A comprehensive model for the evaluation of the local governments expenditure needs and the essential level of local services
Preliminary and incomplete

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Abstract

This paper aims at implementing a statistical model which simultaneously estimates the standard level of local government expenditure needs and the standard level of local government output. First we introduce a theoretical model which highlights the ultimate goal of the process of expenditure needs equalisation. Subsequently we develop a methodology that simultaneously estimates local governments expenditure needs and standard (minimal) level of output. Data from English local authorities over the period 1997-2007 have been used to test the model. In particular the choice of using English data is motivated by the possibility of checking the validity of the model against the ranking of local authorities provided by the Comprehensive Performance Assessment (CPA) adopted in England. On the econometric side, we propose robust methods to compute composite indicators by the way of frontier efficiency methods, and robust methods to estimate production frontiers using Order-*m* and Multivariate Adaptive Regression splines "two stage" estimators. Finally, we propose to estimate expenditure needs using quantile regression.

Keywords: Expenditure needs, Local government evaluation, Productive efficiency, Level of service
JEL classification: *H72, C22, C14* .

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1. Introduction

This research is focused on the construction of a statistical model which estimates local governments expenditure needs together with the standard level (minimal and optimal) of the local services which must be provided by the local authorities.

The construction of a system based on quantitative methods which simultaneously evaluates local expenditure needs and local output standards is motivated by a void in the literature on the measurement of local government expenditure needs. In fact, all of the main research papers [[see e.g.OECD (1981), Reschovsky (2006), Blochliger et al. (2007), Dafflon and Mischler (2007)], do not consider the intimate link that exists between the standard level of local expenditure and the standard level of local services, since the attention is focused mainly on the financial side of the problem.

It is important to stress that without considering the standard level of services which local authorities must provide in relation to their standard level of expenditure, the whole process of intergovernmental fiscal equalisation may not achieve its final end: the production of local services which provide the minimum level of welfare to all citizens independently on the socio-economic characteristics of the jurisdiction in which they live.

Usually, the problem of the standard level of local services is viewed exclusively as a political problem, and as a result is rarely treated with the support of statistical methods. This paper aims to contrast this view, showing that in line with the computation of local expenditure needs, the standard level of local services can also be determined using quantitative methods.

The introduction of the concept of standard output, in line with the concept of standard expenditure is vital in order to avoid the limited discretion achieved on the financial side being neutralised by the discretion left on the output side. Indeed, were this to occur, it would produce a dangerous misalignment between local output and local expenditure. In the end, only the provision of services above the standard should be left exclusively to politics in line with the degree of local government autonomy. Moreover, this analysis can create a bridge between the literature on intergovernmental fiscal equalisation and the literature on local government performance indicators and incentives in the public sector (see e.g.Mizell (2008), Burgess and Ratto (2003), Lockwood and Porcelli (2011)).

The first step in our analysis will be the construction of a theoretical model which highlights the ultimate goal of the process of standardisation with regards to local expenditure and local output; that is, a theoretical model of expenditure needs equalisation. A model that, as far as we could verify is still missing in the literature.

The second step will be the construction of a statistical model that allows to overcome three main problems in the simultaneous estimation of expenditure needs and standard levels of output. The first problem is that one cannot directly estimate the relationship between expenditure and output which keeps constant citizens welfare because both values are determined simultaneously from the interaction between supply and demand of local public goods and seldom one can find good instrumental variables to overcome this issue in a classical two stage framework; the second problem is that the evaluation of the standard (minimal) level of output must be at the same time feasible in terms of central government budget constraint and efficient from the technical and financial point of view; finally the third problem is related to the presence of multi-output production function which makes the estimation of demand functions not always possible.

The rest of the paper is structure as follows: section 2 develops the theoretical model of expenditure needs equalisation, section 3 summarises the main features of the statistical model, section 4 illustrate the strategy followed for the simulation, section 5 presents the results obtained using English data, section 6 concludes.

2. A theoretical model of expenditure needs equalist

This section develops a simple theoretical model which highlights the ultimate goal of the process of standardization with regards to local expenditure and local output; that is, a theoretical model of expenditure needs equalization.

The main features of the model are reported in Figure 1. Graph 1, on the left, shows the indifference curves between the standard level of local services and the degree of deprivation which keep constant the welfare level of the representative citizen. The welfare level increases toward the north-west. The standard level of the representative citizen's welfare is set in correspondence to the highest indifference curve (W_2) tangent to the central government budget constraint, whose slope is represented by the average tax-price necessary to finance the standard level of local public services. Point F represents the average standard level of services which corresponds to the average standard expenditure reported on the horizontal axis of Graph 2. Indeed, Graph 2 on the right displays the relationship between the standard level of output and the standard level of expenditure which keeps the representative citizen welfare fixed. Output-expenditure combinations such as A, below the minimum level of output, should move toward B. Finally, local authorities characterized by output-expenditure combinations like E are beyond the maximum expenditure standard, hence they should finance their extra level of local services using their own fiscal revenue.

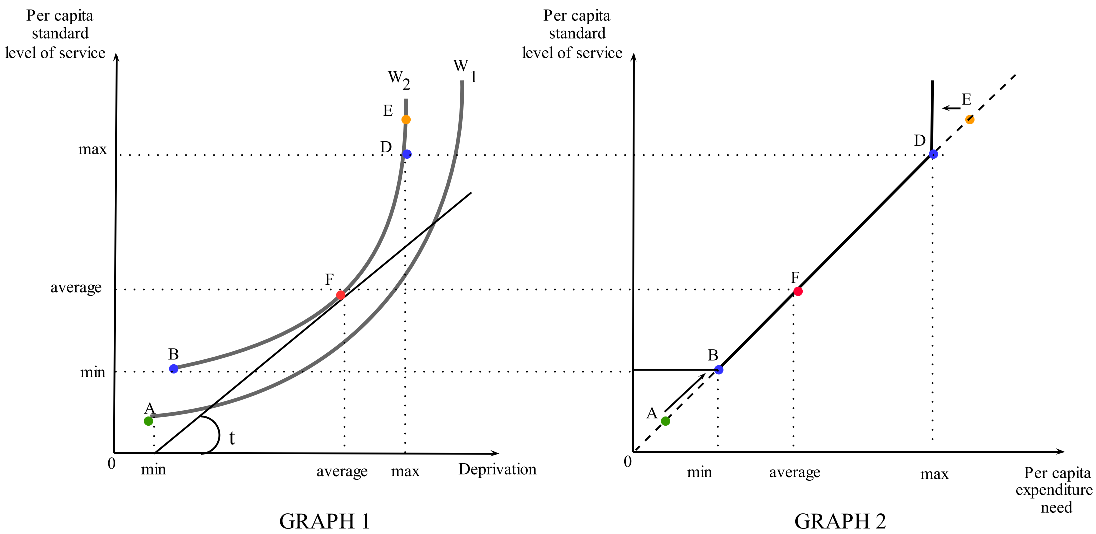

Figure 1: Theoretical model of expenditure needs equalization

The theoretical model demonstrates that the standard level of output which equalizes citizens welfare (also called the essential level of local services) cannot be the same across jurisdictions. Only the minimum threshold can be set at a unique level. Thus far this issue has never been addressed properly in the literature, as a result, erroneously, the essential level of services and the minimum level of services have often been treated as the same measure.

[THE ANALYTICAL VERSION OF THE MODEL IS IN PROGRESS]

3. The statistical model

The main problem that the statistical model described in this section tries to address is that one cannot directly estimate the relationship between expenditure and output which keeps constant citizens welfare reported in Graph 2 of Figure 1. This is due to the fact that both values are determined simultaneously from the interaction between supply and demand of local public goods and seldom one can find good instrumental variables to overcome this issue in a classical two stage framework. A second issue addressed by the model is that the evaluation of the standard (minimal) level of output reported in Graph 2 of Figure 1 must be at the same time feasible in terms of central government budget constraint and efficient from the technical and financial point of view. Finally the last problem addressed by the model is related to the presence of multi-output production function which makes the classical estimation of demand functions not always possible.

This paper, therefore, aims to suggest a solution to this problems and implements an methodology which simultaneously estimates the standard level of local government expenditure and the standard level of local government output.

Figure 2 is helpful in terms of visualizing the main features of the statistical model. In an ideal world the central government possesses perfect information with regards to the following aspects: the exact level of local services necessary to provide all citizens with the same minimum level of welfare, independently on the socio-economic characteristics of their jurisdiction; the exact structure of the local government production function; the cost of each input. Given these assumptions, and assuming the absence of a rigid budget constraint, the graph in Figure 2 can be read clockwise starting from quadrant n°4. First the central government establishes the level of local output which corresponds to different levels of deprivation (quadrant n°4), then given the structure of the production function determines the efficient quantity of inputs (quadrant n°3). Finally, given the cost of inputs, it is possible to compute the standard level of the local expenditure (quadrant n°2). Indeed, quadrant n°1 eventually provides the residual relation between the local context (measured in terms of deprivation) and the local expenditure needs.

In reality the central government is subject to rigid budget constraint and does not possess any of the information listed above, however this information can be derived using econometric tools and collecting data regarding the following variables: the characteristics of the local context to measure the level of deprivation of each jurisdiction, the quantity of output and input employed by each local government, and finally data regarding the expenditures of the local authorities. The empirical model, which will be developed by this research, aims to combine these variables and to eventually estimate the function which links the standard amount of local expenditure with the standard level of local services.

A first insight into the mechanics of the model can be obtained by reading Figure 2 anticlockwise starting from quadrants 1. First, you must estimate the expenditure need function; this can be done using the regression cost approach (RCA) or by using the representative expenditure system (RES) reported in quadrant. The choice between RCA and RES depends on the available data, however RCA should be preferred since it requires less data and involves less discretionality than the RES approach. Quadrant 2 can be used to estimate the level of inputs compatible with the standard expenditure needs.

Subsequently, using information on input and output it is possible to estimate the local services production frontier. In order to derive the relationship between standard expenditure and standard output, the final step will be the elaboration of the rule which will link the estimated expenditure needs with the estimated optimal level of output considering only efficient local authorities. As a result the output need function in quadrant 4 will be derived as a result of the entire procedure and will provided also a robustness test for the validity of the previous analysis.

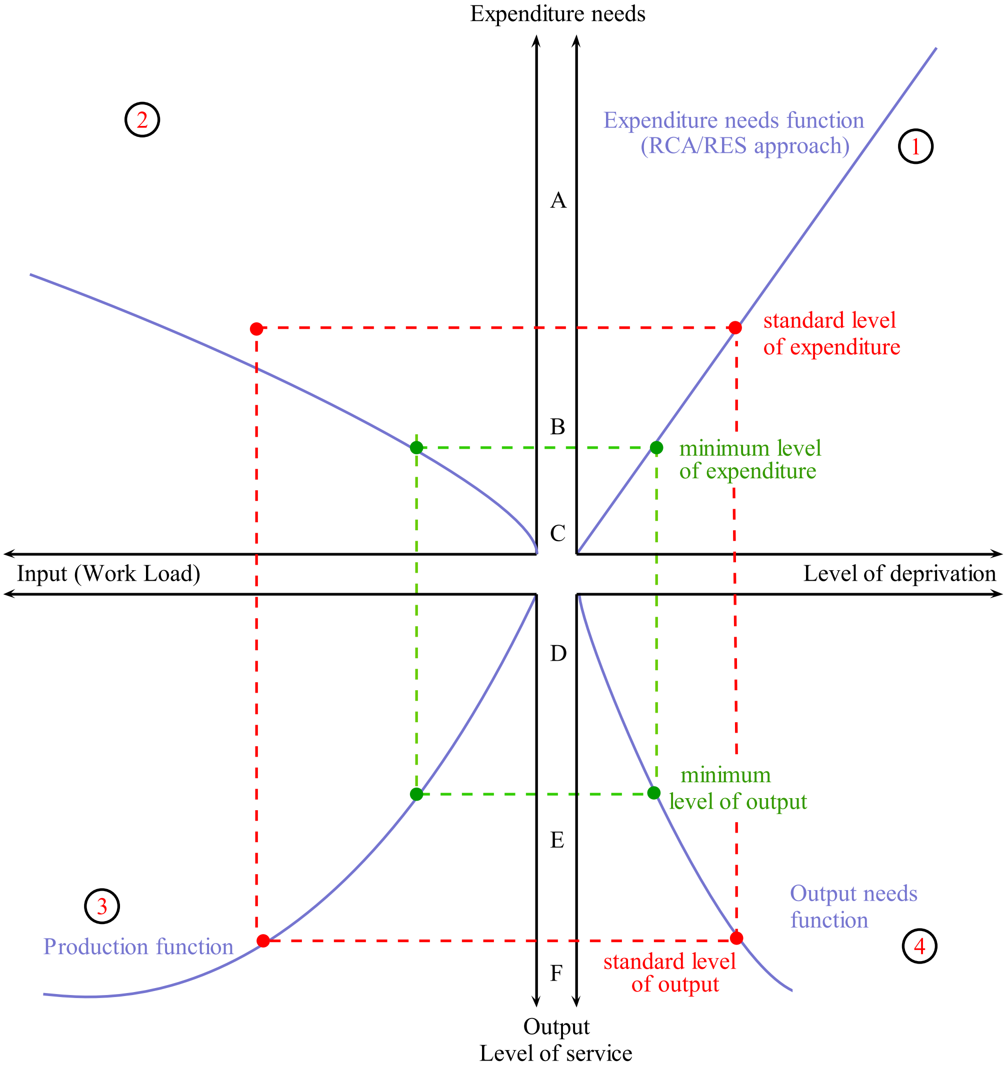

Figure 2: Relations among deprivation, standard expenditure and standard level of output

In sum, and as reported in Figure 2 in red and green, for each local authority it should be possible to compute four per capita benchmark values: the standard level of output and the corresponding standard level of expenditure (different for each jurisdiction); the minimum level of output and the corresponding minimum level of expenditure (the same for all jurisdictions). According to the position of each local authority with respect to its benchmark values of expenditure and output (always in the case of full technical efficiency) as reported in Figure 2 with the letters from A to F, it will be possible to classify the local governments into the different categories with which they correspond, eventually resulting in different recommended actions. For example some of these are reported in Table 1.

POSITION IN FIGURE 2	TYPE OF LOCAL AUTHORITY	ACTION REQUIRED
AF	Output and expenditure above the standard	Use own tax revenue to finance the extra services
BE	Output and expenditure below the standard	Move toward the standard
CD	Output and expenditure below the minimum	Move toward the minimum even if it lies above the standard

Table 1: Types of Local Authorities with respect to their benchmark values, some examples

The model described in Figure 2, can be estimated following two methodologies:

1. first by estimating the relationship between spending, output and inputs considering average values;
2. second using frontier techniques that can be divided into three main categories: parametric ones like Stochastic Frontier Models (SFM), semi-nonparametric ones like Stochastic Nonparametric Envelopment of Data (StoNED) (see e.g. Porcelli (2011), Kuosmanen and Kortelainen (2012)), robust two stage techniques (Vidoli (2011)).

4. Simulation strategy

The simulation based on the logical framework proposed in Figure 2 can be conducted following two opposite paths:

- The first path, reported in Figure 3, must be followed in the presences of rigid budget constraint or if the central government main goal is to revise past level of local government expenditure. In this case, after setting short, medium, or long term expenditure needs (quadrant n°1), it's possible to compute the equivalent number of inputs in correspondence with the average level of input costs (quadrant n°2); after identifying the production frontier (quadrant n°3), then, it's easy to find the standard level of output that is feasible ad efficient at the same time, and that equalises citizens' welfare. This is the path[1] that will be followed in the rest of the simulation.

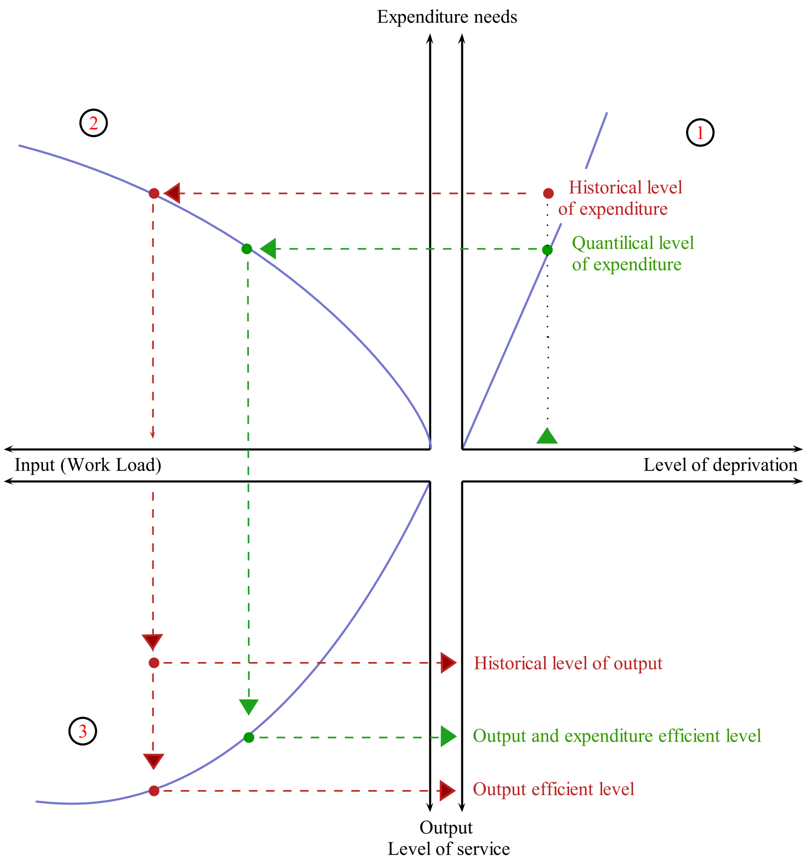

Figure 3: Estimation path: from the level of deprivation to the efficient output

- The second path is the inverse of the first one and can be followed in the absence of a rigid budget constraint when the central authority primary

[1]Historical levels is shown in red, the estimated ones in green.

goal is to reach a desired level of service in relation to a certain level of deprivation. In this case using the estimated production function it's possible to estimate the efficient level of input and consequently the level of expenditure to be financed. This second path will not be taken into account for the simulation because the assumption of unconstrained budget is far away from the reality of the public sector.

The simulation will be based on data from English local authorities (unitary and two-tier). In particular the choice of using English data is motivated by the possibility of checking the validity of our model against the ranking of local authorities provided by the Comprehensive Performance Assessment (CPA). CPA is an evaluation scheme introduced in 2001 and repealed in 2008 that rated local governments in England on the quality and quantity of local services produced in six major areas: education, housing; social care; environment; libraries and leisure; use of resources. Hundreds of performance indicators and a variety of audit and inspection reports were collected, summarized, weighted, and categorized so as to arrive at final star ratings between 0 and 4[2].

The set of environmental variables used to measured local expenditure needs, can be subdivided as follows. First, there are demographic variables, such as the percentage of the total population below the age of 16 and above the age of 75, the number of band D equivalent dwellings per capita that correspond to the tax base of the council tax and has been included as a proxy of the demand for local public services, the population density and the total resident population. The second category includes a set of dummy variables to capture the impact of the ruling party and the features of the electoral system ("all out" election every four years, or "by thirds" system which involves more frequent elections). The third group of variables is related to the structure of the local economy and includes: average household disposable income, the percentage of the workforce claiming unemployment-related benefits, the percentage of people below 65 claiming disability living allowance, the gross weekly pay registered for employees in the private sector.

Variables used in the analysis and their abbreviations can be found in Table 6.

[2]In fact, from 2002-5, the rankings were designated: excellent", "good", "fair", "weak", and "poor", changing to zero to four stars during the latter part of CPA - see Tables A1 and A3 below. But, for simplicity, we refer to star ratings throughout.

5. Results

5.1. First quadrant: Expenditure needs and the level of deprivation

Standard expenditure needs are driven by territorial and socio-demographic characteristics of the population. Different aspects that affect both citizens' needs and local services demand producing a direct impact on production costs. These relationships can be estimated on the average levels (using OLS, GLS or GMM techniques) or on the frontier (via SFA or better via more recent semi-parametric techniques like StoNEZD [Kuosmanen and Kortelainen (2012)]).

In empirical analysis the frontier estimates, while having the desired criteria of public spending rationalisation, provide very extreme results. For this reason standard values obtained trough frontier methods can be considered only as long-term target. With the aim to endow short or medium term target, we have chosen to use the quantile regression (Koenker (2005)) with the goal of reaching an equilibrium point between estimated coefficients robustness and expenditure efficiency. To that end we move towards the minimum spending frontier until the regression coefficients do not deviate "too much" from the overall average case; the trade-off between frontier target and coefficients stability can be interpreted as a medium-term goal.

In other words, whereas the *"average"* methods like OLS approximate the conditional mean of the response variable given covariate values and the *"frontier"* methods approximate the *quasi-maximum*[3] of the independent variables, quantile regression estimate the expected response variable quantile. As a result quantile regression can be considered as a flexible model, in which it's possible *tuning* a parameter according to the Central Government macro-budget.

In order to make clearer the estimation model underlying the quantile regression, in analogy with sample average which can be defined[4] as the solution of the minimization of the sum of squared deviations, we can define each quantile φ_q as the solution of the following minimization problem:

$$Q_q = \arg\min_{\varphi_q \in R} \left\{ \sum_{i \in (i|y_i \geq \varphi_q)} q|y_i - \varphi_q| + \sum_{i \in (i|y_i < \varphi_q)} (1-q)|y_i - \varphi_q| \right\} \tag{1}$$

Moreover, considering the classical linear regression model:

$$y_i = x_i'\beta + u_i \text{ with } E(u_i|x_i) = 0 \tag{2}$$

[3]For *quasi-maximum* we mean the upper quantile in the production framework (or lower in a cost framework) minus (or plus) the stochastic term.

[4]For a more complete discussion, please see Koenker and Bassett (1978).

where $E(y_i|x_i)$ corresponds to the vector $x_i'\beta$. The quantile regression model can be formulated in a similar way as:

$$y_i = x_i'\beta + u_i \text{ with } Q_q(u_{i,q}|x_{i,q}) = 0 \tag{3}$$

where the conditional quantile can be expressed as $Q_q(y_i|x_i) = x_i'\beta$.

Moreover, it is also important to highlight that quantile regression does not represent an estimate for sub-samples of the population, but a more general method that uses all available observations minimizing errors by quantiles, instead of the sample mean.

However, as a drawback of the greater flexibility, in the quantile regression framework researchers are faced with a crucial decision: the choice of the *"right"* quantile q. In this paper, we propose to adopt a criterion similar to Forward Search analysis (Atkinson et al. (2010)), in which, in order to evaluate the coefficients robustness, different models are generated varying units in the sample.

But unlike Atkinson et al. (2010)'s framework, we have not sampled units to study coefficients variations in a single model, instead we have analysed the coefficients variations varying the minimum quantile; practically, we are varying quantile regression model in a range $[0.5-0.01]$ studying the variation of the each standardised estimated parameter as reported in Figure 4.

In conclusion, the estimated relationship, in the first quadrant, can be expressed in a general form as:

$$Expense(q) = \arg\min_q (f(X_k, P_j, I, D_Y)) \tag{4}$$

where X_k are the supply and demand covariates, P_j are the inputs prices, I is the mean income, D_Y the year dummies and q the chosen quantile.

Given the panel structure of our dataset, equation (4) has been estimated through a fixed effect panel quantile regression model subtracting group means from the regressors and including dummy variables for the different years (from 1997 to 2005).

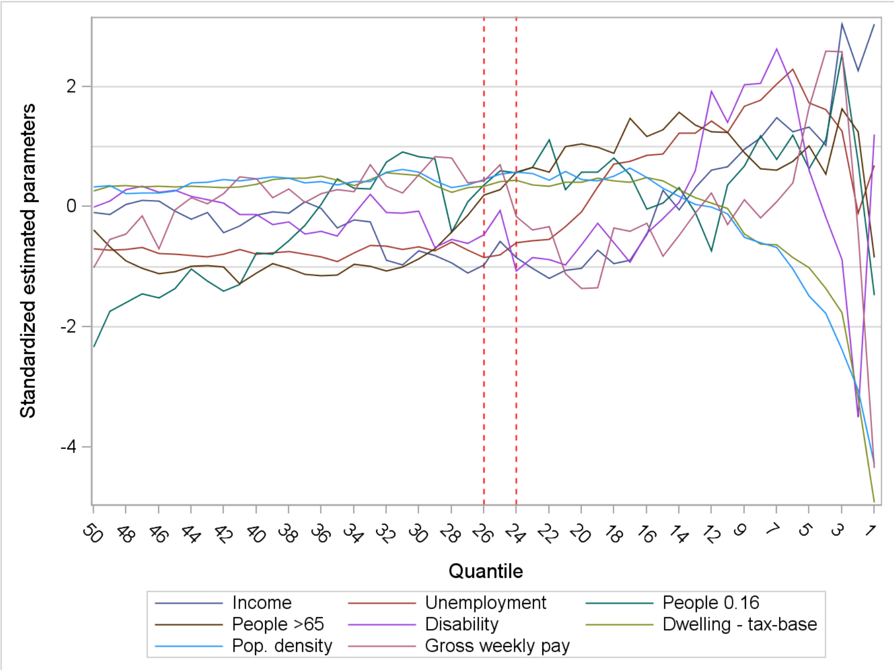

Figure 4: Standardized estimated quantile regression parameters changing reference quantile

Figure 4 shows a good parameters stability until $q = 0.26$, while, for instance, at quantile 0.24 the disability and the gross weekly pay coefficients tend to become quickly unstable. Note also how the relations between the variables change in the distribution tail; this is due to the presence of a few units, and also because the frontier cost function is usually structurally different from the average function.

Table 2 shows the 26° quantile estimates, showing a discrete significance of almost all the variables.

Parameter	**Estimate**	**Std error**	***t* value**	$Pr > \|t\|$
Intercept	-21.6017	1.7866	-12.09	<.0001
Total resident population	-0.003	0.0005	-5.49	<.0001
Total resident population - square	1.006E-9	2.794E-10	3.78	0.0002
Perc. of unemployment related benefit	9.7749	4.8964	2	0.0461
Perc. of people age 0 - 16	7.2828	5.5197	1.32	0.1872
Perc. of people age over 65	2.3227	4.3567	0.53	0.594
Perc. of attendance allowance for people below 65 y	31.9764	9.7036	3.3	0.001
band D equivalent dwelling (percentage per capita)	5.1407	1.5739	3.27	0.0011
Population density (# per hectare)	4.4657	1.3442	3.32	0.0009
Other party percentage of votes	1.0707	0.3321	3.22	0.0013
Gross weekly pay in main job - private firm	0.0473	0.0347	1.36	0.1728
Disposable income (real £ per capita)	0.0039	0.0059	0.66	0.5122
D_{1997}	-340.011	27.3629	-12.43	<.0001
D_{1998}	-336.759	32.3356	-10.41	<.0001
D_{1999}	-228.562	21.6535	-10.56	<.0001
D_{2000}	-180.001	18.4204	-9.77	<.0001
D_{2001}	-154.436	14.4491	-10.69	<.0001
D_{2002}	-56.7892	12.4711	-4.55	<.0001
D_{2003}	-102.209	8.5305	-11.98	<.0001
D_{2004}	-56.5231	7.0072	-8.07	<.0001
D_{2005}	-24.1535	5.364	-4.5	<.0001

Table 2: Quantile regression estimates - quantile = 0.26

After estimating the relationship between per capita expenditure and the level of deprivation on the 0.26 quantile, the fitted values cost can be used for two purposes:

- Improving overall spending efficiency by assigning to each jurisdiction the obtained values;
- re-proportioning for each jurisdiction the obtained values using the ratio between the sum of the fitted values and the historical values, with the aim of keeping fixed the overall budget constraint.

In this simulation we are following the second option computing, for each local authority, the standard expenditure needs over the medium term keeping, at the same time, fixed the total amount of historical local expenditure. In the rest of the paper we will denote this measure as Exp_{q26}, carrying forward the analysis only for the year 2006[5].

[5]For this reason the year dummies D_Y have not been considered in the computation of the fitted values

5.2. *Second quadrant: Average level cost of inputs*

The purpose of the second step is to estimate the average level cost of inputs, with the aim to derive the inputs level corresponding to the minimum expenditure Exp_{q26}. In the present application, in order to derive an average cost level, we first have estimated a function that bind expenses to input and subsequently we have used the inverse of it.

This is relatively straightforward because we consider in our analysis only one input; when two or more inputs are available, it's possible to include technical coefficients of production and compute inputs' combination productively feasible.

$$Expense = f(Inputs, P_j) \tag{5}$$

In our application we consider as input the number of local government employees and as input quality proxy the percentage of high qualified workforce working for local government. The OLS estimate[6] has been reported in Table 3.

Parameter	**Estimate**	**Std error**	t **value**	$Pr > \|t\|$
Intercept	69,374,647	23,131,415	3	0.0034
Local gov. employees	9,820.6896	3,092.4177	3.18	0.002
Local gov. employees - square	-0.1084	0.08401	-1.29	0.1997
High_q	2,162.3047	3,290.0617	0.66	0.5125

Table 3: Average level cost of inputs

Once estimated the average cost function, it is possible to compute the corresponding level of input (standard number of local employees) both in relation to historical expenditure (we called it $Input_{hist}$) and the standard level of expenditure found on the 26° quantile (we called it $Input_{q26}$), using the inverse of the estimated function[7].

$$\begin{aligned} Input_{q26} &= (Exp_{q26} - 69,374,647)/9,820) \\ Input_{hist} &= (Exp_{Hist} - 69,374,647)/9,820) \end{aligned} \tag{6}$$

[6] R^2 equal to 0.39.

[7] In equation 6 we have not considered the input quadratic term and high qualification people term because not statistically significant.

5.3. Third quadrant: Production function

In the third quadrant we are faced with a classic production frontier model. In general terms, the relations can be modeled as:

$$(Output_1, \ldots, Output_i) = f(Input_1, \ldots, Input_j) \quad (7)$$

Productive efficiency methods can be classified on the basis of whether an average-practice or best-practice technology is estimated. The best-practice technologies can be further classified as non-stochastic and stochastic technologies, depending on whether a stochastic noise term is included or not. However, the methods can be classified as being parametric and nonparametric in their orientation.

Parametric methods assume a specific functional form of the production function, which is usually linear in its parameters. Nonparametric methods do not assume a particular functional form, but estimate the benchmark technology based on a minimal set of axioms.

In literature, specification and estimation of production frontier functions are usually carried out by two different approaches:

- Stochastic Frontier Analysis (SFA)(Aigner et al., 1977; Meeusen and van den Broeck, 1977) - Deterministic Frontier Analysis (DFA) (Aigner and Chu, 1968);
- Data Envelopment Analysis (DEA) (Farrell, 1957; Charnes et al., 1978) - Free Disposal Hull (FDH) (Deprins et al., 1984; Grosskopf, 1996).

A third way, proposed in recent years (Vidoli (2011)), attempts to combine the advantages of the robust nonparametric techniques (absence of functional form in the estimation phase) to those of the parametric techniques (presence of functional form in the application phase).

From a methodological point of view, we have enhanced a "two stage" method, based on ideas suggested by Florens and Simar (2005), which estimates the efficiency frontier through robust nonparametric models (Order-*m*, Daraio and Simar (2007)) and bypasses, at the same time, the choice of a specific functional form in the second stage; the MARS (Multivariate Adaptive Regression splines, Friedman (1991)) method, in fact, provides for approximate production function using linear splines without any assumption of a functional form.

In MARS models, the non-linear relationships that generally links output and inputs are described by a set of linear segments characterized by different inclinations, each of which is estimated by means of a basis function in a completely *"data driven"*. MARS method can be interpreted as based on estimation strategies of type *"divide and conquer"*, in which the input space is divided into subspaces each with its own regression equation: this makes the MARS models particularly appropriate in applications with many inputs where the *"curse of dimensionality"* (Bellman (1961)) could create problems with other estimation techniques.

Given the availability of many outputs, we have chosen to compute a composite indicator[8] using the related expenditure per capita as weights.

$$\begin{aligned} CI\_output = & Out_{educ} \cdot Exp_{edu} + \\ & (Out_{social\_care} + Out_{health65}) \cdot Exp_{social\_care} + \\ & Out_{waste} \cdot Exp_{waste} + Out_{paid\_time} \cdot Exp_{paid\_time} \end{aligned} \tag{8}$$

To identify the frontier production function, we have first identified the efficient units using Order-*m* nonparametric method and then, for the efficient units subset, we have fitted the functional relationship between output and input via MARS model, obtaining estimate shown in equation 9 and plotted in Figure 5 (in red)[9].

$$\begin{aligned} CI_{Output} = & 93,457 - 3.9 \cdot max(0; 7,120 - Input) - \\ & 1.9 \cdot max(0; 10,766 - Input) - \\ & 0.1 \cdot max(0; 22,987 - Input) \end{aligned} \tag{9}$$

[8]Without loss of generality, it is always possible to estimate a model multi-input multi-output, but only in the nonparametric form.

[9]In Figure 5 we also reported DEA efficiency estimates - enveloping all data it's influenced by extreme data - and GAM estimate showing that, also in a nonparametric framework, MARS show a similar good fit.

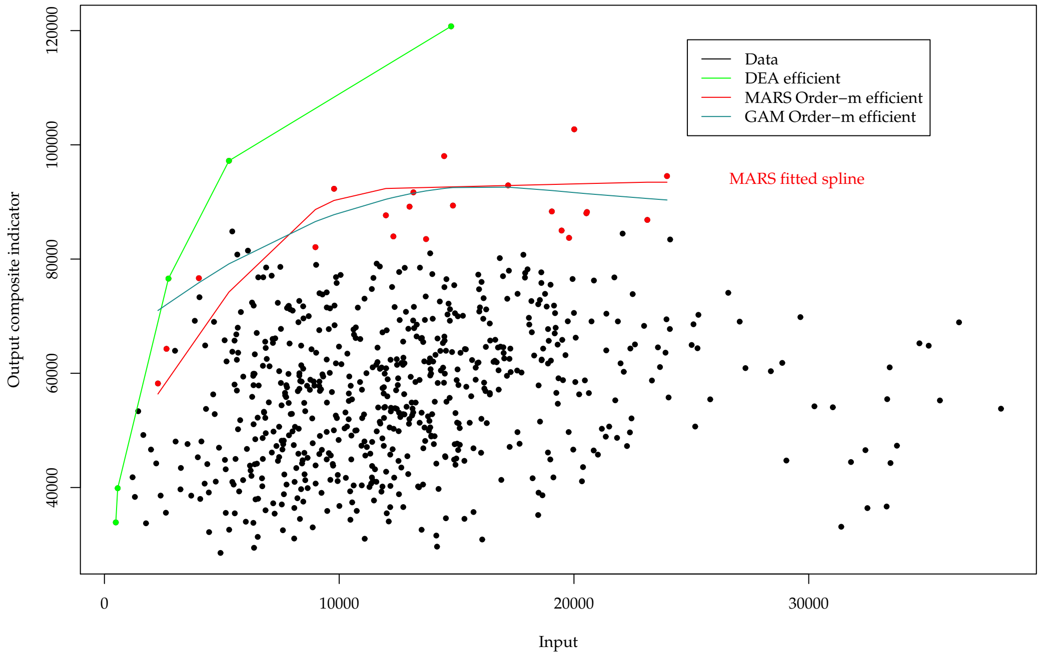

Figure 5: Estimated production function by the way robust techniques

Replacing $Input_{q26}$ in equation (9), we obtain the estimate service level CI^{q26}_{Output} in optimal conditions both from a production and from expenditure point of view.

5.4. *Fourth quadrant: Output and level of deprivation*

The fourth quadrant relates the efficient and feasible levels of output with the local level of deprivation. This last relationship is not estimated but is obtained as a result of the entire exercise providing the standard level of output that each local authority should provide to equalise citizens' welfare. Therefore, the presence of the expected positive relationship between deprivation and output provides also a robustness check for the whole procedure.

To calculate a composite indicator of the level of deprivation, in the absence of explicit weights as for the outputs, several methods can be used: we propose to use an interesting generalisation[10] of the classical BoD approach (see e.g. Nardo et al. (2005),Witte and Rogge (2009)) described further in Appendix.

The composite indicator for deprivation has been compared then with the output composite indicator. Figure 6 show a good correlation (0.462) between

[10]This approach was developed by Vidoli and Mazziotta and presented in September 2012 at the XXXIII conference AISRE, Rome.

the estimated level of service and the level of needs of English local authorities validating the estimates obtained in the previous quadrants.

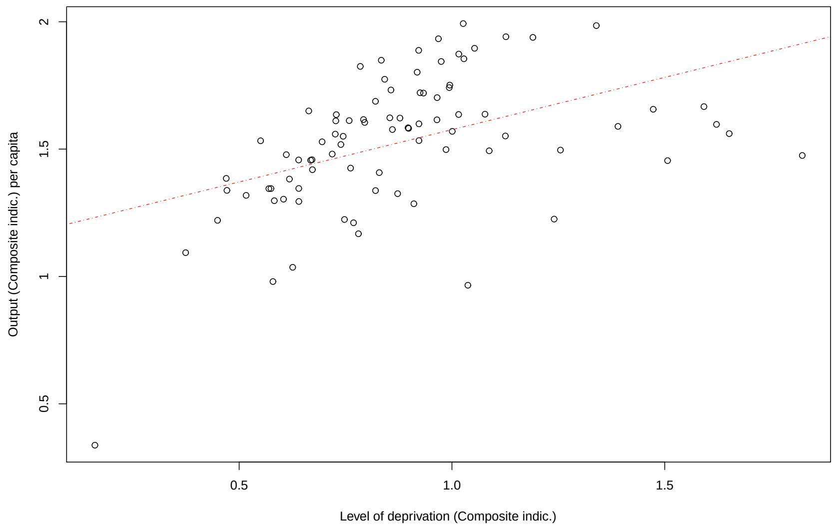

Figure 6: Estimated level of output pro capita vs Level of deprivation

As further, and more important test for the validity of the results, the differences between standard and historical values of output have been compared with CPA scores obtained by English local authorities in the same year (2006)[11]. Table 4 shows how higher CPA scores are associated to a smaller difference between standard and historical level of output. As a result, we find that, on average, those local authorities that performed very bad in terms of CPA have to increase, according to our model, their output by 9%. Instead, those locale authorities that performed very well are producing a level of output almost 2% higher than the standard level.

[11]Comprehensive Performance Assessment (CPA), introduced for single tier and county councils in 2002 and for district councils in 2003, measures how well councils were delivering services using a five scale grade performance indicator that ranged from 0 (poor performance) to 4 (excellent performance).

CPA	Output percentage increase
Class 1	+9.015%
Class 2	+5.933%
Class 3	+2.449%
Class 4	-1,767%

Table 4: CPA and output percentage increase, (class 0 is merged with class 1 because of its small numerosity)

5.5. A comprehensive view of the results

To sum up, for each English local authority, we have obtained two classes of measures using a single and integrated consistent framework:

1. first the standard expenditure needs and the corresponding efficient level of standard service;
2. second the deviations from standard measures, both in terms of expenditure and level of service.

Final results are summarised in Figure 7, where standard values are reported in red and historical values are reported in green. Using the London borough of Lewisham as an example, it can be easily seen that while both expenditure and input are above standard levels, the output is below the standard.

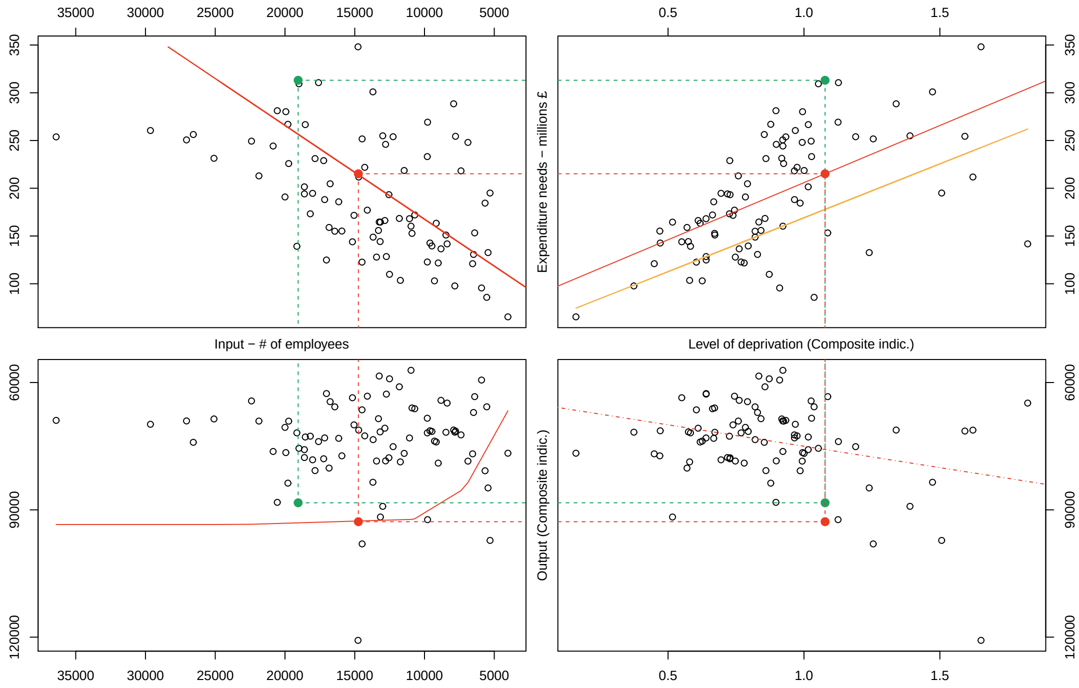

Figure 7: Estimated functions

The entire methodology can be easily extended to include a multi-input and a multi-output production function and other econometric techniques. In Table 5 we present a non-exhaustive proposal of possible extensions in this direction.

In our application			In general terms	
Quadrant	Dimensionality	Estimation methods	Dimensionality	Estimation methods
1	$Y(1) \rightarrow X(k)$	Quantile regression	$Y(1) \rightarrow X(k)$	Quantile regression, GMM on average, STONEZD or Order-*m* - MARS on frontier
2	$Inverse(Y(1) \rightarrow X(1))$	OLS	$Inverse(Y(1) \rightarrow X(n))$	OLS and technical coefficient of production
3	$Y(1) \rightarrow X(1)$	Order-*m* MARS	$Y(m) \rightarrow X(n)$	Productive non parametric methods
4	$Y(1) \rightarrow X(k)$	OLS	$Y(m) \rightarrow X(k)$	Simultaneous equation models

Table 5: Quadrants and econometric estimate methods

6. Conclusion

Appendix 1: Robust composite indicators by means of frontier methods

Benefit of the Doubt approach (DEA application to CI)

"The Benefit of the Doubt approach is formally tantamount to the original input-oriented CCR-DEA model of Charnes et al. (1978), with all questionnaire items considered as outputs and a dummy input equal to one for all observations" Witte and Rogge (2009).

The Farrell-Debreu efficiency scores (input oriented) for a given production scenario $(x, y) \in \Psi$ when x is constant and equal to 1 may be defined as:

$$\theta(x,y) = \inf\{\theta | (\theta, y) \in \Psi\} \tag{10}$$

So the Free Disposal Hull (FDH) estimator is provided by the particular free disposal hull of the sample points:

$$\widehat{\Psi}_{FDH} = \{(\mathbb{1}, y) \in \mathbb{R}^{1+q} | y < Y_i, i = 1, ..., n\} \tag{11}$$

if Ψ is convex, take the convex hull of $\widehat{\Psi}_{FDH}$ called Benefit of Doubt (BoD) in accordance with Cherchye and Kuosmanen (2002):

$$\begin{aligned} \widehat{\Psi}_{BoD} = \{(\mathbb{1}, y) \in \mathbb{R}^{1+q} | y < \sum_{i=1}^{n} \gamma_i y i \text{ for } (\gamma_1, ..\gamma_n) \\ \text{such that } \sum_{i=1}^{n} \gamma_i = 1; \gamma_i \geq 0, i = 1, ..., n\} \end{aligned} \tag{12}$$

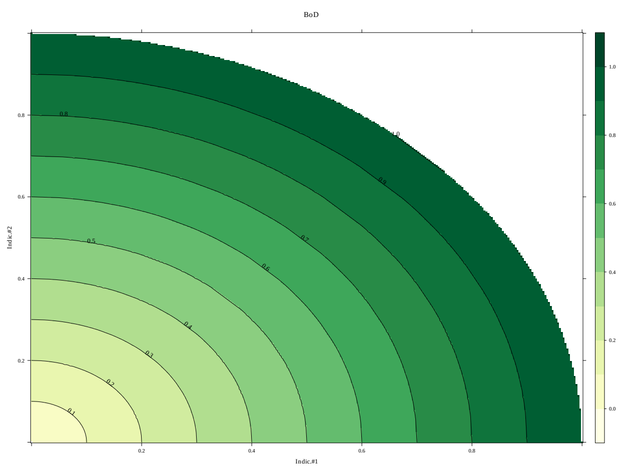

Figure 8: BoD

The main drawbacks are directly linked with the DEA problem solution: since the weights are unit specific, cross-unit comparisons are not possible and the values of the scoreboard depend on the benchmark performance.

There are also two other drawbacks: the multiplicity of equilibria and the robustness.

Robust BoD (Order-m methods application to CI)

One of the main drawbacks of DEA/FDH nonparametric estimators is their sensitivity to extreme values and outliers. To introduce order-*m* we first expose the simplified idea and then we more precisely formalize the model.

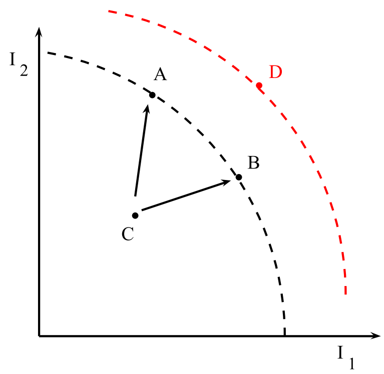

Figure 9: Presence of outliers in a frontier framework

In this context, Cazals et al. (2002) proposed a more robust nonparametric estimator of the frontier. It is based on the concept of the expected minimum input function of order-*m*.
Extending these ideas to the full multivariate case, Daraio and Simar (2005) defined the concept of the expected order-*m* input efficiency score. Daraio and Simar (2005) affirmed that: *"in place of looking for the lower boundary of the support of $F_X(x|y)$, as was typically the case for the full-frontier (DEA or FDH), the order-m efficiency score can be viewed as the expectation of the minimal input efficiency score of the unit (x,y), when compared to m units randomly drawn from the population of units producing more outputs than the level y.*

We extend Daraio and Simar (2005)'s idea into CI's framework by drawing repeatedly and with replacement *m* observations from the original sample of *n* observations and choosing only from those observations which are obtaining higher performance scores (I_1, I_2) - *red lines* - than the evaluated observation C.

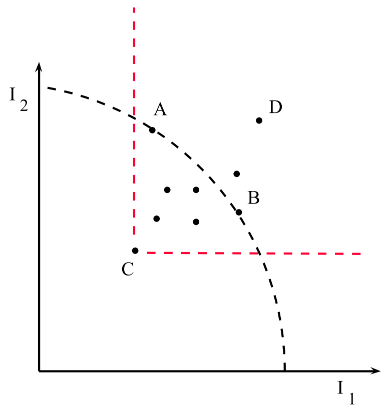

Figure 10: The support of unit C

In other words and practically speaking:

- We draw *m* observation only from those observations which are obtaining higher performance scores than the evaluated observation C;
- We label this set as SET_{bm};
- We estimate BoD scores relative to this subsample SET_{bm} for B times;
- Having obtained the B scores, we compute the arithmetic average.

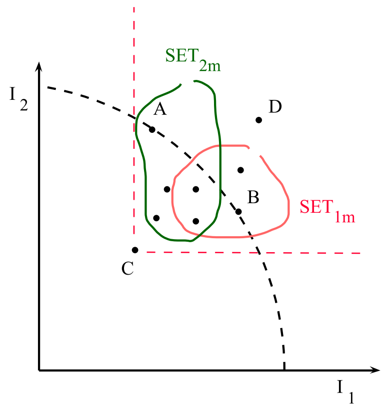

Figure 11: Subsamples

This is certainly a less extreme benchmark for the unit C than the *"absolute"* maximum achievable level of output.

C is compared to a set of m peers (potential competitors) producing more than its level and we take as a benchmark, the expectation of the maximum achievable CI in place of the absolute maximum CI.

More accurately, Daraio and Simar (2005) propose (for a more complete theoretical summary see Fried and Lovell (2008) and Daraio and Simar (2007)) a probabilistic formulation of efficiency concepts, assuming that the Data Generating Process (DGP) of (X,Y) is completely characterized by:

$$H_{XY}(x,y) = Prob(X \leq x, Y \geq y) \tag{13}$$

so $H_{XY}(x,y)$ can be interpreted as the probability for a unit operating at the level (x,y) of being dominated. Note that it is monotone non-decreasing with x and monotone non-increasing with y. This joint probability can be decomposed as follows (yet in a input-oriented framework):

$$H_{XY}(x,y) = Prob(X \leq x|Y \geq y)Prob(Y \geq y) = S_{X|Y}(x|y)F_Y(y) \tag{14}$$

An input oriented efficiency score $\widehat{\theta}(x,y)$ for $(x,y) \in \Psi$ is defined for all y values with $F_Y(y) > 0$ as

$$\widehat{\theta}(x,y) = \inf\{\theta|(\theta x_0, y_0) \in \Psi\} = \inf\{\theta|H(\theta x, y) > 0\} \tag{15}$$

Applying Daraio and Simar (2005)'s ideas to the particular case of composite indicators, equation (13) can be written:

$$H(x,y) = Prob(X \equiv 1, Y \geq y) \tag{16}$$

$$\text{where } \Psi \text{ is the support of } H(x,y) \tag{17}$$

So Farrel-Debreu (input) efficiency score, since $Prob(X \equiv 1|Y \geq y) = 1$ can be written as:

$$H(x,y) = Prob(X \equiv 1|Y \geq y)Prob(Y \geq y) = F_Y(y) \tag{18}$$

$$\theta(1,y_0) = \inf\{\theta|(\theta,y_0) \in \Psi\} = \inf\{\theta|H(\theta,y) > 0\} \tag{19}$$

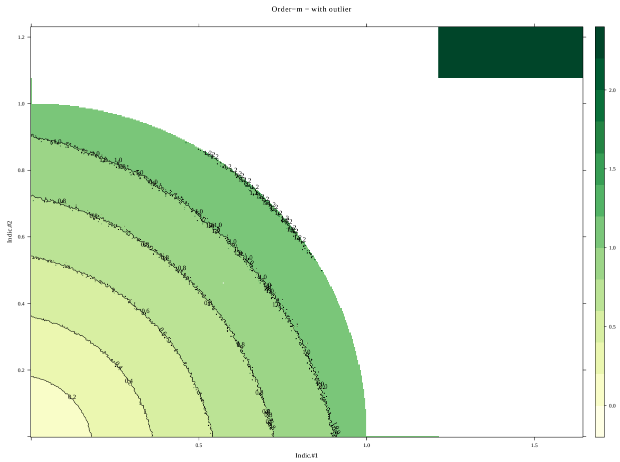

Figure 12: Order-m in presence of outliers

Appendix 2: English education services dataset

Variabile	**Label**
YEARCODE	Unique code
YEAR	1997-2008
CODE	Local authority code
AUTH	Local authority name
REG	Region
LATYPE	Type of local authority
GRSSWK_LA	Gross weekly pay in main job - local authorities (real £)
GRSSWK_PRIVAT	Gross weekly pay in main job - private firm (real £)
HOURPAY_LA	Gross hourly pay - local authorities (real £)
HOURPAY_PRIVAT	Gross hourly pay - private firm (real £)
LOCALGOV_EMP_PRC	Percentage of people working for local government (estimated)
LOCALGOV_EMP	People working for local government (estimated)
HIGHQUAL_LA	People working for local government with high degree (estimated)
HIGH_LA_PRC	Percentage of local gov. employees with high degree (estimated)
TOTPOP	Total resident population
ENGLAND	Dummy=1 England, 0 = Wales
DUMMYCPA	Dummy=1 after 2001, CPA introduction
DUMMYCPAENGLAND	Dummy=1 England · dummy CPA
GRANTRP	Revenue support grants (real £per capita)
GDHIR	Disposable income (real £per capita)
JOBSEEK	Percentage of unemployment related benefit
AGE_0_16	Percentage of people age 0 - 16
AGE_OVER65	Percentage of people age over 65
VATFINANCIAL	Percentage of firms in the financial sector
DISABILITY	Percentage of attendance allowance for people below age 65
TAXBASEP	Band D equivalent dwelling (percentage per capita), council tax-base

Table 6: English education services dataset - 1^{st} part

Variabile	Label
POPDENS	Population density (number per hectare)
RELIGIOUS	Percentage of religious people
WHITE	Percentage of white people
HIGHQUALIFIED	Percentage of high qualified workforce
TENURE	Percentage of tenure (house ownership)
SELFEMPLOYED	Percentage of self employed work force
COUNCILTAXRD	Real council tax per equivalent band D dwelling (effective tax rate)
TAXREQRP	Tax requirement (real £per capita)
TAXREQ_PRC	Tax requirement (% of budget requirement)
OUT_EDUC	% A* - C grades three years moving average window (0 1 3)
OUT_HEALTH65	N. Health & Social Care Over 65s Helped to Live at Home
OUT_SOCIAL_CARE	% Health & Social Care Stability of Placements for Looked After Children
OUT_WASTE	% Waste & Cleanliness Percentage household waste recycled
OUT_PAID_TIME	% Corporate Health Percentage of invoices paid on time
SERVICE_EXPENDRP	Total service expenditure - per capita real
NET_CURR_EXPENDRP	Net current expenditure - per capita real
BUDGET_REQURP	Budget requirement - per capita real
SSA	Standard spending assessment - per capita real
EXP_EDU	Secondary education expenditure (real £ per capita)
EXP_SOCIAL_CARE	Social service expenditure (real £ per capita)
EXP_WASTE	Waste disposal expenditure (real £ per capita)
EXP_PAID_TIME	Central services expenditure (real £ per capita)
CON	Conservative party percentage of votes
LD	Libdem Party percentage of votes
OTHER	Other party percentage of votes
TURNOUT	Latest local election turnout
CPI05	Consumption price index 2005 base

Table 7: English education services dataset - 2^{nd} part

References

Aigner, D., Lovell, C. A. K., Schmidt, P., 1977. Formulation and estimation of stochastic frontier production function models. Journal of Econometrics 6, 21–37.

Aigner, D. J., Chu, S. F., 1968. On estimating the industry production function. American Economic Review 58, 826–839.

Atkinson, A. C., Riani, M., Cerioli, A., 2010. The forward search: Theory and data analysis. Journal of the Korean Statistical Society 39, 117–134.

Bellman, R. E., 1961. Adaptive control processes: a guided tour. Princeton University Press.

Blochliger, H., Merk, O., Charbit, C., Mizell, L., 2007. Fiscal equalization in oecd countries. Tech. rep., OECD network on fiscal relations across levels of government, Working paper , n. 4.

Burgess, S., Ratto, M., May 2003. The role of incentives in the public sector: Issues and evidence. The Centre for Market and Public Organisation 03/071, Department of Economics, University of Bristol, UK.

Cazals, C., Florens, J., Simar, L., 2002. Nonparametric frontier estimation: A robust approach. Journal of Econometrics 106 (1), 1–25.

Charnes, A., Cooper, W., Rhodes, W., 1978. Measuring the efficiency of decision making units. European journal of operational research 2(4), 429 – 444.

Cherchye, L., Kuosmanen, T., 2002. Benchmarking sustainable development: A synthetic metaindex approach. Tech. rep., EconWPA, Others 0210001.

Dafflon, B., Mischler, P., 2007. Measuring Local Government Expenditure Needs. The Copenhagen Workshop 2007, Ch. Expenditure needs equalisation at the local level: methods and practice.

Daraio, C., Simar, L., 2005. Introducing environmental variables in nonparametric frontier models: a probabilistic approach. Journal of Productivity Analysis 24(1), 93–121.

Daraio, C., Simar, L., 2007. Advanced Robust and Nonparametric Methods in Efficiency Analysis. Springer.

Deprins, D., Simar, L., Tulkens, H., 1984. Measuring labor inefficiency in post offices. In: M. Marchand, P. Pestieau, H. Tulkens: (ed.) The Performance of Public Enterprises: Concepts and Measurements. North-Holland, Amsterdam.

Farrell, M. J., 1957. The Measurement of Productive Efficiency. Journal of the Royal Statistic Society 120 (III), 253–281.

Florens, J., Simar, L., 2005. Parametric approximations of nonparametric frontiers. Journal of Econometrics 124, 91–116.

Fried, H. O., Lovell, C. A. K., 2008. The Measurement of Productive Efficiency and Productivity Growth. Oxford University Press, USA.

Friedman, J. H., 1991. Multivariate adaptive regression splines. Annals of Statistics 19 (1), 1 – 67.

Grosskopf, S., 1996. Statistical Inference and Nonparametric Efficiency: A Selective Survey. Journal of Productivity Analysis 7, 161–176.

Koenker, R., 2005. Quantile Regression. Cambridge Books, Cambridge University Press.

Koenker, R., Bassett, G., 1978. Regression quantiles. Econometrica 46 (1), 33–50.

Kuosmanen, T., Kortelainen, M., 2012. Stochastic non-smooth envelopment of data: semi-parametric frontier estimation subject to shape constraints. Journal of Productivity Analysis, 1–18.

Lockwood, B., Porcelli, F., 2011. Incentive schemes for local government: Theory and evidence from comprehensive performance assessment in england. Tech. rep., Warwick Economic Research Papers, n. 960.

Meeusen, W., van den Broeck, J., 1977. Efficiency estimation from cobb−douglas production functions with composed error. International Economic Review 18, 435 – 444.

Mizell, L., 2008. Promoting performance: using indicators to enhance the effectiveness of sub central spending. Tech. rep., OECD Network on Fiscal Relations Across Levels of Government, working paper no. 5.

Nardo, M., Saisana, M., Saltelli, A., Tarantola, S., Hoffman, A., Giovannini, E., 2005. Handbook on constructing composite indicators: Methodology and user guide. OECD Statistics Working Papers 2005/3, OECD, Statistics Directorate.

OECD, 1981. Measuring Local Government Expenditure Needs: The Copenhagen Workshop. OECD.

Porcelli, F., 2011. Electoral accountability and local government efficiency: Quasi-experimental evidence from the italian health care sector reforms. Tech. rep., University of Bari, Working Paper n. 36.

Reschovsky, A., 2006. Local Governance in Industrial Countries. Public Sector Governance and Accountability Series. The World Bank, Ch. Compensating Local Governments for Differences in Expenditure Needs in a Horizontal Fiscal Equalization Program.

Vidoli, F., 2011. Evaluating the water sector in italy through a two stage method using the conditional robust nonparametric frontier and multivariate adaptive regression splines. European Journal of Operational Research 212, Issue 3, 1 August 2011, 583–595.

Witte, K. D., Rogge, N., 2009. Accounting for exogenous influences in a benevolent performance evaluation of teachers. Tech. rep., Working Paper Series ces0913, Katholieke Universiteit Leuven, Centrum voor Economische Studien.