

# Animal Spirits in Entrepreneurial Innovation: Theory and Evidence

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## Abstract

This paper proposes and empirically tests a theory of entrepreneurial innovation that explains its high degree of concentration in space and time. In the model, a successful entrepreneurial project is the result of a search and matching process between entrepreneurs looking for funds and capitalists looking for new ideas to finance. The resulting strategic complementarity between them gives rise to a multiplier effect, whereby any exogenous shock has a magnified effect on the process of innovation. Moreover, if complementarity is sufficiently strong, multiple equilibria arise, which are characterized by different levels of entrepreneurial activity. Using data from the European and the US business angel (BA) market for the period 1996-2010, (i) we show that a complementarity exists between BAs and the entrepreneurial projects submitted to them, and (ii) we estimate the matching function in the BA market and verify the empirical plausibility of the equilibrium multiplicity result.

**Keywords:** Financing of innovation, search and matching, strategic complementarities, business angels.

JEL Classification: 031, C78, L26.

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# 1 Introduction

Suppose you think you have a promising idea for a new business venture but you find it hard to finance your project from banks or other conventional sources of capital, because of the high degree of uncertainty associated with your project and because of the lack of good collateral. You might then want to turn to other agents, specialized in screening and evaluating innovative business projects exactly like yours. If they judge your project valuable, these agents decide to provide you with the necessary capital, as well as technical and managerial advice, in exchange for an equity stake in the project. In the standard economics terminology, you are referred to as the entrepreneur, the specialized agents as the capitalists (such as venture capitalists or business angels), and the whole process is usually described as one of *entrepreneurial innovation*.

Two characteristics are salient when we observe the process of entrepreneurial innovation. The first, well-known, characteristic is its high degree of geographic *clustering*. In the US, for example, roughly half of firms financed by venture capitalists are located in three cities only, San Francisco, Boston, and New York (Chen et Al., 2009). Similar patterns of concentration can be documented for Europe and Asia: think, for instance, of the entrepreneurial clusters in Herzliya (Israel) or in the Guangdong province (China).

The second, probably less known, feature is the higher *volatility* over time of entrepreneurial investments with respect to all other investments. In Figure 1, we have depicted the volatility of investments in fixed capital and in R&D for the period 1995-2010 in both the US and Europe.<sup>1</sup> The pronounced volatility that we observe in Figure 1 almost *disappears* when we compare it with the one observed on the investments provided by venture capitalists (VCs) and private equity (PE) funds devoted to seed and start-ups, which is what we show in Figure 2 for both the US and Europe in the same period.<sup>2,3</sup>

INSERT FIGURES 1 AND 2 HERE

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<sup>1</sup>Volatility is measured as percentage deviation of investment components from their Hodrick-Prescott trends with smoothing parameters set at 6.25 (see Ravn and Uhlig, 2002).

<sup>2</sup>Since we are here interested in investments devoted to innovative projects, we limit our attention to the fraction of VC investments on seed and start-ups.

<sup>3</sup>In both figures, the aggregate "Europe" refers to EU15 plus Poland, Norway, Switzerland, Russia, Estonia, Czech Republic, Hungary, Slovakia, Slovenia.

A look at Figure 2 clearly suggests a pattern of "boom and bust" of entrepreneurial investments. In particular, both the dotcom bubble (and the subsequent bubble burst), as well as the effect of the economic crisis initiated in 2007 are clearly visible from the figure. The higher volatility of entrepreneurial investments arises even when we restrict the attention to those provided by business angels (BAs), as in Figure 3.<sup>4,5</sup>

INSERT FIGURE 3 HERE

The goal of this paper is twofold. On the one hand, we propose a theory of entrepreneurial innovation which is able to explain both the space clusterization and the high volatility over time that we have documented in the previous figures. On the other hand, we try to validate empirically our main theoretical claims against the available data on entrepreneurial innovation.

We construct a dynamic, partial-equilibrium model where an entrepreneurial project (or an innovative, start-up firm) is the outcome of a process of *search and matching* between the two main actors of the innovative process: those who come up with new ideas, that we call entrepreneurs (or simply innovators); and those who screen and select the most valuable ideas deserving financing funds, that we call capitalists (or simply financiers). An innovation is the result of a successful matching between an entrepreneur and a capitalist.<sup>6</sup>

In our model, entrepreneurs are willing to spend their time and intellectual resources to discover a new idea only if they have a chance to meet a capitalist. On the other hand, capitalists are willing to spend their time and intellectual resources to evaluate the profitability of ideas only if they have the chance to meet valuable entrepreneurs. More generally, the return to becoming an entrepreneur (capitalist) is higher, the higher the number of capitalists (entrepreneurs) in the market. Hence, and as it may happen

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<sup>4</sup>Business angels refer to wealthy individuals that invest *their own funds* in entrepreneurial ventures, differently from VCs, which instead gather funds from insitutional investors, such as pension funds. Because of that, the amount invested in each project by a BA is, on average, considerably lower than the one invested by a VC. BAs and VCs, however, share the following crucial feature: they are expected to contribute to the project not only with financial investments but also with managerial and technical expertise (Gompers and Lerner, 1999).

<sup>5</sup>For lack of data on BAs, we have restricted the time span of the series to the decade 2001-2010. Moreover, and for the same reason, Europe here refers to EU15 plus Norway, Poland, Switzerland and Russia.

<sup>6</sup>A more thorough justification of this modeling strategy is provided in Section 2.

in the class of search and matching models (Diamond, 1982, Kiyotaki and Wright, 1993), a strategic *complementarity* exists between entrepreneurs and capitalists, in that the number of entrepreneurs devoting to innovation is an increasing function in the number of capitalists, and viceversa.

The presence of a *thick market externality* across entrepreneurs and capitalists has a number of interesting theoretical implications. The first is the existence of a *multiplier effect* in entrepreneurial innovation, whereby the effect of an exogenous shock on the pace of innovation is magnified by the self-reinforcing nature of the interaction between the two sides of the market for ideas. This multiplier effect may contribute to explain the relatively higher volatility of entrepreneurial investments that we have documented in Figures 2 and 3. In this respect, *animal spirits* matter in the process of entrepreneurial innovation, in the sense that this process may be affected by waves of enthusiasm and/or pessimism.

A second implication is that, if strategic complementarities across entrepreneurs and capitalists are sufficiently strong (which occurs when the assumed aggregate matching function exhibits increasing returns to scale), the model may admit a *multiplicity of equilibria*, each characterized by a different pace of entrepreneurial activity. These equilibria can be ranked from the lowest to the highest number of innovations (matches) produced by the economy. Welfare is maximized at the equilibrium characterized by the highest number of matches. All other equilibria are sub-optimal and are the result of a coordination failure across the market participants (Diamond, 1982, Cooper and John, 1988). Moreover, a "degenerate" equilibrium always exists in this economy, in which the resources devoted to innovation are null. We call it "no-innovation trap".

The possibility of multiple equilibria may contribute to explain the well known phenomenon of geographic *clusterization* of entrepreneurial innovation that we have mentioned above. In principle, even two identical economies in terms of fundamentals may persistently diverge in their innovative performance: some economies may converge towards a "high entrepreneurial activity" equilibrium, others may be trapped into a slow or even stagnant pattern of entrepreneurial activity. Again, the model suggests that *animal spirits* matter in the process of innovation, in the sense that, whether a high or a low activity equilibrium is reached may depend on a self-fulfilling mechanism triggered by entrepreneurs' and capitalists' expectations.

In the second part of the paper, we test empirically the main theoretical predictions of the model using data on innovative projects financed by business angels. We hand-collect yearly data for the period 1996-2010 across a number of European coun-

tries plus the US, on (i) the number of business angels (capitalists), (ii) the number of projects submitted to them (potential entrepreneurs), (iii) the number of deals (successful matches). We first estimate a simultaneous equation model that confirms the existence of a statistically and economically significant complementarity between the number of business angels looking for innovative projects and the number of projects submitted to them. We then estimate the aggregate *matching function* for the BA market, which takes the number of projects and angels as inputs and the number of deals as output. The estimated function has all the technological characteristics assumed in the theoretical model (positive and decreasing marginal returns to both inputs). We finally verify the empirical plausibility of the equilibrium multiplicity by testing the returns to scale of the matching function. In the most reliable estimated model, the scale elasticity of the matching function is slightly but not significantly above unity, roughly suggesting that, in the BA market, multiple equilibria are unlikely.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the theoretical framework, characterizes the stationary equilibrium and proves the strategic complementarity between entrepreneurs and capitalists. Section 4 derives the multiplier effect. Section 5 discusses the possibility of coordination failures in the innovation process and constructs an illustrative example. Section 6 carries out the empirical analysis. Section 7 concludes and discusses the policy implications of the model. All proofs are relegated to a technical appendix at the end of the manuscript.

## 2 Related Literature

This paper is related to the literature, initiated by Arrow (1962), on the market failures associated with the process of innovation, and more particularly, with the process of innovation financing. Both microeconomic theory and empirical evidence have long recognized the potential obstacles hidden in the process of financing innovation, suggesting that innovators may well be financially constrained (for a review of the literature see, for instance, Hall and Lerner, 2010, and Hall, 2005). Theoretical arguments, proposed to explain financial market imperfections in this sector, range from transaction costs to agency problems due to informational asymmetries between the innovator (agent) and the financier (principal).<sup>7</sup>

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<sup>7</sup>While these aspects are common to any financing relationship, a number of additional elements suggest that financing problems can be even more severe for innovative investments: innovations are

In analogy with the labor market literature (Pissarides, 2000), we capture financial market imperfections via search theory. In the words of Phelps (2009, p. 50), "the classical supply-and-demand apparatus does not apply to the *core market* of capitalist economies - the capital market, particularly the market for capital going to entrepreneurs' innovative projects". In fact, it takes time and resources for an entrepreneur to find and convince a financier about the profitability of her business venture; and it takes time and resources for a financier to select the innovative project that she believes is worth financing. The financial market for innovation is, probably, better represented as a *decentralized* market where heterogeneous entrepreneurs and financiers meet bilaterally according to a matching technology. The matching process is then a modeling tool that captures succinctly all frictions characterizing the process of financing innovation - such as information imperfections, or the entrepreneurs' and capitalists' heterogeneities in skills, location, beliefs etc. Moreover, a decentralized market for innovation allows us to describe the realistic situation in which there may exist, at the same time, promising ideas which are not financed ("unemployed" ideas) and unutilized capital searching for good ideas to finance.

The idea of modeling entrepreneurial innovation as a process of search and matching is not new in the entrepreneurial finance literature. A few papers go deeper into the microeconomic foundations of the market frictions in the financing of innovation (Silveira and Wright, 2007, Silveira and Wright, 2010, Chiu, Meh and Wright, 2011). Others emphasize the contractual content of the relationship between entrepreneurs and capitalists (Boadway et al., 2005, Keuschnigg, 2003). Neither of these papers, however, identify and empirically test the complementary nature of this relationship, nor do they explore its implications to explain the two stylized facts on entrepreneurial finance highlighted above. Two other contributions deserve special mention inside this stream of literature. The first is the work by Inderst and Muller (2004), who construct a search and matching model of entrepreneurial innovation that may generate a pattern of boom and bust of entrepreneurial investments. Their theory, however, does not rely on the strategic complementarity across the two sides of the market of innovation:<sup>8</sup> ups and downs are instead due to a self-reinforcing relationship between the profitability of entrepreneurial investments, the entry of venture capitalists and the market valuation

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"unique" events, and the process aimed at producing them is an uncertain and largely unpredictable economic activity.

<sup>8</sup>Indeed in their model, due to the hypothesis of a constant flow of new ideas over each unit of time, entrepreneurs and capitalists are inversely related.

of start-ups. The second contribution is the one by Michelacci and Suarez (2004), who analyze the relationship between entrepreneurial finance and the stock market development in the framework of an endogenous growth model. Interestingly for our purposes, the model may generate multiple Pareto-rankable equilibria. This result, however, relies on a different mechanism from ours, namely, the complementarity across the "going public" decisions of start-up firms.

Finally, this paper is close to the literature attempting to explain the observed geographic clusterization of the entrepreneurial process. Several explanations have been proposed which are based on the existence of a network externality, such as input sharing, labor market pooling or knowledge spillovers (see, for instance, Jaffe et al., 1993; Audretsch and Feldman, 1996; Chen et al., 2009). While comparing the efficacy of the alternative explanations is not in the scope of this paper, we signal the presence of a different positive network externality that could rationalize the same phenomenon: the one between entrepreneurs and capitalists.

## 3 The Model

### 3.1 Innovation as a Search and Matching Process

The world is populated by a measure  $E$  of entrepreneurs and a measure  $K$  of capitalists who must decide whether to participate or not in a *fair of ideas*.<sup>9</sup> For simplicity, but without any loss of generality, both agents have an identical linear utility function,  $u(x) = x \forall x \in R_+$ . Time is continuous, and new ideas arrive randomly to the entrepreneurs according to a Poisson process with (exogenous) instantaneous probability  $\sigma$ . In order for these raw entrepreneurial ideas to become marketable innovations however, entrepreneurs need the (financial and managerial) support of capitalists.

Once an entrepreneur has come up with a new idea, she has to decide whether to pursue it by participating in the fair, or abandon it and wait for the next idea. To pursue it, each entrepreneur has to pay a cost  $c_E$ , representing the cost of developing and submitting the project to the financiers. This cost is idiosyncratically drawn from a (twice continuously differentiable) cumulative distribution function  $F(c_E)$  in the support  $[0, \overline{c_E}]$ . If the entrepreneur pays her own  $c_E$ , she acquires the right to participate in the fair and hence, as we will see, the *chance* of matching the "right"

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<sup>9</sup>Probably, the theoretical framework closest to ours is the now classical "coconut model" by Diamond (1982), the main difference being that ours is a two-sided search model.

capitalist and implement her project. Cost  $c_E$  can also be interpreted as an (inverse) measure of the *quality* of the entrepreneur's project.

On the other hand, each capitalist sustains an entry cost  $c_K$  to participate in the fair of ideas (the cost of screening, evaluating and selecting the entrepreneurial projects). This cost, which can be thought of as an inverse measure of each capitalist's *talent*, is idiosyncratic across capitalists and distributed according to a (twice continuously differentiable) cumulative distribution function  $G(c_K)$  in the support  $[0, \overline{c_K}]$ .<sup>10</sup>

To analyze the entry decisions of entrepreneurs and capitalists in the fair of ideas, we now need to specify the potential benefits that accrue to them if they pay the entry fee. Let  $L_E \leq E$  and  $L_K \leq K$  denote, respectively, the endogenous number of entrepreneurs and capitalists participating in the fair (that is, those that have paid their respective entry cost). An entrepreneurial venture is the result of a process of successful *search and matching* between an entrepreneur and a capitalist both attending the fair. We capture this production process of new ideas via the following aggregate matching function:

$$M = M(L_E, L_K), \quad (1)$$

with  $\partial M / \partial L_j > 0$  and  $\partial^2 M / \partial L_j^2 < 0$  for  $j = E, K$ , implying positive and decreasing marginal returns to both inputs. We also impose  $M(L_E, 0) = M(0, L_K) = 0$ , that is, the absence of entrepreneurs or capitalists implies zero successful matches. No further structure is imposed on the returns to scale of the matching function. The instantaneous probability of matching for, respectively, entrepreneurs and capitalists attending the fair, is then given by

$$\alpha_E = \frac{M}{L_E} \text{ and } \alpha_K = \frac{M}{L_K}. \quad (2)$$

The standard assumptions on the first two derivatives of the matching function (that will be verified empirically in Section 6) imply that  $\partial \alpha_j / \partial L_j < 0$  and  $\partial \alpha_j / \partial L_{-j} > 0$  for  $j = E, K$ . That is to say, the matching probability for an entrepreneur decreases with the number of entrepreneurs and increases with the number of capitalists (and the same holds for capitalists).

For an entrepreneur, the value of waiting for a new idea is denoted by  $V_E^0$  and defined by the following asset equation:

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<sup>10</sup>One might alternatively interpret  $c_E$  and  $c_K$  as outside options, that is, as the opportunity costs of devoting to entrepreneurship.

$$rV_E^0 = \sigma \int_0^{c_E^*} (V_E^1 - V_E^0 - c_E) dF(c_E), \quad (3)$$

where  $r$  is the exogenous riskless interest rate,  $c_E^*$  is the highest cost for which there is still entry (to be determined at equilibrium), and  $V_E^1$  represents the expected payoff associated with the entrepreneurial venture for an entrepreneur (the lifetime return to an entrepreneur attending the fair). This latter value is defined by

$$rV_E^1 = \alpha_E \theta \pi + V_E^0 - V_E^1, \quad (4)$$

where  $\pi$  represents total instantaneous profits originating from the innovation, and  $\theta \in (0, 1)$  is the entrepreneurs' fraction of these profits. These asset equations have the usual interpretations. Equation (3) tells us that, for an entrepreneur, the flow of utility from waiting for a new idea is equal to the instantaneous probability of a new idea times the corresponding payoff, which is given by the capital gain associated with participating in the fair minus the entry cost. Equation (4) instead, says that the flow of utility from venturing into innovation is equal to the probability of a successful matching with a capitalist times the payoff associated with this chance, plus the capital gain (or loss) deriving from exiting from the fair. Values  $V_E^0$  and  $V_E^1$  can be alternatively interpreted as the value to an entrepreneur of being, respectively, *outside* and *inside* the fair of ideas. Note that, in the two expressions above, we have decided to focus directly on the steady state, as we have imposed  $\dot{V}_E^h = 0$  for  $h = 0, 1$ .

Two implicit assumptions in (4) are worth noticing. First, every match becomes a successful innovation, that is to say, every venture-backed firm raises positive profits. Indeed, observation suggests that only a small fraction of funded projects reaches that stage (anecdotal evidence suggests that this fraction is below 20%). The second assumption is that, whether or not a successful matching has occurred, all entrepreneurs go back to the initial "inventive" stage (say, by selling their idea -or patent- to a firm which will start production in case of successful matching, or coming up with a new, better idea in case of failure).<sup>11</sup> This endless circular process is meant to represent the so called *venture capital cycle* described by Gompers and Lerner (1999). None of these assumptions is necessary for any of our results.

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<sup>11</sup>In the business literature, an individual with such characteristics is sometimes referred to as a *serial* entrepreneur. We prefer to portray him/her as a *Schumpeterian* entrepreneur, given that he/she dedicates him/herself exclusively to innovation.

Let us now turn to capitalists. The expected payoff associated with being a capitalist outside the fair is denoted by  $V_K^0$  and defined by the following asset equation:

$$rV_K^0 = \int_0^{c_K^*} (V_K^1 - V_K^0 - c_K) dG(c_K), \quad (5)$$

where  $c_K^*$  is the highest cost for which there is still entry for capitalists, and  $V_K^1$  represents the expected value from participating in the fair of ideas. This value is defined by<sup>12</sup>

$$rV_K^1 = \alpha_K (1 - \theta) \pi + V_K^0 - V_K^1, \quad (6)$$

where  $(1 - \theta) \pi$  is the capitalists' fraction of the profits prevailing in the market (again, along the steady state it is  $\dot{V}_K^h = 0$  for  $h = 0, 1$ ).

Before analyzing the choice behavior of entrepreneurs and capitalists, let us briefly comment on two issues. First, the allocation of the innovation profits across entrepreneurs and capitalists, as captured by the parameter  $\theta$ , is here taken as exogenous. This is not because we believe the contractual arrangement between entrepreneurs and capitalists is uninteresting but simply because our focus is different.<sup>13</sup> Secondly, in expression (6) we have implicitly supposed that the cost of financing the entrepreneurial project is null (so that the capitalists' contribution to the venture is technical and/or managerial but not financial). This is only to economize on parameters and simplify calculations.

### 3.2 Complementarities in the Innovation Process

In general, two activities are complementary whenever the return from one activity increases as the intensity of the other activity increases. Bringing this definition to our context, we say that entrepreneurs and capitalists are complementary if the return from attending the fair of ideas for an entrepreneur (capitalist) is increasing in the number of capitalists (entrepreneurs) attending the fair. We now prove that this statement is true in our model.

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<sup>12</sup>The implicit assumption here is that each capitalist can enter into one and only one project at a time, and that each entrepreneur needs one and only one capitalist.

<sup>13</sup>An extensive literature has focused on optimal contracts between capitalists and entrepreneurs (for instance in the presence of moral hazard and adverse selection): see, among others, Keuschmigg (2003), Inderst and Muller (2004), Michelacci and Suarez (2004), Silveira and Wright (2007).

Let us focus on entrepreneurs first. Their cost from attending the fair is distributed according to  $F(c_E)$  and is independent of the number of capitalists. Their benefit, associated with the chance of a successful matching with a capitalist, is instead measured by the difference  $V_E^1 - V_E^0$ . Subtracting (3) from (4), and solving the resulting equation for  $V_E^1 - V_E^0$ , we obtain

$$V_E^1 - V_E^0 = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF(c_E)}{1 + r + \sigma F(c_E^*)}. \quad (7)$$

It is immediate to prove that the expression above is increasing in  $\alpha_E$ , and thus in  $L_K$ . The intuition is straightforward: the higher the number of capitalists, the higher the matching probability for an entrepreneur, and hence the higher her return from participating in the innovation process.

The same argument holds for capitalists. Their cost is independent of the number of entrepreneurs, while their benefit is measured by the difference  $V_K^1 - V_K^0$ . Again, solving the system made up of (5) and (6) for  $V_K^1 - V_K^0$ , we obtain

$$V_K^1 - V_K^0 = \frac{\alpha_K (1 - \theta) \pi + \int_0^{c_K^*} c_K dG(c_K)}{1 + r + G(c_K^*)}, \quad (8)$$

which is increasing in  $\alpha_K$ , and thus in  $L_E$ . The two previous results are summarized in the following

**Lemma 1.** *The return from attending the fair of ideas for an entrepreneur (capitalist) is increasing in the number of capitalists (entrepreneurs) attending the fair.*

To characterize a stationary equilibrium for this economy, we now consider the optimal entry decisions for both entrepreneurs and capitalists. At each point in time, the choice of the  $E - L_E$  entrepreneurs who are outside the fair, as to whether to pursue their project or abandon it, depends on the relative costs and benefits of the project. The cost  $c_E$  is distributed according to  $F(c_E)$ , while the benefit is measured by the difference  $V_E^1 - V_E^0$ . There exists an *inframarginal* entrepreneur for whom  $c_E^* = V_E^1 - V_E^0$ .

Substituting for the expression given in (7), we obtain

$$c_E^* = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF(c_E)}{1 + r + \sigma F(c_E^*)}. \quad (9)$$

All entrepreneurs whose entry cost is lower than  $c_E^*$  find it profitable to participate in the fair. The expression above links the threshold cost  $c_E^*$  to the probability of successful matching for entrepreneurs  $\alpha_E$ , and hence to the number of entrepreneurs and capitalists attending the fair,  $L_E, L_K$ : as an immediate implication of Lemma 1, a higher  $L_K$  and/or a lower  $L_E$  leads to an increase in the probability of a successful matching with a capitalist ( $\alpha_E$ ), which in turn causes an increase in the cutoff value of the entry cost  $c_E^*$ .<sup>14</sup>

In analogy to the previous case, the chance of a successful matching with an entrepreneur is worth  $V_K^1 - V_K^0$  to a capitalist. Given that the cost of this chance  $c_K$  is distributed according to  $G(c_K)$ , there exists an inframarginal capitalist for whom  $c_K^* = V_K^1 - V_K^0$ . Substituting for the expression given in (8), we obtain

$$c_K^* = \frac{\alpha_K (1 - \theta) \pi + \int_0^{c_K^*} c_K dG(c_K)}{1 + r + G(c_K^*)}. \quad (10)$$

All capitalists whose entry cost is lower than  $c_K^*$  find it profitable to participate in the fair. This expression captures the positive relationship between  $\alpha_K$  and  $c_K^*$ .

Finally remind that, for both entrepreneurs and capitalists, the inflows into the fair of innovation must be equal to the outflows along the *steady state*, that is

$$\dot{L}_E = \sigma (E - L_E) F(c_E^*) - L_E = 0, \quad (11)$$

and

$$\dot{L}_K = (K - L_K) G(c_K^*) - L_K = 0. \quad (12)$$

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<sup>14</sup>Define

$$Z(c_E^*, \alpha_E) = \frac{\alpha_E \theta \pi + \sigma \int_0^{c_E^*} c_E dF(c_E)}{1 + r + \sigma F(c_E^*)} - c_E^*$$

as the implicit function of  $c_E^*$  with respect to  $\alpha_E$ . It is immediate to prove, via the implicit function theorem, that  $dc_E^*/d\alpha_E > 0$ .

Equation (11) captures the evolution of entrepreneurs over time. Along the steady state, the number of entrepreneurs deciding to participate in the fair ( $\sigma(E - L_E) F(c_E^*)$ ) must equalize the number of entrepreneurs who exit from the fair (whether successfully or not) and return to the waiting stage ( $L_E$ ). An analogous interpretation can be given to (12).

Equation (11) can be interpreted as a positive relationship between  $L_E$  and  $c_E^*$ .<sup>15</sup> A higher value of  $c_E^*$  implies greater entry in the market of innovation. To maintain the steady state, the number of exits must correspondingly increase. Hence, a higher value of  $L_E$  is required for equation (11) to hold. The same is true, *mutatis mutandis*, for equation (12) capturing  $L_K$  as a positive function of  $c_K^*$ .

Before defining a stationary equilibrium for this economy, let us characterize the type of interdependence between entrepreneurs and capitalists that is implied by this model. The number of entrepreneurs venturing in innovative projects depends on the number of capitalists deciding to back these projects, as this affects the chance of a successful matching. This relationship is captured by a "best-response" function for entrepreneurs,  $L_E = f_E(L_K)$ , that is implicitly defined by the system made up of (9) and (11). On the other hand, the number of capitalists devoting their time and resources to screening and evaluating innovative projects depends on the chances of encountering good potential entrepreneurs. This relationship is given by the capitalists's best-response function,  $L_K = f_K(L_E)$ , defined by the system made up of (10) and (12). We now prove that these two best-response functions are positively sloped.

**Theorem 1** *Entrepreneurs and capitalists are complementary, in that the number of entrepreneurs attending the fair of ideas is an increasing function of the number of capitalists attending the fair, and viceversa:  $df_j/dL_{-j} > 0 \forall j = E, K$ .*

Intuitively, a higher number of capitalists participating in the fair raises the chance of a successful matching for an entrepreneur, it makes her participation to the fair more profitable, and thus it brings about an increase in the number of entrepreneurs (and viceversa). We can call this complementarity *strategic*, as it is the result of endogenous and interdependent entry choices of the two types of agents. The existence of this strategic complementarity will be verified empirically in Section 6. We are ready for the following

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<sup>15</sup>This will be explicitly shown in the proof of Theorem 1.

**Definition.** A stationary equilibrium for this economy is any 4-tuple  $(L_E, L_K, c_E^*, c_K^*)$  that solves the four equations (9), (10), (11) and (12).

Finally, we close the section with a result "disciplining" the number of equilibria admitted by this economy. We prove the following<sup>16</sup>

**Theorem 2** *If the matching function (1) is homogeneous of degree 1, the economy admits one and only one stationary equilibrium.*

In the next two sections, we will investigate the theoretical implications of the strategic complementarity between entrepreneurs and capitalists both when the economy admits a multiplicity of stationary equilibria and when the equilibrium is unique. We start with the latter.

## 4 The Multiplier Effect

Suppose that the matching function is homogenous of degree 1, and hence that the equilibrium is unique. We now prove that the presence of strategic complementarities makes this equilibrium highly sensitive to disturbances. For illustrative purposes, say that a negative shock hits this economy so that, for instance,  $\Delta\sigma < 0$  (meaning that, for a whatever reason, entrepreneurs become less "creative"). This negative shock reduces the payoff associated with the entrepreneurial activity and thus reduces the number of entrepreneurs. This, however, is not the end of the story. The lower number of entrepreneurs weakens the incentive to become capitalist, which in turn further lowers the incentive to entrepreneurship. This process continues *ad infinitum*, describing a vicious circle whereby the aggregate response to the shock is stronger than the initial partial response. The strategic complementarity across the two main actors of the innovation process magnifies the initial effect of the shock and gives rise to what is usually referred to as a *multiplier effect*.

More formally, define  $L_j = f_j(L_{-j}, \rho)$  as the (positively sloped) reaction function of agents of type  $j$  with respect to the agents of type  $-j$  (for  $j = E, K$ ), parameterized by  $\rho$  capturing any feature that affects  $L_j$  other than changes in  $L_{-j}$ . We are now ready to state the following

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<sup>16</sup>A similar result in a different model is provided by Diamond (1984).

**Theorem 3** *A multiplier effect characterizes the process of entrepreneurial innovation, in that the total equilibrium response of entrepreneurs and capitalists to an exogenous shock is greater than the partial response:*

$$\left| \frac{dL_j^*}{d\rho} \right| = \left| \frac{\partial f_j}{\partial \rho} \right|_{(L_E^*, L_K^*)} + \left| \frac{\partial f_j}{\partial L_{-j}} \right|_{(L_E^*, L_K^*)} \left| \frac{dL_{-j}^*}{d\rho} \right| > \left| \frac{\partial f_j}{\partial \rho} \right| \quad \forall j = E, K.$$

The effect of an exogenous shock on the market of ideas is amplified by the strategic complementarity across the two sides of the market for ideas. Hence, any factor that affects the entrepreneurs' or the capitalists' payoff may have a big impact on the level of innovative activity. This mutual, self-reinforcing, interaction between entrepreneurs and capitalists contributes to explain the extremely high volatility of entrepreneurial investments that we have documented in Figures 2 and 3. To use a phrase from Summers (1988), our entrepreneurial equilibrium is *fragile*, in the sense that it is potentially subject to large fluctuations in the level of activity. This suggests that *animal spirits* may play a role in explaining the dynamics of entrepreneurial innovation. It also suggests that even small temporary shocks can have long-lasting consequences on the innovative process and, hence, on the overall economic performance.

## 5 Coordination Failures in Entrepreneurial Activity

The presence of a thick market externality always raises the possibility of *coordination failures* across the market participants (see for instance Diamond, 1982, 1984, Cooper and John, 1988). In particular, if the complementarity between entrepreneurs and capitalists is strong enough – which occurs when the matching function exhibits increasing returns to scale –, the model may generate multiple equilibria.

How do we interpret equilibrium multiplicity? For the sake of illustration, suppose that the economy admits two (non-degenerate) equilibria, respectively denoted by superscripts  $O, P$ , with  $(L_j)^O > (L_j)^P$  for  $j = E, K$  (an example of this kind is developed at the end of this section). It is easy to interpret these two equilibria as *self-fulfilling* equilibria triggered, respectively, by optimistic or pessimistic expectations. Whenever entrepreneurs expect a high number of capitalists to be matched with  $(L_K^e = (L_K)^O)$  where the superscript  $e$  stands for "expected", their number will be high as well,  $(L_E)^O$ . Similarly, whenever capitalists expect a high number of entrepreneurs  $(L_E^e = (L_E)^O)$ ,

their number will also be high,  $(L_K)^O$ . Equilibrium  $O$  can be labelled as the optimistic (or *thick*) equilibrium. Via a totally symmetric argument, expecting few entrepreneurs and capitalists entering the market makes the agents converge towards the low-entry equilibrium  $P$ , which can be referred to as the pessimistic (or *thin*) equilibrium.

Given that in our model only profitable innovations are pursued, whenever multiple equilibria exist, they can be *Pareto-ordered* from the lowest to the highest number of innovations (matches) produced by the economy. Welfare is thus maximized at the equilibrium characterized by the highest number of matches: all other equilibria are sub-optimal and are the result of a coordination failure between entrepreneurs and capitalists.

Our economy is also potentially subject to a most disruptive coordination failure. If entrepreneurs expect *no* capitalist participating in the fair ( $L_K^e = 0$ ), the number of expected matches and thus the probability of matching a capitalist for an entrepreneur are both null ( $M(L_E, 0) = 0$ ,  $\alpha_E = 0$ ). As a result, the expected value from participating in the fair is negative (as entry costs are positive while benefits are null), implying no entrepreneur entering into the market of innovation at equilibrium ( $c_E^* = 0$ ,  $(L_E)^T = 0$ ). Symmetrically, if capitalists expect no entrepreneur at the fair of ideas ( $L_E^e = 0$ ), none of them will participate either ( $(L_K)^T = 0$ ). The result of this extreme form of miscoordination is a (degenerate) equilibrium in which  $(L_E)^T = (L_K)^T = 0$ . We call this equilibrium a *no-innovation trap*, as no innovation ever takes place in this economy. Note that our economy *always* admits this kind of equilibrium (hence Theorem 2 has established uniqueness of non-degenerate equilibria only).

The possibility of multiple equilibria provides an alternative explanation to the strong space clusterization that we observe in entrepreneurial innovation. This is not to deny the importance of fundamentals in explaining the different patterns of entrepreneurial behavior that we observe across different regions: for economic, institutional, or even cultural reasons, some regions may simply provide more powerful incentives to entrepreneurship. What we claim here is that, all other things equal, *animal spirits* matter in entrepreneurial innovation, in the sense that, to some extent, a favourable entrepreneurial climate (or the lack of it) may turn out to be self-fulfilling. In this respect, the role of the policy maker might be crucial in providing a coordination device towards a path of bouncing entrepreneurial activity. Evidence on public intervention across the developed world confirms this claim (Lerner, 2010). We come back to this issue in the concluding section.

**Example.** Consider the model developed in Section 3 and further suppose that (i) the matching function (1) is Cobb-Douglas with increasing returns to scale:  $M = \delta L_E^{\beta_E} L_K^{\beta_K}$  with  $\delta \in R_+$ ,  $\beta_E, \beta_K < 1$  and  $\beta_E + \beta_K > 1$ , and that (ii) entry costs are the same for every entrepreneur and every capitalist,  $\underline{c}_E$  and  $\underline{c}_K$ .<sup>17</sup> This economy admits three stationary equilibria. The first (thin) equilibrium is given by the pair  $\left((L_E)^P, (L_K)^P\right)$  that solves the following system:<sup>18</sup>

$$\begin{cases} \underline{c}_E = \frac{\frac{\delta L_E^{\beta_E} L_K^{\beta_K}}{L_E} \theta \pi + \sigma \underline{c}_E}{1+r+\sigma} \\ \underline{c}_K = \frac{\frac{\delta L_E^{\beta_E} L_K^{\beta_K}}{L_K} (1-\theta) \pi + \underline{c}_K}{2+r}. \end{cases} \quad (13)$$

The second (thick) equilibrium is instead given by the pair  $\left((L_E)^O, (L_K)^O\right)$  that maximizes market entry and thus the size of the innovation fair:

$$\begin{cases} (L_E)^O = \sigma \left( E - (L_E)^O \right) \\ (L_K)^O = K - (L_K)^O. \end{cases}$$

Finally, the third equilibrium is the no-innovation trap,  $\left((L_E)^T, (L_K)^T\right) = (0, 0)$ . It is possible to prove that the trap and the thick equilibrium are stable, while the thin equilibrium is unstable.<sup>19</sup>

For illustrative purposes, let us carry out a numerical simulation of this economy. First fix these numerical values for the following parameters:<sup>20</sup>  $\beta_E = 0.726$ ,  $\beta_K = 0.438$ ,  $\delta = 0.046$ . Further assume  $r = 0.05$ ,  $\pi = 100$ ,  $\theta = 0.5$ ,  $\sigma = 1$ ,  $\underline{c}_E = 5$ ,  $\underline{c}_K = 2$ ,  $K = 400$ ,  $E = 180$ .<sup>21</sup>

The first system defining the thin equilibrium becomes

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<sup>17</sup>In some respects, this example resembles the one developed by Diamond (1982) in Section IX.

<sup>18</sup>Under constant returns to scale, this system is impossible (as it is made up of two equations in one unknown,  $L_K/L_E$ ), and hence this equilibrium disappears.

<sup>19</sup>The two best response functions in (13) are in fact strictly convex whenever the matching function is characterized by increasing returns to scale. The algebraic proof of this statement, as well as a graphical intuition of the three equilibria, are available upon request from the authors. We omit them for brevity.

<sup>20</sup>These numbers are drawn from a non-linear estimation of a Cobb-Douglas matching function obtained using the same business angels dataset of Section 6 (but restricted to Europe only). The estimates table is available upon request from the authors.

<sup>21</sup>The last two values are the average number of business angels and yearly submitted projects in Europe.

$$\begin{cases} 5 = \frac{0.046(L_E)^{-0.274}(L_K)^{0.438} \cdot 50 + 5}{0.05 + 2} \\ 2 = \frac{0.046(L_E)^{0.726}(L_K)^{-0.562} \cdot 50 + 2}{0.05 + 2} \end{cases}$$

whose solution is given by  $\left((L_E)^P, (L_K)^P\right) \simeq (13, 33)$ .

The second system defining the thick equilibrium is instead given by

$$\begin{cases} 180 - (L_E)^O = (L_E)^O \\ 400 - (L_K)^O = (L_K)^O \end{cases}$$

whose solution is  $\left((L_E)^O, (L_K)^O\right) = (90, 200)$ .

## 6 Empirical Evidence from the Business Angel Market

This section is devoted to the empirical validation of the main theoretical claims made in the previous sections. We first test the complementarity between entrepreneurs and capitalists in the business angel market.<sup>22</sup> We then analyze the technological characteristics of the innovation process in this market by directly estimating the aggregate matching function given in (1). We finally verify the empirical plausibility of the result of multiple equilibria by analyzing the returns to scale of the estimated matching function.

### 6.1 Data

The matching function expresses the output -the number of innovations- as a positive function of two inputs -the number of potential entrepreneurs and the number of capitalists. A key challenge of our analysis is the search of suitable data to estimate this function. One of the two inputs, the "number of potential entrepreneurs", is particularly difficult to measure. Usually, we observe the number of actual entrepreneurs, which is a proper subset of the group of those who are willing to become entrepreneurs but may or may have not been financed yet. The European Association for Business Angels (EBAN) and the US Center for Venture Research (CVR) at the University of New Hampshire, however, have recently started to collect more detailed yearly data, at macro level, across angel investors. In particular, they both record the total number

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<sup>22</sup>A definition of business angels has been provided in footnote 4.

of entrepreneurial projects *submitted* to each business angel. We then use this number of projects as a proxy for the number of potential entrepreneurs. Moreover, EBAN and CVR collect yearly data on two other dimensions capturing our remaining two variables of interest: the number of business angels (as a proxy for the input "number of capitalists"), and the number of deals (as a proxy for the output "number of undertaken entrepreneurial projects"). We hand-collect yearly data over the three mentioned dimensions of the business angels activity across EU-15 countries, plus Norway, Poland, Switzerland, Russia and the US over the period 1996-2010.<sup>23</sup> A summary description of these data is provided in Table 1.

INSERT TABLE 1 HERE

## 6.2 Complementarities between Entrepreneurs and Capitalists

Theorem 1 has established that the number of entrepreneurs attending the fair of ideas is an increasing function of the number of capitalists attending the fair, and viceversa. This section estimates a simultaneous equation model to verify this statement in the business angel market. In our data, the pairwise correlation coefficient between the number of business angels and the number of projects submitted to them is significantly high and approximately equal to 0.95.

We estimate the following system:

$$\begin{cases} (l_E)_{i,t} = \alpha_0 + \alpha_K (l_K)_{i,t} + \alpha_{E-1} (l_E)_{i,t-1} + \alpha_{DC} (DC)_{i,t} + \boldsymbol{\alpha}_c \mathbf{c}_{i,t} + \varepsilon_{i,t} \\ (l_K)_{i,t} = \beta_0 + \beta_E (l_E)_{i,t} + \beta_{K-1} (l_K)_{i,t-1} + \beta_{MC} (MC)_{i,t} + \boldsymbol{\beta}_c \mathbf{c}_{i,t} + \eta_{i,t} \end{cases} \quad (14)$$

where  $(l_E)_{i,t}$ ,  $(l_K)_{i,t}$  and  $(l_E)_{i,t-1}$ ,  $(l_K)_{i,t-1}$  are the logs of the number of projects submitted and of the business angels in country  $i$  at time  $t$  and  $t - 1$ , respectively;  $\mathbf{c}_{i,t}$  is a vector of controls including country dummies and a trend variable;  $DC$  and  $MC$  are two World Bank Indicators indicating, respectively, the amount of the domestic credit

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<sup>23</sup>Data for European countries and the US are recorded in the annual reports compiled by, respectively, EBAN and CVR (in particular, EBAN Annual Reports from 2005 to 2010, and CVR Angel Market Activity Reports from 2003 to 2010). Note that these data cover most but not all the business angels activity across the countries considered. The reason is that angel networks are not obliged to release any data.

provided by the banking sector in percentage of GDP and the value of the market capitalization of listed companies in percentage of GDP.

The key parameters capturing the nature of the relationship between business angels and entrepreneurial projects are  $\alpha_K$  and  $\beta_E$ .  $\alpha_K$  is the elasticity of the number of projects submitted to the contemporaneous number of business angels. Conversely,  $\beta_E$  is the elasticity of the number of business angels to the contemporaneous number of submitted projects. If both  $\alpha_K$  and  $\beta_E$  are significantly positive, we conclude that  $(l_E)_{i,t}$  and  $(l_K)_{i,t}$  are complementary, thereby verifying Theorem 1.

System identification relies on the availability of instrumental variables for both equations. In the first equation of system (14), the one-period lagged variable of the number of projects submitted,  $(l_E)_{i,t-1}$ , and the amount of the domestic credit provided by the banking sector in percentage of GDP,  $(DC)_{i,t}$ , are supposed to affect the current number of projects submitted without directly affecting the number of business angels. In particular, the use of  $(DC)_{i,t}$  as instrument is justified under the conviction that a well developed banking sector stimulates the emergence of entrepreneurial ideas by facilitating the access to credit for novel or established entrepreneurs. On the other hand, as business angels are private affluent individuals willing to invest their own funds on risky entrepreneurial ventures, their number is assumed not to be directly affected by the degree of development of the banking system. The second equation of system (14) is instead identified by the simultaneous use of  $(l_K)_{i,t-1}$  and  $(MC)_{i,t}$  as instruments for  $(l_K)_{i,t}$ . In this case, stock market capitalization is supposed to have a positive effect on the number of business angels because a well-developed stock market facilitates their exit through initial public offerings (IPOs).<sup>24</sup> This procedure identifies both  $(l_E)_{i,t}$  and  $(l_K)_{i,t}$  with one overidentifying restriction.

Since there is no theoretical reason to exclude *a priori* the existence of correlation between the error terms  $\varepsilon$  and  $\eta$ , the system is estimated through the three-stage least squares method (3SLS).<sup>25</sup> Results are presented in Table 2.

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<sup>24</sup>This is not to deny that a well-developed stock market facilitates the entrepreneurial activity as well. Here the implicit assumption is that, at the very early stages of development of their business ventures, that is, when deciding whether or not to submit their projects to the investors, entrepreneurs are much more focused on the market success of their initiatives rather than on their eventual exit strategy.

<sup>25</sup>The 3SLS method, developed by Zellner and Theil (1962), is a full information method that extends the two-stage least squares (2SLS) through the additional estimation of the variance-covariance matrix of cross-equations error terms ( $\hat{\Sigma}$ ). 3SLS provides therefore consistent and more efficient estimates than 2SLS. If  $\hat{\Sigma}$  is diagonal, 2SLS equation by equation and 3SLS are identical.

INSERT TABLE 2 HERE

The Sargan test of overidentifying restrictions over the simultaneous equations system signals a non rejection of the null hypothesis that the instruments are uncorrelated with the error term. Moreover, the rank condition over the entire system of equations is satisfied. This suggests that our estimates are statistically valid.<sup>26</sup>

The 3SLS estimates of system (14) point towards the existence of a significantly positive relationship between the number of potential entrepreneurs and the number of investors. In particular, a 1% increase in the number of business angels leads to a 0.30% rise in the number of submitted projects. On the other hand, a 1% increase in the number of projects produces a 0.20% positive change in the number of business angels. These estimates indicate that entrepreneurs and capitalists are complements in the business angel market.

## 6.3 Estimating the Matching Function

### 6.3.1 A CES-Type Matching Function

Using the data illustrated above, we now carry out a pooled regression estimation of different specifications for the matching function starting with the following CES-type function:

$$M_{i,t} = A(\beta_E (L_E)_{i,t}^\Psi + \beta_K (L_K)_{i,t}^\Psi)^{\frac{v}{\Psi}} \exp(\boldsymbol{\beta}_c \mathbf{c}_{i,t} + \varepsilon_{i,t}), \quad (15)$$

where  $M_{i,t}$  is the number of deals in country  $i$  at time  $t$ ;  $(L_E)_{i,t}$  and  $(L_K)_{i,t}$  are the number of projects submitted and of business angels in country  $i$  at time  $t$ , respectively;  $\mathbf{c}_{i,t}$  is a vector of controls;  $v$  is the return-to-scale parameter;  $\beta_E$  and  $\beta_K$  are share parameters,  $A$  is a scale technology parameter. For this function, the (constant) Hicks elasticity of substitution between the two input factors is given by  $\eta = 1/(1 - \Psi)$ . The CES collapses to a Cobb-Douglas function when  $\eta \rightarrow 1$  (or,  $\Psi \rightarrow 0$ ).

In the theoretical part of the paper, we have assumed positive marginal productivities to both inputs. This was a necessary condition to obtain strategic complementarity between entrepreneurs and capitalists (as shown in the proofs of Lemma 1 and of Theorem 1). In specification (15), the marginal return to  $L_j$  is given by  $\partial M_{i,t} / \partial (L_j)_{i,t} = Av\beta_j L_j^{\Psi-1} (\beta_j (L_j)_{i,t}^\Psi + \beta_{-j} (L_{-j})_{i,t}^\Psi)^{\frac{v}{\Psi}-1} \exp(\boldsymbol{\beta}_c \mathbf{c}_{i,t} + \varepsilon_{i,t})$  (for  $j = E, K$ ).

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<sup>26</sup>See, among the others, Murray (2006).

Positive marginal productivities to both inputs require  $\beta_E, \beta_K, v, A > 0$ . We are now going to verify empirically these parameter restrictions.

A second feature that is worth verifying is the degree of *technological* complementarity between the two inputs in (15). Two inputs are technological complements when the marginal productivity of one input is increasing in the use of the other input, that is, when the cross-partial derivative of the matching function is strictly positive. In our context, this would imply that the impact of one additional entrepreneur on the innovation output is increasing in the number of capitalists, and viceversa. The cross-partial derivative of (15) is given by

$$\frac{\partial M_{i,t}}{\partial (L_j)_{i,t} \partial (L_{-j})_{i,t}} = Av(v - \Psi) \beta_j \beta_{-j} \left( (L_j)_{i,t} (L_{-j})_{i,t} \right)^{\Psi-1} (\beta_i (L_j)_{i,t}^\Psi + \beta_{-j} (L_{-j})_{i,t}^\Psi)^{\frac{v}{\Psi}-2} \cdot \exp(\beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t}),$$

which is strictly positive if  $(v - \Psi) > 0$ . Hence, a positive difference between  $v$  and  $\Psi$  would signal the existence of a technological complementarity between the two input levels.<sup>27</sup>

The results from the nonlinear estimation of the log-CES matching function are shown in Table 3.<sup>28</sup> Estimates are in line with our theoretical predictions. Both  $A$  and  $v$ , and the share parameters are all significantly positive. The same is true for the estimated difference between  $v$  and  $\Psi$ . Hence, the higher the number of business angels, the greater (more positive) the effect of the number of entrepreneurial projects submitted on the innovation process, and viceversa. Or equivalently, the impact of one additional project submitted on the number of deals is positive and increasing in the number of business angels, and viceversa.

### INSERT TABLE 3 HERE

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<sup>27</sup>The notion of technological complementarity is distinct from (and stronger than) the notion of strategic complementarity identified in Section 3. Entrepreneurs and capitalists can be strategic complements even when they are not complementary inputs in the matching function. The proof of Lemma 1 shows that, in order for the entrepreneurs' return from entering into the fair to be increasing in  $L_K$ , it must only be that  $\partial M / \partial L_K > 0$  (and the same holds when we consider the capitalists' return). In other words, strategic complementarity only requires function (1) to exhibit positive marginal productivities in both inputs, while technological complementarity further requires that these marginal productivities be increasing in the use of the other input.

<sup>28</sup>Estimates are computed using nonlinear least squares, and the residuals have an approximately normal distribution.

Our estimates also suggest a unitary elasticity of substitution between the two inputs, because the  $\Psi$  parameter is positive but not significantly different from zero. This brings us to consider, in the next subsection, a Cobb-Douglas specification of the matching function.

### 6.3.2 A Cobb-Douglas Matching Function

Given a Cobb-Douglas (CD) matching function of the form  $M_{it} = A(L_E)_{i,t}^{\beta_E} (L_K)_{i,t}^{\beta_K} \exp(\beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t})$ , we now estimate the following log-transformation:

$$m_{i,t} = \beta_0 + \beta_E (l_E)_{i,t} + \beta_K (l_K)_{i,t} + \beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t} \quad (16)$$

where  $m_{i,t}$  is the log of the number of deals in country  $i$  at time  $t$ ;  $(l_E)_{i,t}$  and  $(l_K)_{i,t}$  are the logs of the number of projects submitted and of the business angels in country  $i$  at time  $t$ , respectively;  $\mathbf{c}_{i,t}$  is a vector of controls.

Model (16) is estimated via a *robust* regression to deal with the presence, in the dataset, of outliers that can distort the ordinary least squares estimator (OLS). By considering squared residuals, OLS tend to give an excessive importance to observations with very large residuals and, consequently, distort the parameters' estimation in presence of outliers. Adopting the graphical tool proposed by Rousseeuw and Van Zomeren (1990), Figure 4 shows that several outliers are present, suggesting that there is a serious risk that the OLS estimator be strongly attracted by outliers (Rousseeuw and Leroy, 1987).<sup>29</sup> To tackle this issue, and following the recent literature (Verardi and Croux, 2009), we adopt the MM-estimators method which has been found suitable to combine a high resistance to outliers and high efficiency.<sup>30</sup>

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<sup>29</sup>In particular, two observations for Belgium and Norway are *bad leverage points*, meaning that their explanatory variables are slightly different from those of the rest of data and their outcomes are higher than they should be according to the fitted model. The collected data for US are large *good leverage points*, suggesting that the characteristics of the US business angels market are rather different from the other countries but that the number of deals is consistent with what the model predicts. Finally, few other observations (i.e., for Italy, Portugal, Denmark, Netherlands and Poland) are *vertical outliers*, being standard in their characteristics but more or less successful in terms of number of deals than the model would suggest.

<sup>30</sup>The intuition behind the method is simple. In the classical OLS estimation, the objective is to minimize the variance of the residuals. Given that the variance is sensitive to outliers, this may result in distorted OLS estimates. The class of robust S- and MM-estimators instead minimize a measure of dispersion of the residuals that is less sensitive than the variance to extreme values.

INSERT FIGURE 4 HERE

Estimation results of the log-linear CD matching function are shown in the first column of Table 4. The share parameters are both significantly positive: a 1% increase in the number of submitted projects (business angels) leads to a 0.53% (0.55%) increase in the number of deals.

INSERT TABLE 4 HERE

We have so far verified the existence of a complementarity between the *levels* of the two inputs: a 1 unit increase in  $L_i$  leads to an increase in the output which is increasing in the level of the other input.<sup>31</sup> By construction however, the log-linear Cobb-Douglas specification in (16) implicitly assumes a *constant elasticity* of the output with respect to each input, that is to say: a 1% increase in  $L_i$  leads to a constant increase in the output. In the next section, we consider two generalizations of model (16) that allow us to verify the existence of a technological complementarity in *elasticities*.

### 6.3.3 A Translog Matching Function

The first generalization that we consider is a log-linear Cobb-Douglas with a *log-interaction term* between the demand and the supply of financial funds:

$$m_{i,t} = \beta_0 + \beta_E (l_E)_{i,t} + \beta_K (l_K)_{i,t} + \beta_{EK} \left[ (l_E)_{i,t} \cdot (l_K)_{i,t} \right] + \beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t}, \quad (17)$$

This model allows us to verify whether the two inputs are complementary in *elasticities*, that is, to explicitly test whether a 1% increase in  $L_i$  leads to an increase in the output which depends positively on the other input. This complementarity is captured by the positive sign of coefficient  $\beta_{EK}$ .<sup>32</sup>

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<sup>31</sup>Two inputs in a CD function are always complementary to the extent that the input shares ( $\beta_E, \beta_K$ ) are strictly positive. In fact, the cross-partial derivative of the CD function writes as  $dM / (dL_j dL_{-j}) = A \beta_j \beta_{-j} M / (L_j L_{-j}) \exp(\beta_{\mathbf{c}} \mathbf{c}_{it} + \varepsilon_{it})$ , which is higher than zero as long as  $\beta_j > 0$  for  $j = E, K$ .

<sup>32</sup>The cross-partial derivative is given by  $dm / dl_j dl_{-j} = \beta_{EK}$ , which is strictly positive if and only if  $\beta_{EK} > 0$ .

The second model we estimate is a transcendental logarithmic (translog) function, which generalizes the log-Cobb-Douglas form by allowing the output elasticity with respect to each input to vary with the size of both input shares:<sup>33</sup>

$$m_{i,t} = \beta_0 + \beta_E (l_E)_{i,t} + \beta_K (l_K)_{i,t} + \beta_{EK} \left[ (l_E)_{i,t} \cdot (l_K)_{i,t} \right] + \beta_{EE} \left[ (l_E)_{i,t} \right]^2 + \beta_{KK} \left[ (l_K)_{i,t} \right]^2 + \beta_{\mathbf{c}} \mathbf{c}_{i,t} + \varepsilon_{i,t} \quad (18)$$

The translog considers the squares of the two log-inputs. Decreasing elasticity for both inputs would imply  $\beta_{EE}, \beta_{KK} < 0$ .

The four models that we have estimated are all closely related to each other. The log-linear CD matching function (16) is nested into the interaction-augmented log-linear CD matching function (17), which is nested into the translog specification (18). In particular, (16) and (17) are directly obtained from (18) by applying the following restrictions, respectively:  $\beta_{EK} = \beta_{EE} = \beta_{KK} = 0$  and  $\beta_{EE} = \beta_{KK} = 0$ . Finally, it can be demonstrated that the translog specification can be obtained from a second-order Taylor approximation of the logarithmic transformation of the CES specification (15).<sup>34</sup>

As with model (16), also (17) and (18) are estimated via *robust* regressions. Results are shown in columns (2) and (3) of Table 4. The estimated elasticities of the matching function all have the expected signs and are highly statistically significant.

In column 2, the interaction term between the logarithms of the two explanatory variables is positive and highly statistically significant, which suggests the existence of an input complementarity, not only between the levels of BAs and entrepreneurial projects, but also between their elasticities. In other words, the impact of a 1% increase in the number of business angels on the number of deals is positive and *increasing* with the number of entrepreneurial projects, and viceversa.

Given that the log-linear CD specification is nested in the interaction-augmented log-linear CD specification and that the latter's parameter estimates are all significant, we conclude that model (17) is to be preferred to model (16). Notice also that, moving from the first to the second specification leads to a significant reduction in the estimated

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<sup>33</sup>The scale elasticity of a translog is defined by  $\epsilon = \epsilon_E + \epsilon_K$ , where  $\epsilon_E = \beta_E + \beta_{EK} (l_K)_{it} + 2\beta_{EE} (l_E)_{it}$  is the elasticity of new deals with respect to the number of submitted projects, and  $\epsilon_K = \beta_K + \beta_{EK} (l_E)_{it} + 2\beta_{KK} (l_K)_{it}$  is the elasticity of new deals with respect to the number of BAs.

<sup>34</sup>When the elasticity of substitution is in the neighborhood of unity, a two-input CES function may be approximated by a Taylor expansion which has the form of (18) under the following restrictions:  $\beta_{EK} = -2\beta_{EE} = -2\beta_{KK}$  (Kmenta, 1967).

first-order elasticity of the number of deals to the number of entrepreneurs (which drops to 0.27%). This means that, in the simplest specification, the estimated elasticity of  $m_{it}$  to  $l_E$  erroneously captures the positive role of the omitted interaction term.

The best fit is obtained under model (18), which estimates the more general translog matching function. Moving from model (17) to model (18), the role of the first-order terms on the outcome of interest remains substantially unchanged, while the impact of the interaction term significantly improves, going from 0.02% to 0.31%. This change is accompanied by a significant and negative impact of the squared values of the two inputs, thus confirming our conjecture of diminishing elasticities for both inputs.

Finally, we test the robustness of our previous empirical findings by introducing a few control variables in the  $\mathbf{c}_{it}$  vector of controls. We consider the two World Bank Indicators introduced in Subsection 6.2: i) the value of the market capitalization of listed companies in percentage of GDP (MC); (ii) the amount of the domestic credit provided by the banking sector in percentage of GDP (DC). As before, we expect stock market capitalization to affect positively the BA activity because it facilitates investors' exit. On the other hand, a well developed banking sector may crowd out the investment activity of business angels and thus lower the overall number of deals in the BA market. Results, shown in Table 5, are substantially similar to our previous findings. The coefficients of the controls all have the expected signs, and the stock market capitalization seems to significantly affect the (log-)number of deals.

INSERT TABLE 5 HERE

## 6.4 Equilibrium Multiplicity in the Business Angel Market

For reasons of tractability, most models of entrepreneurial finance (such as Inderst and Muller, 2004 or Michelacci and Suarez, 2004) *assume* constant returns to scale (CRS) of the matching function. The empirical estimation carried out above gives us the possibility to verify the plausibility of this hypothesis by testing explicitly the returns to scale (RTS) of the estimated matching function. Moreover and more importantly for our purposes, as Theorem 2 states that CRS imply a unique equilibrium, this test allows us to evaluate whether one or more than one equilibrium is to be expected in the BA market.

Table 3 presents the estimated returns to scale for the CES matching function and the results of an F-test for the null hypothesis of constant returns to scale ( $v = 1$ ).

Following Yashiv (2000) and Warren (1996), the last rows of both tables 4 and 5 present the estimated RTS at the sample mean of the explanatory variables and the results of the F-test for the null hypothesis of CRS of the respective log-linear matching function specification (for the translog model, the null hypothesis implies the following three linear restrictions on the parameters:  $\beta_E + \beta_K = 1$ ,  $\beta_{EE} + \beta_{EK} = 0$ ,  $\beta_{KK} + \beta_{EK} = 0$ ).

Overall, our results suggest that returns to scale are either constant or slightly increasing. The nonlinear estimation of the log-CES matching function (Table 3) gives us an estimated scale elasticity significantly positive and in the neighborhood of 1. The null hypothesis of constant returns to scale cannot be rejected. Moving to the log-linear matching function specifications without further control variables (Table 4), our evidence supports the presence of increasing returns to scale of the estimated matching function. In particular, in the translog model the output elasticity evaluated at the sample mean of explanatory variables is around 1.06, and the null hypothesis of CRS is rejected at 99%. When the selected World Bank Indicators are added among the control variables (Table 5), the translog model continues to exhibit mildly increasing returns to scale of the order of 1.01, but the null hypothesis of CRS cannot be rejected.

## 7 Conclusions and Policy Implications

This paper has built a model of the market for innovation that focuses on the relationship between innovators and financiers. An innovation is the outcome of a search and matching process between an innovator with a new project and a financier backing that project. The model has investigated the choice of innovators and financiers as to whether or not to participate in a "fair of innovation" and has determined the equilibrium number of innovators and financiers contributing to the innovation process along the steady state. The main purpose of the modeling strategy that we have followed has been the one of representing the "venture capital cycle" described in the literature on entrepreneurial finance (Gompers and Lerner, 1999).

We have shown that a strategic complementarity exists between innovators and financiers, in that an increase in participation of the former induces an increase in participation of the latter (and viceversa). Two main implications are drawn on this basis. First, the innovation process is subject to a multiplier effect which magnifies the effects of any shock on the innovative performance of the system. Secondly, coordination failures between innovators and financiers may occur, which are driven by

pessimistic beliefs about the attendance to the fair of innovation. These two results may contribute to explain the concentration of the entrepreneurial activity in both space and time that we observe in the real world.

The second part of the paper has been devoted to an empirical analysis of the dynamics of the innovation process in the BA market, in order to verify the consistency of this dynamics with the theoretical predictions of the model. Using data on the business angel market for the period 1996-2010 across a group of European countries plus the US, our empirical analysis has confirmed that the number of angel investors looking for promising entrepreneurial projects to finance and the number of projects submitted to them are complementary. We have then estimated the aggregate matching function between angel investors and would-be entrepreneurs. All estimated function specifications confirm the assumptions made in the theoretical model. The best fit is obtained with a translog specification. We have finally verified the empirical plausibility of equilibrium multiplicity by testing the returns to scale of the matching function. In the most reliable model (the translog specification), the estimated scale elasticity of the matching function is slightly but not significantly above unity, suggesting that multiple equilibria are unlikely but not impossible.

Two main policy implications may be drawn from our analysis. First, government intervention may be useful in the form of a "stabilization policy", that is, in order to attenuate the pronounced cyclicity of entrepreneurial innovation that we have documented above and that, according to our theory, originates from the presence of a multiplier effect in this process. Secondly, given that our model of innovation admits the possibility of coordination failures -that is, of equilibria characterized by sub-optimally low paces of innovation-, an effective government intervention might be able to initiate a *virtuous* cycle, that is, to favor the coordination of economic agents towards a path of faster innovation. For instance, public policy could in principle help drive the economy out of "bad equilibria" (such as the "no-innovation trap"). This task may not be as easy as it appears from a theoretical model: the policy maker might be incompetent or captured by special interests. Yet, empirical evidence confirms that, behind every successful story of entrepreneurial innovation (from the Silicon Valley to the Singapore VC industry), the role of public policy has always been crucial at the very early stages of development. In the words of Lerner (2010, p.42), "every hub of cutting-edge entrepreneurial activity in the world today had its origins in proactive government intervention. Similarly, the venture capital industry in many nations has been profoundly shaped by government intervention".

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## A Proofs

**Proof of Lemma 1.** Differentiating (7) with respect to  $L_K$ , we obtain

$$\frac{d(V_E^1 - V_E^0)}{dL_K} = \frac{\partial(V_E^1 - V_E^0)}{\partial\alpha_E} \cdot \frac{\partial\alpha_E}{\partial L_K} = \frac{\theta\pi}{1+r+\sigma F(c_E^*)} \cdot \frac{1}{L_E} \frac{\partial M}{\partial L_K},$$

which is always strictly positive, given that the marginal productivity of capitalists is strictly positive ( $\partial M/\partial L_K > 0$ ). The same reasoning applies to the capitalists' returns to the number of entrepreneurs.

**Proof of Theorem 1.** The entrepreneurs' reaction function with respect to capitalists,  $f_E(L_K)$ , is implicitly defined by the steady state condition

$$H(L_E, L_K) \equiv \sigma(E - L_E)F(c_E^*(L_E, L_K)) - L_E = 0$$

where  $c_E^*(L_E, L_K)$  is itself implicitly defined in (9). It is

$$\frac{dH}{dL_K} = \sigma (E - L_E) F' \frac{dc_E^*}{d\alpha_E} \frac{d\alpha_E}{dL_K},$$

which is strictly positive given that  $dc_E^*/d\alpha_E > 0$ , and that  $d\alpha_E/dL_K = (1/L_E) (\partial M/\partial L_K) > 0$ . On the other hand, it is

$$\frac{dH}{dL_E} = -\sigma F + \sigma (E - L_E) F' \frac{dc_E^*}{d\alpha_E} \frac{d\alpha_E}{dL_E} - 1,$$

which is strictly negative given that  $dc_E^*/d\alpha_E > 0$  and that  $d\alpha_E/dL_E = (1/L_E) (\partial M/\partial L_E - M/L_E) < 0$ .

Hence, by the implicit function theorem, we have

$$\frac{df_E}{dL_K} = -\frac{\frac{dH}{dL_K}}{\frac{dH}{dL_E}} > 0$$

To prove that  $df_K/dL_E > 0$ , a totally symmetric argument can be developed starting from the total differentiation of (12) with respect to  $L_E$ .

**Proof of Theorem 2.** We here prove that, if the matching function exhibits constant returns to scale (CRS), the stationary equilibrium is unique. First pose  $\Omega \equiv L_K/L_E$ . Given that (1) has CRS, we can write  $\alpha_E \equiv M/L_E = m(\Omega)$ , and  $\alpha_K \equiv M/L_K = (1/\Omega) m(\Omega)$ . The entry conditions, (9) and (10), are then both functions of  $\Omega$  only, the former increasing, the latter decreasing, that is,  $c_E^* \left( \overset{+}{\Omega} \right)$  and  $c_K^* \left( \overset{-}{\Omega} \right)$ . By substituting these functions respectively into (11) and (12), we obtain

$$L_E(\Omega) = \frac{\sigma E F(c_E^*(\Omega))}{1 + \sigma F(c_E^*(\Omega))} \quad (19)$$

and

$$L_K(\Omega) = \frac{KG(c_K^*(\Omega))}{1 + G(c_K^*(\Omega))}. \quad (20)$$

Standard differential calculus proves that  $L_E(\Omega)$  defined in (19) is monotone increasing in  $\Omega$ , while  $L_K(\Omega)$  defined in (20) is monotone decreasing in  $\Omega$ . Hence, the function defined as their ratio,  $L_K/L_E(\Omega)$ , is unambiguously decreasing in  $\Omega$ . Given that it is  $\Omega \equiv L_K/L_E$ , a stationary equilibrium is a fixed point of this function. We now prove that this function admits one and only one fixed point.

Define  $g(\Omega) \equiv L_K/L_E(\Omega) - \Omega$ . There exist sufficiently low values of  $\Omega$  such that  $g(\Omega) > 0$ , as well as sufficiently high values of  $\Omega$  such that  $g(\Omega) < 0$ .<sup>35</sup> Given that  $g(\Omega)$  is a continuous and monotone decreasing function in  $\Omega$ , the intermediate value theorem guarantees the existence of one and only one  $\Omega^*$  such that  $g(\Omega^*) = 0$ , that is, such that  $L_K/L_E(\Omega^*) = \Omega^*$ . Finally, it might still be the case that multiple equilibria exist, even though they are all characterized by a unique ratio  $\Omega^*$ . This instance, however, can be excluded once we realize that  $L_E(\Omega)$  and  $L_K(\Omega)$ , defined in (19) and (20), are monotone functions of  $\Omega$ .

**Proof of Theorem 3.** Define  $L_j = f_j(L_{-j}, \rho)$  as the reaction function of  $L_j$  to  $L_{-j}$ , for  $j = E, K$  and where  $\rho$  captures all the model's parameters. By convention, suppose that  $\partial f_j/\partial \rho > 0$ . Then it is

$$\frac{dL_E^*}{d\rho} = \frac{\partial f_E}{\partial \rho} + \frac{\partial f_E}{\partial L_K} \frac{dL_K^*}{d\rho}.$$

On the other hand,

$$\frac{dL_K^*}{d\rho} = \frac{\partial f_K}{\partial \rho} + \frac{\partial f_K}{\partial L_E} \frac{dL_E^*}{d\rho}.$$

Substituting the second expression into the first, we obtain

$$\frac{dL_E^*}{d\rho} = \frac{1}{1 - \frac{\partial f_E}{\partial L_K} \frac{\partial f_K}{\partial L_E}} \left( \frac{\partial f_E}{\partial \rho} + \frac{\partial f_E}{\partial L_K} \frac{\partial f_K}{\partial \rho} \right) > \frac{\partial f_E}{\partial \rho},$$

given that  $\partial f_K/\partial \rho > 0$  and that -as ensured in Theorem 1-  $\partial f_j/\partial L_{-j} > 0$  for  $j = E, K$ .

---

<sup>35</sup>The standard assumptions on the matching function imply that

$$\lim_{\Omega \rightarrow 0} \frac{L_K}{L_E}(\Omega) = +\infty$$

and

$$\lim_{\Omega \rightarrow +\infty} \frac{L_K}{L_E}(\Omega) = 0.$$

Even though they are not necessary, these two results ensure the existence of the two regions where  $g(\Omega) > 0$  and  $g(\Omega) < 0$ .

|                   | No. Deals |       | No. Angels |       | No. Projects |        | Obs. |
|-------------------|-----------|-------|------------|-------|--------------|--------|------|
|                   | mean      | sd    | mean       | sd    | mean         | sd     |      |
| Austria           | 5         | 3     | 79         | 30    | 63           | 21     | 10   |
| Belgium           | 29        | 17    | 142        | 136   | 197          | 105    | 9    |
| Switzerland       | 6         | 2     | 206        | 162   | 160          | 108    | 4    |
| Germany           | 18        | 10    | 145        | 59    | 147          | 109    | 7    |
| Denmark           | 17        | 29    | 76         | 86    | 38           | 30     | 4    |
| Catalonia (Spain) | 16        | 10    | 251        | 173   | 227          | 109    | 8    |
| Finland           | 8         | 5     | 185        | 127   | 35           | 13     | 10   |
| France            | 206       | 69    | 2563       | 877   | 833          | 259    | 4    |
| Greece            | 1         | 1     | 11         | 3     | 8            | 4      | 5    |
| Italy             | 8         | 9     | 252        | 96    | 153          | 139    | 7    |
| Luxembourg        | 1         | 0     | 8          | 0     | 30           | 0      | 1    |
| Netherlands       | 48        | 26    | 367        | 554   | 174          | 82     | 10   |
| Norway            | 3         | 1     | 101        | 72    | 32           | 13     | 4    |
| Poland            | 4         | 2     | 56         | 28    | 100          | 83     | 4    |
| Portugal          | 3         | 4     | 117        | 151   | 137          | 258    | 6    |
| Russia            | 3         | 1     | 88         | 46    | 35           | 7      | 2    |
| Sweden            | 43        | 28    | 284        | 157   | 358          | 306    | 4    |
| United Kingdom    | 78        | 73    | 1444       | 1663  | 369          | 214    | 13   |
| USA               | 50914     | 8157  | 238820     | 22876 | 370787       | 115773 | 9    |
| All countries     | 3815      | 13571 | 18140      | 63099 | 27746        | 102121 | 121  |

Table 1: Summary statistics.

|                                 | $l_E$                 | $l_K$                 |
|---------------------------------|-----------------------|-----------------------|
| $\alpha_K$                      | 0.3021**<br>(0.1296)  |                       |
| $\alpha_{E-1}$                  | 0.6378***<br>(0.0922) |                       |
| $\beta_E$                       |                       | 0.2014**<br>(0.0873)  |
| $\beta_{K-1}$                   |                       | 0.7098***<br>(0.1009) |
| $\alpha_{DC}$                   | 0.0074***<br>(0.0026) |                       |
| $\beta_{MC}$                    |                       | 0.0023**<br>(0.0011)  |
| Year controls                   | YES                   | YES                   |
| Country controls                | YES                   | YES                   |
| N                               | 101                   | 101                   |
| R-squared                       | 0.9325                | 0.9525                |
| Hansen-Sargan statistic         |                       | 2.668                 |
| Hansen-Sargan statistic p-value |                       | 0.2634                |

Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 2: 3SLS Results.

|                                       | log-CES               |
|---------------------------------------|-----------------------|
| $\ln(A)$                              | 0.9358<br>(0.3136)    |
| $\beta_K$                             | 0.0967**<br>(0.0057)  |
| $\beta_E$                             | 0.9033***<br>(0.0057) |
| $v$                                   | 0.9200**<br>(0.0258)  |
| $\Psi$                                | 0.5161<br>(0.1799)    |
| Year controls                         | YES                   |
| Country controls                      | YES                   |
| N                                     | 116                   |
| R-squared                             | 0.95                  |
| $(v - \Psi) > 0$ p-value              | 0.0147                |
| $\Psi \rightarrow 0$ p-value          | 0.2135                |
| CRS p-value                           | 0.1987                |
| Robust standard errors in parentheses |                       |
| *** p<0.01, ** p<0.05, * p<0.1        |                       |

Table 3: Results of the nonlinear estimation of the log-CES matching function.

|                  | (1)                   | (2)                        | (3)                    |
|------------------|-----------------------|----------------------------|------------------------|
|                  | log-CD                | log-CD with<br>interaction | translog               |
|                  | Elasticities          | Elasticities               | Elasticities           |
| $\beta_K$        | 0.5517**<br>(0.2214)  | 0.5672***<br>(0.0380)      | 0.6087***<br>(0.0896)  |
| $\beta_E$        | 0.5362***<br>(0.1843) | 0.2689***<br>(0.0535)      | 0.3264***<br>(0.0609)  |
| $\beta_{EK}$     |                       | 0.0154***<br>(0.0030)      | 0.3074***<br>(0.0425)  |
| $\beta_{KK}$     |                       |                            | -0.1356***<br>(0.0246) |
| $\beta_{EE}$     |                       |                            | -0.1612***<br>(0.0204) |
| Year controls    | YES                   | YES                        | YES                    |
| Country controls | YES                   | YES                        | YES                    |
| N                | 116                   | 116                        | 116                    |
| R-squared        | 0.92                  | 0.92                       | 0.90                   |
| RTS              | 1.09                  | 1.00                       | 1.06                   |
| CRS p-value      | 0.0372                | 0.8925                     | 0.0017                 |

Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Table 4: Results of the estimations of the log-log matching function specifications.

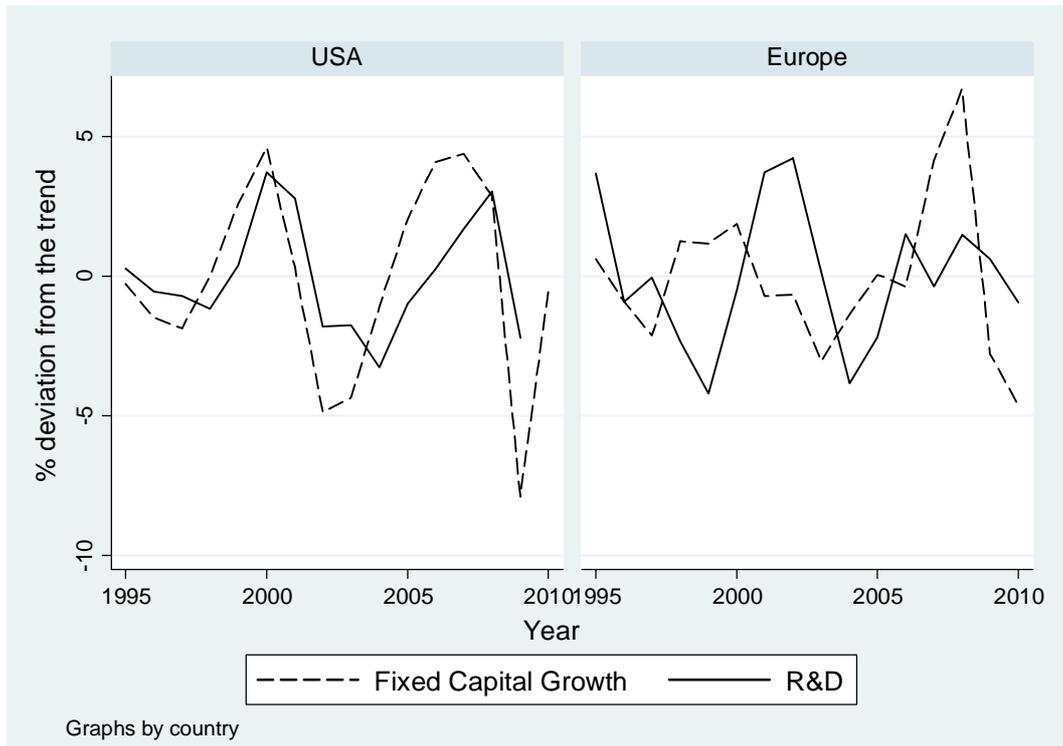
|                  | (1)                   | (2)                        | (3)                    |
|------------------|-----------------------|----------------------------|------------------------|
|                  | log-CD                | log-CD with<br>interaction | translog               |
|                  | Elasticities          | Elasticities               | Elasticities           |
| $\beta_K$        | 0.1690*<br>(0.0909)   | 0.4242***<br>(0.1488)      | 0.5304***<br>(0.1748)  |
| $\beta_E$        | 0.8198***<br>(0.0761) | 0.4403***<br>(0.1340)      | 0.3326**<br>(0.1360)   |
| $\beta_{EK}$     |                       | 0.0101***<br>(0.0031)      | 0.2627***<br>(0.0460)  |
| $\beta_{KK}$     |                       |                            | -0.1156***<br>(0.0375) |
| $\beta_{EE}$     |                       |                            | -0.1347***<br>(0.0208) |
| $\beta_{DC}$     | -0.0006<br>(0.0014)   | 0.0025<br>(0.0017)         | 0.0021<br>(0.0021)     |
| $\beta_{MC}$     | 0.0065***<br>(0.0021) | 0.0042***<br>(0.0014)      | 0.0034***<br>(0.0007)  |
| Year controls    | YES                   | YES                        | YES                    |
| Country controls | YES                   | YES                        | YES                    |
| N                | 114                   | 114                        | 114                    |
| R-squared        | 0.92                  | 0.93                       | 0.92                   |
| RTS              | 0.99                  | 0.97                       | 1.01                   |
| CRS p-value      | 0.6971                | 0.4483                     | 0.8907                 |

Robust standard errors in parentheses

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

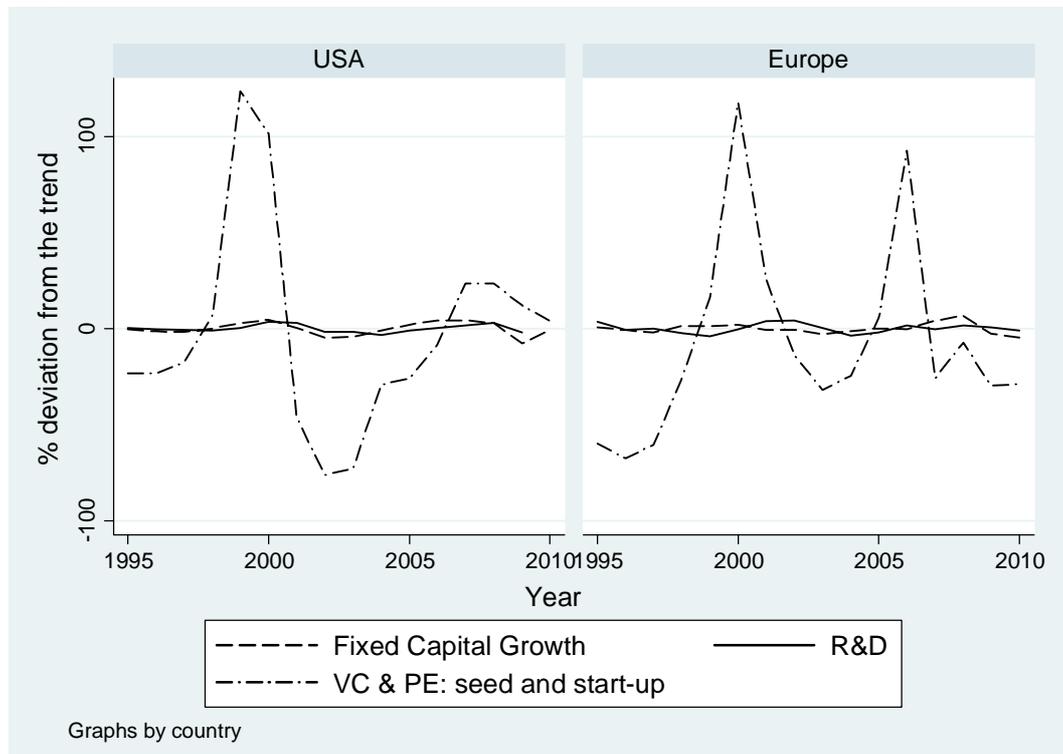
Table 5: Results of the estimations of the log-log matching function specifications (continued).

Figure 1. Volatility of investments in fixed capital and in R&D, years 1995-2010.



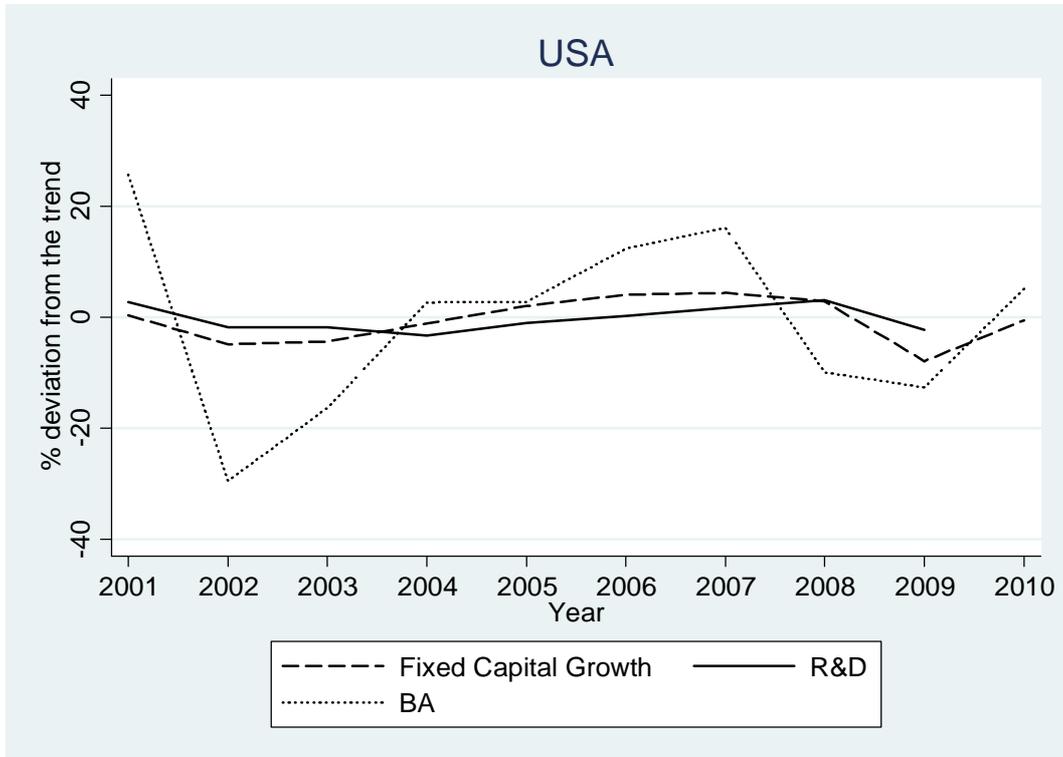
Note: The value of US investments in R&D for the year 2010 is missing from the OECD database.  
 Source: Own elaborations from OECD.Stat.

Figure 2. Volatility of investments in fixed capital and in R&D, and volatility of investments in seed-and start-ups provided by Venture Capitalists (VC) and Private Equity (PE) funds, years 1995-2010.



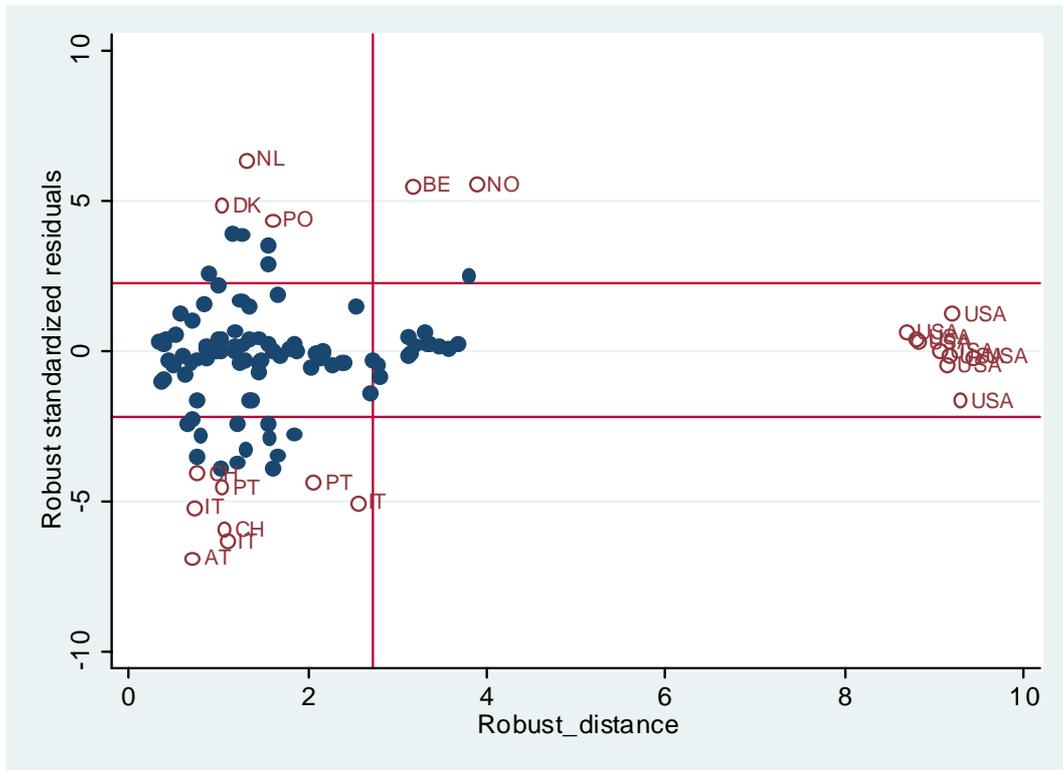
Note: The value of US investments in R&D for the year 2010 is missing from the OECD database.  
 Source: Own elaborations from OECD.Stat, MoneyTree Report, EBAN Annual Reports.

Figure 3. Volatility of investments in fixed capital and in R&D, and volatility of investment funds provided by business angels (BA), years 2001-2010.



Note: The value of US investments in R&D for the year 2010 is missing from the OECD database.  
 Source: Own elaborations from OECD.Stat, MoneyTree Report, CVR Angel Market Activity Reports.

Figure 4. Diagnostic plot of standardized robust residuals versus robust Mahalanobis distance of the vector of covariates from the vector of their means.



Note. The Mahalanobis distance of a multivariate vector  $\mathbf{x}$  of  $1 \times p$  dimension with mean vector  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}$  is defined as:  $D(x) = \sqrt{(\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})}$ , which follows a chi-squared distribution with  $p$  degree of freedom under normality. Observations lying at the right hand side of the vertical limit (set at  $\sqrt{X_{p,0.975}^2}$ ) are defined as *good leverage points*. Their presence does not affect the OLS-estimation but it affects the statistical inference since they do deflate the estimated standard errors. Observations lying above or below the area delimited by the two horizontal limits (set at -2.25 and +2.25, respectively) are defined as *vertical outliers* and affect the estimated intercept of an OLS-estimation. Observations lying both at the right hand side of the vertical limit and outside the 95% confidence interval of the Standard Normal are considered *bad leverage points*. Their presence significantly affects the OLS-estimates of both the intercept and the slope.