Fertility-related pensions, tax-*cum*-subsidy and $\mathcal{A}$–Pareto efficiency

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Preliminary version – Please do not quote

Abstract An overlapping generations small open economy with endogenous fertility, time cost of children and child pensions is analysed to show that the command optimum can be decentralised in a market setting using a PAYG transfer from the young to the old and either a tax-*cum*-subsidy policy (i.e., a linear wage tax on labour income collected and rebated in a lump-sum way within the younger working-age generation), or using a child factor into the pension formula. Indeed, the second and third instruments affect both fertility and the opportunity cost of children. Moreover, by applying the generalised notion of Pareto efficiency introduced by Golosov et al. (2007) in a context of endogenous population, some normative conclusions can actually be drawn: since only the utilities of those who are born are evaluated, we apply the concept of $\mathcal{A}$–efficiency and conclude that when PAYG pensions are in existence, either the tax-*cum*-subsidy policy or the child factor can effectively be used as an alternative to the child allowance to internalise the externality of children, while also representing an $\mathcal{A}$–Pareto improvement. Moreover, we note that only whether a tax-*cum*-subsidy policy is absent, it is optimal to introduce a PAYG system fully related to individual fertility.

Keywords $\mathcal{A}$–efficiency; Fertility-related pensions; Overlapping generations; Small open economy; Tax-*cum*-subsidy

JEL Classification H24; H55; J13; J26

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1. Introduction

The debate on the existence of an "interior" golden rule of procreation in the neoclassical growth model with overlapping generations (OLG) and exogenous demography (Diamond, 1965) is long lasting (Phelps, 1968; Samuelson, 1975; Deardorff, 1976; Jaeger and Kuhle, 2009; de la Croix et al., 2011). Although such a debate is undoubtedly important in macroeconomics, only recently has Abio (2003) shown that a socially optimal population growth rate can exist in the Diamond OLG model with endogenous fertility. Therefore, policies aiming at achieving the first best in a market setting may be highly valuable.

The achievement of a golden rule of procreation is even more important in economies with public systems of social security (e.g. PAYG pensions), because of the existence of external effects that children create on society as a whole (see Cigno, 1993). This implies that the *laissez-faire* fertility rate may be different from the golden rule one.[1]

Moreover, population ageing and below-replacement fertility, experienced especially in developed countries (see, e.g., Chesnais, 1998; Kohler, 2002; Livi-Bacci, 2006), have exacerbated problems about offspring externalities and even stimulated some recent analyses of welfare implications of public pensions in an OLG small open economy environment with endogenous fertility, see van Groezen et al. (2003) and Fenge and Meier (2005).[2] The former authors show that PAYG pensions and child allowances act as Siamese twins to replicate the *first best* in a market economy with *fixed* costs of children (represented by expenditures on commodities necessary for their upbringing). The latter authors compare the substitutability of the child allowance with an instrument that captures the relative importance of the individual number of children on PAYG pensions (namely, the child factor), to replicate the *second best* allocation in an economy with *time* cost of children (i.e., raising a child reduces the time available for parents to earn labour income in the market).

This paper discusses an alternative way to deal with the external effects of children in an OLG small open economy with endogenous fertility and PAYG pensions, namely the tax-*cum*-subsidy (T/S) policy[3] (defined here as the case of a linear wage tax collected and rebated as a lump-sum subsidy within the working generation), and extends Fanti and Gori (2012). In particular, contrary to van Groezen et al. (2003) and similarly to Fenge and Meier (2005), we define the cost of child upbringing in terms of time. First, we give necessary and sufficient conditions for the existence of the first best, namely, when the market interest rate is fairly low (high), the planner should be patient (impatient) enough, i.e. it should discount the utilities of future generations slightly (heavily), otherwise the command optimum cannot exist. Second, we show that PAYG pensions (tax on the elderly) and the T/S (S/T) policy act as Siamese twins

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[1] Indeed, with PAYG pensions in existence, children imply a positive externality in the economy which, however, is not taken into account by each single individual in the market because the contribution to the PAYG system is shared among all the members of that generation, and thus the benefit of having a child is too small to be internalised. In fact, as van Groezen et al. (2003, p. 245) claimed: "a PAYG-pension system implies that part of the social benefits of having a child reveals itself in a growing tax base and are imperceptible to the individual parent."

[2] Other contributions that deal with similar topics are Kolmar (1997), Abio et al. (2004), Cigno and Werding (2007), Fenge and Meier (2009) and Fenge and von Weizsäcker (2010).

[3] See Atkinson and Stiglitz (1980) and Gahvari (1993).

to realise the command optimum in a market setting.[4] Indeed, the latter instrument can be used effectively as an alternative to child allowances to correct offspring externalities because it stimulates fertility and reduces the opportunity cost of children. Moreover, and most important, with PAYG pensions in existence the T/S scheme represents an A–Pareto improvement because the rise in fertility overcompensates the reduced consumption, and thus implies a welfare gain. Indeed, in their pioneering work Golosov et al. (2007) have introduced a generalised notion of Pareto efficiency that can be applied in environments with endogenous population to compare alternatives, i.e. to compare either the welfare of the same groups of people with respect to different allocations or the welfare of different groups of people, i.e. different generations in different time periods, with respect to one allocation, as constant population is required for its standard concept to be properly used. In that paper, a distinction is made between A–efficiency (i.e., Pareto efficiency applied to people *alive* in each state) and P–efficiency (i.e., Pareto efficiency applied to agents who are alive in each state and to *potential* agents too, namely those who are not yet born). In the former case, therefore, efficiency is defined by comparing states only between born agents, and thus it is unnecessary to specify utility functions for the unborn. In the latter, unborn descendants are treated symmetrically with those who are already born, and a utility function for them exists in turn.[5]

Since in the present paper population is endogenous because individuals choose the desired family size, the standard concept of Pareto efficiency cannot be applied to draw valid normative conclusions. Though the treatment of the unborn as economic agents is a relevant issue, we follow Conde-Ruiz et al. (2010) and exclusively evaluate the preference profiles of agents who are alive in each state and, hence, we use the concept of A–efficiency throughout.

The rest of the paper is organised as follows. Section 2 presents the market economy. Section 3 shows that the command optimum can be realised by a government in a market that has both PAYG transfers from the young to the old and the T/S scheme at its disposal. Section 4 discusses the welfare effects of the T/S policy in an economy with PAYG pensions by applying the notion of A–Pareto efficiency. Section 5 concludes.

2. The market economy

Consider an OLG small open economy with perfect capital mobility that faces an exogenously given (constant) interest rate r. Production takes place according to the neoclassical constant-returns-to-scale technology $f(k_t, h_t)$, where k_t and h_t are, respectively, the stock of capital per young person and the labour time. Since capital is perfectly mobile and the labour time is variable, a constant interest rate implies a fixed ratio of capital to the total volume of labour as well as a constant wage rate w per unit of labour, but the stock of capital per young varies with h_t in the long run.

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[4] Note that we provide a formal proof of which combination of instruments should actually be used to realise the first best in a market setting only in the case when both the world interest rate and social discount rate are fairly low, the other case being symmetrical.

[5] Other two important papers that deal with the modified notion of Pareto efficiency with endogenous population in models with overlapping generations are Michel and Wigniolle (2007) and Conde-Ruiz et al. (2010).

At every date $t = 0, 1, 2, \ldots$, the government finances two policies with separate (balanced) budgets, which are detailed below.

(*i*) Fertility-related PAYG pensions $P_t = p_t N_{t-1}$ to redistribute across generations, which are constrained by the amount of tax revenues $\eta\left[(1-\omega)\bar{n}_{t-1} + \omega n_{t-1}\right] N_t$, where p_t represents the per old pension expenditure in period t, $\eta \in (0, w)$ is a constant lump-sum contribution to the fertility-related PAYG system paid by young workers, $\bar{n}_{t-1}$ is the average number of children in the overall economy at time $t-1$, n_{t-1} is the individual number of children in the same period, N_i is the number of young adults of generation i, and n_{t-1} is the child factor (see, Kolmar, 1997; Abio et al., 2004; Fenge and Meier, 2005, 2009; Fenge and von Weizsäcker, 2010), which captures the importance of the individual number of children relative to the average number of children into the PAYG system. Since $N_t = \bar{n}_{t-1} N_{t-1}$, the *per pensioner* budget constraint of the government in period t reads as:

$$p_t = \eta\left[(1-\omega)\bar{n}_{t-1} + \omega n_{t-1}\right]. \tag{1.1}$$

Eq. (1.1) therefore shows that at time t PAYG pensions depend on (*a*) the individual rate of fertility at time $t-1$ (n_{t-1}) with a share ω of the contribution, and (*b*) the average rate of fertility in the overall economy at time $t-1$ ($\bar{n}_{t-1}$) with a share $1-\omega$ of the contribution. The case $\omega = 0$ defines the standard PAYG system; the case $0 < \omega < 1$ defines the partially fertility-related PAYG system; the case $\omega = 1$ defines the fully fertility-related PAYG system; the case $\omega > 1$ defines a *hyper-child* pension system; the case $\omega < 0$ defines a *under-child* pension system which, therefore, includes a negative child pension component into the pension formula (see Fenge and von Weizsäcker, 2010).

(*ii*) a T/S scheme $\tau_t N_t = \sigma w h_t N_t$ to redistribute within the working-age generation, where $\tau_t > 0$ and $0 < \sigma < 1$ are a lump-sum subsidy and a constant labour income tax rate, respectively.[6] The *per young* government budget of the T/S policy can therefore be expressed as:

$$\tau_t = \sigma w h_t. \tag{1.2}$$

The economy is populated by perfectly rational and identical three-period lived individuals. Life is divided into childhood and adulthood, the latter period being in turn divided into working/child-rearing time (youth) and retirement time (old age). We assume that only those who are born have utility functions. Children do not take economic decisions. Young adults of generation t (N_t) draw utility (U_t) from young- and old-age material consumption ($c_{1,t}$ and $c_{2,t+1}$, respectively) and the number of children (n_t), see Eckstein and Wolpin (1985), Eckstein et al. (1988) and Galor and Weil (1996), which are assumed to be a normal good. This is the so-called weak form of altruism towards children (see Zhang and Zhang, 1998), because parents derive utility directly from the number of children they have but do not enjoy the utility derived from their descendants. Each young agent devotes a fraction $h_t = 1 - q n_t$ of time to work on the labour market and earns the wage w per unit of labour, with $q n_t$ being the

[6] The T/S policy, therefore, is defined here as the case of a linear wage tax collected and rebated in a lump-sum fashion within the working generation. Note also that the opposite case of PAYG tax on the old and subsidy-*cum*-tax (S/T) are defined as $\eta < 0$ and $\sigma < 0$, respectively.

share of time spent raising n descendants in period t and $0<q<1$.[7] Therefore, the budget constraint of the young of generation t reads as follows:

$$c_{1,t}+s_t=w(1-q\,n_t)(1-\sigma)+\tau_t-\eta\,, \tag{2.1}$$

i.e., the disposable income is divided between material consumption and savings (s_t). Old individuals retire and live on the proceeds of savings plus the interest accrued from t to $t+1$ at the constant rate r that prevails in the world capital market, and the expected pension benefit, $p^e{}_{t+1}$, that is:

$$c_{2,t+1}=(1+r)s_t+\eta\left[(1-\omega)\bar{n}_t+\omega n_t\right]. \tag{2.2}$$

By taking factor prices and Eqs. (1.2) and the average number of children as given, the individual representative of generation t chooses material consumption over the life cycle and fertility to maximise the lifetime logarithmic utility function

$$U_t=\ln(c_{1,t})+\beta\ln(c_{2,t+1})+\gamma\ln(n_t)\,, \tag{3}$$

subject to Eqs. (2.1) and (2.2), where $0<\beta<1$ is the subjective discount factor and $\gamma>0$ the parents' relative taste for children. The first order conditions for an interior solution are:

$$\frac{c_{2,t+1}}{c_{1,t}}\cdot\frac{1}{\beta}=1+r\,, \tag{4}$$

$$\frac{c_{1,t}}{n_t}\cdot\gamma=q\,w(1-\sigma)-\omega\frac{\eta}{1+r}. \tag{5}$$

Eq. (4) equates the marginal rate of substitution between consumption when young and when old to the constant world interest factor; Eq. (5) equates the marginal rate of substitution between consumption when young and the number of children to the marginal cost of raising an additional child. Notice that the labour income tax rate σ, as part of the T/S policy, and the child factor $\omega>0$ both act as a child allowance by reducing the marginal cost of children.

Combining Eqs. (1), (2), (4) and (5) gives:

$$n^*=\frac{\gamma(w-\eta)}{q\,w[(1+\beta)(1-\sigma)+\gamma]-[(1+\beta)\omega+\gamma]\frac{\eta}{1+r}}\,, \tag{6}$$

$$c_1^*=\frac{(w-\eta)\left[q\,w(1-\sigma)-\omega\frac{\eta}{1+r}\right]}{q\,w[(1+\beta)(1-\sigma)+\gamma]-[(1+\beta)\omega+\gamma]\frac{\eta}{1+r}}. \tag{7}$$

From Eq. (6) the following proposition holds:

Proposition 1. *The T/S policy and the existence of a positive child factor into the pension formula always stimulate fertility.*

Proof. Since $\frac{\partial n^*}{\partial\sigma}>0$ for every $0<\sigma<1$, and $\frac{\partial n^*}{\partial\omega}>0$ for every $\omega>0$, Proposition 1 follows. **Q.E.D.**

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[7] Note that $h_t>0$ implies $n_t<1/q:=\hat{n}$, which is the maximum number of children that an individual can give birth to. Indeed, the higher the time spent raising a child, the lower $\hat{n}$.

Proposition 1 shows that the use of a T/S scheme reduces the opportunity cost of children (i.e., the net labour income earned by the young), because it stimulates fertility thereby reducing the supply of labour. Indeed, as can be seen from Eq. (5), the existence either of the term $(1-\sigma)$ or the term ω in the right-hand side of it implies, when PAYG pensions are in existence, that the (endogenously determined) number of children is too low in the absence of an appropriate policy as they give rise to a positive externality that is not taken into account by each single individual in the market.

3. The first best solution

In this section we derive the first best solution and analyse how the government can use the intra- and inter-generational instruments discussed above to make the first best feasible in a market setting.

Following van Groezen et al. (2003), we assume that the social planner maximises the discounted flow of individual (lifetime) utilities over an infinite horizon

$$W_t = \sum_{i=t}^{+\infty} \delta^{i-t} \cdot U\left(c_{1,i-1}, c_{2,i}, n_{i-1}\right), \tag{8}$$

subject to the economy's resource constraint $f\left[k, h_i(n_i)\right] = c_{1,i} + \frac{c_{2,i}}{n_{i-1}} + n_i k - n_i d_{i+1} + (1+r) d_i$,

where $0 < \delta < 1$ is the social discount factor, d is the amount of per young foreign debt and capital is assumed to fully depreciate at the end of every period.[8] Maximisation of Eq. (8) gives the following first order conditions for the command optimum:

$$\frac{c_{1,t+1}}{c_{1,t}} = \frac{\delta(1+r)}{n_t}, \tag{9}$$

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1}{\beta} = 1+r, \tag{10}$$

$$\frac{c_{1,t}}{n_t} \cdot (\beta + \gamma) = q\,w + k - d_{t+1}. \tag{11}$$

Comparing Eqs. (5) and (11) makes clear the reasons why the individual number of children may differ from its socially optimal value: the planner, in fact, takes into account both the *inter-generational transfer effect* (the marginal benefit) and the *capital dilution effect* (the marginal cost) of raising an additional child. The former effect is captured by the subjective discount factor β in Eq. (11): it implies that the number of children in the market is too low in comparison with the social optimum.

[8] It should be noted that in the case of fixed cost of children (see van Groezen et al., 2003), the labour time is constant and, thus, with a linearly homogenous aggregate production function defined on capital and aggregate labour, the stock of capital per young person is constant as well. In contrast, in this model raising children is time-consuming and this, in turn, affects the labour supply h of the young parents. Therefore, in a small open economy with a constant interest rate the capital stock per young depends on h and, hence, on n, because the marginal product of capital is influenced by the amount of labour in the long run. This would significantly change the analysis of the first best respect to van Groezen et al. (2003). Indeed, the economy's resource constraint would no longer be concave and, hence, corner solutions for n may appear in such a case. Therefore, in order to make the model tractable, we need a further assumption to keep the stock of capital per young constant in the small open economy: i.e., we assume a sort of short-medium run where the speed of adjustment of the labour supply is higher than that of capital, so that the former does not influence the latter.

The latter describes how fertility affects savings: it is measured by the term $k-d$ in Eq. (11) and implies that the number of children in the market is too high in comparison with the social optimum. Only if these two opposite forces exactly cancel each other out is the fertility rate in the market optimal (see van Groezen et al., 2003).

Exploiting Eqs. (9)-(11) and the economy's resource constraint, we get the optimal amount of per young foreign debt as:

$$d^{**}=k+q\,w\cdot\left[\frac{\beta+\delta(1+\beta+\gamma)}{\delta(1+\beta+\gamma)-\gamma}\right]-\frac{w}{1+r}\cdot\frac{\beta+\gamma}{\delta(1+\beta+\gamma)-\gamma}. \tag{12}$$

Combining Eqs. (9), (11) and (12), the first best fertility rate and young-age consumption are respectively given by:

$$n^{**}=\delta(1+r), \tag{13}$$

$$c_1^{**}=\frac{\delta\,w[1-q(1+r)]}{\delta(1+\beta+\gamma)-\gamma}. \tag{14}$$

From Eq. (13) the condition $n<1/q:=\hat{n}$ implies $\delta<\hat{\delta}:=1/q(1+r)$ must hold to ensure $h(n)>0$.

Now, define *Case (a)* as $r<\bar{r}$ and $\bar{\delta}<\delta<1$, and *Case (b)* as $r>\bar{r}$ and $0<\delta<\bar{\delta}$, where $\bar{r}:=(1-q)/q$ and $\bar{\delta}:=\gamma/(1+\beta+\gamma)$. Moreover, let $\bar{\bar{\delta}}:=\hat{\delta}\cdot\bar{\delta}$ and $\bar{\bar{r}}:=(q\cdot\bar{\delta})^{-1}-1$ be two threshold values of the social discount factor and the constant world interest rate, respectively, with $\bar{r}<\bar{\bar{r}}$ and $\hat{\delta}<\bar{\delta}$ ($\hat{\delta}>\bar{\delta}$) for every $r>\bar{\bar{r}}$ ($r<\bar{\bar{r}}$). Therefore, we have the following proposition as regards the existence of the command optimum.

Proposition 2. *The first best solution exists if and only if either Case (a) or Case (b) holds*.

Proof. From Eq. (14) it is straightforward to show that $c_1^{**}>0$ if and only if *Case (a)* or *Case (b)* is alternatively fulfilled. Moreover, $U''_{c_1c_1}<0$ and $U''_{nn}<0$ always hold. **Q.E.D.**

Proposition 2 reveals the necessary and sufficient (technical) conditions for the existence of the first best, showing that the planner should either be patient (far-seeing), if the constant world interest factor is fairly low, *Case (a)*, or impatient (myopic), if the constant world interest factor is fairly high, *Case (b)*.[9] Unfortunately, it is difficult to precisely know what values the social discount rate takes in real economies, and then what conditions are likely to be fulfilled. Therefore, we have chosen only to present a theoretical model without embarking in empirical investigations of what values the social discount rate should actually take. Moreover, it is interesting to note that in economic theoretical literature there exists a long lasting debate, based on a plethora of different arguments presented by leading philosophers, economists and mathematicians, about whether the future generations' well-being should be discounted (e.g., Koopmans, 1967; Dasgupta and Heal, 1979) or not (e.g., Ramsey, 1928; Harrod, 1948).

[9] This implies that if the market interest is low (high) enough and the planner heavily (slightly) discounts the utilities of future generations, the first best solution does not exist.

Depending on whether the planner is patient or impatient, however, our model implies the existence of a policy mix that should alternatively be used to make the first best feasible in a market setting, as the following proposition shows.[10]

Proposition 3. *Let Case (a) hold. Then* $\hat{\delta} > 1$ *and* $\bar{\bar{\delta}} > \bar{\delta}$. *Therefore, for any* $r < \bar{r}$ *the command optimum can be decentralised by a government in a market economy: (1) without intervention if, and only if,* $\delta = \bar{\bar{\delta}}$*; (2) for any* $-\tilde{\omega} < \omega < 1$ *(*$\omega > 1$*), using the couple* $0 < \eta_{GR}(\delta) < w$ *and* $0 < \sigma_{GR}(\delta) < 1$ *(*$0 < \eta_{GR}(\delta) < w$ *and* $\sigma_{GR}(\delta) < 0$*) if* $\bar{\bar{\delta}} < \delta < 1$*; (3) for any* $\omega < 1$ *(*$1 < \omega < \tilde{\omega}$*) using the couple* $\eta_{GR}(\delta) < 0$ *and* $\sigma_{GR}(\delta) < 0$ *(*$\eta_{GR}(\delta) < 0$ *and* $0 < \sigma_{GR}(\delta) < 1$*) if* $\bar{\delta} < \delta < \bar{\bar{\delta}}$*; (4) when* $\omega = 1$*, using the couple* $0 < \eta_{GR}(\delta) < w$ *and* $\sigma_{GR}(\delta) = 0$ *(*$\eta_{GR}(\delta) < 0$ *and* $\sigma_{GR}(\delta) = 0$*) if* $\bar{\bar{\delta}} < \delta < 1$ *(*$\bar{\delta} < \delta < \bar{\bar{\delta}}$*); (5) for any* $\sigma < 0$ *(*$0 < \sigma < 1$*), using the couple* $0 < \eta_{GR}(\delta) < w$ *and* $\omega_{GR}(\delta) > 1$ *(*$0 < \eta_{GR}(\delta) < w$ *and* $1 > \omega_{GR}(\delta) > -\tilde{\omega}$*) if* $\bar{\bar{\delta}} < \delta < 1$ *(6) for any* $\sigma < 0$ *(*$0 < \sigma < 1$*), using the couple* $\eta_{GR}(\delta) < 0$ *and* $\omega_{GR}(\delta) < 1$ *(*$0 < \eta_{GR}(\delta) < w$ *and* $1 < \omega_{GR}(\delta) < \tilde{\omega}$*) if* $\bar{\delta} < \delta < \bar{\bar{\delta}}$*, where*

$$\eta_{GR}(\delta) := \frac{w[\delta q(1+r)(1+\beta+\gamma)-\gamma]}{\delta(1+\beta+\gamma)-\gamma}, \tag{15}$$

$$\sigma_{GR}(\delta) := \frac{(1-\omega)[\delta q(1+r)(1+\beta+\gamma)-\gamma]}{q(1+r)[\delta(1+\beta+\gamma)-\gamma]}, \tag{16}$$

$$\omega_{GR}(\delta) := 1 - \frac{\sigma q(1+r)[\delta(1+\beta+\gamma)-\gamma]}{\delta q(1+r)(1+\beta+\gamma)-\gamma}. \tag{17}$$

Proof. To be provided.

Proposition 3 reveals that both the direction and extent of redistribution across and within generations to implement the first best solution depend on the key parameters of the problem. In particular, if the social discount rate is low enough (i.e., high values of δ, $\bar{\bar{\delta}} < \delta < 1$), the command optimum can be replicated with the use of both PAYG tax and either the T/S policy or child pensions by a government in the market. Indeed, in an economy with PAYG pensions the fertility (saving) rate is too low (high) if compared with the social optimum, because children imply a positive external effect on society as a whole and the inter-generational transfer effect is higher than the capital dilution effect. A rise in the PAYG tax, therefore, is necessary to increase redistribution towards the elderly. This in turn causes a rise in the amount of per young foreign debt while also implying a reduction in the market fertility rate. However, an appropriate use of the T/S policy (i.e., a rise in the linear wage tax rate σ) or the child factor increases fertility and reduces saving up to the point in which the externality of children is completely eliminated and the social optimum is achieved.

[10] Note that for reasons of expository clarity we exclusively concentrate on *Case (a)* and discuss the policy implications of it, while leaving *Case (b)* untreated. This because the recommended optimal policy mix is symmetrical depending on the mutual relationship between the market interest rate and the social discount rate.

Only for a specific set of parameter values, i.e., $\delta = \overline{\overline{\delta}}$, the market solution automatically coincides with the first best and thus no government intervention is indeed necessary in such a case.

Therefore, using the words of van Groezen et al. (2003), we can conclude that when fertility is endogenous and raising children is time-consuming, a PAYG tax and either a T/S scheme or child pensions act as "Siamese twins" to realise the first best in a competitive market setting.

4. Welfare effects of the tax-*cum*-subsidy policy

In the previous section we analysed and discussed an alternative way to child allowance, namely the T/S policy, to deal with the external effects of children in economies with PAYG inter-generational transfers. We showed how to decentralise the first best solution in a market economy starting from the case in which the fertility rate is in general suboptimal if no instruments exist to correct the externalities of children.

There exists an extensive literature that deals with the problem of reforming social security, especially in the light of problems of population ageing that developed countries are still facing (see, amongst others, Breyer, 1989; Homburg, 1990; Belan et al., 1998; Corneo and Marquardt, 2000; Boeri et al., 2001, 2002; van Groezen et al., 2003), as the market interest rate tends to become higher than the rate of return of the PAYG system (namely, the fertility rate). Indeed, when PAYG pensions are in existence and fertility is endogenous, van Groezen et al., (2003) have shown that starting from the case of absence of child allowance (a situation which is not Pareto efficient given the positive external effects that children create on society as a whole in such a case), introducing a subsidy per child at not too high a level generates a Pareto improvement, while merely reducing the PAYG tax together with a debt policy does not.

Our objective in this section is to study the welfare effects of, alternatively, the T/S policy and child pensions when PAYG transfers from young workers to old retirees exist, and the number of children represents a choice variable for parents. More precisely, we go one step further and apply the notion of A–efficiency (i.e., Pareto efficiency applied to people alive in each state) introduced by Golosov et al. (2007) since we do not evaluate utilities of potential agents, namely those who are yet unborn. The results of our analysis are summarised in the following proposition.

Proposition 4. *(1) Let $0 < \eta < w$ hold. Then, for any $0 < \omega < 1$ a T/S policy $0 < \sigma < \hat{\sigma}$ is A–Pareto-improving and*

$$\sigma = \sigma^* := \frac{(1-\omega)\eta}{q\,w(1+r)} \tag{18}$$

is A–Pareto efficient. (2) Let $0 < \eta < w$ hold. Then, for any $0 < \sigma < 1$ a child pension policy $0 < \omega < \hat{\omega}$ (for low values of σ) or $-\hat{\omega} < \omega < 0$ (for high values of σ) is A–Pareto-improving and

$$\omega = \omega^* := 1 - \sigma q\, w \frac{1+r}{\eta} \tag{19}$$

is A–Pareto efficient.

Proof. To be provided.

Proposition 4 reveals that when PAYG pensions exist the use of either the T/S scheme or child pensions can make all individuals better off, while also implying an A–Pareto improvement. The economic reason is the following: the tax on labour income (or the child factor) has the task of performing redistribution[11] in such a way to increase the incentives for child-rearing for all people in the economy and this, in turn, increases fertility which implies (*i*) a direct utility gain and (*ii*) an indirect positive effect on the size of the pension benefit "of the parents' generation" (see van Groezen et al., 2003, p. 248). Since the rise in fertility overcompensates the reduced consumption, social welfare increases. Indeed, to achieve the A–Pareto efficient allocation it is sufficient to perform the intra-generational redistribution in such a way as to let the labour income tax in the T/S scheme be equal to the present value of the contribution to the (inter-generational) PAYG scheme divided by the cost of raising an additional child. Moreover, to realise the first best the redistribution from the young to the old should be performed according to the rule dictated by Eq. (15) and used together with σ^* (Eq. 17), which equals the golden rule value $\sigma_{GR}(\delta)$ (Eq. 16) in such a case.

5. Conclusions

In this paper we analysed a small open economy with overlapping generations, endogenous fertility and time cost of children. We give necessary and sufficient conditions for the existence of the first best and show, when it exists, that the government can let the market solution coincide with the command optimum through the use of PAYG transfers from the young to the old and either a tax-*cum*-subsidy policy (i.e., a linear wage tax rebated as a lump-sum subsidy within the working generation), or child pensions. The second and third instrument, in fact, increase the incentives for children's care and thus acts as a fertility-enhancing device, while also representing an A–Pareto improvement (see Golosov et al., 2007).

A natural extension of the present paper is to include longevity as an endogenous variable, determined either by public or private investments in health (see, amongst others, Chakraborty, 2004; Chakraborty and Das, 2005; Bhattacharya and Qiao, 2007; Pestieau et al., 2008; de la Croix and Ponthière, 2010; Leung and Wang, 2010), to study whether and how the first best can be realised in a competitive economy with both endogenous fertility and endogenous longevity.

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[11] If such a tax performs redistribution across persons with different incomes, its introduction is unlikely to be a Pareto improvement.

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