Public Private Partnerships: Information Externality in Sequential Investments

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Abstract

This paper studies the benefit coming from bundling two sequential activities in a context of Public Private Partnerships (PPPs). Differently from the previous literature, I introduce a source of asymmetric information regarding the parameter linking the building phase with the operational activity (externality parameter).

Within this framework, PPPs allow the government to extract the private information about the externality parameter and to minimize the informational rents needed to incentivize the builder’s effort.

Our results suggest how PPPs can become those commitment devices that force governments to define more coherent and informed plans that optimally set the first period welfare improving investment in order to reduce the unexpected ex post costs.

Keywords: complete contracts; agency theory; information externality; sequential investment; bundling

JEL classification: D86, L33, H11, H57, C61
1 Introduction

The realization of a public infrastructure with the aim of providing services to the citizens represents a long term project characterized by several complexities that must be adequately taken into account to achieve satisfactory results. The decision about the optimal strategy to realize these investments more and more came to coincide with a choice between Traditional Procurement (TP) and Public and Private Partnerships (PPPs). In case of TP, the public institution allocates the several phases of the project to different private firms, but it remains the owner of the infrastructure, the only financer and it is fully accountable for poor results. In case of PPPs, a single private consortium made of several firms is in charge of realizing and managing the infrastructure, it can assume the role of financer and the risks are optimally shared between public and private partners.

The literature has highlighted several advantages of PPPs on TP mechanisms. First, the bundling mechanism can allow a stronger incentive to innovate and to invest in the quality of the infrastructure during the building phase (Martimort and Pouyet 2008; Iossa and Martimort 2008). Second, a PPP optimally allocates risks between the two sectors; consequentially, the private partner has a stronger incentive to make more effort during the operational phase. Third, this option is able to attract private financers that can act as monitors reducing the information asymmetry between the government and the private agent (Iossa and Martimort 2011).

Since the 90’s and in later years real world applications of PPPs became common in most developed and developing countries. Such examples covered a wide set of sectors and typologies; from standard projects where the main source of revenues were users fees (e.g., motorways, parkings, public transports etc.) to very complex projects where the private profits came essentially from government subsidies (e.g., hospital, schools, prisons etc.).

Differences between theory and practice come from the interpretation of the bundling effect. The theory emphasizes the potential benefits of PPPs in exploiting the presence of ex ante positive externalities (Iossa & Martimort 2008, Martimort & Pouyet 2008). The practice reveals, instead, the ability of PPPs to discover potential innovative channels linking the different stages of the project. This paper restores a connection between the theoretical background and the practical evidence, indeed the
analysis describes a model where the working agents privately know how and how much are correlated two sequential investment phases (differently from Martimort & Pouyet 2008 and Iossa & Martimort 2008). Applications of this theoretical proposition to real world cases are represented by all those contractible innovations related to the building phase of the project whose impact to the following managerial activity is not ex ante perfectly recognized by the government, i.e.: the development of a metro without driver, the use of new materials for the construction of a motorway, the application of a new energy technology to a public building etc. In such cases, long term commitment devices, as PPPs, could grant institutions, politicians and administrative workers more useful decision-making information.

The model is developed in an asymmetric information framework and it is based on the “new economics of regulation” and contract theory approach. The theoretical methodology assumes that the principal (government) is able to write complete contracts contingent to the realization of contractible variables that are verifiable ex post. The project is made of two sequential phases that are connected through a production externality that links the operational costs with the first phase outcomes. The sources of asymmetric information are not limited to the externality parameter (hidden information), but they include also the efforts of the private agents that are not directly verifiable by the principal (hidden action). The government must choose between TP and PPPs with the aim of maximizing the net social welfare produced by the investment taking into account the shadow cost of public funds induced by the transfer of rents from the public to the private agents. The final results detect a potential ex ante advantage from choosing PPPs that increases more the future private information is uncertain for the public buyer. The analysis has been furtherly enlarged considering, as a robustness check, the possibility of the government to write, under TP, simultaneous contracts with both agents at the beginning of the project. The benefit of PPPs partially holds in the new framework if the shadow cost of public funds is positive.

\(^2\)This strand of literature applied to the procurement context is connected with the book of Laffont and Tirole (1993).

\(^3\) Even if this hypothesis seems too strong for very complex PPPs, real world experience shows that verification of quality/efficiency may be relatively easier in some sectors (e.g., highways quality) than in others (e.g., hospitals).

\(^4\) More the variance of the private information parameter is higher.
2 Related literature review

The literature on PPPs is focused on identifying how problems, like contracts incompleteness and asymmetric information, influence the organizational management of a multiperiod public investment.

The incomplete contract approach is the more developed and it is based on a setting where a contract is not able to cope with every aspect of the economic relation between the partners. Starting from this insight, the model of Hart (2003) studies the pros and cons associated with a PPP, focusing on the builder’s investment decisions in the current period about a public infrastructure, given his commitment of running the infrastructure in a future period. Hart concludes that the PPP is the best solution if the quality of the service can be well identified, while the quality of the building shouldn’t be explicitly stated in the contract.

Following the incomplete contract methodology, Martimort & Pouyet (2008) together with Iossa & Martimort (2008) develop a two stage model introducing an externality parameter as connection between the phases of the project. This parameter is known from the beginning and it is negative in cases where the first phase investment increases the second phase costs, while it is positive otherwise. Their conclusion are driven by the externality variable inasmuch as the bundling mechanism (PPPs), leading to the internalization of the cost and benefit related with the second period activity, is socially preferable only when the externality is positive. Martimort & Pouyet (2008) expand their basic model allowing for general schemes, more complete contracts and introducing an adverse selection issue concerning the operating costs. They conclude that with a benevolent decision-maker and a privately informed operator the optimal organizational form is still bundling (PPPs) when the externality is positive.

The potential advantages of PPPs highlighted by the literature can be partially or totally neutralized in a context of future uncertainty (exogenous shocks) or by considering agency problems within the private consortium. In the first case, the PPP option implies an excessive transfer of risks to a risk adverse consortium and a lack of flexibility induced by the early commitment (Iossa and Martimort

\footnote{The builder's payment depends on the operator's cost.}
In the second case, the imperfect bundling determines a suboptimal privately negotiated incentive structure within the consortium and reduces the scope for welfare-improving PPPs (as compared to Traditional Procurement) (Greco 2013).

Most of the existing literature assumes that the government is able to commit for a multi-period contractual relationship. When it is not the case, the government wants to renegotiate the contract at the second stage of the project affecting, as a consequence, the first period system of incentives⁶ (Guash et al. 2007, Engel et al. 2009 and Valéro 2012). This problem creates contract distortions that directly affect investment costs and successful probabilities.

With the introduction in the UK of the Private Finance Initiative (PFI) program in 1992 much emphasis has been set to the financial aspect of PPPs. The achievement of the “value for money” as well as the attraction of different sources of financing became the main goals of practitioners and public institutions when starting PPPs. Nevertheless, the theoretical literature mainly analyse the contractual aspects and implications of these investment, while it isn’t common to approach the analysis from a financial point of view. The main related contributions come from Engel et al. (2007, 2010). In the first paper they state the “irrelevance result” according to which there are no public financial advantages from PPPs with respect to TP due to the participation of the private financing. Indeed, PPPs cannot be justified because they free public funds inasmuch as the public sector current saving in terms of distortionary taxation is perfectly balanced by the future public budget costs in terms of potential revenues losses. Engel et al. (2010) contrasts the conventional perception regarding the higher PPPs cost of capital compared to the TP procedures. They sustain that, with adequate contracting, PPPs can replicate the intertemporal risk profile of public provision. Therefore, their risk premium reflects either the inefficiently assigned exogenous risks or the optimal transfer of endogenous risks that must be discounted in order to compute the correct cost of capital.

Differently from the previous literature, I developed a theoretical model with the presence of both moral hazard and adverse selection. The hidden information problem is related to the externality parameter that connects the two investment phases, while the hidden action issue concerns the non

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⁶At the first period contractual stage the good firm has the incentive to mimic the bad one knowing that the informed principal will be able to extract its surplus at the second period. This problem is known as ratchet effect
verifiable builder effort.

The study detects a potential benefit from doing PPPs in a context of contractible outcomes and incremental innovations. As contributions to the literature of reference, the paper permits to better understand how the government investment decision is affected when the externality parameter introduced by Iossa & Martimort (2008) and Martimort & Pouyet (2008) becomes uncertain. Furthermore, the analysis goes further with respect to Engel (2007) and shows how the shadow cost of public funds comes to be relevant in driving the government choice between TP and PPPs in a context of sequential investments with multiple sources of asymmetric information.

The following paragraphs are organized as follows. Section 3 lays out the model. Sections 4 and 5 discuss the unbundling and the bundling scenarios. Section 6 analyze the net surplus produced by the different scenarios through a welfare analysis. Section 7 concludes.

3 The Model

The government aims at the realization of a public infrastructure able to provide services for the citizens; the project is made by two phases: the construction of the public asset and the provision of services.

The realized facility generates a social surplus equal to $CS = S_0 + S * I(e_1)$. The surplus can be divided into two components: a constant term ($S_0$) that depends on the realization of the basic infrastructure and a second part that linearly depends on an incremental innovative investment $I(e_1)$ that is contractible and increasing with the builder’s effort ($I(e_1) = e_1 + \epsilon$ where $\epsilon \sim g(0, \sigma^2_\epsilon)$ and $g(\epsilon) \sim [\epsilon^l, \epsilon^h]$). The effort ($e_1$) is not verifiable by the government and it implies a non monetary disutility for the builder equal to $\psi(e_1)$ that, by assumption, satisfies the following properties: $\psi' \geq 0$, $\psi'' \geq 0$ and $\psi(0) = 0$.

In the analysis the government is assumed benevolent and able to commit for a long term project. It acts as a principal and sets the contracts in order to maximize the social welfare function.

7Following the examples presented before, $I(e_1)$ can be represented by the development of a metro without driver, the use of new materials for the construction of a motorway, the application of a new energy technology to a public building etc.
The function is composed by the sum of social surplus \((CS = S_0 + S * I(e_1))\) and the firms’ utilities \((U)\) net of the government’s expenses \((T)\) weighted with the shadow cost of public funds \((\lambda)\) that captures the distortion imposed to taxpayer in order to collect the money needed for the investment\(^8\).

The first phase of the project is entrusted to a builder whose utility is defined as follows:

\[
U_1 = T_1 - \psi(e_1)
\]  

(2)

The builder is in charge of the construction of the basic infrastructure, that entails a fixed cost that is totally reimbursed and a non monetary disutility of effort. As compensation, he receive a transfer that increases with the level of innovativeness introduced in the project.

The second phase activity is assigned to an operator that receives, as return for his services, the following utility.

\[
U_2 = T_2 - \left[O - e_2(\theta) - I(e_1)\theta\right] - \psi(e_2)
\]  

(3)

The return of the operator is composed by the gross transfer from the government net of the monetary cost \((C_2(\theta) = O - e_2(\theta) - I(e_1)\theta)\) and the non monetary disutility of effort \((\psi(e_2))\). The monetary cost is verifiable and observable, it is determined by: the fixed part \(O\), the cost reducing effort \(e_2\) and the impact of the first stage investment in the second phase. This last effect is driven by \(\theta\) that reflects the private information of the operator. This parameter defines whether or not the builder’s investment in innovation increases (negative externality \(\theta < 0\)) or reduces (positive externality \(\theta > 0\)) the operational costs\(^9\). The agent is able to acquire this information during the building phase when the main features of the infrastructure become observable\(^10\). The government cannot directly detect

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\(^8\)The model follows the approach used in the procurement model of Laffont & Tirole (1986)

\(^9\)For instance, an automatic without driver may reduce the need for drivers (positive externality). Nevertheless, innovative designs or materials for the construction of public buildings can increase the social surplus, but also the maintenance costs (negative externality).

\(^10\)This parameter can capture, for example, the impact of the development of an automatic metro into the operational costs. The government can forecast what will be the effect of this innovation, but only the operator is able to perfectly compare the saving of costs in terms of lower drivers’ salaries with the potential increase of expenses in terms of
the private information of the operator, but he can observe the distribution of the variable over a range of values: 
\[ f(\theta) \sim [\theta_l, \theta_h] \]
where \( \int_{\theta_l}^{\theta_h} \theta f(\theta)d\theta = \bar{\theta} \). For the purpose of this analysis, the possible forms that \( f(\theta) \) can take have been restricted to the class of piecewise differentiable functions that allow the use of the optimal control theory. A further standard requirement regards the hazard rate (\( \frac{F(\theta)}{f(\theta)} \)) that is assumed monotonic with respect to \( \theta \): \( d\left(\frac{F(\theta)}{f(\theta)}\right)/d\theta \geq 0 \) In addition to the monetary expenses, the operator experiences a non monetary cost for providing effort captured by the function \( \psi(e_2) \) that, by assumption, satisfies the same properties already defined for the builder effort: \( \psi' \geq 0, \psi'' \geq 0 \) and \( \psi(0) = 0 \).

For the achievement of the project the government can choose between two possibilities: unbundling and bundling. In the first case the two phases are managed by different firms, while in the second case there is a single private consortium that takes care of both stages.

4 Unbundling

Within the unbundling scenario the government chooses to undertake the two parts of the project through different agents: the builder and the operator. These players act autonomously and the government offers two distinct contracts.

In the first stage the government wants to maximize the builder’s effort, but it must cope with a problem hidden action; therefore, it can only offer an incentive contract based on the level of observable and verifiable outcome, i.e., a proportional transfer \( \{T_1 = I(e_1)t_1\} \) linking the builder’s compensation with the investment output \( I(e_1) \).

In the second stage the relation between the principal and the agent is influenced both by a problem of hidden information and hidden action. The government can offer a menu a incentive feasible contracts based on the verifiable outcome and able to induce the truthful revelation of the firm’s cost parameter, i.e., a revelation mechanism \( \{t(\hat{\theta}), C(\hat{\theta})\} \) that defines the cost that the

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11 this assumption implies the decreasing of effort with the increasing of the agent inefficiency.
firm has to realize and the net transfer it will receive when the cost parameter \( \hat{\theta} \) is announced\(^{12}\):

\[ \text{Within this scenario the contractual agreements are signed according to the following timeline.} \]

\[
\begin{array}{ccc}
\text{Contract with the} & \text{Builder implements} & \text{Operator implements} \\
\text{builder} & \text{effort} & \text{effort} \\
\hline
\text{\( \theta \) is realized} & \text{Contract with the} & \text{Social benefit} \\
& \text{operator} & \text{and costs are realized}
\end{array}
\]

The purpose of the government is to characterize the perfect Bayesian equilibria of the overall game; the solution of this problem is standard and is made through a backward induction strategy.

### Second stage of the game

In the second stage the principal set a contract in order to enhance the operator’s incentives to make effort with the minimum transfer of money: rent-efficiency trade off.

\[
\begin{align*}
\max_{e_2(\theta)} W^G_2 &= \int_{\hat{\theta}}^{\theta_b} \{ U_2(\theta) - (1 + \lambda)[t_2(\theta) + C_2(\theta)] \} dF(\theta) \\
\text{s.t.} & \quad 1- \quad \frac{dU_2}{d\theta} = -\psi'(e_2(\theta)) \\
2- U_2(\theta^b) &= 0
\end{align*}
\]

The government maximizes the net social function related to the second period taking into account the agent’s information constraints. The contract is offered at the ex post stage, once the agent already

\(^{12}\) As usual, we know from the revelation principle that any regulatory mechanism is equivalent to a direct revelation mechanisms that induce a truthful revelation of the firm’s cost parameter. This optimal regulatory mechanism can then be implemented at the optimum through a menu of linear contracts (Laffont & Tirole 1993)
knows his type, as a consequence the participation constraint is binding for the most inefficient operator that must receive at least his reservation utility (normalized to 0) in order to accept the contract [2]. Besides, the government must structure the transfers to optimally induce the truthful revelation of the private agent’s information [1]. The problem solution (appendix A.2) leads to the following optimal level of effort:

\[ e_2^U(\theta) = \psi'^{-1}[1 - \frac{\lambda}{1 + \frac{F(\theta)}{f(\theta)}} \psi''(e_2^U(\theta))] \tag{4} \]

The result is standard for the literature of reference, indeed, it underlines the role played by the asymmetric information that affects downward the optimal value of effort with respect to the first best results (appendix 1). If the monotone hazard rate property \( \frac{d(F(\theta)/f(\theta))}{d\theta} \geq 0 \) holds, the solution is decreasing with \( \theta \), therefore the distortion is lower more the firm is efficient. On the other hand, all the firms, except the least productive, receive a positive utility and more the agent is efficient more his information rent is high.

**First stage of the game**

At the beginning of the first period the government is able to propose a contract that includes both the parameters defining the first period social welfare and the expected second stage value function \((V_2)\) - see appendix 2) that considers the discounted surplus related to the managerial activity.

\[
\max_{e_1} \int_{e^l}^{e^h} \left\{ \left[ (S_0 + I(e_1) \cdot S) + U_1(e_1) - (1 + \lambda)T_1 \right] + [V_2] \right\} g(e)de \\
\text{s.t.} \quad 1- e_1 = \arg\max_{e_1} E[U_3] \\
2- E_\epsilon [T_1 - \psi(e_1)] \geq 0
\]

\[13\text{Otherwise, the efficient type would have the incentive to mimic the inefficient agent.} \]
The function of the government is composed by the social surplus deriving from the realization of the infrastructure, the expected second stage value function and the builder’s payoff net of the government’s costs weighted with the shadow cost of public fund. The random shock \( \epsilon \) is realized after the conclusion of the contract, therefore the participation constraint is defined ex ante [2]. The incentive compatibility constraint [1] takes into account the optimal operator’s effort choice for a given contract that comes from the maximization of his ex ante utility. The problem solution (appendix 2) leads to the following result:

\[
\psi'(e_U^1) + \lambda \frac{dT^*}{dc_1} = S + (1 + \lambda)\bar{\theta} \tag{5}
\]

where \( \frac{dT^*}{dc_1} = \psi'(e_U^1) + \psi''(e_U^1)e_U^1 \).

The first order condition equalizes the expected marginal benefit (right hand side) with the marginal cost (left hand size). Increasing the level of effort creates a current benefit for the society as well as a possible future saving of operation costs when the expected externality between the two phases is positive (\( \theta > 0 \))\(^{14} \). On the other hand, a stronger level of effort makes the operator suffer from a higher non monetary disutility and marginally increase the transfer at the optimum to the private agent. The main parameters entering equation 5 are the expected externality value (\( \bar{\theta} \)) and the shadow cost of public funds (\( \lambda \)). \( \bar{\theta} \) affects negatively or positively the marginal benefit depending on whether the investment realized during the first phase increases or decreases the costs needed to manage the infrastructure. \( \lambda \) captures the distortion imposed to taxpayers when public money are transferred to the private builder (\( \lambda \frac{dT^*}{dc_1} \)); moreover, it enlarges the expected positive or negative impact of the externality parameter ((1 + \( \lambda \))\( \bar{\theta} \)).

\(^{14} \) If the expected externality is negative (\( \theta < 0 \)), the total level of expected marginal benefit decreases.
4.1 A Robustness check: simultaneous contracts

The previous analysis describes what normally happens in real world situations where public procurement contracts related to multi-stage projects are signed when outcomes of previous phases are already observable by both partners. As an alternative to this strategy, the government could define ex ante all the contracts with the multiple agents, i.e., before the investment start. This option is theoretically feasible, but practically it is not implemented. Indeed, linking an agent’s obligations with future outcomes of different contracts is normally not allowed by the legal system and it is not considered by the economic context.

In this section I expand the previous framework allowing for the possibility to write simultaneous contracts from the beginning. This development represents a theoretical exercise that works as a robustness check in the analysis of the unbundling structure. Within this scenario, the timeline of the game is the following:

![Timeline Diagram]

The solution strategy is left to the appendix 2. The results finally obtained are described by the following equations.

\[
\psi'(e_1^S) + \lambda \frac{dT^*}{de_1} = S + (1 + \lambda)\theta \tag{6}
\]

where \( \frac{dT^*}{de_1} = \psi'(e_1^S) + \psi''(e_1^S)e_1^S \)

\[
e_2^S(\theta) = \psi'^{-1}[1 - \frac{\lambda}{1 + \lambda\frac{F(\theta)}{f(\theta)}} \psi''(e_2^S(\theta))] \tag{7}
\]

Equation 6 reports the optimal builder’s effort. With respect to the sequential contracts scenario,
the marginal benefit changes. Indeed, the positive or negative impact of the first phase investment to the second phase costs is driven by the real value of $\theta$ that is actually defined with the ex ante contract between the government and the operator. Equation 7 reports the optimal operator’s effort that doesn’t change with respect to the sequential contracts case\textsuperscript{15}.

5 Bundling

Within this setting the approach and the initial assumptions are very similar compared to the unbundling scenario; there is a single private agent (consortium) that sustains a cost over the two periods dependent on the same parameters as before and based on an ex ante information structure that doesn’t change with the new environment. The consortium receives a compensation for his activities that is defined as the sum of the builder and the operator utilities:

$$U_B = T_1(e_1) + T_2(\theta) - C(\theta) - \psi(e_1) - \psi(e_2(\theta))$$

(8)

The government can offer, in this case, a menu of incentive feasible contracts based on verifiable outcomes that must be able to induce the truthful revelation of the operator’s cost parameter and to enhance the first period investment, i.e., a triplet: $\{t_1(\hat{\theta}), t_2(\hat{\theta}), C_2(\hat{\theta})\}$ that respects the incentive constraints and defines costs and transfers when the private parameter $\hat{\theta}$ is announced.

Within this scenario, the timeline of the game takes the following form:

\textsuperscript{15}In this section I implicitly assume that the government cannot commit to leave the agent with a negative ex post utility; this hypothesis is plausible given that the operator gets the private information before the activity starts. Nevertheless, if it is not the case, the government’s optimal strategy consists in offering a fixed price contract: $t_2(C_2) = a - C_2$, where $a = \int_{\theta_l}^{\theta_h} \{\psi(e_2(\theta)) + C_2(\theta)\} d\theta$. The operator, as residual claimant, makes the efficient decision, receives no rents in expectation and takes the risk of having a negative ex post utility. Developing the analysis in this direction doesn’t change the core results of the paper.
The screening strategy proposed by the government forces the consortium to truthfully reveal his private information when it becomes observable. The government maximizes the net social welfare produced by the project over the two periods taking into account the incentive feasible constraints.

\[
\max_{e_1, e_2(\theta)} \int_{e_1}^{b} \left\{ \int_{0}^{\theta} \left\{ \left[ S_0 + I(e_1) \ast S + U_B - (1 + \lambda)[I(e_1)t_1 + t_2(\theta) + C_2(\theta)] \right] f(\theta)d\theta \right\} f(\epsilon)d\epsilon \right\}
\]

s.t.

1- \[ e_1 = \arg \max_{\epsilon_1} E[U_B(e_1)] \]

2- \[ \frac{dE[U_B]}{d\theta} = -\psi'(e_2(\theta)) \]

3- \[ E[\psi(e_1) - t_1 - \psi(e_1) - t_2(\theta) - \psi(e_2(\theta))] \geq 0 \]

The government goals are the maximization of the social surplus and the agents’ rents extraction. On the other hand, it must take into consideration the firms’ incentives and interests that are embodied in the three constraints. The first equation incentivizes the agent to make more of an effort during the building phase [1], the second equation represents the mechanism needed to obtain a truthfully revelation of the private information parameter [2], while the third equation reflects the participation constraint that is binding for the more costly type [3]. The maximization solution (appendix 2) leads to the following outcomes:

\[
\psi'(e_1^B) + \lambda \frac{dT_B}{de_1} = S + (1 + \lambda)\theta
\]

where \( \frac{dT_B}{de_1} = \psi'(e_1^B) \)
\[ e_2^B(\theta) = \psi^\prime [1 - \frac{\lambda}{1 + \lambda f(\theta) \psi''(e_2(\theta))] \]  

(10)

The operator’s level of effort (Equation 9) doesn’t change at the optimum with respect to the unbundling case solution\(^{16}\). Equation 10 reports the builder’s effort. Differently from the unbundling scenario, the contract with the consortium is made before the investment start. Therefore, thanks to the revelation mechanism, the information becomes contractible since the first stage and the optimal level of builder’s effort can be set on the base of the real value of \( \theta \) announced by the operator. A further difference with the unbundling context (both sequential and simultaneous contracts) comes from the distortionary cost that the society supports for a marginal transfer from the government to the private agent. In case of PPPs the principal has the opportunity to leave the consortium with no ex ante private rent at the optimum affecting downward the total marginal cost of effort. This is possible because the first period rent that the government transfers to the agent for inducing an optimal level of effort can be recovered with the reduction of the second stage transfer, while the optimal incentive scheme created by the contract remains not affected (appendix 3).

Similarly to the previous case, the main parameters entering the outcomes equations are the shadow cost of public funds \( \lambda \) and the externality parameter \( \theta \). \( \lambda \) increases the marginal cost of effort and it enlarged the effect of the externality. \( \theta \) captures the positive or negative impact of the first phase investment in the second period costs; differently from the unbundling scenario, in this case, the builder’s investment decision is directly affected by \( \theta \) and not only by its average value.

\(^{16}\) As I made for the unbundling case with simultaneous contracts, in this section I implicitly assume that the agent cannot commit not to exit the contract when he discovers \( \theta \); this hypothesis is plausible given that the operator gets the private information before the activity starts. Nevertheless, if it is not the case, the government’s optimal strategy consists in offering a second term fixed price contract: \( t_2(C_2) = a - C_2 \), where \( a = \int_{\theta_1}^{\theta_2} (\psi(e_2(\theta)) + C_2(\theta))d\theta \). In such a situation, the bundling mechanism could furtherly increase the builder’s effort inasmuch as the distortion induces by the adverse selection issue will disappear. Saying differently, if the analysis is enlarged relaxing this assumption, the potential benefits of PPPs will increase.
6 Welfare analysis

In this section we compare the bundling and the unbundling scenarios in terms of ex ante social welfare with the purpose of detecting which are the determinants that drive the choice of a government facing an informational setting similar to the one proposed in this paper.

The Welfare analysis is performed using the expected objective function of the government within the two periods: $E_{\bar{e},c}[S_0 + S * I(e_1) + U_1 + U_2 - (1 + \lambda)(t_1 + t_2 + C_2)]$. In this paragraphs it is assumed that $\psi(e_1) = \frac{e_1^2}{2}$ and $\psi(e_2) = \frac{e_2^2}{2}$. These functions respect the initial hypotheses of the model and allow us to compute the value of the agents’ efforts at the optimum. The analysis is reported in appendix 4; the result is summarized by the following formula that describes the difference in value functions between the social welfares produced under bundling and unbundling.

$$V_B - V_U = \{RS\} + \{IE\} = \left\{ \frac{\lambda}{2(1 + \lambda)(1 + 2\lambda)}(S + (1 + \lambda)\bar{\theta})^2 \right\} + \left\{ \frac{(1 + \lambda)}{2}\sigma^2 \right\}$$  \hspace{1cm} (11)

where

$RS = \frac{\lambda}{2(1 + \lambda)(1 + 2\lambda)}(S + (1 + \lambda)\bar{\theta})^2$ - Rent Saving

$IE = \frac{(1 + \lambda)}{2}\sigma^2$ - Information Externality

The result can be decomposed in two effects that are explained in details through the following definitions

**Rent Saving effect (RS):** This effect is always positive or equal to zero. It reflects the ex ante marginal benefit of the society $(S + (1 + \lambda)\bar{\theta})$ coming from the increasing builder’s investment under the bundling scenario, when $\lambda > 0$. PPPs allow the principal to recover the first period incentive rents during the operational phase without affecting the second period incentive compatibility constraint. As a consequence, the government can optimally align the agent’s incentives maximizing the total rents’ extraction\textsuperscript{17}. This effect is meaningful inasmuch as the performance contracts are costly for the society $(\lambda \geq 0)$. Moreover, the additional transfer of risks to the private consortium is not costly due to the context of risk neutrality; the introduction of the agents’ risk aversion would have changed the final results.

**Information Externality effect (IE):** This effect is always positive or equal to zero. PPPs commit the government to define a more informed investment plan taking into account every short term and long term correlations between the builder’s investment and the future phases of the project. Precisely, bundling the two tasks allows the government to internalize the operator’s private

\textsuperscript{17}The rent efficiency trade off is more slack.
information in the builder’s innovative investment. This effect increases with the uncertainty of the private information parameter; therefore, if the variance decreases, the private information is less worthy for the operator and there is a lower benefit from choosing PPPs.

These effects lead to the following proposition that summarizes the main result of the paper:

**Proposition 1:** In a context of risk neutrality and when complete contracts based on verifiable outcomes are feasible, Bundling strictly dominates Unbundling.

The results described by propositions 1 and 2 are driven by two parameters: $\theta$ and $\lambda$. $\theta$ reflects the private information parameter of the operator and it is the key variable explaining the IE effect. On the other hand, besides being the main determinant of the RS effect, $\lambda$ is also able to influence the IE effect that increases where the shadow cost of public funds is higher. This last observation highlights an important result: the more the government is constrained in collecting funds the more it can benefit from a more informed investment plan that could allow the ex post saving of costs.

From a comparative statics analysis (appendix 5), we can compute the dynamics of the RS and The IE effects when the main parameters entering the two formulas change.

The RS effect depends from $\lambda$ and $\bar{\theta}$. With respect to the shadow cost of public funds, the dynamics of the RS effect are the following:

- when $\lambda = 0$, the RS effect is equal to 0; indeed, the transfer of funds from the public to the private agent is costless for the society,
- when $\lambda$ increases, the RS effect follows a positive trend for lower values of $\lambda$ and for a positive $\bar{\theta}$; nevertheless, the trend is negative.

With respect to the expected externality value, the dynamics of the RS effect are the following:

- when $S > -(1 + \lambda)\bar{\theta}$, the RS effect increases with $\bar{\theta}$, i.e., when there is a positive expected externality or a sufficiently low negative externality,
• when $S < -(1 + \lambda)\bar{\theta}$, the RS effect decreases with $\bar{\theta}$, i.e., when the marginal impact in terms of higher costs of the negative externality is higher than the marginal short term benefit of the innovative investment.

On the other hand, both $\lambda$ and $\sigma^2_\theta$ positively affect the trend of the IE effect.

As robustness check, I discuss in the appendix 4 the welfare analysis under simultaneous contracts. It is interesting to see how the benefit of bundling still holds. The RS effect doesn’t change with the new environment, while the IE effect is lower and strictly dependent on $\lambda$, but still positive.

7 Conclusion

This paper can be introduced within the strand of literature that emphasizes a particular feature of a PPP: the bundling mechanism. Differently from most of the existing works, in this model the government is able to write complete contracts and the externality between the two phases is privately known to the operator and can be positive or negative. Furthermore, the investment is uncertain and the final surplus is related with the builder’s effort. The results highlight three effects: the production externality, the information externality and the rent saving effects. These impacts derive from the interconnection between the moral hazard issue related with the first stage and the hidden information problem affecting the relation between the principal and the operator. This type of setting is quite common in real world experiences inasmuch during the first period a problem of hidden action normally arises, while the variables related with future determinants are, in most of the cases, uncertain and cause of asymmetric information among agents.

The existing literature highlights how bundling two tasks involves a benefit only when the externality between the two phases is positive, therefore only when the first stage investment decreases the operational costs (Iossa and Martimort 2008, Martimort and Pouyet 2008). This paper provides further explanations for choosing a bundling governance besides the described literature advantage;
indeed, it exists a potential benefit even when there is a negative externality. More precisely, bundling
the two tasks allows the government to internalize, through the contract, the future impact of the
investment into the second phase; considering this externality, the principal has the possibility to set
a more coherent investment plan able to either incentivize the builder’s effort, when the externality is
positive, or avoiding excessive operational costs and rents, when the externality is negative. This effect
can explain the classical empirical evidence according to which ex post costs are lower in case of PPPs
with respect to TP despite the ex ante predictions. The result derives from the optimal transfer of risks
in a context of complete contracts and it is driven by the private information parameter and by the
shadow cost of public funds. The latter variable is able to influence the organizational choice inasmuch
as bundling the two phases with the presence of endogenous risks allows the principal to create an
implicit incentive to invest due to the longer time perspective of the private agent. As a consequence,
the government, offering adequate performance contracts, is able to save money both in the first phase
that is characterized by a problem of hidden action and in the second phase where the informational
rents can be reduced keeping the participation and the incentive compatibility constraints satisfied.
The result provides an efficiency reason for suggesting PPPs in a context of public financial disruption;
this explanation doesn’t contrast the “irrelevance theorem” of Engel et al. (2007) because the benefit
is not due to the participation of the private financing, but from the optimal transfer of risks in a
context of interrelated asymmetric information problems. In such an environment, PPPs allow the
government to optimally set his investment plan creating the right incentives for the private agents
and considering every future potential cost connected with the current investment level.

As a conclusion, The adoption of a PPP is particularly suggested, according to this paper, when
the government is able to write complete contracts and when there are investments with uncertain
future determinants and implications.

Several extensions can enrich the current analysis.

First of all, agents are considered risk neutral in the paper; the introduction of some degrees of risk
aversion is able to capture the possible asymmetric cost (between bundling and unbundling) caused
by the transfer of risks from the principal to the agents.

As a second point, it has been assumed that there are no asymmetry of objectives within the
bundled consortium. Greco (2012) introduces the concept of imperfect bundling under an incomplete contracts framework. It would be interesting to test the impact of this distortion following a complete contract approach similar to the one proposed in this paper.

As further developments of the model, the possibility of ex post adaptation investments could be introduced or, as an alternative, the abilities of the government to write complete contracts and to perfectly distinguish the two phases of the project could be mitigated. In the first case, PPPs could lack the needed degree of flexibility and the space for the bundled choice could be restricted (Iossa and Martimort 2011); On the other hand, the development of the second point could generate a possibility of collusion within the consortium that determines a cost from bundling the two tasks.

Finally, it is important to highlight that the previous study is based on the assumption that the government can commit for a long term period. If this is not the case, the principal has the incentive to extract all the rent concerning the second stage of the game after the revelation of the operator type with the initial contract. Anticipating this strategy, the consortium wants to maximise its first period payoff and the separating contract is not always implementable. This problem has been already investigated by Valero (2012) that shows as, even under government opportunism, there is space for welfare improving PPPs. The analysis of this issue within the presented model could be an interesting test to understand the extension of the final results.
References


Appendix 1: First Best Benchmark

The government maximizes the total welfare function, there are no problems of hidden information or hidden action

\[
\max_{e_1, e_2(\theta)} \int_{-\epsilon}^{\epsilon} \{S_0 + I(e_1) \ast S - (1 + \lambda) [I(e_1)t_1 + t_2(\theta) + C_2(\theta)] \\
+ I(e_1)t_1 + t_2(\theta) - \psi(e_1) - \psi(e_2(\theta))\} g(\epsilon) d\epsilon
\]

The government can totally extract agents’ rents

\[
\max_{e_1, e_2(\theta)} \int_{-\epsilon}^{\epsilon} \{S_0 + I(e_1) \ast S - (1 + \lambda) [\psi(e_1) + \psi(e_2(\theta)) + C_2(\theta)]\} g(\epsilon) d\epsilon
\]

Optimizing w.r.t. \(e_1\) and \(e_2(\theta)\) yields respectively

\[
\psi'(e_{FB1})(1 + \lambda) = S + (1 + \lambda) \theta
\]

\[
\psi'(e_{FB2}(\theta)) = 1
\]

Appendix 2: Proof of the Unbundling Problem

Let us solve the problem backward

Second stage of the game

The government maximizes the second stage welfare function taking into account the operator’s incentive constraints

\[
\max_{e_2(\theta)} \int_{(\theta)}^{\phi} \{t_2(\theta) - \psi(e_2(\theta)) - (1 + \lambda) [t_2(\theta) + O - e_1(\theta) - e_2(\theta)]\} dF(\theta)
\]

s.t.
\[ \frac{dU_2}{de_2} = -\psi'(e_2(\theta)) \]

\[ U_2(\theta^h) = 0 \]

The ex post participation constraint is binding for the least efficient agent. The incentive compatibility constraint that derives from the application of the envelope theorem allows us to compute the agent’s utility
\[ U(\theta) = \int_{\theta^h}^{\theta_l} \psi(e_2(\tilde{\theta}))d\tilde{\theta} + U(\theta^h) \]

Integrating by parts we can compute the expected rent granted to the operator by the principal
\[ \int_{\theta^h}^{\theta_l} \{ U(\theta) \} f(\theta)d\theta = \int_{\theta^h}^{\theta_l} \{ f(\theta) \} f(\theta)d\theta = \int_{\theta^h}^{\theta_l} \{ \frac{F(\theta)}{f(\theta)} \psi'(e_2(\theta)) \} f(\theta)d\theta \]

Substituting the constraints into the government’s function, we obtain the principal’s optimization problem

\[ \max_{e_2(\theta)} W_2 = \int_{\theta^h}^{\theta_l} \{ -(1 + \lambda)[O - e_1 \theta - e_2(\theta) + \psi(e_2(\theta))] - \lambda \left( \frac{F(\theta)}{f(\theta)} \psi'(e_2(\theta)) \right) \} f(\theta)d\theta \]

Optimizing w.r.t. \( e_2 \) yields to the optimal level of effort, just like is reported in the text

\[ \psi'(e_2^U(\theta)) = 1 - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \psi''(e_2^U(\theta)) \]

Substituting the optimal level of effort in the objective function of the government and solving the integral, we obtain the value function for the government

\[ V_2 = -O + (1 + \lambda)e_1 \bar{\theta} - \int_{\theta^h}^{\theta_l} \{ (1 + \lambda)[-e_2^U(\theta) + \frac{(e_2^U(\theta))^2}{2} + \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \psi''(e_2^U(\theta))] \} f(\theta)d\theta \]

**First stage of the game**

The government maximizes the sum of the first stage welfare function and the second stage value function taking into account the builder’s incentive constraints.
\[
\max_{e_1} \int_{\delta}^{\theta} \{ S_0 + I(e_1) * S - (1 + \lambda)I[\psi(e_1)T] + I(e_1)T_1 - \psi(e_1) + V_2 \} g(\epsilon) d\epsilon
\]

s.t.
\[
1- \quad \frac{dE_1[U_1]}{de_1} = 0
\]
\[
2- \quad E_1[I(e_1)t_1 - C_1 - \psi(e_1)] \geq 0
\]

From the incentive compatibility constraint we can compute the marginal transfer at the equilibrium \( t_1 = \psi'(e_1) \); from the participation constraint we obtain the ex ante expected utility: \( E_1[U_1] = e_1T_1 - \psi(e_1) \). Substituting into the government’s function, we get the principal’s optimization problem.

\[
\max_{e_1} W_1 = e_1 + S - (1 + \lambda)[e_1\psi'(e_1)] + e_1\psi'(e_1) - \psi(e_1) + V_2
\]

Optimizing w.r.t. \( e_1 \) yields
\[
\psi'(e_1^U) + \lambda[\psi'(e_1^U) + \psi''(e_1^U)e_1^U] = S + (1 + \lambda)\theta
\]

where \([\psi'(e_1^U) + \psi''(e_1^U)e_1^U] = \frac{dT}{de_1}\).

**A Robustness check: simultaneous contracts**

From a theoretical point of view, the government can offer both the operator’s and the builder’s contracts before the investment start. Within this framework, the government maximizes the total welfare function taking into account the incentive constraints

\[
\max_{e_1, e_2(\theta)} \int_{-\epsilon}^{+\epsilon} \{ \int_{\delta}^{\theta} \{ S_0 + I(e_1) * S - (1 + \lambda)[I(e_1)T_1 + t_2(\theta) + C_2(\theta)] + I(e_1)t_1 + t_2(\theta) - \psi(e_1) - \psi(e_2(\theta)) \} f(\theta)d\theta \} g(\epsilon) d\epsilon
\]
s.t.

1- \( \frac{dE_1[U_1]}{de_1} = 0 \)

2- \( E_1[U_1] \geq 0 \)

3- \( \frac{dU_2}{dp} = -\psi'(e_2(\theta)) \)

4- \( U^h_2 \geq 0 \)

Substituting the constraints into the government’s objective function we obtain the principal’s maximization problem.

\[
max_{e_1, e_2(\theta)} WS = \int_{\theta} \{ -\epsilon_1 \theta + \epsilon_2(\theta) \} f(\theta) d\theta
\]

Optimizing w.r.t. \( e_1 \) and \( e_2(\theta) \) yields respectively

\[
\psi'(e_1^S) + \lambda \frac{dT^*}{de_1} = S + (1 + \lambda) \theta
\]

\[
\psi'(e_2^S(\theta)) = 1 - \frac{\lambda F(\theta)}{1 + \lambda} \psi''(e_2^S(\theta))
\]

**Appendix 3: Proof of the Bundling Problem**

The government maximizes the total welfare function taking into account the consortium’s incentive constraints.

\[
max_{e_1, e_2(\theta)} \int_{\theta} \{ \int_{\theta} \{ -\epsilon_1 \theta + \epsilon_2(\theta) \} f(\theta) d\theta \} g(\epsilon) d\epsilon
\]
s.t.

1- \[ \frac{dE[U_B]}{de_1} = 0 \]

2- \[ \frac{dE[U_B]}{de_2} = -\psi'(e_2(\theta)) \]

3- \[ E[I(e_1)t_1 + t_2(\theta^h) - \psi(e_1) - \psi(e_2(\theta^h))] \geq 0 \]

From the incentive compatibility constraint related to the first phase of the project we obtain \( t_1 = \psi'(e_1) \). Substituting in the participation constraint we get the following ex ante utility: \( E[U_B] = e_1\psi'(e_1) + t_2(\theta^h) - \psi(e_1) + \psi(e_2(\theta^h)) = 0 \). Given that the government aims at the agent’s rent extraction and considering that the consortium is not protected by a limited liability constraint, the principal can set the second period transfer equal to \( t_2(\theta^h) = \psi(e_1) + \psi(e_2(\theta^h)) - e_1\psi'(e_1) \). As a consequence, the participation constraint is ex ante binding for the least efficient operator, while the private incentive compatibility constraints remain effective. The obtained government’s welfare function is defined as follows

\[
\max_{e_1, e_2(\theta)} W_B = \int_{\theta_l}^{\theta_h} \left\{ S_0 + e_1S - (1 + \lambda)[\psi(e_1) + \psi(e_2(\theta)) + O - e_1\theta - e_2(\theta)] \\
- \lambda\left[ \frac{F(\theta)}{f(\theta)} \psi'(e_2(\theta)) \right] \right\} f(\theta) d\theta
\]

Optimizing w.r.t. \( e_1 \) and \( e_2(\theta) \) yields respectively

\[ \psi'(e_1^B) + \lambda \frac{dT_B}{de_1} = S + (1 + \lambda)\theta \]

where \( \frac{dT_B}{de_1} = \psi'(e_1^B) \)

\[ \psi'(e_2^B(\theta)) = 1 - \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \psi''(e_2^B(\theta)) \]

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Appendix 4 - Welfare Analysis

The expected function that is used to perform the comparative statics analysis is the following one:

\[
\int_{\theta'} \left\{ S_0 + e_1 S - (1 + \lambda)[e_1 t_1 + t_2(\theta) + O - e_1 \theta - e_2(\theta)] \\
+ (e_1 t_1 - \psi(e_1)) + (t_2(\theta) - \psi(e_2(\theta))) \right\} f(\theta) \, d\theta
\]

Using the new effort functions the first order conditions in the bundling case become:

\[
e^{B}_1 = \frac{S + (1 + \lambda)\theta}{1 + \lambda} \\
e^{B}_2(\theta) = 1 - \frac{\lambda}{1 + \lambda} \frac{f(\theta)}{F(\theta)}
\]

Substituting in the government objective formula we obtain the value function under bundling

\[
V^{B} = \int_{\theta'} \left\{ S_0 + S\epsilon^{B}_1 - (1 + \lambda)\left[\frac{(\epsilon^{B}_1)^2}{2} + \frac{(\epsilon^{B}_2)^2}{2} + O - \epsilon^{B}_1 \theta - \epsilon^{B}_2(\theta)\right] - \lambda \frac{F(\theta)}{f(\theta)} \psi''(e_2(\theta)) \right\} f(\theta) \, d\theta
\]

Differences between the two scenarios come from the builder’s effort; hence the analysis is developed using only factors dependent on \(e^{B}_1\)

\[
V^{B} = \int_{\theta'} \left\{ S_0 + S\epsilon^{B}_1 - (1 + \lambda)\left[\frac{(\epsilon^{B}_1)^2}{2} + \frac{(\epsilon^{B}_2)^2}{2} + O - \epsilon^{B}_1 \theta - \epsilon^{B}_2(\theta)\right] - \lambda \frac{F(\theta)}{f(\theta)} \psi''(e_2(\theta)) \right\} f(\theta) \, d\theta
\]

The new efforts functions applied to the unbundling case yields respectively

\[
e^{U}_1 = \frac{S + (1 + \lambda)\theta}{1 + 2\lambda} \\
e^{U}_2(\theta) = 1 - \frac{\lambda}{1 + 2\lambda} \frac{f(\theta)}{F(\theta)}
\]

Substituting in the government objective formula we obtain the value function under unbundling

\[
V^{U} = \int_{\theta'} \left\{ S_0 + S\epsilon^{U}_1 - \left(1 + 2\lambda\right) - (1 + \lambda)\left[\frac{(\epsilon^{U}_1)^2}{2} + O - \epsilon^{U}_1 \theta - \epsilon^{U}_2(\theta)\right] - \lambda \frac{F(\theta)}{f(\theta)} \psi''(e_2(\theta)) \right\} f(\theta) \, d\theta
\]

Differences between the two scenarios come from the builder’s effort; hence the analysis is developed using only factors dependent on \(e^{U}_1\)
\[ V_U = \int_\theta^\pi \left\{ \frac{S^2}{1+2\lambda} - \frac{\theta S(1+\lambda)}{1+2\lambda} - \frac{\theta^2 S^2(1+2\lambda)}{2(1+2\lambda)} + \frac{\theta^2 (1+\lambda)^2}{2(1+2\lambda)} - (1 + \lambda)[-\frac{\theta S}{1+2\lambda} - \frac{\theta^2 (1+\lambda)}{1+2\lambda}] \right\} f(\theta) d\theta \]

\[ V_B - V_U = \frac{\lambda S^2}{2(1+\lambda)(1+2\lambda)} + \frac{\lambda S}{1+2\lambda} + \frac{(1+\lambda)\sigma^2_\theta}{2(1+2\lambda)} + \frac{\lambda (1+\lambda)}{2(1+2\lambda)} \]

\[ V_B - V_U = \frac{\lambda}{2(1+\lambda)(1+2\lambda)} (S^2 + (1 + \lambda)\theta^2) + 2(1 + \lambda)\theta S + \frac{(1+\lambda)\sigma^2_\theta}{2} \]

\[ V_B - V_U = \frac{\lambda}{2(1+\lambda)(1+2\lambda)} (S + (1 + \lambda)\theta)^2 + \frac{(1+\lambda)\sigma^2_\theta}{2} \]

A Robustness check: simultaneous contracts

\[ V_S^B = \int_\theta^\pi \left\{ \frac{S^2}{1+2\lambda} - \frac{S\theta(1+\lambda)}{1+2\lambda} - \frac{S^2(1+\lambda)\theta S}{2(1+2\lambda)} + \frac{\theta^2 (1+\lambda)^2}{2(1+2\lambda)} - (1 + \lambda)[-\frac{S}{1+2\lambda} - \frac{\theta S}{1+2\lambda}] \right\} f(\theta) d\theta \]

\[ V_S^U = \int_\theta^\pi \left\{ \frac{S^2}{2(1+2\lambda)} + \frac{(1+\lambda)\theta S}{1+2\lambda} - \frac{\theta^2 S^2(1+2\lambda)}{2(1+2\lambda)} + \frac{\theta^2 (1+\lambda)^2}{1+2\lambda} \right\} f(\theta) d\theta \]

\[ V_S^U = \frac{S^2}{2(1+2\lambda)} + \frac{(1+\lambda)\theta S}{1+2\lambda} + \frac{(1+\lambda)^2}{2(1+2\lambda)} E[\theta^2] \]

\[ V_B - V_S^B = \frac{\lambda S^2}{2(1+\lambda)(1+2\lambda)} + \frac{\lambda S}{1+2\lambda} + \frac{\lambda (1+\lambda)}{2(1+2\lambda)} E[\theta^2] \]

\[ V_B - V_S^U = \frac{\lambda}{2(1+\lambda)(1+2\lambda)} (S^2 + (1 + \lambda)\theta^2) + 2(1 + \lambda)\theta S + \frac{\lambda (1+\lambda)}{2(1+2\lambda)} \sigma^2_\theta \]

\[ V_B - V_S^U = RS + \frac{\lambda}{(1+2\lambda)} IE \]

Appendix 5 - Comparative Statics Analysis

Comparative statics analysis of the RS effect with respect to \( \lambda \)
\[ R_S = \frac{\lambda}{2(1+\lambda)(1+2\lambda)} (S + (1 + \lambda)\bar{\theta})^2 \]

\[ \frac{dR_S}{d\lambda} = (S + (1 + \lambda)\bar{\theta})^2 (\frac{2(1+\lambda)(1+2\lambda)-2(3+4\lambda)\lambda}{4(1+\lambda)^2(1+2\lambda)^2}) + \frac{\lambda}{2(1+\lambda)(1+2\lambda)} 2\bar{\theta}(S + (1 + \lambda)\bar{\theta}) \]

\[ \frac{dR_S}{d\theta} = (S + (1 + \lambda)\bar{\theta})^2 (\frac{2(1+3\lambda+2\lambda^2)-6(6+8\lambda)\lambda}{4(1+\lambda)^2(1+2\lambda)^2}) + \frac{\lambda}{2(1+\lambda)(1+2\lambda)} 2\bar{\theta}(S + (1 + \lambda)\bar{\theta}) \]

\[ \frac{dR_S}{d\lambda} = (S + (1 + \lambda)\bar{\theta}) \left[ \frac{(1-2\lambda^2)}{2(1+\lambda)^2(1+2\lambda)^2} \right] (S + (1 + \lambda)\bar{\theta}) + \frac{\lambda}{2(1+\lambda)(1+2\lambda)} 2\bar{\theta} \]

Comparative statics analysis of the RS effect with respect to \( \bar{\theta} \)

\[ R_S = \frac{\lambda}{2(1+\lambda)(1+2\lambda)} (S + (1 + \lambda)\bar{\theta})^2 \]

\[ \frac{dR_S}{d\theta} = \frac{\lambda^2}{2(1+\lambda)(1+2\lambda)^2} 2(S + (1 + \lambda)\bar{\theta}) \]

Comparative statics analysis of the IE effect with respect to \( \lambda \)

\[ IE = \frac{(1+\lambda)}{2} \sigma^2_{\theta} \]

\[ \frac{dIE}{d\lambda} = \frac{\sigma^2_{\theta}}{2} \]

Comparative statics analysis of the IE effect with respect to \( \sigma^2_{\theta} \)

\[ IE = \frac{(1+\lambda)}{2} \sigma^2_{\theta} \]

\[ \frac{dIE}{d\sigma^2_{\theta}} = \frac{(1+\lambda)}{2} \]