<sup>13</sup> If they materialise *ex post*, they can easily be accommodated as shocks  $z_{t+1} < 0$  that deviate the nominal growth rate from trend (see par. 3.5). It should also be pointed out that, as will be seen, the existence of ME and the main features of this model *do not depend on these effects*.

Coming to the default decision, the key point, as explained above, is that this decision is not uniquely dictated by "objective" financial factors, but it essentially depends on the comparative costs of the various options that the government faces. Many models are available, usually based on the optimization of some objective function of the government (see Introduction). For present purposes, we do not need a detailed model. Also, default may in

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<sup>11</sup> The same applies if we expand equation (2) into the usual net present value formulation of the intertemporal primary budget constraint (e.g. Buiter (2012)).

<sup>12</sup> Interestingly, however, the extended studies by Reinhart and Rogoff (2009) on historical public debt crises show that major solvency problems were resolved by a combination of monetization, inflation and extraordinary operations (often concomitantly with major external events such as wars, revolutions, and political changes). Relaxation of constraints on  $m$  and  $s$  will be introduced in the context of specific non-government interventions in debt management.

<sup>13</sup> On reason being that such effects are notoriously controversial, though in the most recent empirical research "Keynesian effects" seem to prevail (e.g. Coenen et al. (2010), Burriel et al. (2011), Blanchard and Leigh (2013)).

practice take a variety of forms and extensions; these technicalities would complexify the analysis in a substantial way, and I shall keep them to a minimum.

It is natural and sufficient to assume that the cost of solvency is increasing and convex in  $b^*_{t+1}$ ,  $C(b^*_{t+1}) > 0$ ,  $C'(b^*_{t+1}) > 0$ ,  $C''(b^*_{t+1}) \geq 0$ . On the other hand, the government also perceives costs from default, essentially in the form of reputation loss towards electors and creditors that may thwart future re-election and access to borrowing. These costs are likely to be perceived as independent of the size of the budget (debt),  $D(b^*_{t+1}) > 0$ ,  $D'(b^*_{t+1}) = 0$ , as well as of the size of default or of other technicalities<sup>14</sup>. This comparative-cost framework is sufficient to obtain a default rule.

In fact, given  $b^*_{t+1}$ , the government will always choose  $\min(C(b^*_{t+1}), D(b^*_{t+1}))$ . Yet there exists a single value  $\bar{b}$  such that (i)  $C(\bar{b}) = D(\bar{b})$ , and (ii)  $C(b^*_{t+1}) >_< D(b^*_{t+1})$  for any  $b^*_{t+1} >_< \bar{b}$ . Hence the government will comply with the solvency constraint only up to the limit primary-surplus  $\bar{b}$  beyond which the cost of solvency exceeds the cost of default. Note that  $\bar{b}$  is increasing in the cost of default and decreasing in the cost of solvency.

It should also be borne in mind that in reality the government's options include not only solvency/default but also partial fiscal adjustment, i.e. a primary surplus  $b_{t+1} < b^*_{t+1}$ . In this (frequent) case, the consequence is usually not immediate default, but rather an increase in outstanding debt at a higher interest rate that defers either full solvency with greater fiscal effort or default. This entails an intertemporal cost assessment that I will not consider here. However, the model can also accommodate temporary deviations from solvency, as will be seen in due time.

A crucial point of the present model is that  $b^*_{t+1}$  is common knowledge, whereas  $\bar{b}$  cannot be assessed with certainty by investors. The main reason being that  $\bar{b}$  is the result of government's preferences, information and other decision inputs which are typically not accessible to external subjects. On the other hand, as will be clear later (see par. 3.1), the government has no incentive to disclose the true value of  $\bar{b}$  *ex ante*.

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<sup>14</sup> Later in paragraph 2.3 I will introduce the specification of default relevant to investors. Gros (2012) presents a model where the size of default, controlled by the "haircut" rate imposed on creditors, is a choice variable.

### 2.3. Probability of default and sovereign risk premium

I now move to the investors' side and I first introduce the basic model of sovereign risk premium leading to the determination of the interest rate on outstanding debt. Investors are risk neutral, and as explained above, they operate under uncertainty about the government's default decision, namely the unobservable primary surplus  $\bar{b}$ . Hence the key variable is some measure of the probability of default  $p$ , which I shall analyse below. It is known that, if default occurs, it consists of two measures: (i) cessation of interest payments, (ii) a possible percentage  $h$  of "haircut" on outstanding capital. A particular form of haircut, to which we shall return later, affects foreign investors by way of devaluation of the denomination currency of the debt. Investors have also access to an alternative safe asset yielding a constant return  $\bar{i}$ . Given  $p$ , arbitrage will determine, for each unit of capital,

$$(1 + i_{t+1})(1 - p) + (1 - h)p = 1 + \bar{i}$$

and

$$(4) \quad i_{t+1} = \frac{\bar{i} + hp}{1 - p}$$

Note that, as commonly expected,  $i_{t+1}$  is increasing in  $p$  and  $h$ .

In this framework,  $p$  is the probability that the solvency primary surplus  $b^*_{t+1}$  exceeds the government's limit value  $\bar{b}$ . The standard treatment is one with the representative agent who holds "the" probability distribution of possible values  $f(\tilde{b})$ . The rational expectations hypothesis also posits that  $E(\tilde{b}) = \bar{b}$ . Here I propose a different interpretation and measurement of the default probability by departing from the representative agent shortcut and allowing for heterogeneous agents in their beliefs about  $\bar{b}$ .

My proposed characterization falls into the category of "rational beliefs" (Kurz (1994, 1996, 2011))<sup>15</sup>. The intuition is that investors correctly understand the government's default choice model, but no one possesses full knowledge and information of all the specific inputs leading to the actual level of  $\bar{b}$ , which are open to subjective, diversified assessment. Let  $S$  be the

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<sup>15</sup> Rational beliefs models "specify beliefs about *exogenous variables* at the micro foundation level and then deduce forecast functions about *endogenous variables* from an equilibrium analysis" (Kurz (2011, p. 191)). A related concept is that of "model-consistent expectations" (see e.g. the seminal papers in Frydman and Phelps ()). This is the same foundational principle of the rational expectations hypothesis, which however relaxes the constraint that all agents come to know the single "true" model that generates the data.

complete set of determinants of  $\bar{b}$ . Let  $S$  be decomposable in subsets  $s_n \in S$ , with mappings  $F_n$  from  $s_n$  to  $\bar{b}_n$ . Subsets  $s_n$  may differ either because each contains (some) different elements from the others and/or because each contains different measures of the same elements. Hence, individual beliefs (and decisions) are all consistent with the process generating the default event except the exact dimension of  $\bar{b}$ .<sup>16</sup> An implication, as will be seen in paragraph 3.1, is that as (some) determinants of  $\bar{b}$  in  $S$  change (or are supposed to change), also the *distribution* of beliefs changes accordingly.

To be faithful to the competitive paradigm, investors are not allowed to communicate or exchange or come to know each others' beliefs.<sup>17</sup> However imagine that we (as meta-observers) have the investors' opinion poll about the level of  $\bar{b}$  so that we can construct the relative frequency distribution of such beliefs as surveys of professional forecasts are generally presented.<sup>18</sup> For  $N$  beliefs  $\bar{b}_n$  with relative frequency  $f_n$ , the average or "market belief" is  $\bar{b}_M = \sum_n \bar{b}_n f_n$ . As an implication of the hypothesis that beliefs are rational, we may posit the *cross-sectional* restriction  $\bar{b}_M = \bar{b}$  – i.e. the market is right as belief aggregator. But we shall see that what does matter for interest-rate determination are the first two moments of the frequency distribution of beliefs. This means that two different distributions with the same  $\bar{b}_M$  determine two different interest rates, which has the important implication that no individual investor can consistently infer  $\bar{b}_M$  from observed interest rates.

Since investors are risk neutral, each invests or not upon his/her individual belief of  $\bar{b}$ , given  $b^*_{t+1}$ . Hence a consistent representation of the

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<sup>16</sup> The latter may well be the expected value of a subjective probability distribution, but this is immaterial here.

<sup>17</sup> A long-standing theoretical literature has also shown that even in the presence of communication or coordination devices of beliefs, heterogeneity may persist (for a recent reappraisal see Xiong (2013, pp. 16-ff.), Guesnerie and Jara-Moroni (2011)). Using available data, Poplawsky-Ribeiro and Rülke (2011) test the degree of accuracy, bias, and convergence of fiscal forecasts. They find, similarly to results on other macroeconomic forecasts, persistence of biases and heterogeneity, though these have been reduced for countries subject to the SGP rules, which have probably restricted the degree of variability of fiscal policies across countries.

<sup>18</sup> See e.g. the ECB Survey of Inflation Forecasts ([www.ecb.org](http://www.ecb.org)). Unlike other macroeconomic key variables, fiscal forecasts are seldom surveyed. Consensus Economics Forecasts ([www.consensuseconomics.com](http://www.consensuseconomics.com)) on a monthly basis elaborates professional forecasts of fiscal deficits for major European countries for the current year and one year ahead. See also Poplawsky-Ribeiro and Rülke (2011).

market's default probability  $p$  is the cumulated frequency of the subset of individual beliefs  $d \in N$  such that  $\bar{b}_d \leq b^*_{t+1}$ , i.e.  $p = \sum_d f_d$ . A feature of cumulated frequency is that it increases monotonically with  $b^*_{t+1}$ , so that we can posit  $p(b^*_{t+1}) \in [0, 1]$ ,  $p'(b^*_{t+1}) > 0$ . This grants that also the risk premium will be increasing in  $b^*_{t+1}$ . The underlying mechanism is that low-tail investors withdraw from the market so that excess supply arises, the bond price falls, and the risk premium increases. Yet, contrary to the no-trade implication of the representative agent, these sellers have a counterpart in the investors who remain in the market. This has further relevant implication regarding the relationship between  $b^*_{t+1}$  and  $i_{t+1}$  that will be discussed below.

The frequency distribution of heterogeneous beliefs may also be given a different interpretation with substantial practical consequences that are precluded by the representative agent. In fact,  $f_n$  can also be counted in terms of investment capacity rather than per head, that is to say, larger  $f_n$  correspond to larger investment shares that may enter or leave the market. Consequently, market concentration becomes a relevant variable. For instance, it is quite natural that the beliefs of large investors have a stronger impact on the relationship between  $b^*_{t+1}$  and  $p$ . Moreover, a market with fewer large investors may display less variance of "weighted beliefs" than a market with many small investors, with the consequences that will be studied in paragraph 3.5.

#### 2.4. "Good" and "bad" equilibria

The gist of the previous treatment consists of three elements: (i) the government's threshold value of the primary surplus  $\bar{b}$  that triggers the default decision, (ii) the government's reaction function (GR),<sup>19</sup> which complies with the solvency primary balance  $b^*_{t+1}$  up to  $\bar{b}$

$$(5) \quad b^*_{t+1} = \begin{cases} \frac{d_t}{1+n_{t+1}} i_{t+1} < \bar{b} \\ \text{default otherwise} \end{cases}$$

and (iii) the equation that determines the interest rate on outstanding debt (IR)

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<sup>19</sup> This reaction function with limit "fiscal effort"  $\bar{b}$  can also be interpreted as a stepwise "all-or-nothing" case of the S-shaped function of "fiscal fatigue" employed by Ghosh et al. (2013), which instead postulates a smooth transition from compliance to non-compliance with the solvency condition.

$$(6) \quad i_{t+1} = \frac{\bar{i} + hp(b^*_{t+1})}{1 - p(b^*_{t+1})}$$

For the readers' convenience, Table 1 in the Appendix reports the values of  $b^*$ , spreads, and other relevant model variables for the EMU12 countries from 2010 to 2012.

The key feature of the GR-IR system is that  $b^*_{t+1}$  and  $i_{t+1}$  are interdependent via the market's probability of default  $p$ . Given the properties of the function  $p(b^*_{t+1})$ , the result is the typical positive feedback mechanism such that the higher is  $b^*_{t+1}$ , the higher is  $i_{t+1}$ , and so forth.

The GR function is linear in  $i_{t+1}$  with slope determined by outstanding debt  $d_t$  (which increases the slope) and nominal growth  $n_{t+1}$  (which decreases the slope). As long as the governments fulfils  $b^*_{t+1}$ , the outstanding debt remains constant (if the government misses the target, this can be treated as a shock to  $d$ : see below, par. 3.1).

Let us now examine the IR function. Given  $\bar{i}$  and  $h$ , its shape depends on the function  $p(b^*_{t+1})$ , i. e. on the distribution of investors' beliefs. Figure 1 exemplifies the IR function generated by a continuous Normal distribution of beliefs and  $\bar{i} = 2\%$ ,  $h = 0$  (for notational simplicity the time subscript is dropped). Normality is a (sensible) case where opinions are relatively concentrated around the market belief  $\bar{b}_M$  (the vertical dotted line) with tails of optimists (high  $\bar{b}$ ) and pessimists (low  $\bar{b}$ ); yet the resulting IR function displays some noteworthy features of general value.

[Figure 1]

First, the function is almost flat (the risk premium is negligible) for a relative wide range of low values of  $b^*$ ;  $p(b^*)$  and  $i$  increase faster as  $b^*$  approaches and then exceeds  $\bar{b}_M$ . This is a noteworthy feature that can shed some light on one of the several puzzles that have recently emerged in the standard theory of risk premia (see Introduction). Why did spreads across euro-sovereign debts remain so small until 2009 regardless of differences in debt stocks and deficits? Why do the United States or the United Kingdom or Japan pay negligible spreads in comparison with not so fiscally worse (or even better) euro-sovereigns?<sup>20</sup> This model suggests that debt stocks and deficits do not matter *per se* but in relation to the configuration of the IR function, and in particular the risk premium depends on the *distance* of  $b^*$  from the market belief  $\bar{b}_M$ . Market confidence in a

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<sup>20</sup> See De Grauwe and Ji (2012a) for evidence about these phenomena.

high  $\bar{b}$  allows the government to sustain a larger  $b^*$  with lower risk premium.

Second, the risk premium does increase at news on a worse fiscal outlook (higher  $b^*$ ), but it does not just jump from zero to infinity as  $b^*$  grows larger. This pattern is consistent with the recent experience of euro-sovereigns under financial distress.<sup>21</sup> The reason is that, as explained above, the market mechanism underlying the slope of the IR function is that as  $b^*$  grows, a greater fraction of investors (or investments in value) withdraw from the market, but they find a counterpart in investors who wish to keep a long position. In other words, the upward movement along the IR describes the phase of a debt crisis in which the government faces a growing liquidity problem, which eventually may turn into a solvency problem.

The shape and location of the IR function are clearly crucial factors. They depend on the mean and variance of the distribution of beliefs, a typical important feature of heterogeneous beliefs models (Kurz (2011)). The effects on IR of different mean and variance of beliefs will be discussed below in the context of market shocks (par. 3.1).

The two functions GR and IR are plotted in the single space  $(b^*, i)$  in Figure 2. As in the baseline case, the IR is plotted for  $h = 0$ . It is also assumed that  $\bar{b}_M = \bar{b}$ .

[Figure 2]

The geometry of these functions allows for multiple intersections or "equilibria". Their meaning is that of points where the value of  $b^*$  that determines  $i$  via IR is the value of  $b^*$  determined by  $i$  via GR. They can therefore be understood as fixed points of the debt market. As long as  $\bar{i} > 0$  and  $p(\bar{b}) < 1$ , which I regard as the normal cases, equilibria can be up to three. Figure 2 represents this case:  $G$  is a "good" equilibrium (low  $b^*$  and  $i$ ),  $B$  is a "bad" equilibrium (high  $b^*$  and  $i$ ), and  $D$  is the default equilibrium. Note that, unlike ME models with representative agent and rational expectations, the bad equilibrium is not necessarily the default state, which may be regarded as a special case when IR intersects GR exactly at  $i^D$ , the

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<sup>21</sup> The relationship between  $b^*$  and the spread for the EMU12 countries can be seen in Table 1, and in Figure 7 for the financially distressed countries. The choice of  $b^*$  according to our GR function is somewhat arbitrary, because each country might have a specific target either self-imposed or imposed by external agencies. However, our definition of  $b^*$  represents the minimal requirement that keeps the debt/GDP ratio constant, whereas some countries under debt distress may have "more ambitious programmes" of debt reduction implying larger targets.

interest rate that triggers default. Nonetheless, default remains a possible event driven by investors' beliefs, whereas the distinction between  $B$  and  $D$  has interesting implications that will be discussed shortly.

Let us examine points  $G$  and  $B$ .<sup>22</sup> If  $G$  exists, it should be that  $\partial\text{IR}/\partial b^*|_G < \partial\text{GR}/\partial b^*|_G$ , which means that  $G$  is an attractor. Let us consider an initial arbitrary value of  $i_0$  to the right of  $G$  as in Figure 2. On the GR function we can read the value of  $b^*_0$  consistent with  $i_0$ . But the IR function indicates that for  $b^*_0$  the market would demand a lower  $i_1$ , which would allow for a lower  $b^*_1$  so that the only equilibrium is  $G$ . The same happens if we start to the left of  $G$ , with  $i$  and  $b^*$  increasing up to  $G$ . On the other hand, if  $B$  exists, it should be that  $\partial\text{IR}/\partial b^*|_G > \partial\text{GR}/\partial b^*|_G$ , which means that  $B$  is not an attractor. The reader can easily see that for any arbitrary  $i_0$  to the right of  $B$  the subsequent values along IR and GR would deviate from  $B$  because the market would want a higher  $i$  which would require a higher  $b^*$  and so on. Hence, to the right of  $B$  the government is bound to default. Therefore, we can thus far establish that

(P1) a) *If a good and bad equilibrium exist, the good equilibrium is an attractor for any initial condition below the bad equilibrium.* b) *For any initial condition above the bad equilibrium, the government is bound to default.*

The analytical properties of the model encapsulated in proposition (P1) indicate that the system actually has two domains of attraction, one towards the good equilibrium (on the left of  $B$ ), the other towards the default event (on the right of  $B$ ). Note that default occurs for any  $i \geq i^D$ , that is, before  $D$  is reached. Hence,  $B$  and  $D$  are "virtual" equilibria, and when default occurs a fraction of investors may be taken by surprise, which cannot happen under rational expectations but is actually a rationale for strategic default (Reinhart and Rogoff (2009)).

In sum, the interesting issue is to establish the extent of the good-equilibrium domain, or recalling President Draghi's words, how good the fiscal outlook should be in order to remain within the good-equilibrium domain. Our previous analysis shows that there is no clear-cut answer: the

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<sup>22</sup> As standard practice in ME models, though limitative, I examine the local properties of fixed points independently of a formal analysis of the system's behaviour out of fixed points. This would require additional assumptions, for instance that the government and the market react sequentially to the respective decisions  $b^*$  and  $i$ .

extension of the good- (or bad-) equilibrium domain depends on the location of *both* the GR *and* IR functions, i.e. *fiscal fundamentals and market beliefs cannot be disentangled*.

Having established the basic properties of the model we can now examine some issues that are currently under discussion, especially in connection with the euro-sovereign crisis.

### 3. The model at work

#### 3.1. Fiscal and market shocks, fundamentals and non-fundamentals

To begin with, we shall examine how, given an initial good equilibrium, the system reacts to shocks. The model can deal with both *fiscal* and *market* shocks. The former affect the GR function, the latter affect the IR function. Concomitantly, the model, albeit stylised, may help shed some light on the much-debated issue of the role of fundamentals *vs.* non-fundamentals. The fundamentals are captured by the GR function: the initial debt stock, the nominal growth rate, the interest rate. Market assessment is captured by the IR function, given the risk-free benchmark interest rate  $\bar{i}$  and the haircut  $h$ , which may react to fundamental as well as non-fundamental news. Fiscal and market shocks can, of course, compound.

The outcome of any type of shock eventually depends on the initial position and on the new configuration of the two functions. The key issue is whether or not a new set of fixed points exists. If it exists, then the system possess a new good equilibrium; otherwise it is bound to default.

#### Fiscal shocks

We can distinguish between two types of fiscal shocks. The first type changes the slope of the GR function, the second shifts it. The first type is due to changes in the nominal growth  $n_{t+1}$ , the second is due to additional factors that change the value of  $b^*_{t+1}$  *coeteris paribus*. The latter may consist of extraordinary operations  $(m_{t+1}, s_{t+1})$  that affect the solvency condition (see equation (3)). These shocks should be understood as news at time  $t$  about the relevant variables at  $t+1$ . A particular shock that is worth mentioning occurs when the government misses the solvency primary surplus. Actually, this is an *ex-post* shock, but it can easily be accommodated within the model. If at any point in time  $t$   $b_t < b^*_t$ , the

consequence is that the outstanding debt stock  $d_t$  rises, the GR function becomes flatter so that the new solvency condition requires a higher  $b^*_{t+1}$ .

To exemplify, let us examine the two cases portrayed in Figure 3 (again the time subscript is suppressed).<sup>23</sup> Case a) exemplifies a "small" negative fiscal shock due to bad news about lower  $n$ ; the new (solid) GR function is flatter. Starting at the good equilibrium  $G$ , the government should plan a higher  $b^*$  against which the market sets a higher  $i$ . The new good equilibrium is  $G_1$ . Note that both  $b^*$  and  $i$  are eventually higher than they would be in the absence of an increase in the market's default probability (the movement along the IR function). However, in the region of low  $b^*$  and flat IR this effect may be small or negligible. If instead the government starts at the bad equilibrium  $B$ , the small shock is sufficient to lead to default.

[Figure 3a]

Case b) exemplifies a "large" negative fiscal shock  $s > 0$ , e.g. a bailout of banks as in Ireland or Spain, that shifts the GR function below the IR function so that no new fixed points exist. The government plans the commensurate increase in the primary surplus  $b^*_1$ , while the market attaches a higher probability of default to it that raises the interest rate to  $i_1$  where the government should implement an additional primary surplus, and so on up to the actual default decision of the government. This is a typical case of a self-fulfilling, non-fundamental attack, because  $b^*_1$  is sustainable by the government, and from that point onwards nothing changes in fundamentals that justifies the attack except self-generated higher interest payments. The dotted GR function further rotated downwards exemplifies the presence of Keynesian effects of the greater primary surplus  $b^*_1$ . The reader can easily verify that, along the dotted line, the self-fulfilling process is amplified.

[Figure 3b]

### Market shocks

The model can also deal with market shocks, that is, changes in the IR function. For a given state of GR, and given  $\bar{i}$ , the good-equilibrium domain may shrink owing to a steeper IR. This, in turn, depends on the distribution of investors' beliefs. It is interesting to note that the IR function reduces the

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<sup>23</sup> The same caveat as in fn. 22 applies.

good-equilibrium domain (i) the lower is the market belief  $\bar{b}_M$  with respect to the true  $\bar{b}$  or (ii) the lower is the variance of the beliefs ("belief coagulation"). The first case is a violation of the cross-sectional rational beliefs hypothesis, but it may nonetheless be of some relevance, for beliefs may prevail over reality as the government, *coeteris paribus*, finds itself in the attraction domain of default in spite of the fact that its true  $\bar{b}$  is higher than believed by the market. The second case amounts to the reverse of what is commonly called a "mean preserving spread" of the distribution. As an example, Figure 4 shows the IR functions generated by continuous Normal distributions that differ in their mean and variance. Take  $N(7, 1.4)$  as benchmark. A lower  $\bar{b}_M$  makes the IR steeper; a lower variance has an interesting two-faceted effect: the IR is flatter below  $\bar{b}_M$  and steeper above  $\bar{b}_M$ . Anyway, for a given GR function, the bad-equilibrium point shifts downwards, and the attraction domain of the good equilibrium shrinks.

[Figure 4]

These effects cast a problematic light on factors that may foster reshuffling or coagulation of beliefs, such as the role of opinion makers, official institutions, rating agencies, *gurus*, etc. These, too, may be related to fundamentals or not. Since investors form their beliefs rationally, news about changes in the determinants of  $\bar{b}$  will be reflected by  $\bar{b}_M$  consistently. Yet, if news that Greece is closer to default feed the same belief as to Italy so that its  $\bar{b}_M$  is lowered, this can be classified as contagion of non-fundamentals.

These effects of shifts in the mean and variance of beliefs also explain why governments have no incentive to, and usually do not, communicate their true  $\bar{b}$ . On the one hand, dispersed beliefs create more favourable conditions when  $b^*$  is relatively high. On the other hand, governments always have an incentive to communicate a value of  $\bar{b}$  greater than the true one, or that they will *never* default, which makes their communication worthless to investors.

In sum, with regard to market shocks, a sufficiently large displacement of the IR function may suddenly push the government from an initial equilibrium, even a good equilibrium, into the default domain.

### 3.2. Foreign debt vs. domestic debt

How the composition of debt affects investors' appetite is matter of extensive research, but the issue has recently been raised in the context of

the crisis of the euro-sovereign debt, pointing out a relationship among persistent current account deficits, accumulation of foreign debt and higher risk premia (Gros (2011), Gros and Alcidi (2011), Alessandrini et al. (2012))<sup>24</sup>. This relationship is, however, controversial (Obstfeld (2012)). My aim here is not to take a position but to show how the discussion can be clarified within the present framework.

One controversial issue is why a larger share of foreign debt should come with a higher risk premium. According to the present model, higher risk premium may be the result of either worse fundamentals or worse market beliefs. As to fundamentals, e.g. growth capacity, there is no clear connection with the composition of debt. Hence the problem lies in the way the foreign component of debt affects the IR function.

The problem can be addressed from two different viewpoints: that of investors in general, and that of foreign investors in particular. As to investors in general, one argument is that the presence of foreign debt restricts the government's ability to service its debt because foreign investors cannot be taxed. Since the tax burden would fall on the sole shoulders of domestic taxpayers, the government's solvency costs, both economic and political, would be higher. As a consequence, the market belief  $\bar{b}_M$  would be lower, which, as seen above, would determine a higher risk premium *coeteris paribus*.

As to foreign investors, an oft-heard argument is that they may fear the so-called "selective default". If the government could default on foreign debt only, the default costs would be reduced by their domestic component. This conjecture, too, lowers  $\bar{b}_M$  and concentrates the risk on foreigners at the one and same time. A complementary argument is that foreign capital is typically more volatile than domestic capital<sup>25</sup>, so that fire sales of the government bonds would be fast and large. However, selective default in a highly integrated financial system of cross-border private investors mixed up with large multinational entities is technically and legally quite problematic (e.g. private foreign investors may hold shares in resident investment funds holding domestic debt).

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<sup>24</sup> Tomz and Wright (2013) provide a wide set of stylized facts about foreign sovereign debts and defaults.

<sup>25</sup> Recall for instance the literature on the so-called "sudden stops" of capital inflows (Calvo and Reinhart (2000))

As already said, a particular specific risk faced by foreign investors is currency devaluation, which we may treat in the model as a haircut  $h$  on the capital value. If  $h > 0$ , the IR function shifts upwards by the same amount weighted by the share of foreign debt, generating higher risk premium and a reduction of the good-equilibrium domain. Euro-membership, however, implies that euro-denominated debt offers total protection to *all euro-resident investors*, so what is relevant to each euro-country's IR function is only the share of *non-euro-resident investors*, and the chance of a devaluation of the euro, which is not under direct control of any single country. In normal times, the compound effect of these two factors is small. In fact, data suggest that until 2008, interest-rate differentials across the subsequent euro-countries were largely driven by the currency risk component, which almost disappeared with the monetary union (Wyplosz (2006)). Therefore, a critical factor, as it turned out to be during the debt crisis, is the *belief in the irreversibility of the euro*. If it trembles, the IR function of distressed countries worsens via the  $h$  factor of the foreign debt component because (i) the redenomination of debt in devalued domestic currency has no longer zero probability, and (ii) *all* foreign debt holders are affected. As a matter of fact, the ECB's complaints for excessive spread for specific countries is mostly attributed to rumours about their exit from the EMU (Draghi (2012)).<sup>26</sup>

Overall, the present model suggests that a high share of foreign debt may act adversely, but the actual effect depends not so much on foreign debt *per se* as on its interaction with other economic and institutional factors that shape the investors' beliefs.

### **3.3. Why "austerity" may not work: A model of the "Greek tragedy"**

The traditional "shock therapy" of front-loaded, "ambitious" fiscal consolidation plans, also known in Europe as "austerity" has now become highly controversial well beyond the circles of traditional opponents.<sup>27</sup> Greece, Portugal and Ireland have been subject to shock therapies as conditionality for access to rescue funds. The governments of Italy and

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<sup>26</sup> However, Favero and Missale (2001) find that this risk component in current spreads is not so large.

<sup>27</sup> See among others the *Forum* organized by the website *Vox* ([www.voxeu.org](http://www.voxeu.org)) and Corsetti (ed., 2012)

Spain in power since 2011 have sought to follow the same strategy pre-emptively. The persistence of high spreads notwithstanding hard austerity plans *implemented* by countries like Italy and Spain, or the self-defeating effect of conditionality plans in Greece and Portugal (and partly in Ireland), raise the thorny issue of whether such plans were too small (non credible) or too large (non sustainable). Data over the debt crisis years 2010-12 reproduced in Figure 5 show that the issue concerns euro-sovereigns in general.

[Figure 5]

The horizontal axis measures fiscal consolidation as a positive change in the primary-balance/GDP ratio over the previous year; the vertical axis measures year averages of monthly spreads above the German ten-year *Bund*. The best fit is non-linear and is provided by the second-order polynomial plotted in the figure.<sup>28</sup> The fitness is poor and with the wrong sign, suggesting that larger fiscal efforts have not been rewarded with smaller spreads, and/or that persistence of high spreads has forced larger fiscal consolidations in a vicious circle. The fever of high spreads started to recede in the last quarter of 2012 only after the ECB launched the new OMT programme.

The GR-IR model can provide an analytical treatment of this problem. Let us consider a country whose  $GR_0$  function falls into the default domain as in Figure 6.

[Figure 6]

At the market rate  $i_0$ , the government is ready to stay solvent with  $b^*_0$ . However, at  $b^*_0$  the market rate would rise to  $i_1$ , and the related primary surplus  $\bar{b}$  would be unsustainable by the government. Then the government files for a rescue package, say some loan  $s < 0$  with conditionality, which shifts the  $GR_0$  function leftwards to  $GR_1$ .<sup>29</sup>

The model clarifies that whether the loan + austerity package is good or bad cannot be judged independently of the context. One critical factor is

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<sup>28</sup> The fitting equation for each country  $c$  and year  $t$  is:

$$spread_{mt} = 2.56 + 0.22\Delta b_{mt} + 0.01\Delta b_{mt}^2, \quad R^2 = 0.06$$

where  $\Delta b_{mt}$  is the year change in the primary balance.

<sup>29</sup> Different formats of the rescue package can be accommodated in the model. Direct conditional loans, such as those granted by the IMF or the newly created European Stability Mechanism (ESM), are captured by  $s < 0$ , which cuts  $b^*$  (see equation (3)). The same effect obtains with an ECB intervention in the primary debt market,  $m > 0$ , e.g an OMT under the ESM conditionality.

whether the loan is sufficient to reach the good-equilibrium domain. Suppose it is not, as shown in the figure. Given the market rate  $i_1$ , the conditionality commits the government to achieving the new solvency primary surplus  $b^*_1$ . This is less than the non-sustainable  $\bar{b}$  but greater than  $b^*_0$ . Then we observe the following notable events. First, the package *per se* has no effect on  $i_1$ . Second, as the government commits itself to  $b^*_1$ , the market responds with an increase in the interest rate to  $i_2$ , which again sets the government on an unsustainable path. Hence, it is the combination of austerity with an insufficient loan that condemns the rescue package to failure. Note that we have obtained this outcome with no deflation effects of the austerity on  $n$ , which would exacerbate the problem (the  $GR_1$  function would tilt rightwards). The problem would also be exacerbated by the so-called plans of private investors' participation in losses (that is  $h > 0$  and a steeper IR). This sequence of events is remarkably resemblant to what happened with the so-called "Greek tragedy".

The lesson of this model is twofold: first, the market response to the plan is not part of the solution but part of the problem; second, a successful rescue plan should be *large* and *concessional* enough to pull the government out of the default domain. This is by no means easy to engineer, not least because the IR function is not easily detectable. Note, however, that a rescue plan with these characteristics is tantamount to setting a ceiling on the interest rate, e.g.  $i_1$ . Ideally, the smallest plan that achieves this result consists of shifting the GR up to the tangency point on the IR, where  $(b^*_0, i_1)$  is the new (single) equilibrium.<sup>30</sup> Therefore, the lending institution should not seek to mimic the market, but to substitute the market by charging administered concessional interest rates with sustainable conditionality to the extent that is necessary to regain the good-equilibrium domain. This is in fact the same policy recommendation provided by Corsetti and Dedola (2011) and Cooper (2012). The same point is made by De Grauwe (2011) who consequently criticizes the operation rules of the EFSF as well as of the forthcoming ESM. From this viewpoint, the OMT programme, to the extent that it implies an interest-rate ceiling on the applicant's sovereign debt, appears as a superior solution.

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<sup>30</sup> Technically, this would not be a stable equilibrium. However, a small additional leftward shift of GR would provide a new good equilibrium.

## 4. Conclusions

The ongoing dramatic euro-sovereign debt crisis has prompted a new generation of models of debt dynamics and management characterized by multiple equilibria (ME) due to interactions between fiscal fundamental variables and investors' assessment of default probability. Typically, these interactions may give rise to self-fulfilling attacks on the sovereign debt, leading to default in spite of initial sustainable conditions. In this paper I have presented a ME model in this vein, whose main novel feature is that default probability is not attributed to a single representative investor but is measured as the cumulated frequency of the investors' heterogeneous beliefs regarding the sustainability of the solvency primary surplus to be achieved by the government. The model identifies an attraction domain of default within which the government is bound to default although initial solvency conditions are sustainable. The extent of this domain may be larger or smaller depending on the interplay between fiscal fundamentals and the *distribution* of investors' belief.

By means of this model I have addressed some controversial issues in the current debate on the euro-sovereign debt crisis, such as puzzles concerning the pattern of risk premia before and after the crisis, the identification of non-fundamental and contagion components in risk premia, the role of the foreign component of debt, pitfalls in "austerity" therapies. Some relevant policy implications also ensue. First, it is crucial that fundamental as well as non-fundamental cross-country interdependencies are taken into account in the policy design. Second, rescue systems should be in place against the default attraction domain. In fact, it is hard for a government to escape from this domain by its own means. In particular, in this domain the so-called fiscal "austerity" is not the right response, even ignoring possible contractionary effects on nominal growth. For "ambitious" fiscal plans are assessed as unsustainable by a larger share of investors leading to higher, not lower, interest rate. Among rescue systems, both central banks' interventions in the sovereign-debt market and bailout packages may be effective provided that they are large enough to remove the country's fiscal outlook from the default domain. Both instruments, implicitly or explicitly, entail the charging of a non-market interest rate as long as necessary.

Against this background, the EMU institutional framework based on the country-by-country "rules + sanctions" approach is ill suited to giving

guidance in the current crisis. The "self-fulfilling prophecies" problem, and the role played by non-fundamental and contagion phenomena, are not contemplated, and the EMU provides neither *ex-ante* protection nor *ex-post* rescue. To date the EMU institutional setup has been part of the problem rather than of the solution.

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## Appendix

Table 1. Fiscal targets, fiscal consolidation and spreads in the EMU, 2010-12

		Austria	Belgium	Finland	France	Germany	Greece
2010	$b$	-1.8	-0.4	-1.5	-4.7	-1.8	-4.7
	$b^*$	2.2	3.2	1.3	2.4	2.0	12.0
	$\Delta b$	-0.5	1.5	-0.1	0.5	-1.3	5.7
	$sp$	0.48	0.72	0.27	0.38	0.00	6.35
2011	$b$	0.0	-0.4	0.6	-2.6	1.6	-2.2
	$b^*$	2.3	3.9	1.4	2.6	2.1	24.1
	$\Delta b$	1.8	0.0	2.0	2.1	3.4	2.5
	$sp$	0.71	1.63	0.40	0.71	0.00	13.14
2012	$b$	-0.3	0.4	0.5	-1.9	1.7	-1.0
	$b^*$	1.8	3.2	0.9	2.3	1.2	43.8
	$\Delta b$	-0.3	0.9	-0.1	0.7	0.0	1.2
	$sp$	1.06	1.78	0.43	1.19	0.00	23.53
2010		Ireland	Italy	Luxemb.	Netherl.	Portugal	Spain
	$b$	-28.0	0.0	-0.4	-3.1	-7.0	-7.4
	$b^*$	3.8	4.6	0.4	1.8	4.4	2.3
	$\Delta b$	-16.0	0.8	0.0	0.3	0.4	2.0
2011	$sp$	3.00	1.29	-0.13	0.27	3.43	1.51
	$b$	-9.7	1.0	-0.1	-2.6	-0.4	-6.1
	$b^*$	8.9	6.3	0.5	1.8	9.6	3.3
	$\Delta b$	18.4	1.0	0.3	0.5	6.6	1.3
2012	$sp$	6.99	2.82	0.32	0.38	7.63	2.83
	$b$	-4.2	3.4	-1.2	-2.3	0.1	-3.3
	$b^*$	7.3	6.9	0.3	1.3	13.0	4.1
	$\Delta b$	5.5	2.4	-1.1	0.3	0.5	2.9
	$sp$	5.30	4.25	0.41	0.51	10.23	4.40

$b$  = primary balance, % of GDP;  $b^*$  = solvency primary-balance (equation (5)),  $\Delta b$  = year change in the primary balance;  $sp$  = year average of monthly spreads over ten-year German bonds.

Source: AMECO database; ECB, *Interest rate statistics*.

Figure 1. The IR function with a continuous Normal distribution  $N(-7, 1.4)$  of the investors' beliefs,  $\bar{i} = 2\%$  and  $h = 0$

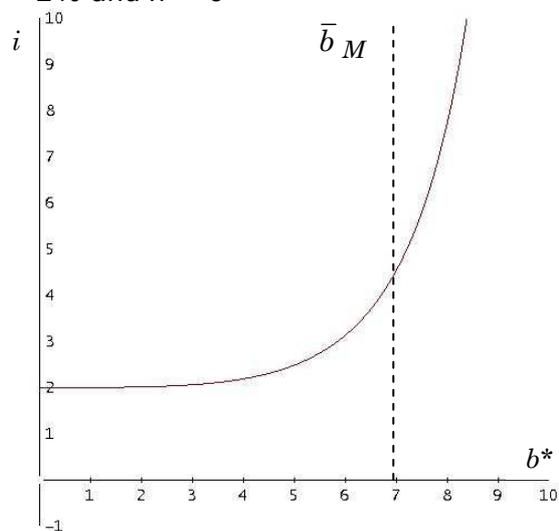


Figure 2. The GR-IR model

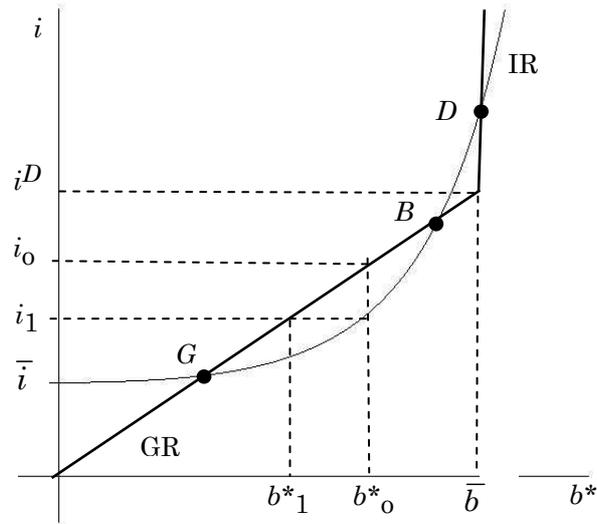


Figure 3a. A small fiscal shock

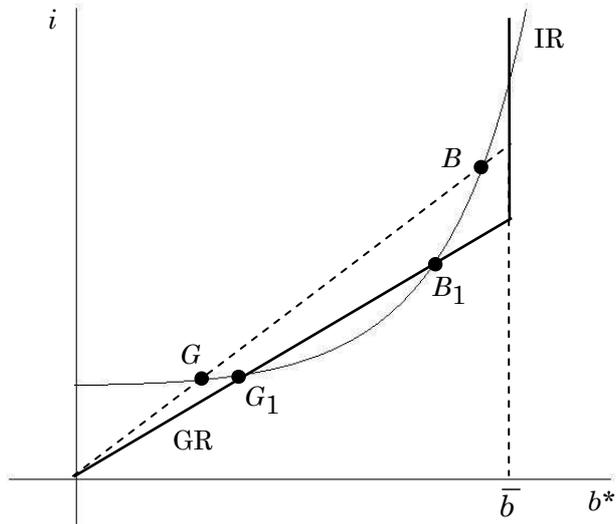


Figure 3b. A large fiscal shock

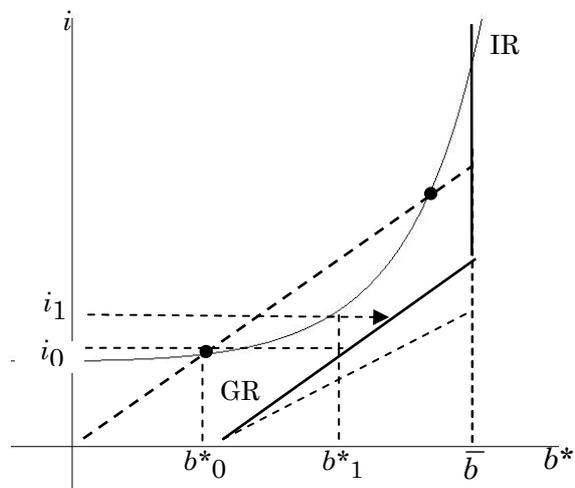


Figure 4. The IR function with different Normal distributions of investors' opinions.

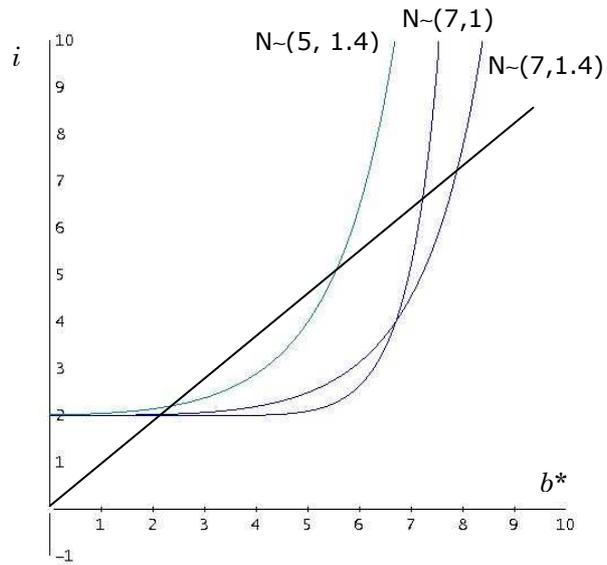
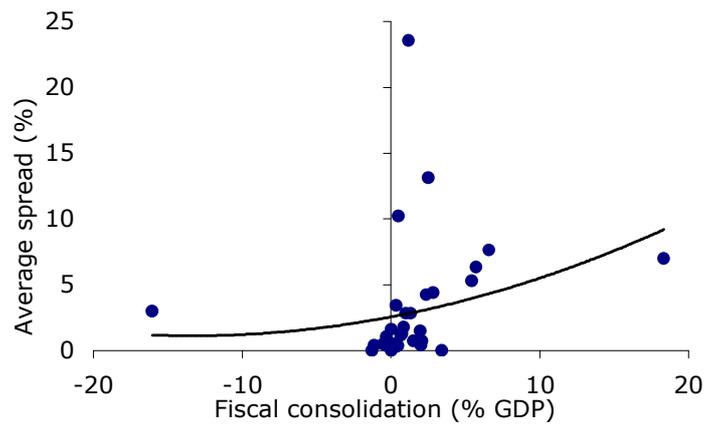


Figure 5. Fiscal consolidation and spreads. EMU12 countries, 2010-12



Source: Table 1.

Figure 6. A model of the "Greek tragedy"

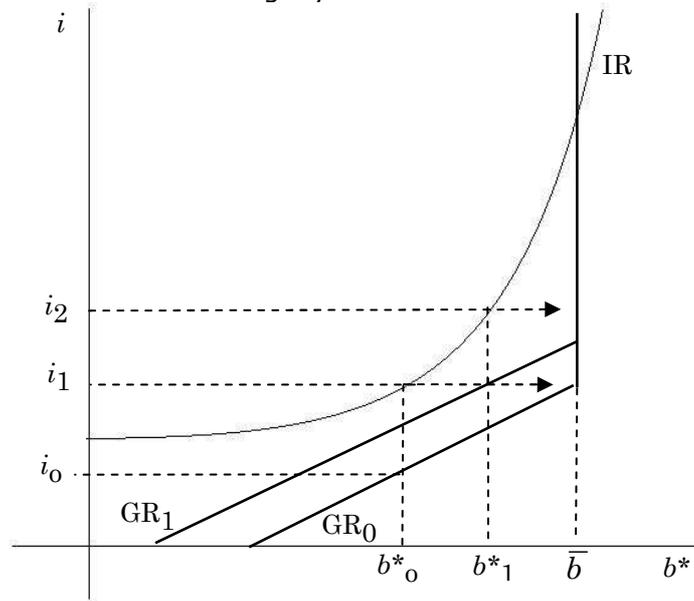
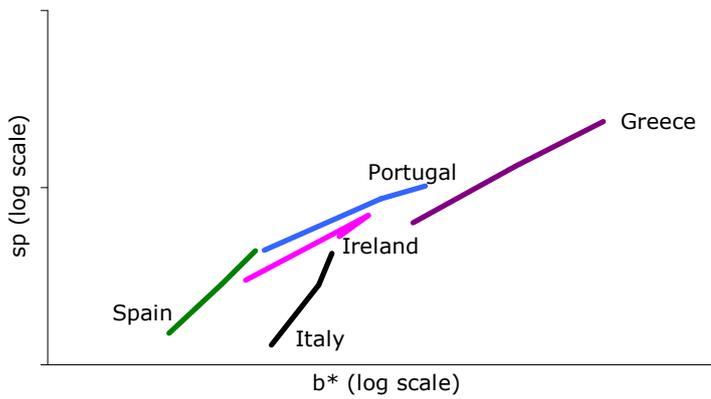


Figure 7. Solvency primary-balance ( $b^*$ ) and spread ( $sp$ ), EMU selected countries, 2010-12



Source: Table 1.