Education race, supply of skills and the wage skill premium*

Luca G. Deidda

Università di Sassari, CRENoS, and CeFiMS (Un. London)

Dimitri Paolini

Università di Sassari, CRENoS and Université catholique de Louvain, CORE

April 2012

Abstract

We model a competitive labor market populated by workers who are heterogeneous in wealth and skills, in which education plays a signaling role. We show that whenever the accumulation of factors of production such has technology results in a wider wage premium for skills over time – as it might happen under skill biased technological progress – the investment in education needed to sustain a talent separating equilibrium, in which skilled workers are able to perfectly signal their skills, also increases. Hence, increases in the wage skill premium induce an education race as skilled individuals try and invest more to signal themselves. However, if due to imperfect capital markets, the borrowing capacity of poor individuals is lower than that of rich ones, such race will eventually come to an end as poor and skilled individuals are no longer able to finance the amount of investment needed to signal their talent, and end up pooled together with unskilled and rich at a lower level of education. Hence, the behavior of the long run supply of skills with respect to an increase in the wage-skill premium is sluggish. Such mechanism supports a supply side explanation –which complements the skill bias technological change hypothesis – for the long run trends of (i) The wage-skill differentials and (ii) The relative supply of postgraduates and college graduates in the US labor market.

JEL CODES: D4, D8, L15
KEYWORDS: Signaling, Wage-skill premium, supply of skills

* We thank Andreas Irmen, Francesco Lippi, Pierre Picard and, particularly, Luisito Bertinelli for helpful comments and suggestions. Moreover, we thank seminar participants at CREA (University of Luxembourg), CRENoS-DiSEA (University of Sassari) and DSE (University of Firenze) for comments. Part of this paper has been written while the second author was hosted in CREA (University of Luxembourg). Financial support from the Luxemburgish Fond National de la Recherche (FNR; AM2c measure) is gratefully acknowledged. Both authors gratefully acknowledge financial support by the Italian Ministero dell' Università (PRIN) and by the Regione Autonoma Della Sardegna (Legge n. 7).

1 Introduction

There is a vast literature documenting both the rise of wage inequality across educational groups of American workers and the evolution of the supply of skills over the last forty years[1].

Many economists have proposed a demand driven explanation for the observed increase in wage inequality across skill groups, based upon the idea of skill bias technical progress (see Acemoglu, 2002). In this paper, we put forward a theory of supply of skills under imperfect capital markets and asymmetric information, which provides a complementary explanation.

Dividing American workers – into the following five educational groups: i. Postgraduate degree holders (PGs), ii. College Graduates (CGs); iii. Workers with some college (SCs); iv. High school graduates (HSGs); v. High school dropouts (HSDs), over the period 1963-2002[2] the following trends emerge[3]

i. Wage premium for education over time (figure 1): Over the whole period, inequality across educational groups has increased over time. More educated groups have been generally gaining on less-educated ones;

ii. Educational composition of workforce (see figure 2) as well as 8, 9, and 10 in appendix): The percentage of PGs and CGs and SCs in the overall workforce has increased over time, while the percentage of HSGs and HSDs have been decreasing over time.[4]

Particularly relevant to this paper is the fact that:

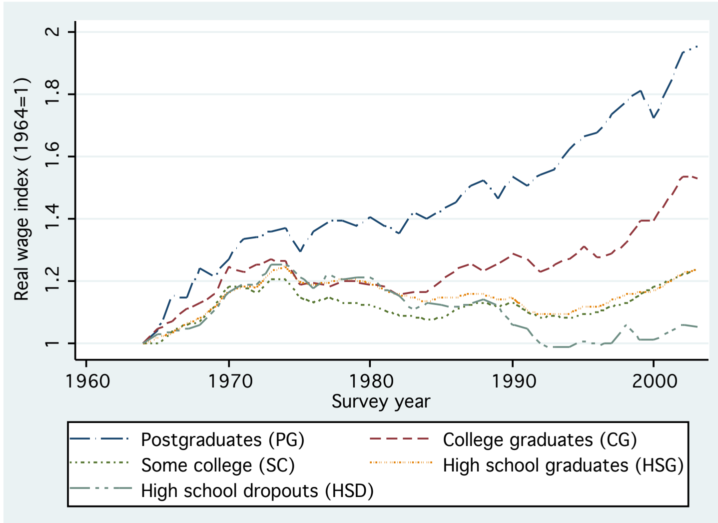
[1]Acemoglu, 2002, Goldin and Katz, 1998, 2007, Katz and Murphy, 1992, Autor, Katz and Kearny, 2008, Autor, Katz and Krueger, 1997

[2]We acknowledge the use of data made publicly available by Zvi Eckstain and Eva Nagypal, and we actually follow the same procedure they used in Eckstein Nagypal (2004).

[3]This is the standard classification adopted in the relevant literature.

[4]Note that: i. For white collars and professional workers the educational composition matches that for the overall workforce; ii. For blue collars, the percentage of workers with SCs, CGs or HSGs have grown over time, while PGs have stayed more or less constant, and finally, the percentage of workers with HSDs has been decreasing.

Figure 1: Wage premium for education



i. A substantial part of skill-wage premium is accounted for by the growth of salaries paid to PGs vs those paid to SGs or less. That is, salaries of PGs have been growing substantially faster than salaries of CGs (see figure 3).[5]

ii. Yet – in spite of the fact that the wage skill premium has grown more for PGs than for CGs – over the same time period – the relative supply of PGs has increased less than that of CGs (see figure 2 and 3).

If the return to postgraduate education has grown more than the return to college education, why hasn't the relative supply of PGs increased more than the supply of CGs?

We provide an answer to this question by developing a model of the supply of skills in the labor market where –close in spirit to the Spence (1974) – education serves as a signal. We show that –other things equal– with imperfect capital markets, an increase in the wage skill differential –either caused by exogenous changes in some state variable such as capital or technological progress – could be associated with a reduction in the relative supply of observable skills associated with the highest level of education compared to the relative supply of skills associated with lower levels of education. Such sluggish behavior of the supply of skills reinforces the effect of demand driven factors such as skill biased

[5]Barrow and Rouse (2005) calculate that the hourly wage gap between college and non college educated workers which had grown by 25%, grew only by 10% in the 1990s.

Figure 2: Educational composition of workforce

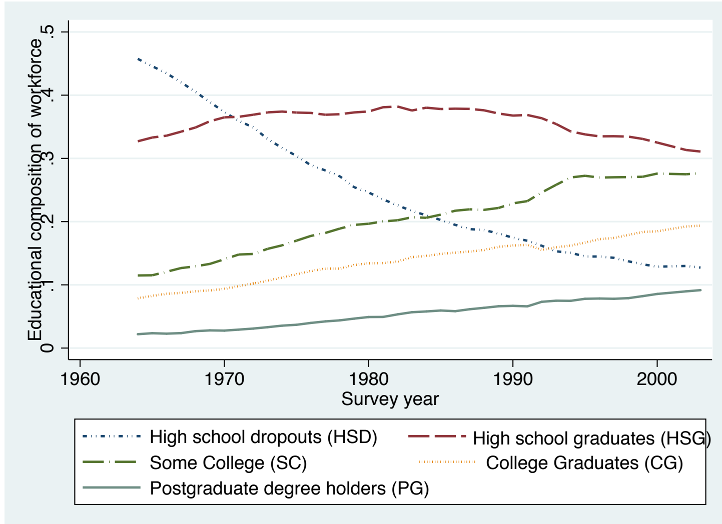
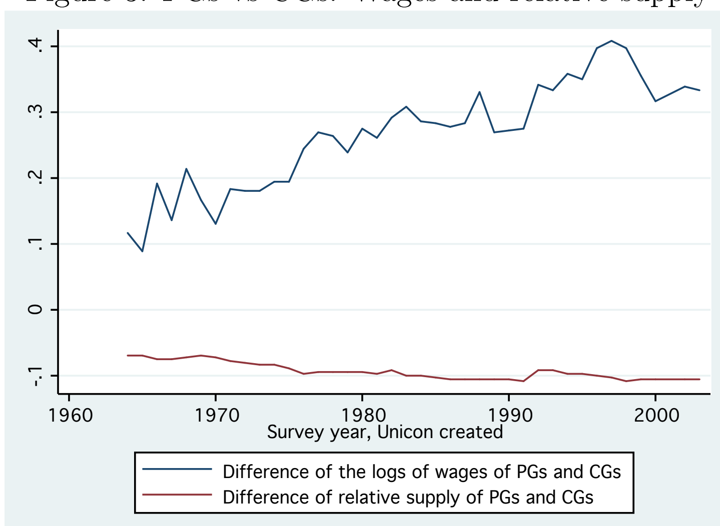


Figure 3: PGs vs CGs: Wages and relative supply



technical progress, in determining the evolution of the wage skill premium for PGs.

We develop a static model of a labor market populated by competitive firms and heterogeneous workers. Workers are heterogeneous along two dimensions: initial wealth and skills (i.e. talent). Some individuals are poor and some other are rich; some individuals are talented and some others are untalented. Both individual skills and wealth are private information. Firms demand labor in order to produce. The marginal productivity of labor depends positively on the skills of the worker(s), and, on the stock of other accumulable factors such as (physical) capital and/or technology. Firms are competitive and take prices as given. Since they hire workers not knowing their productivity, equilibrium salaries equal expected marginal productivity of workers conditional on education.

Workers use their time endowment to get education and work. Other things equal, skilled workers benefit more from education than unskilled workers. This sorting condition implies the possibility of a talent separating equilibrium (TSE) whereby, only talented workers invest in education. Under this equilibrium education is a perfect signal, so that individual skills can be perfectly inferred from the individual level of education. Supply of equilibrium observable high skill labor is maximum in that all talented workers are able to reveal their talent in the labor market. Since all workers are paid their expected productivity, in this TSE educated workers, who are all talented, are paid a salary greater than the salary paid to the uneducated ones, who are all untalented. The wage difference between the two groups measures the market skill premium.

However, investing in education requires financial resources, and, with imperfect capital markets, poor agents face a higher cost of financial capital compared to rich individuals. We model imperfect capital market by introducing the extreme assumption that no capital market where agents can borrow actually exists. Accordingly, the TSE described above can only exist if the investment in education that sustains such equilibrium implies an amount of financial resources that poor individuals can self-finance.

Assume the economy finds itself in theTSE described above, and consider the effect of an increase in the endowment of accumulable inputs, by means of comparative statics. The marginal productivity of workers should go up. However, it could be the case that the marginal productivity of skilled workers goes up more than that of low skill workers. This would happen for instance in the case of skill biased technological progress (Acemoglu, 2002). Suppose, this is the case. Then, the wage premium between educated (skilled) and uneducated (unskilled) workers associated with the TSE should go up. This would mean that the reward to the investment in education has increased; Which, in turns, implies that unskilled workers have a stronger motive to mimic skilled workers by investing in education. Accordingly, the level of investment in education by skilled workers necessary to sustain a TSE must increase. As the wage-skill premium increases, individuals have to engage in an education race, by investing more in education in order to signal their skills.

The above mechanism would eventually lead to a situation in which as the wage premium for education grows large enough, the level of investment in education necessary to sustain a TSE becomes greater than the maximum amount that poor individuals can self-finance. At this stage, if there are no capital markets where poor individual can borrow, the TSE cannot longer exist. We show that –under these circumstances – the equilibrium that prevails is a talent pooling equilibrium (TPE) in which only rich and talented individuals are able to perfectly signal their high skills, while rich and untalented and poor and talented are pooled together at a lower (intermediate) level of education.

Under this TPE, the relative supply of equilibrium observable high skills –as revealed by the highest equilibrium level of education – is reduced even compared with the relative supply of equilibrium observable intermediate skills – associated with the intermediate education level — compared to what happens in the TSE. This is because in the TPE "Poor and talented" are pooled together with "rich and untalented" at a lower level of education compared to that played by the "rich and skilled". The education level played by this pool of heterogeneous agents is still a signal of skills, but the expected skill level of the pool is lower than the skill level supplied by rich and talented individuals who are investing more in education. Hence, on overall the supply of high skills displays a sluggish behavior with respect to the increase in the wage skill premium. Education goes up, but the relative supply of (equilibrium observable) skills does not go up at all levels of education, as not all skilled individuals are able to invest enough in education to signal themselves. More precisely, even though the wage skill premium grows in favor of high skills, the relatively supply of high skills – associated with the highest level of education – goes down compared to the relative supply of intermediate skills – associated with the intermediate level of education.

The model provides a micro foundation for the sluggish behavior of the supply of skilled labor at highest levels of education (PGs), which could complement the effect of demand-driven models, such as the skill biased technological progress hypothesis, in explaining (i) The long run widening of the wage gap between high skill (PGs) and intermediate skill (CGs) labor, together with (ii) The higher increase in relative supply of CGs compared to PGs.

We note that as the economy moves from a TSE to a PSE, the pool of workers who get intermediate levels of education, made by "rich and unskilled" and "poor and skilled" is characterized by a higher degree of skill heterogeneity compared to the group of "skilled and rich" who take high levels of education. Accordingly, wage dispersion (not controlling for tenure) within the first of the two groups should be higher than the second, so long as wages gradually reflect true productivity as the employment relationship evolves over time. Indeed, this matches the evidence reported in figure 12 according to which while within group wage dispersion has gone up for all educational groups, it has gone up more for CGs than it has for PGs.

Also, the model's results are consistent with the evidence provided by Carneiro and Lee (2008) according to which the increase in college enrollment has been associated with a reduction in average quality over the period 1960-2000.[6]

The paper is organized as follows. Section 2 presents the baseline model. Section 3 presents the results in the case of exogenous skill biased technological change. Section 4 concludes the paper.

2 Baseline Model

We consider a market is populated by a continuum of size 1 of risk-neutral individuals and a continuum of size 1 of competitive firms. Each individual is endowed with an amount ω of initial wealth and an amount N of time. At the beginning of the economy, nature assigns a level of wealth and a level of talent to each individual. Individuals are heterogeneous along two dimensions: initial wealth, $\omega \in \Omega$, where $\Omega = \{\underline{\omega}, \overline{\omega}\}$, with

[6]Related to that, Walker and Zhu (2007) find evidence for the UK that while the average mean return to a degree did not drop in response to the large increase in the flow of graduate that took place in the during 1993-2003, there are large drops in the returns when one compares cohorts that went to university before and after the rapid expansion.

$0 \leq \underline{\omega} < \overline{\omega}$, and talent $\theta \in \Theta$, where $\Theta = \{\underline{\theta}, \overline{\theta}\}$, with $\overline{\theta} > \underline{\theta}$. A pair $\{\theta, \omega\}$ defines an individual of type $\tau = (\theta, \omega)$, and $\Gamma = \Theta \times \Omega$ is the set of all possible types. An individual is assigned talent $\overline{\theta}$, ($\underline{\theta}$), with probability π, $(1 - \pi)$, and a level of wealth, $\overline{\omega}$, ($\underline{\omega}$), with probability δ, (1-δ). To simplify notation, we define the function $i = i(\tau)$, which assigns a numeric value i to each type τ with:

i. $i = 1$ indicates rich and talented: $i(\overline{\omega}, \overline{\theta}) = 1$;

ii. $i = 2$ indicates rich and untalented: $i(\overline{\omega}, \underline{\theta}) = 2$;

iii. $i = 3$ indicates poor and talented: $i(\underline{\omega}, \overline{\theta}) = 3$;

iv. $i = 4$ indicates poor and untalented: $i(\underline{\omega}, \underline{\theta}) = 4$.

Agents are informed about the distribution of types and the distribution of wealth. Information about individual talent and wealth is private;

Firms are homogeneous. A firm hiring l units of labor from a pool of workers of average talent θ produce output y according to

$$y = \phi l, \tag{1}$$

where

$$\phi \equiv g(\theta, x) \tag{2}$$

is the marginal product of a pool of workers of average talent θ. Note that, according to equation (2), ϕ depends on x, which measures other inputs at firm-level, possibly including capital and/or technical knowledge. We assume that, $\overline{\phi} > \underline{\phi}$, where $\overline{\phi} = g(x, \overline{\theta})$, and $\overline{\phi} = g(x, \underline{\theta})$ would be the values of the marginal productivity of pools of workers of talent equal to $\overline{\theta}$ and $\underline{\theta}$, respectively.

Individuals choose how to allocate time between working and educating themselves. Let n the amount of time spent in education by an individual. Then, $N - n$ is the quantity of labor that the individual can supply in the labor market; where a degree of level n is feasible only if $n \leq N$. Obtaining a degree of level n, gives to an individual the option to earn the salary that the labor market pays for workers with such level of education, $w(n)$. Alternatively, the individual could choose not to disclose his level of education and earn the salary that the market pays to uneducated workers, $w(0)$. Note that in both case, he can supply at most an amount $N - n$ of labor, measured in units of time. An individual who does not engage in education, will get a salary $w(0)$, and he can supply at most an amount of labor equal to N.

Investing in a degree (of length) n requires an amount of financial resources $c = c(n)$ where c is strictly increasing in n. There is no capital market where individuals can borrow to finance investment in education. Hence, an individual can undertake an investment n if and only if her wealth-endowment, ω, weakly exceeds, $c(n)$, that is $\omega \geq c(n)$.

Define

$$n_i^{\max} : c(n_i^{\max}) = \omega \tag{3}$$

as the maximum investment in education that can be self-financed by an individual ndowed with an amount ω of wealth. Note that, $n_1^{\max} = n_2^{\max} \equiv n_{1,2}^{\max}$, and $n_3^{\max} = n_4^{\max} \equiv n_{3,4}^{\max}$, with $n_{1,2}^{\max} > n_{3,4}^{\max}$. We assume that,

$$n_{1,2}^{\max} > N > n_{3,4}^{\max} \tag{4}$$

so that while rich individuals can self-finance any possible investment in education, the same is not true for poor individuals.

2.1 Wages

Wages are set at the beginning of the working relationship in a Walrasian fashion – both firms and workers are price-takers, and wages equal marginal productivity of hired workers. Firms hire workers not knowing their individual talent. They observe the education of each worker, which might be informative about the talent of the worker. Hence, in equilibrium, the wage paid to workers will be equal to the expected marginal productivity of hired workers conditional on their level of education n, that is,

$$w(n) = \phi(E(\theta|n)). \tag{5}$$

where $E(\theta|n)$ is the average level of talent of workers with education n.

2.2 Individual payoff function V_i

Given a wage schedule $w(n)$ assigning a wage to each possible value of n, we define $V_i = V_i(n, w(n), w(0), c(n))$ the payoff function of an individual of type $i = 1, 2, 3, 4$, who engages in a degree n. We assume that V_i satisfies the following properties:

A1. **Payoff as a function of** n. V_i is a continuous and differentiable in n. we assume V_i is net of the value of the outside option of a worker with a level n to earn the salary $w(0)$ that uneducated workers earn, by declaring himself uneducated. Accordingly, for an uneducated worker,

$$V_i(0, w(0), w(0), c(0)) = 0 \tag{6}$$

holds. Furthermore, consider a wage schedule, $w(n) = \overline{w}$ for all $n > 0$, with $w(0) \geq 0$ and $\overline{w} > w(0)$. Then, V_i is strictly decreasing in n, with

$$\lim_{n \to N} V_i(n, w(n), w(0), c(n)) \quad < \quad 0 \tag{7}$$

$$\lim_{n \to 0+} V_i(n, w(n), w(0), c(n)) \quad > \quad 0. \tag{8}$$

A2. **Payoff as a function of** $w(n)$. For any given $n \in (0, N)$, V_i is continuous and strictly increasing in $w(n)$, with

$$\lim_{w(n) \to \infty} V_i(n, w(n), w(0), c(n)) \quad = \quad \infty \tag{9}$$

$$\lim_{w(n) \to w(0)} V_i(n, w(n), w(0), c(n)) \quad < \quad 0. \tag{10}$$

A3. **Payoff as a function of wealth,** ω. For a given level of talent, θ, V_i is, other things equal, increasing with ω, so that

$$V_1 > V_3 \tag{11}$$

$$V_2 > V_4 \tag{12}$$

A4. **Payoff differences across agents homogeneous in talent**. The net payoff from an investment, $n > 0$, in education is the same across individuals who are homogeneous in talent. Given a wage function $w(n)$, and two levels of education, n', and n'', with $n' \neq n''$, define

$$\Delta_i = V_i(n'', w(n''), w(0), c(n'')) - V_i(n', w(n'), w(0), c(n')). \tag{13}$$

We impose

$$\Delta_1 = \Delta_3 \equiv \Delta_{13} \tag{14}$$

$$\Delta_2 = \Delta_4 \equiv \Delta_{24} \tag{15}$$

Note that the above assumption implies additive separability of V_i in wealth, ω.

A5. **Payoff difference across individuals heterogeneous in talent**. The payoff from an investment in education is higher for talented than for untalented individuals. Given two levels of education n', n'', with $n'' > n' \geq 0$, such that $w(n'') \geq w(n')$, we impose

$$\Delta_{13} \geq \Delta_{24}, \tag{16}$$

with strict inequality if $w(n'') > w(n')$.

2.3 Sorting condition

Given the properties of V_i, the following sorting condition applies:

Lemma 1 (Sorting condition)**.** *Let $w(n')$ and $w(n'')$ two levels of salary, associated with levels of education n' and n'', respectively, where $n'' > n' \geq 0$ and $w(n'') > w(n') \geq 0$. Then, if $\Delta_{24} \geq 0$, then $\Delta_{13} > 0$ holds.*

Proof. See appendix.

2.4 Timing, equilibrium concept and characterization

The time sequence of events in the economy is the following:

Stage 0. Nature decides each worker's individual type i;

Stage 1. Given $w(n)$, workers simultaneously decide education levels;

Stage 2. Workers decide whether to supply labor or not;

Stage 3. Firms observe education levels of workers, and, given $w(n)$, firms' decide whether to demand labor or not;

Stage 4. Labor market clears and exchange (if any) takes place.

2.4.1 Equilibrium definition and candidate equilibria

Let $\mu(n)$ be a belief function that assigns a probability $\mu(n) \in [0, 1]$ that the talent of an individual who chooses a level n of education equals $\overline{\theta}$. Then,

Definition 1. *An equilibrium is a set of strategies for workers and firms, a wage function $w(n)$, and a belief function $\mu(n)$ such that:*

i. Firms and workers' strategies are optimal based upon the available information;

ii. Beliefs are derived from the strategy profiles using Bayes' rule whenever possible;

iii. The wage function, $w(n)$, is consistent with agents' optimal strategies and clears the market for labor.

Given the above definition, there are sixteen candidate equilibria in pure strategies:

1. Four equilibria with perfect talent separation: i. $1-2-3-4$; ii. $13-24$; iii. $1-3-24$; iv. $13-2-4$;

2. Pooling equilibria: i. $12-3-4$; ii. $14-23$; iii. $1-2-34$; iv. $14-2-3$; v. $134-2$; vi. $124-3$; vii. $12-34$; viii. $1-234$; ix. $123-4$; x. $1-23-4$; xi. 1234;

2.5 Characterization and existence of Talent Separating Equilibria (TSE)

Define a TSE an equilibrium in which individuals of talent $\overline{\theta}$ play $n \in \mathcal{N}_{\overline{\theta}}$ and individuals of talent $\underline{\theta}$ play $n \in \mathcal{N}_{\underline{\theta}}$, and $\mathcal{N}_{\overline{\theta}} \cap \mathcal{N}_{\underline{\theta}} = \emptyset$. Then,

Lemma 2. *In any TSE, $\mathcal{N}_{\overline{\theta}}$ and $\mathcal{N}_{\underline{\theta}}$ must be singletons.*

Proof. See appendix.

The above result directly implies that the following candidate equilibria:

1. $1-2-3-4$;

2. $13-2-4$;

3. $1-3-24$

never exist. The unique candidate TSE left is therefore $13-24$.

Lemma 3 (Characterization and Existence of TSE)**.** *A TSE is always characterized as follows: Types* 1 *and* 3 *play* $n_{13}>0$ *and receive a salary* $w_{13}=\overline{\phi}$*; types* 2 *and* 4 *play* $n_{24}=0$ *and receive a salary* $w_{24}=\underline{\phi}$*, where* $\overline{\phi}>\underline{\phi}>0$*. A TSE exists if and only if – other things equal – the marginal productivity of a talented worker,* $\overline{\phi}$*, is not too large:*

$$\overline{\phi}\leq\overline{\phi}^{\max} \tag{17}$$

where

$$\overline{\phi}^{\max}:V_i(n_{34}^{\max},\overline{\phi}^{\max},\underline{\phi},c(n_{34}^{\max}))=V_i(0,\underline{\phi},\underline{\phi},c(0)),\quad i=2,4 \tag{18}$$

Proof. See appendix.

The intuition is as follows. $\overline{\phi}$ is the wage paid to types 1 and 3 in a TSE. If $\overline{\phi}$ exceeds $\overline{\phi}^{\max}$, then the investment in education required to signal talent exceeds the maximum investment that poor and talented can afford to finance; and the TSE cannot exist. Lemma 3 also implies that –given the value of $\underline{\phi}$– a TSE exists only if there is not too much wage dispersion among skill groups, i.e. if the wage paid to high skill workers is not too different (large) than that paid to low skill workers.

2.6 Characterization and existence of Talent Pooling Equilibria (TPE)

A talent pooling equilibrium (TPE) is defined as an equilibrium where individuals of different talent, play the same level(s) of n with some positive probability.

The following constitute Pooling equilibria: i. $12-3-4$; ii. $14-23$; iii. $1-2-34$; iv. $14-2-3$; v. $134-2$; vi. $124-3$; vii. $12-34$; viii. $1-234$; ix. $123-4$; x. $1-23-4$; xi. 1234;

The following result holds

Lemma 4 (Monotonicity)**.** *Let* n' *and* n'' *two levels of* n *played with positive probability in equilibrium, so that* $w(n'')$*, and* $w(n')$ *are the corresponding wages. Then, if* $w''(n'')>(=)w'(n')$*,* $n''>(=)n'$ *must hold.*

Proof. See appendix.

We note that the distribution of talents across rich individuals is the same as that across poor individuals. Accordingly, the above lemma implies that the following candidate TPE: i. 12-34; ii. 14-23, never exist.

2.6.1 TPE equilibrium outcomes

We are left with a number of possible TPE equilibria: 1. 1234; 2. $1-234$; 3. $123-4$; 4. $124-3$; 5.$134-2$; 6. $12-3-4$; 7. $1-2-34$; 8. $1-4-23$; 9. $1-23-4$; 10. $14-2-3$.

Define,

$$\theta_{1234} \equiv \pi\overline{\theta} + (1-\pi)\underline{\theta} \tag{19}$$

$$\theta_{23} \equiv \left[\frac{\pi(1-\delta)}{\pi(1-\delta)+(1-\pi)\delta}\right]\overline{\theta} + \left[\frac{(1-\pi)\delta}{\pi(1-\delta)+(1-\pi)\delta}\right]\underline{\theta} \tag{20}$$

$$\theta_{34} \equiv \pi\overline{\theta} + (1-\pi)\underline{\theta} \tag{21}$$

$$\theta_{234} \equiv \left[\frac{(1-\delta)\pi}{(1-\delta)+(1-\pi)\delta}\right]\overline{\theta} + \left[\frac{(1-\pi)}{(1-\delta)+(1-\pi)\delta}\right]\underline{\theta}. \tag{22}$$

as the expected of pools of workers of types: (i) 2 and 3, (ii) 3 and 4, (iii) 2, 3, and 4 respectively. We provide the following result for a subset of the possible TPE:

Lemma 5 (Characterization and existence of TPE)**.**

1. *TPEs such that:*

   i. *All individuals play the same level of education,* $n_{1234} \geq 0$*;*

   ii. *Type* 1 *individuals play* $n_1 > 0$ *and types* $2, 3, 4$ *play* $n_{234} \geq 0$

   always exist. Such equilibria are characterized by salaries (i) $w_{1234} = \phi_{1234}$*, where* $\phi_{1234} \equiv \phi(\theta_{1234}, x)$*, and; (ii)* $w_1 = \overline{\phi}$*, and* $w_{234} = \phi_{234}$*, where* $\phi_{234} \equiv \phi(\theta_{234}, x)$*;*

2. *TPEs such that: Type* 1 *plays* n_1*, type* 4 *plays* n_4 *and types* $2, 3$ *play* n_{23} *are characterized by levels of education* $n_4 = 0$*,* $n_1 > n_{23} > 0$*, and associated salaries,* $w_1 = \overline{\phi}$*,* $w_{23} = \phi_{23}$*, where* $\phi_{23} \equiv \phi(\theta_{23}, x)$*, and* $w_4 = \underline{\phi}$*. They exist if and only if there exist* n_{23} *such that* $n_{23} \leq n_{34}^{\max}$ *and*

$$V_i(0, \underline{\phi}, \underline{\phi}, c(0)) = V_i(n_{23}, w(n_{23}), \underline{\phi}, c(n_{23})) \qquad i = 2, 4 \tag{23}$$

3. *TPEs such that: Type* 1 *plays* n_1*, types* $3, 4$ *play* n_{34} *and type* 2 *plays* n_2 *are characterized by investments in education,* $n_2 = 0$*,* $n_1 > n_{34} > 0$*, and salaries,* $w_1 = \overline{\phi}$*,* $w_{34} = \phi_{34}$*, where* $\phi_{34} \equiv \phi(\theta_{34}, x)$*,* $w_2 = \underline{\phi}$*. They exist if and only if there exist* n_{34} *such that* $n_{34} \leq n_{34}^{\max}$ *and*

$$V_i(0, \underline{\phi}, \underline{\phi}, c(0)) = V_i(n_{34}, w(n_{34}), \underline{\phi}, c(n_{34})) \qquad i = 2, 4 \tag{24}$$

Proof. See appendix.

2.7 Equilibrium refinement and robust equilibria

We now identify the set of equilibria robust, starting with TSE, according to the following forward induction argument.

Definition 2 (Intutitive criterion (IC))**.** *Consider a candidate equilibrium E. Let Θ_1 and Θ_2 be two subsets of the set of possible talents, Θ, such that $\Theta_1 \bigcup \Theta_2 = \Theta$ and $\Theta_1 \bigcap \Theta_2 = \emptyset$. Let Θ_1 be the subset of talents such that individuals with talent $\theta \in \Theta_1$ are worse off from a deviation, d, no matter what the beliefs of the firms observing d are. Let Θ_2 be the subset of talents such that individuals with talent $\theta \in \Theta_2$ always strictly benefit from the same deviation, d, provided that firms assign probability zero to the event that an individual of talent $\theta \in \Theta_1$ has deviated. In other words, any individual with talent $\theta \in \Theta_2$ strictly benefits from the deviation, d, for any system of firms' beliefs that assign probability zero to the event that an individual of talent $\theta \in \Theta_1$ has deviated. Then, if the subset Θ_2 is non-empty, E is not robust.*

2.7.1 Robust TSE

Lemma 6 (Unique robust TSE)**.** *The unique robust TSE outcome, is characterized as follows. Types 1 and 3 play $n_{13} = \underline{n}_{13}$, where*

$$\underline{n}_{13} : V_i(\underline{n}_{13}, \overline{\phi}, \underline{\phi}, c(\underline{n}_{13})) = V_i(0, \underline{\phi}, \underline{\phi}, c(0)), \quad i = 2, 4. \tag{25}$$

and receive a salary $w_{13} = \overline{\phi}$, while types 2 and 4 play $n_{24} = 0$, and receive a salary $w_{24} = \underline{\phi}$.

Proof See appendix.

Note that the robust TSE is the Riley outcome, that is the equilibrium associated with the lowest investment in education that allows types 1,3 to separate themselves from types 2,4.

2.7.2 Robust pooling equilibria

Lemma 7 (Non-robust TPE)**.** *Let Γ_p^E the subset of types heterogeneous in talent θ who are pooled together in a TPE, E. Then, E is not-robust if any of the pooled types is talented, i.e. if the talent of any $\tau \in \Gamma_p^E$ is $\overline{\theta}$.*

Proof. The proof is immediate.

Given the above lemma, the only TPE that could be robust are $1-34-2$, $1-32-4$, and $1-234$. With respect to these candidate equilibria, the following lemma applies.

Lemma 8 (Robust TPE)**.** *A TPE where type 1 separates and type(s) 2 and/or 4 pool with type 3 is robust if and only if the marginal productivity of a talented worker, $\overline{\phi}$, is large enough:*

$$\overline{\phi} > \overline{\phi}^{\max} \tag{26}$$

where $\overline{\phi}^{\max}$ is defined by equation (18).

Proof. See appendix. □

The intuition is as follows. $\overline{\phi}$ is the wage paid to type 1 (rich and talented) in a candidate robust TPE. If $\overline{\phi}$ is below $\overline{\phi}^{\max}$, then type 3 (poor and talented) agents could always afford the amount of education required to signal talent, so that any pooling equilibrium is not robust. Lemma 8 therefore implicitly suggests that – for given $\underline{\phi}$ – TPEs are robust if only if there is enough equilibrium wage dispersion among skill groups.

2.7.3 Prevailing equilibrium

The above analysis suggests that an increase (reduction) in the wage skill premium – due to an increase in marginal productivity of talented individuals relative to that of untalented individuals – promotes the emergence of TPE (TSE) as opposed to TSE (TPE). More precisely,

Proposition 1 (Prevailing equilibrium)**.** *The prevailing equilibrium is as follows:*

i. If $\overline{\phi} \leq \overline{\phi}^{\max}$, the unique robust equilibrium is a TSE, in which talented individuals play $n_{13} = \underline{n}_{13}$ and receive $w_{13} = \overline{\phi}$, while untalented play $n_{24} = 0$, and receive a salary $w_{24} = \underline{\phi}$;

ii. If $\overline{\phi} > \overline{\phi}^{\max}$, robust equilibria include only TPE. Individuals of type 1 always separate by playing $n_1 > 0$ and get a salary $w_1 = \overline{\phi}$, while for the other types any of the following outcomes are possible:

1. *Individuals of types, 2, 3, and 4, play $n_{234} \in [0, n_1)$, and get a salary: $w_{234} = \phi_{234}$, with $w_{234} < w_1$;*
2. *Types 2 and 3 play $n_{23} \in (0, n_1)$, and get a salary $w_{23} = \phi_{23}$, with $w_{23} < w_1$, while individuals of type 4 play $n_4 = 0$, and get a salary $w_4 = \underline{\phi}$, with $w_4 < w_{23}$;*
3. *Types 3 and 4 play $n_{34} \in (0, n_1)$, , and get a salary $w_{34} = \phi_{34}$, with with $w_{34} < w_1$, while individuals of type 2 play $n_2 = 0$, and get a salary $w_2 = \underline{\phi}$, with with $w_2 < w_{34}$.*

Proof. The proof follows immediately from the combination fo lemmata 1-8.

Proposition 1 summarizes the main result of the paper. Other things equal, if the marginal productivity of talented workers, $\overline{\phi}$ increases above a certain critical value, $\phi^{\max}$, the economy switches from an equilibrium where all talented individuals are able to perfectly signal their skills to an equilibrium where only rich talented individuals manage to do so, while poor and talented stay pooled with untalented individuals.

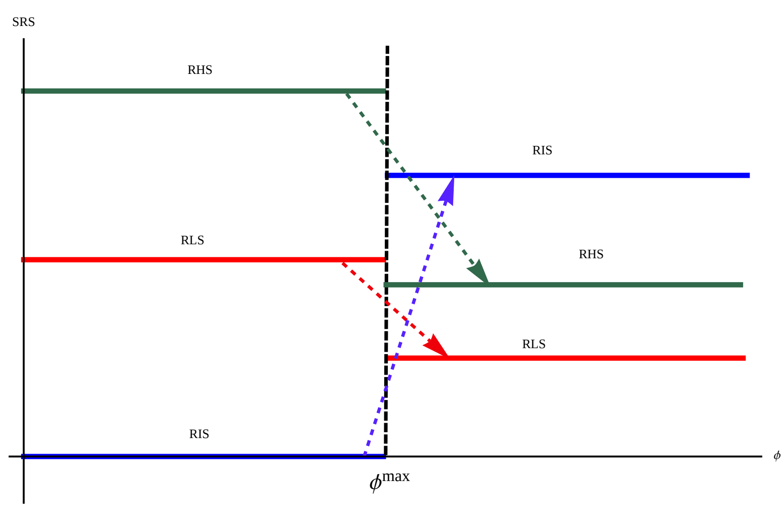
2.8 Aggregate supply of workers by education, expected skill level, wage dispersion

The prevailing equilibrium can be characterized also in terms of relative aggregate supply of workers by education levels, and expected skill levels associated with such educational levels. Table 1 describes aggregate supply depending on the prevailing equilibrium.

Table 1: Aggregate supply of workers and wages by education level

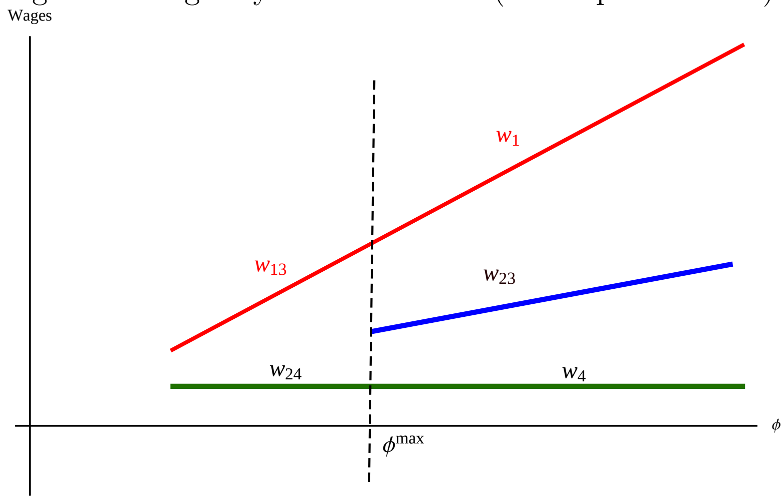
	Labor supply (LS) by education level, wages (w)					
	High		Intermediate		Low	
	LS	w	LS	w	LS	w
$TSE : \overline{\phi} \leq \overline{\phi}^{\max}$	π	$\overline{\phi}$	0	$-$	$1-\pi$	$\underline{\phi}$
$TPE : \overline{\phi} > \overline{\phi}^{\max}$	$\delta\pi$	$\overline{\phi}$	$1-\delta\pi$	ϕ_{234}	0	$-$
			$\delta+\pi-2\delta\pi$	ϕ_{23}	$(1-\delta)(1-\pi)$	$\underline{\phi}$
			$1-\delta$	ϕ_{34}	$\delta(1-\pi)$	$\underline{\phi}$

Figure 4: Labor supply by education level (and expected skills)



Starting from a situation in which, given the marginal productivity of untalented workers, $\underline{\phi}$, the marginal productivity of talented workers, $\overline{\phi}$, is below $\overline{\phi}^{\max}$, if $\overline{\phi}$ increases above $\overline{\phi}^{\max}$, aggregate relative supply of (expected) skills at various levels of education behaves as follows see figure 4

Figure 5: Wages by education level (and expected skills)



1. Relative supply of workers with high levels of education and high expected skills (RHS), drops from π to $\delta\pi$;

2. Relative supply of workers with intermediate levels of education and intermediate expected skills (RIS) increases from 0 to either $1 - \delta\pi$, or $\delta + \pi - 2\delta\pi$, or $1 - \delta$;

3 . Relative supply of workers with low levels of education and low expected skills (RLS) drops from $1 - \pi$ to either 0 or $(1 - \delta)(1 - \pi)$, or $\delta(1 - \pi)$.

Comment. As the wage skill premium increases, due to exogenous factors such as the accumulation of inputs, the supply of high expected skills drops relatively to that of expected intermediate skills. This is consistent with the observed trends in wage skill premium and relative supply of PGs and CGs in the US economy, see figure 3.

The above results also offer a direct interpretation of the sluggish behavior of the relative supply of PGs vs CGs as the wage gap between these two categories of workers has been increasing over time in favor of PGs. Suppose the economy is initially in a TSE equilibrium in which talented get college degree, i.e. $n_{13} = n_{CG}$ and untalented are getting no college education. In this equilibrium, relative supply of PGs equals zero and relative supply of CGs equals π. As $\overline{\phi}$ increases over time above $\overline{\phi}^{max}$, the economy switches to TPEs, in which rich and talented get post-graduate education, i.e. $n_1 = n_{PG} > n_{CG}$, while poor and talented pool with rich and untalented and or poor and untalented at intermediate education, which could for instance be equal n_{CG}. Suppose types 2, 3 and 4 pool together. In this equilibrium, the relative supply of PGs has increased to $\delta\pi$, while the relative supply of CGs is either equal to $1-\delta\pi$. Compared to the TSE the increase in supply of PGs is $\delta\pi$, while the increase in supply of CGs is $1-\delta\pi-\pi$. If

$$\delta < \frac{1-\pi}{2\pi} \tag{27}$$

Then the increase in CGs is greater than the increase in CGs in spite of the increase in PGs salaries being greater than that of CGs.

We next explore a variant in which production technology is subject to exogenous technical change.

3 Exogenous skill bias technical progress

Let $\theta(n)$ be the expected level of talent associated with a pool of workers characterized by a level of education n. Let $\mathcal{N}$ the set of levels of education in the economy. Following Acemoglu (2002), assume a CES production function:

$$Y = \left[\sum_{n\in\mathcal{N}} \left[\psi(\theta(n), x) L(\theta(n))\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{28}$$

where $L(\theta(n))$ is labor by a pool of workers with an homogeneous level of education n - and expected level of talent $\theta(n)$ – and $\psi(\theta(n), x)$ is the related productivity augmenting technology term. We assume that $\psi(\theta(n), x)$ is increasing in $\theta(n)$, and technology, x.

3.0.1 Demand for labor

For any expected level of talent $\theta(n)$, each firm chooses how much labor to demand, $L^d(\theta(n))$, in order to maximize profits, which implies the following implicit demand function:

$$w(\theta(n)) = \left[\psi(\theta(n), x)^{\frac{\sigma-1}{\sigma}} + \sum_{e\neq n}^{\mathcal{N}} \left[\psi(\theta(e), x)\frac{L^d(\theta(e))}{L^d(\theta(n))}\right]^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} \psi(\theta(n), x)^{\frac{\sigma-1}{\sigma}} \tag{29}$$

3.1 Supply of labor

Define δ_θ the fraction of agents of talent θ and δ_ω the fraction of agents of wealth ω. Then, the fraction of agents of type τ is

$$\delta_\tau = \delta_\theta \delta_\omega \tag{30}$$

For a given equilibrium, E, let N^E be the set of education levels played with positive probability and, for any $n \in N^E$, $\Gamma^E(n) \subseteq \Gamma$ the set of types playing n. Then, the supply of labor conditional on level n of education is

$$L^S(\theta(n)) = \sum_{\tau \in \Gamma(n)^E} \delta_\tau (N - n) \tag{31}$$

The expected level of talent conditional on n is

$$\theta(n) = \frac{\sum_{\tau \in \Gamma^E(n)} \delta_\tau \theta}{\sum_{\tau \in \Gamma^E(n)} \delta_\tau} \tag{32}$$

3.2 Equilibrium salaries

For each expected level of talent $\theta(n))$, equilibrium salaries are found imposing the market clearing condition $L^D(\theta(n)) = L^S(\theta(n))$, where $L^D(\theta(n))$ is aggregate demand for labor. Note that, for any given value of $\theta(n))$ for which supply is zero in equilibrium, the associated equilibrium salary would be $w(\theta(n)) \to \infty$.

3.2.1 Extreme off equilibrium beliefs.

Consider a symmetric equilibrium E where the not all the talents are fully revealed in equilibrium. Let Θ^E the set of talents not fully revealed in equilibrium.

Definition 3 (Extreme beliefs)**.** *Given an equilibrium E, we define extreme the beliefs associated with the equilibrium belief function $\mu(.)$, if for any $n \notin \mathcal{N}^E$, $\mu(\theta|n) = 0$ for any $\theta \in \Theta^E$, provided that Θ^E is non-empty.*

Given the above definition,

Lemma 9. *Any equilibrium E such that Θ^E is non-empty exists (and it is robust to D1) if and only if the associated beliefs are extreme.*

proof. See appendix.

In the following discussion we focus on equilibria that do not require extreme beliefs.

3.3 Equilibrium characterization

It is immediate to verify that restricting attention to equilibria such that Θ^E is empty – means that the set of possible equilibria includes only TSE of the type $13-24$ and TPE of the types $1-23-4$, $1-34-2$.

3.3.1 Characterization of theTSE robust to Intuitive criterion.

Given Lemma 6 the unique robust TSE $13-24$ is characterized as follows. Talented individuals, of types 1 and 3, play $n_{13}>0$, while untalented individuals, of type 2 and 4, play n_{24}. Accordingly,

i. The expected level of talents are:

$$\theta(n_{13}) = \overline{\theta} \tag{33}$$

$$\theta(0) = \underline{\theta} \tag{34}$$

ii. The supply of skilled and unskilled labor are respectively equal to:

$$L^S(\overline{\theta}) = \pi(N-n_{13}) \tag{35}$$

$$L^S(\underline{\theta}) = (1-\pi)N \tag{36}$$

iii. The equilibrium salaries are:

$$w_{13} = \left[\psi(\overline{\theta},x)^{\frac{\sigma-1}{\sigma}} + \psi(\underline{\theta},x)^{\frac{\sigma-1}{\sigma}}\left(\frac{(1-\pi)N}{\pi(N-n_{13})}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} \psi(\overline{\theta},x)^{\frac{\sigma-1}{\sigma}} \equiv \overline{\phi} \tag{37}$$

$$w_{24} = \left[\psi(\underline{\theta},x)^{\frac{\sigma-1}{\sigma}} + \psi(\overline{\theta},x)^{\frac{\sigma-1}{\sigma}}\left(\frac{\pi(N-n_{13})}{(1-\pi)N}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} \psi(\underline{\theta},x)^{\frac{\sigma-1}{\sigma}} \equiv \underline{\phi} \tag{38}$$

Accordingly, the wage skill premium is defined as

$$\Phi \equiv \frac{\overline{\phi}}{\underline{\phi}} = \left(\frac{\pi(N-n_{13})}{(1-\pi)N}\right)^{-\frac{1}{\sigma}} \left(\frac{\psi(\overline{\theta},x)}{\psi(\underline{\theta},x)}\right)^{\frac{\sigma-1}{\sigma}} \tag{39}$$

Taking logs:

$$\ln\Phi = \frac{\sigma-1}{\sigma}\ln\left(\frac{\psi(\overline{\theta},x)}{\psi(\underline{\theta},x)}\right) - \frac{1}{\sigma}\ln\left(\frac{\pi(N-n_{13})}{(1-\pi)N}\right) \tag{40}$$

Assuming skill biased technological progress, so that

$$\frac{\psi(\overline{\theta}, x)}{\psi(\underline{\theta}, x)} \tag{41}$$

grows in x, the wage skill premium associated with the equilibrium will increase over time so long as $\sigma > 1$. In turns the analysis developed in section 2 above then suggests that the level of education necessary in order to signal a high level of talent should also increase.

3.3.2 Characterization of TPE.

According to section 2, TPE robust to the intuitive criteria are the following: $1-234$, $1-23-4$, or $1-34-2$. Let us characterize for instance the TPE $1-23-4$. In such equilibrium, type 1 plays $n_1 > 0$, types 2 and 3 play $n_{23} > 0$, with $n_{23} < n_1$, and type 4 plays $n_4 = 0$. Moreover,

i. Expected levels of talent conditional on education are:

$$\theta(n_1) = \overline{\theta} \tag{42}$$

$$\theta(n_{23}) = \theta_{23} \tag{43}$$

$$\theta(0) = \underline{\theta} \tag{44}$$

ii. Supply of labor conditional on education:

$$L^S(\overline{\theta}) = \delta\pi(N - n_1) \tag{45}$$

$$L^S(\theta_{23}) = [\pi(1-\delta) + (1-\pi)\delta](N - n_{23}) \tag{46}$$

$$L^S(\underline{\theta}) = (1-\pi)(1-\delta)N \tag{47}$$

iii. Salaries are:

$$w_1 = \left[\psi(\overline{\theta}, x)^\alpha + \left(\psi(\theta_{23}, x)\frac{L(\theta_{23})}{L(\overline{\theta})}\right)^\alpha + \left(\psi(\underline{\theta}, x)\frac{L(\underline{\theta})}{L(\overline{\theta})}\right)^\alpha\right]^{\frac{1}{\alpha}-1} \psi(\overline{\theta}, x)^\alpha \tag{48}$$

$$w_{23} = \left[\psi(\theta_{23}, x)^\alpha + \left(\psi(\underline{\theta}, x)\frac{L(\underline{\theta})}{L(\tilde{\theta})}\right)^\alpha + \left(\psi(\overline{\theta}, x)\frac{L(\overline{\theta})}{L(\tilde{\theta})}\right)^\alpha\right]^{\frac{1}{\alpha}-1} \psi(\theta_{23}, x)^\alpha \tag{49}$$

$$w_4 = \left[\psi(\underline{\theta}, x)^\alpha + \left(\psi(\theta_{23}, x)\frac{L(\tilde{\theta})}{L(\underline{\theta})}\right)^\alpha + \left(\psi(\overline{\theta}, x)\frac{L(\overline{\theta})}{L(\underline{\theta})}\right)^\alpha\right]^{\frac{1}{\alpha}-1} \psi(\underline{\theta}, x)^\alpha \tag{50}$$

where $\alpha = (\sigma - 1)/\sigma$. Rearranging we obtain the following expression for the wage skill differentials:

$$\Phi = \left(\frac{L^S(\overline{\theta}}{L^S(\underline{\theta})} \right)^{-\frac{1}{\sigma}} \left(\frac{\psi(\overline{\theta}, x)}{\psi(\underline{\theta}, x)} \right)^{\frac{\sigma-1}{\sigma}} \tag{51}$$

$$\Phi_{23} = \left(\frac{L^S(\theta_{23})}{L^S(\underline{\theta})} \right)^{-\frac{1}{\sigma}} \left(\frac{\psi(\theta_{23}, x)}{\psi(\underline{\theta}, x)} \right)^{\frac{\sigma-1}{\sigma}} \tag{52}$$

Taking logs, we obtain

$$ln(\Phi) = \frac{\sigma - 1}{\sigma} \ln \left(\frac{\psi(\overline{\theta}, x)}{\psi(\underline{\theta}, x)} \right) - \frac{1}{\sigma} \ln \left(\frac{L^S(\overline{\theta}}{L^S(\underline{\theta})} \right) \tag{53}$$

$$ln(\Phi_{23}) = \frac{\sigma - 1}{\sigma} \ln \left(\frac{\psi(\theta_{23}, x)}{\psi(\underline{\theta}, x)} \right) - \frac{1}{\sigma} \ln \left(\frac{L^S(\theta_{23})}{L^S(\underline{\theta})} \right) \tag{54}$$

Assuming skill biased technological progress, so that

$$\frac{\psi(\overline{\theta}, x)}{\psi(\underline{\theta}, x)} \tag{55}$$

$$\frac{\psi(\theta_{23}, x)}{\psi(\underline{\theta}, x)} \tag{56}$$

grow in x, as long as $\sigma > 1$, both the wage skill premia, Φ and Φ_{23}, will increase over time. Note that, clearly, Φ_{23} will grow less than Φ.

3.3.3 Prevailing equilibrium.

In particular, assume that technological progress does not affect unskilled labor: $\psi(\underline{\theta}, x) = \psi(\underline{\theta})$ for all x. Then, proposition 1 applies directly so that there exist a critical value of x call it $x^{\max}$ such that $\overline{\phi} = \phi^{\max}$ above which the economy switches from thes TSE, $13-24$, to a TPE such as $1-23-4$. The evolution of the supply of expected skills and wage-skill premia is depicted in figures 6, and 7.

3.4 Wage skill premium, relative supply of skills and education race

According to the skill biased technological progress hypothesis, the wage skill premium has been pushed up by an increase in the demand for skills following an initial increase in the supply of skills due to investments in education by the workforce population. This

Figure 6: Wages by education level (and expected skills)

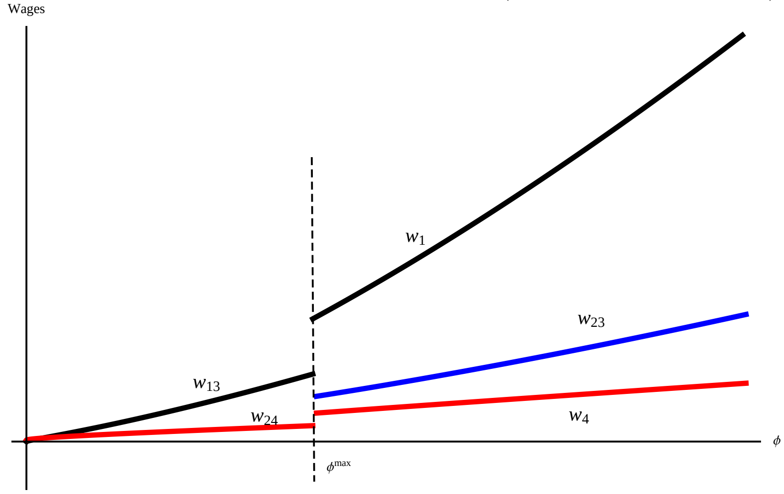
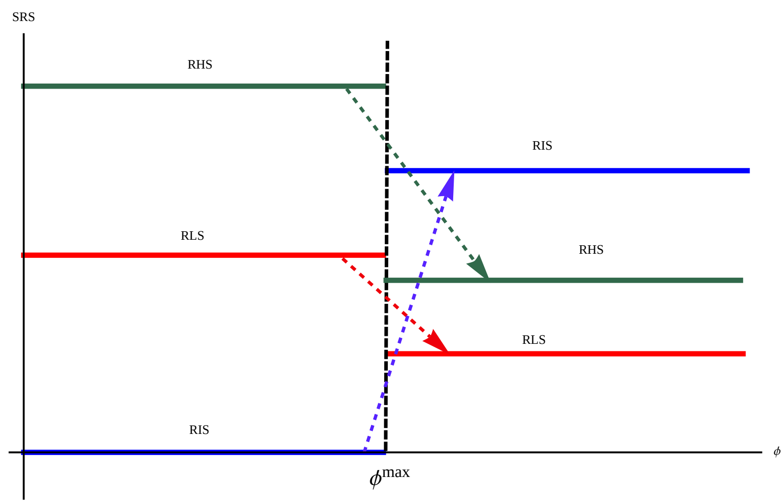


Figure 7: Labor supply by education level (and expected skills)



explanation fits the evidence about the trends in wage-skill differentials and education for the US in the sixties and seventies. However, while most of the skill wage premium is accounted for by salaries of PGs, unexpectedly, the relative supply of PGs has increased less than the relative supply of CGs. The model we propose complements the skill biased technological change hypothesis by providing an explanation for such sluggish behavior of relative supply of CGs with respect to that of CGs. A summary of the argument follows.

Skilled biased technological change generates an increase in the wage premium between educated (skilled) and uneducated (unskilled) workers in a TSE. But, if the wage gap between skilled and unskilled associated with the TSE increases, this means that the reward to the investment in education has increased. This, in turns, implies that –given an TSE– unskilled workers have a stronger motive to mimic skilled workers by investing in education. In turns, this implies that the minimum level of investment in education by skilled workers necessary to sustain a TSE must increase. In other words, as the wage-skill premium increases, individuals engage in an education race.

As the wage premium for education grows large enough the minimum level of investment in education necessary to sustain a TSE will eventually become greater than the maximum amount that poor individuals can self-finance. At this stage, if there are no capital markets where poor individual can borrow, the TSE cannot longer exist. We show that –under these circumstances – the equilibrium that prevails is a talent pooling equilibrium (TPE) in which only rich and talented individuals are able to perfectly signal their high skills by investing enough time in education, while rich and untalented and poor and talented are pooled together at a lower level of education, with poor and untalented also perfectly signal their low skills by not investing in education at all.

Under this TPE, the relative supply of high skills –as revealed by the education level – is reduced compared to that of intermediate skills. This is because, "Poor and skilled" are pooled together with "rich and unskilled" at a lower level of education compared to that played by the "rich and skilled". The education level played by this pool of heterogeneous agents is still a signal of skill, but the expected level of skills in the pool is lower than the level of skills supplied by rich and talented individuals who are investing more in education. In other words, the expected level of skills supplied by the agents pooled together at this lower level of education is lower than that supplied by the rich and talented who are investing more in education. Hence, on overall the supply of high skills displays a sluggish behavior with respect to the increase in the wage skill premium. Education goes up, but the supply of high expected skills conditional on education level does not, as not all skilled individuals are able to invest enough in education to signal themselves.

Furthermore, according to the model, the pool of workers at intermediate levels of education, should be characterized by a higher skill dispersion (variance) than the pool of workers at high and low levels of education. This is consistent with the fact that wages paid in the US exhibit an increasing dispersion at CG, while dispersion of wages at PGs exhibit no such trend.

As such the model provides a microfoundation for the sluggish behavior of the supply of skilled labor, which could complement the effect of demand-driven models, such as the skill biased technological progress hypothesis, in explaining (i) The long run widening of the wage gap between skilled and unskilled labor together with (ii) The widening of the wage gap between postgraduates and college graduates.

4 Conclusion

There is a vast literature documenting the rise of wage inequality across educational groups of American workers and the increase in the supply of skills in the US labor market over the last forty years. Many economists have proposed a demand driven explanation for such phenomenon based upon the idea of skill bias technical progress.

We propose a complementary explanation based on a model of the labor market where workers are heterogeneous with respect to wealth and skills –both unobservable– and costly investment in education might have a role in signaling the level of skills. In equilibrium workers are paid their expected productivity, which depends positively on the level of skills as well as on other factors (for instance technical progress) that combine with labor. We show that if the increase in the endowment of accumulable factors results in a wider wage premium for education, the minimum investment in education needed to sustain a perfectly separating equilibrium (PeSE) in which skilled workers are able to perfectly signal their skills increases. Hence, an increase in the wage skill premium generates an education race with skilled individuals investing more and more to signal themselves. However, if capital markets are imperfect so that the borrowing capacity of poor individuals is lower than that of those who are rich, this race will finally lead to a situation in which –for a sufficiently large increase in the endowment of the accumulable factor– poor and skilled individuals are no longer able to invest enough to signal themselves and end up pooled together with untalented and rich at a lower level of education. Hence, the supply of skills is sluggish with respect to an increase in the wage-skill differential. The model offers a supply side explanation for the widening of the wage-skill differential as well as for the widening of the wage gap between postgraduates and college graduates.

References

[1] Acemoglu, D., 2003. Patterns of Skill Premia. The Review of Economic Studies. 70: 199-230.

[2] Acemoglu, D., 2002. Technical Change, Inequality and the Labor Market. Journal of Economic Literature, XL (March 2002), 772.

[3] Acemoglu, D., 1999. Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence. The American Economic Review. 89(5): 1259-78.

[4] Acemoglu, D., 1998. Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality. Quarterly Journal of Economics. 113: 1055-1089.

[5] Acemoglu, D., and D. H. Autor, 2010. Skills, Tasks and Technologies: Implications for Employment and Earnings. Handbook of Labor Economics Volume 4, Orley Ashenfelter and David E. Card (eds.), Amsterdam: Elsevier.

[6] Autor, D. H., Lawrence F. Katz and Melissa S. Kearney, 2008. Trends in U.S. wage inequality: Revising the revisionists. Review of Economics and Statistics, 90(2): 300323.

[7] Autor D. H., L. F. Katz and A. B. Krueger, 1997. Computing Inequality: Have Computers Changed the Labor Market? NBER Working Papers 5956, National Bureau of Economic Research

[8] Barrow, L., and C. E. Rouse, 2005. Does College Still Pays? The Economist's Voice, V. 2 (4): 1-8.

[9] Carneiro, P., and S. Lee, 2008. Trends in Quality-Adjusted Skill Premia in the United States, 1960-2000. CEMMAP wp n. CWP02/09

[10] Eckstein, Z., and E. Nagypal. 2004. The Evolution of U.S. Earnings Inequality: 1961-2002. Federal Reserve Bank of Minneapolis Quarterly Review, 28-2: 10-29

[11] Goldin C. and L. Katz2007), 2007. Long-Run Changes in the Wage Structure: Narrowing, Widening, Polarizing. Brookings Papers on Economic Activity, 2:2007, 135165.

[12] Goldin, C., and L. F. Katz. 1998. The Origins of Technology-Skill Complementarity. Quarterly Journal of Economics 113, no. 3 (June): 693732

[13] Katz, L. F., and K. M. Murphy. 1992. Changes in Relative Wages, 196387: Supply and Demand Factors. Quarterly Journal of Economics 107, no. 1 (February): 3578.

[14] Spence M., 1974. Market Signaling: Information Transfer in Hiring and Related Screening Processes. Cambridge MA: Harvard University Press

[15] Walker, I., and Y. Zhu, 2007. The College Wage Premium, Overeducation and the Expansion of Higher Education in the UK. UDC Geary Institute DP.

A Appendix

A.1 Proof of lemma 1

From A5, $\Delta_{13} > \Delta_{24}$ if $w(n'') > w(n')$. But then, given $w(n'') > w(n') > 0$, $\Delta_{24} \geq 0$ directly implies $\Delta_{13} > 0$. $\square$

A.2 Proof of lemma 2

Consider a candidate TSE equilibrium in which, $n', n'' \in \mathcal{N}_\phi$, with $n'' > n'$, for some level of talent, θ. Since the equilibrium is talent separating, $w(n'') = w(n') = \phi(\theta, x)$ must hold. But then, given A1, for individuals of talent θ, playing n'' is strictly dominated by playing n', which destroys the candidate equilibrium. Hence, we conclude that $\mathcal{N}_{\overline{\phi}}$ and $\mathcal{N}_{\underline{\phi}}$ must be singletons. $\square$

A.3 Proof of lemma 3

Characterization. The characterization of the TSE follows directly from two consideration. First, by definition, in any TSE, the values expected marginal productivity conditional on $n \in \mathcal{N}_{\underline{\theta}}$ and $n \in \mathcal{N}_{\overline{\theta}}$, are respectively equal to $\underline{\phi} = \phi(\underline{\theta}, x)$, and, $\overline{\phi} = \phi(\overline{\theta}, x)$. Second, any candidate TSE equilibrium where $n_{24} > 0$ would be deviated as for any possible off equilibrium beliefs, agents of type 2 and 4 would be better off by playing $n = 0$.

Existence. By definition, in any candidate TSE talented individuals play $n_{13} > 0$ and receive a salary $w(n_{13}) = \overline{\phi}$, while untalented individuals play $n_{24} = 0$ and receive a salary $w(n_{24}) = \underline{\phi}$. A necessary condition for a TSE is $n_{13} \leq n_{3,4}^{\max}$, so that n_{13} is feasible for types 3. Let us consider a candidate TSE that satisfies such condition. The participation constraints associated with the candidate equilibrium are:

$$PC_{13} \;:\; V_i(n_{13}, \overline{\phi}, \underline{\phi}, c(n_{13})) \geq 0, \quad i = 1, 3 \tag{A.1}$$
$$PC_{24} \;:\; V_i(0, \underline{\phi}, \underline{\phi}, 0) \geq 0, \quad i = 2, 4 \tag{A.2}$$

Given property A1 of V_i, $V_i(0, \underline{\phi}, \underline{\phi}, c(0)) \geq 0$ is always true. Therefore, PC_{24} are always satisfied.

Given, $\overline{\phi} > \underline{\phi}$, Properties $A1$ and $A2$ of the payoff function, V_i, imply $V_i(0, \overline{\phi}, \underline{\phi}, c(0)) > V_i(0, \underline{\phi}, \underline{\phi}, c(0))$ for given i. Accordingly, since $V_i(0, \underline{\phi}, \underline{\phi}, c(0)) \geq 0$, $V_i(0, \overline{\phi}, \underline{\phi}, c(0)) > 0$, follows. Finally, property $A1$ implies that there exist a strictly positive critical value of n_{13}, call it $\overline{n}_{13} > 0$, such that

$$\overline{n}_{13} : V_i(\overline{n}_{13}, \overline{\phi}, \underline{\phi}, c(\overline{n}_{13})) = V_i(0, \underline{\phi}, \underline{\phi}, 0), \quad i = 1, 3 \tag{A.3}$$

such that $PC_{1,3}$ is satisfied for $n_{13} \leq \overline{n}_{13}$.

The Incentive compatibility constraints are:

$$ICC_{13} \;:\; V_i(n_{13}, \overline{\phi}, \underline{\phi}, c(n_{13})) \geq V_i(0, \underline{\phi}, \underline{\phi}, c(0)), \quad i = 1, 3 \tag{A.4}$$
$$ICC_{24} \;:\; V_i(n_{13}, \overline{\phi}, \underline{\phi}, c(n_{13})) \leq V_i(0, \underline{\phi}, \underline{\phi}, c(0)), \quad i = 2, 4. \tag{A.5}$$

Define,

$$\underline{n}_{13} : V_i(\underline{n}_{13}, \overline{\phi}, \underline{\phi}, c(\underline{n}_{13})) = V_i(0, \underline{\phi}, \underline{\phi}, c(0)), \quad i = 2, 4. \tag{A.6}$$

as the minimum value of n_{13} such that the ICC_{24} holds and types 2,4 have no incentive to mimic types 1,3. Then, given property A1 of the payoff function, in any TSE, $n_{13} \geq \underline{n}_{13}$, must hold. Note that, given properties A1 and A2 of V_i, $V_i(0, \underline{\phi}, \underline{\phi}, c(0)) \geq 0$, and $\overline{\phi} > \underline{\phi}$ together imply $\underline{n}_{13} > 0$.

Given that, (i) $n_{13} \geq \underline{n}_{13}$, and (ii) $n_{13} \leq n_{34}^{\max}$ are both necessary for a TSE, then,

$$\underline{n}_{13} \leq n_{3,4}^{\max} \tag{A.7}$$

is also necessary. We now show that the above condition is both necessary and sufficient for a TSE.

Given Property A5, for any $n_{13} > 0$, $w(n_{13}) > 0$, :

$$V_i(n_{13}, \overline{\phi}, \underline{\phi}, c(n_{13})) - V_i(0, \underline{\phi}, \underline{\phi}, c(0)) > V_{i'}(n_{13}, \overline{\phi}, \underline{\phi}, c(n_{13})) - V_{i'}(0, \underline{\phi}, \underline{\phi}, c(0)) \tag{A.8}$$

holds, with $i = 1, 3$ and $i' = 2, 4$. Therefore, given property A1 of V_i, $\underline{n}_{13} < \overline{n}_{13}$ holds, so that we conclude that there exist a non empty set of values of n, $[\underline{n}_{13}, \overline{n}_{13}]$, such that all PCs and ICCs are simultaneously satisfied for any $n_{13} \in [\underline{n}_{13}, \overline{n}_{13}]$. Therefore, if and only if $n_{3,4}^{\max} \geq \underline{n}_{13}$, (see condition (A.7)), there exist some $n_{13} \in [\underline{n}_{13}, \overline{n}_{13}]$ that not only satisfies all constraints, but it is also feasible for type 3 individuals; which is necessary and sufficient for a TSE to exists. Hence, condition (A.7) is both necessary and sufficient for a TSE, and in any TSE, $n_{13} \in [\underline{n}_{13}, n_{34}^{\max}]$.

We then note that, since given property A2 of V_i, for given n_{13}, $V_i(n_{13}, \overline{\phi}, \underline{\phi}, c(n_{13}))$ is strictly increasing in $\overline{\phi}$ for any i, with

$$\lim_{\overline{\phi} \to \infty} V_i(n_{13}, \overline{\phi}, \underline{\phi}, c(n_{13})) \;=\; \infty \tag{A.9}$$

$$\lim_{\overline{\phi} \to \underline{\phi}} V_i(n_{13}, \overline{\phi}, \underline{\phi}, c(n_{13})) \;<\; V_i(0, \underline{\phi}, \underline{\phi}, c(0)). \tag{A.10}$$

Therefore, there exist a critical value of $\overline{\phi}$, call it $\overline{\phi}^{\max}$, such that,

$$V_i(n_{34}^{\max}, \overline{\phi}^{\max}, \underline{\phi}, c(n_{34}^{\max})) = V_i(0, \underline{\phi}, \underline{\phi}, c(0)), \quad i = 2, 4 \tag{A.11}$$

so that $n_{34}^{\max} = \underline{n}_{13}$, while for $\overline{\phi} < (>) \overline{\phi}^{\max}$

$$V_i(n_{34}^{\max}, \overline{\phi}^{\max}, \underline{\phi}, c(n_{34}^{\max})) > (<) V_i(0, \underline{\phi}, \underline{\phi}, c(0)), \quad i = 2, 4 \tag{A.12}$$

so that $n_{34}^{\max} > (<) \underline{n}_{13}$. Therefore, the necessary and sufficient condition for a TSE, $n_{34}^{\max} \geq \underline{n}_{13}$, can be restated as

$$\overline{\phi} \leq \overline{\phi}^{\max} \tag{A.13}$$

□

A.4 Proof of lemma 4

Suppose by contradiction, a candidate equilibrium where two levels of education played with probability n'' and n' satisfy: $w(n'') > (=) w(n')$ and $n' > (\neq) n''$. If $w(n'') > w(n')$ then, property A1 of the payoff function V_i implies that playing $n' > n''$ is dominated by playing n'', which destroys the equilibrium. Similarly, if $w(n'') = w(n')$, playing $\max(n', n'')$ would be equilibrium dominated unless $n' = n''$. $\square$

A.5 Proof of lemma 5

Case 1.i. All individuals play the same level of education, $n_{1234} \geq 0$. A necessary condition for this equilibrium is that $n_{1234} \leq n_{3,4}^{\max}$. Let us consider a candidate equilibrium that satisfies such condition. The average level of talent associated with n_{1234} is, $\theta_{1234} = \pi\overline{\theta}+(1-\pi)\underline{\theta}$. Hence, the equilibrium salary is, $w(n_{1234}) = \phi_{1234}$, where, $\phi_{1234} \equiv \phi(\theta_{1234}, x)$, with $\phi_{1234} \in (\underline{\phi}, \overline{\phi})$. Participation constraints are:

$$PC_{1,2,3,4} : V_i(n_{1234}, w(n_{1234}), w(0), c(n_{1234})) \geq 0, \quad i = 1, 2, 3, 4.$$

Let the off equilibrium beliefs associated with $n = 0$ such that $w(0) < \phi_{1234}$. While this is a necessary condition for the equilibrium, it is obvious that there always exist sufficiently pessimistic off equilibrium beliefs such that the condition is satisfied. Then, property A1 of the V_i function implies that –for any given $i = 2, 4$– there always exist a strictly positive critical value of n_{1234}, call it $\bar{n}_{1234} > 0$, such that

$$\bar{n}_{1234} : V_i(\bar{n}_{1234}, w(n_{1234}), w(0), c(n_{1234}) = 0,$$

so that $PC_{1,2,3,4}$ is satisfied for types $2, 4$, for any $n_{1234} \leq \bar{n}_{1234}$. Moreover, property A5 of V_i ensures that whenever $PC_{1,2,3,4}$ is satisfied, for types $2, 4$, it is also satisfied for types $1, 3$.

Finally, note that there always exist off equilibrium beliefs such that there are no profitable deviations. For instance, let us assume that off equilibrium beliefs imply $w(n) = \underline{\phi}$ for all $n \neq n_{1234}$. Then, so long as n_{1234} satisfies the above participation constraint, there are no profitable deviations. Hence, the type of equilibrium we are analyzing always exists.

Case 1.ii. In a candidate candidate equilibrium where types 1 play $n_1 > 0$, while types 2, 3, and 4, play $n_{234} \geq 0$, with $n_{234} \neq n_1$, types 1 receive a salary $w_1 = \overline{\phi}$, and other types receive a salary $w_{234} = \phi(\theta_{234}) \equiv \phi_{234}$. Suppose $n_{234} = 0$. Furthermore, let n_1 satisfy the following condition:

$$V_{13}(n_1, \overline{\phi}, \phi_{234}, c(n_1)) = V_{13}(0, \phi_{234}, \phi_{234}, c(0)). \tag{A.14}$$

It is immediate to verify that, given the properties of V_i, the above candidate equilibrium satisfied participation constraints and incentive compatibility constraints for all types. Furthermore, given the properties of V_i a value of n_1 satisfying the above condition always exists, which finally proves that the above candidate equilibrium always exist.

Case 2. In a candidate candidate equilibrium where type 1 plays $n_1 > 0$, types 2, 3, play n_{23} and type 4 plays $n_4 = 0$, type 1 receive a salary $w_1 = \overline{\phi}$, types 2 and 3 receive a salary $w_{23} = \phi(\theta_{23}) \equiv \phi_{23}$, and type 4 receives a salary $w_4 \equiv \underline{\phi}$. Clearly, since $\overline{\phi} > \phi_{23} > \underline{\phi}$, $n_1 > n_{23} > 0$ must hold.

First thing we note is that, since $n_2^{\max} = n_3^{\max}$, type 2 incentive compatibility requires:

$$n_{23} : V_2(n_{23}, \phi_{23}, \underline{\phi}, c(n_{23})) = V_2(0, \underline{\phi}, \underline{\phi}, c(0)) = 0 \tag{A.15}$$

Given the properties of the V_i function, there always exist n_{23} such that the above condition holds. Furthermore, provided that phi_{23} is sufficiently close to $\underline{\phi}$, $n_{23} \leq n_{34}^{\max}$, which is necessary for type 3 to be able to play n_{23}. Let us assume this is the case. Then, consider a value of n_1 such that

$$V_i(n_1, \overline{\phi}, \underline{\phi}, c(n_1)) = 0 \qquad i = 1, 3 \tag{A.16}$$

Given the properties of the V_i function, such level of n_1 always exist. Furthermore, it is immediate to verify that such value of n_1 would satisfy all incentive compatibility constraints. Hence, provided that $n_{23} \leq n_{34}^{\max}$, where n_{23} satisfies the above condition, the candidate equilibrium characterized above exists.

Case 3. The same logics of case 2 applies. □

A.6 Proof of lemma 6

Consider candidate TSE, call it E, such that $n_{13}^E > \underline{n}_{13}$, where $\underline{n}_{13}$ is defined by equation (A.6). Consider a deviation n' such that $n' \in (\underline{n}_{13}, n_{13}^E)$ (note that only downward deviations are to be considered, as upward deviation make everyone worse off for any possible beliefs). Recall that $\underline{n}_{13}$ is the maximum level of education that, given a TSE, agents of types 2 and 4 are willing to play in order to mimic types 3 and 1. Then, given $n' > \underline{n}_{13}$, agents of type 2 and 4 are strictly worse off even if the associated off equilibrium beliefs assign probability 1 to the fact that the deviation comes from a talented individual, that is even when $\mu(n') = 1$.

On the other hand, given $n' < n_{13}^E$, individuals of types 1 and 3, talented, strictly benefit from the deviation if firms' off equilibrium beliefs assign probability zero that an untalented as deviated, which implies that such beliefs assign probability one that a talented as deviated, $\mu(n') = 1$. Hence, according E is not robust to the intuitive criterion (see definition 2).

Consider now a candidate TSE, call it E, such that $n_{13}^E = \underline{n}_{13}$. Consider a deviation n' such that $n' < \underline{n}_{13}$. By definition, types 2 and 4 will be strictly benefiting from such deviation if $\mu(n') = 1$ and the same is true for types 1 and 3. Hence E is robust to the IC. □

A.7 Proof of lemma 8

Let E a TPE such that $1 \in \Theta_p^E$, where Θ_p^E is the set of types pooled together. Define

$$\overline{n}_p : V_i(\overline{n}_p, w(\overline{n}_p), \underline{\phi}, c(\overline{n}_p)) = 0, \quad i = 2, 4 \tag{A.17}$$

as the maximum equilibrium level of education played by agents that are pooling together in E. Accordingly,

$$\overline{n}_{24} : V_i(\overline{n}_{24}, \overline{\phi}, \underline{\phi}, c(\overline{n}_{24})) = V_i(0, \underline{\phi}, \underline{\phi}, c(0)), \quad i = 2, 4 \tag{A.18}$$

is maximum level of education that individuals of type $i = 2, 4$ are willing to play deviating from an E, in which they play $\overline{n}_p$, if perceived as individuals of talent $\overline{\theta}$ by doing so. We note that, $\overline{n}_{24}$ is independent of the set of individuals who are pooling. Then, if $\overline{n}_{24} < n_{34}^{\max}$, E is not robust, and it is robust otherwise. It is important to note that $\overline{n}_{24} = \underline{n}_{13}$ (see equation (A.6)), so that, given the definition of $\overline{\phi}^{\max}$ (see equation (18)),

$$\overline{\phi} \geq (<)\overline{\phi}^{\max} \Rightarrow \overline{n}_{24} \geq (<)n_{34}^{\max}. \tag{A.19}$$

Hence, E is (not) robust if $\overline{\phi}(<) \geq \overline{\phi}^{\max}$. □

A.8 Proof of Lemma 9

By definition, for any $\theta \in \Theta^E$, $L^S(\theta)) = 0$. Hence, it follows directly from the equilibrium conditions in the labor market that, $w(\theta) \to \infty$. Hence, if for some deviation n, $\mu(\theta|n) > 0$ for some $\theta \in \Theta^E$, the payoff from the deviation would be infinite, which would destroy the equilibrium. Hence, only extreme beliefs support the equilibrium.□

B Baseline model: Alternative characterizations of the prevailing equilibrium

Proposition 1 offers a characterization of the prevailing equilibrium as a function of $\overline{\phi}$, other things equal. Equilibrium is TSE if $\overline{\phi} \leq \phi^{\max}$ and TPE otherwise.

Alternative characterizations can be provided in terms of $\underline{\phi}$ or $\overline{\phi} - \underline{\phi}$.

Characterization in terms of $\underline{\phi}$. According to its definition (see equation 18), $\overline{\phi}^{\max}$ is a function

$$\overline{\phi}^{\max} = \phi^{\max}(\underline{\phi}) \tag{B.1}$$

Given the properties of V_t, $\overline{\phi}^{\max}$ is continuous, differentiable, and strictly increasing in $\underline{\phi}$, with

$$\phi^{\max}(0) > 0 \tag{B.2}$$

$$\lim_{\underline{\phi} \to \infty} \overline{\phi}^{\max} = \infty \tag{B.3}$$

Then, if

$$\overline{\phi} > \phi^{\max}(0) \tag{B.4}$$

holds, the characterization of the prevailing equilibrium provided in proposition 1 can be provided, equivalently, as a function of $\underline{\phi}$ for given $\overline{\phi}$ in that there exist a critical threshold for the marginal productivity of untalented workers,

$$\underline{\phi}^{\min} : \overline{\phi}^{\max} = \overline{\phi} \tag{B.5}$$

such that, given $\overline{\phi}$, if $\underline{\phi} < \underline{\phi}^{\min}$ then the prevailing equilibrium is a TPE, while if $\underline{\phi} > \underline{\phi}^{\min}$ then the prevailing equilibrium is a TSE.

Characterization in terms $\overline{\phi}-\underline{\phi}$. We now explore the conditions under which the prevailing equilibrium can be characterized for given values of the wage dispersion associated with a candidate TSE, defined as $\Delta w \equiv \overline{\phi} - \underline{\phi}$.

Define $W_t = W(t, N-n, w(n))$ the (expected) present value of labor income of an individual who invests in (and completes) a degree of length n and works for $N-n$. We impose that W_t is linear in $w(n)$:

$$\frac{dW_t}{dw(n)} = const \tag{B.6}$$

and that the marginal effect of an increase in $w(n)$ goes down with n. That is, if $n_1 > (<)n_0$, then

$$\frac{dW_t}{dw(n_1)} < (>) \frac{dW_t}{dw(n_0)} \tag{B.7}$$

Assume that the payoff function V_t satisfies the following property:

A6. For any $w(n)$, and n, the function V_t is additive separable in the (present) value of labor income and investment in education $c(n)$.

Then, the $\overline{\phi}^{\max}$ is linear in $\underline{\phi}$, with

$$\frac{d\overline{\phi}^{\max}}{d\underline{\phi}} = const > 1 \tag{B.8}$$

where $const$ is a function of $n_{34}^{\max}$, which in turns is defined by equation (3). We know that for $\overline{\phi}^{\max}(\underline{\phi}) > \underline{\phi}$ for all x. Hence, given any two values $\overline{\phi}$, $\underline{\phi}$ such that $\overline{\phi} = \underline{\phi}$ so that wage dispersion equals zero $\Delta w = \overline{\phi} - \underline{\phi} = 0$, then the correspondent value of $\overline{\phi}^{\max}(\underline{\phi})$ satisfies,

$$\overline{\phi} < \overline{\phi}^{\max}(\underline{\phi}) \tag{B.9}$$

so that the prevailing equilibrium is TSE.

Starting from $\Delta w = 0$, suppose that both $\overline{\phi}$ and $\underline{\phi}$ increase by $\Delta\overline{\phi}$ and $\Delta\underline{\phi}$ respectively, with $\Delta\overline{\phi} > \Delta\underline{\phi}$, so that the value of wage dispersion goes to

$$\Delta w = \Delta\overline{\phi} - \Delta\underline{\phi} > 0. \tag{B.10}$$

Then, $\overline{\phi}^{\max}$ also increases, by

$$\Delta\overline{\phi}^{\max} = const\Delta\underline{\phi} \tag{B.11}$$

Since $const > 1$, $\Delta\overline{\phi}^{\max}$ could be either greater or smaller than $\Delta\overline{\phi}$. Hence, in principle, we do not know whether the economy will stay in a TSE or switch to TPE as Δw increases.

For any $\Delta\underline{\phi} \geq 0$,

$$\overline{\phi}^{\max}(\underline{\phi} + \Delta\underline{\phi}) = \overline{\phi}^{\max}(\underline{\phi}) + const\Delta\underline{\phi} \tag{B.12}$$

Assume that the increase in marginal productivity of a talented worker is proportional to the increase in marginal productivity of an untalented worker,

$$\Delta\overline{\phi} = const_1\Delta\underline{\phi} \tag{B.13}$$

Then, if

$$const_1 > const \tag{B.14}$$

there exists s critical value of wage dispersion associated with a TSE, call it $\widehat{\Delta} w$, such that the prevailing equilibrium is TSE for $\Delta w \leq \widehat{\Delta} w$ and TPE otherwise.

We note that, for $\Delta w = \widehat{\Delta} w =$, $\overline{\phi} = \overline{\phi}^{\max}$ holds. That is, the prevailing equilibrium characterized by proposition 1 with respect to $\overline{\phi}$, can be equivalently characterized in terms of the wage dispersion $\Delta w = \overline{\phi} - \underline{\phi}$ associated with a TSE.

Characterization of the prevailing equilibrium in terms of the state variable x. According to equation (2), the marginal productivity of workers of talent θ, ϕ, is a function of talent, θ, and of a state variable x.

Let us assume that the effect of talent and of x are multiplicative:

$$g(\theta, x) = g(\theta) f(x) \tag{B.15}$$

where both $g(\theta)$ and $f(x)$ are continuous, positive, and strictly increasing in their arguments. Moreover, we assume that $f(x)$ satisfies:

$$f(0) = 1 \tag{B.16}$$

$$\lim_{x \to \infty} f(x) = +\infty \tag{B.17}$$

It then follows that:

$$\Delta \phi = \left[g(\overline{\theta}) - g(\underline{\theta}) \right] f(x) \tag{B.18}$$

so that $\Delta \phi$ is continuous and strictly increasing in x, $\Delta \phi \equiv \Delta \phi(x)$, with

$$\Delta \phi(0) = \overline{\theta} - \underline{\theta} > 0 \tag{B.19}$$

$$\lim_{x \to \infty} \Delta \phi(x) = +\infty \tag{B.20}$$

Therefore, the prevailing equilibrium can be chacterized with respect to x in a way equivalent to the characterization we provided with respect to the wage dispersion associated with a TSE, $\Delta w = \Delta \phi$. In particular, there exist a critical value of the state variable x , call it $\widehat{x}$, such that the prevailing equilibrium is TSE for $x \leq \widehat{x}$ and TPE otherwise.

Figure 8: Educational composition of professionals

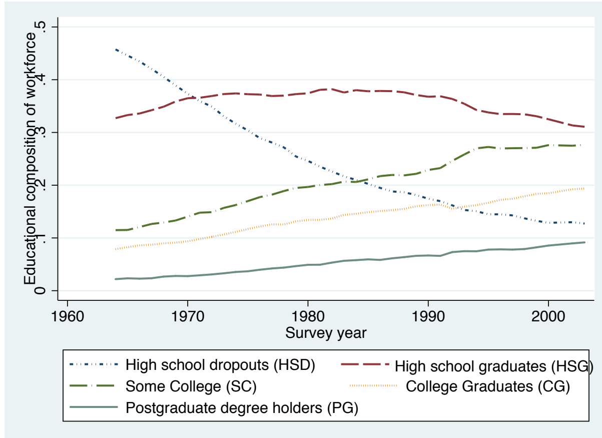


Figure 9: Educational composition of white collars

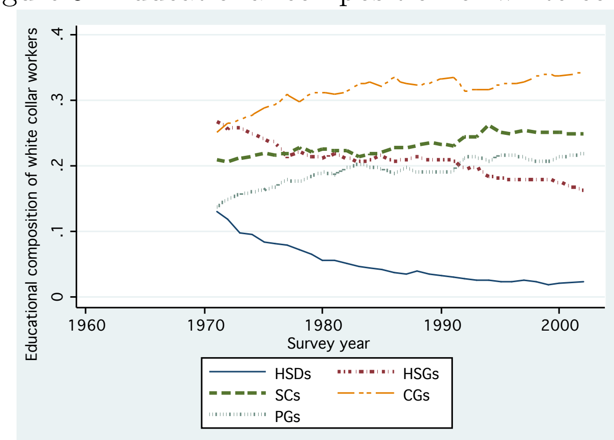


Figure 10: Educational composition of blue collars

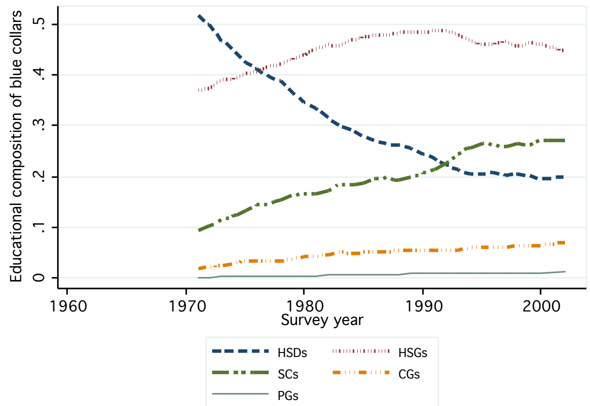
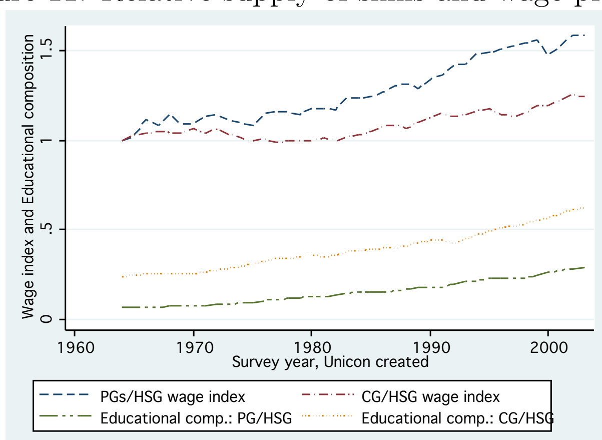


Figure 11: Relative supply of skills and wage premia



B.1 Payoff function and Present Value of Salaries

Define $W_t = W(t, N - n, w(n))$ the (expected) present value of labor income of an individual who invests in (and completes) a degree of length n and works for $N - n$. We impose that W_t is linear in $w(n)$:

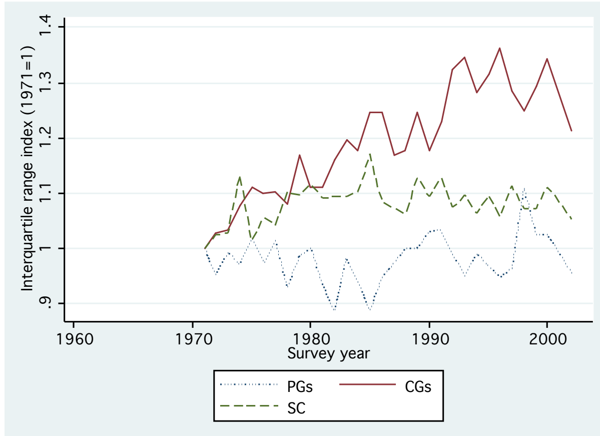
$$\frac{dW_t}{dw(n)} = const \tag{B.21}$$

and that the marginal effect of an increase in $w(n)$ goes down with n. That is, if $n_1 > (<)n_0$, then

$$\frac{dW_t}{dw(n_1)} < (>)\frac{dW_t}{dw(n_0)} \tag{B.22}$$

Usare queste cose nella payoff function

Figure 12: Wage dispersion within educational groups



B.1.1 Labor supply by education level, expected skills and wage-skill differential

Proposition 1 offers a characterization of the prevailing equilibrium as a function of $\overline{\phi}$, other things equal. Equilibrium is TSE if $\overline{\phi} \leq \phi^{\max}$ and TPE otherwise.

According to its definition (see equation 18), $\overline{\phi}^{\max}$ is a function

$$\overline{\phi}^{\max} = \phi^{\max}(\underline{\phi}) \tag{B.23}$$

Given the properties of V_t, $\overline{\phi}^{\max}$ is continuous, differentiable, and strictly increasing in $\underline{\phi}$, with

$$\phi^{\max}(0) > 0 \tag{B.24}$$

$$\lim_{\underline{\phi}\to\infty} \overline{\phi}^{\max} = \infty \tag{B.25}$$

Then, if

$$\overline{\phi} > \phi^{\max}(0) \tag{B.26}$$

holds, the characterization of the prevailing equilibrium provided in proposition 1 can be provided, equivalently, as a function of $\underline{\phi}$ for given $\overline{\phi}$ in that there exist a critical threshold for the marginal productivity of untalented workers,

$$\underline{\phi}^{\min} : \overline{\phi}^{\max} = \overline{\phi} \tag{B.27}$$

such that, given $\overline{\phi}$, if $\underline{\phi} < \underline{\phi}^{\min}$ then the prevailing equilibrium is a TPE, while if $\underline{\phi} > \underline{\phi}^{\min}$ then the prevailing equilibrium is a TSE.

We explore the conditions under which the prevailing equilibrium can be characterized for given values of the wage dispersion associated with a candidate TSE, defined as $\Delta\phi \equiv \overline{\phi} - \underline{\phi}$.

Assume that the (present) value of labor income is linear in the wage rate $w(n)$ and that the payoff function V_t satisfies the following property:

A6. For any $w(n)$, and n, the function V_t is additive separable in the (present) value of labor income and investment in education $c(n)$.

Then, the $\overline{\phi}^{\max}$ is linear in $\underline{\phi}$, $d\overline{\phi}^{\max}/d\underline{\phi} = const$. Suppose that both $\underline{\phi}$ and $\overline{\phi}$ are strictly increasing and differentiable functions, $\underline{\phi}(x)$ and $\overline{\phi}(x)$, of some state variable x, with $\overline{\phi}(0) = \underline{\phi}(0) \geq 0$. Accordingly, Δw will also be a differentiable function $\Delta w(x)$ of x, with $\Delta w(0) = 0$. Given that $\underline{\phi}$ increases in x and $\overline{\phi}^{\max}$ increases in $\underline{\phi}$, $\overline{\phi}^{\max}$will also be an increasing function of x. Furthermore, we know that for $\overline{\phi}^{\max}(\underline{\phi}) > \underline{\phi}$ for all x. Hence, given $\overline{\phi}(0) = \underline{\phi}(0)$, for $\Delta w = 0$,

$$\overline{\phi} < \overline{\phi}^{\max} \tag{B.28}$$

so that the prevailing equilibrium is TSE.

If wage dispersion is to increase in x then, for all $x \geq 0$

$$\frac{d\overline{\phi}}{dx} > \frac{d\underline{\phi}}{dx} \tag{B.29}$$

Assume the above inequality holds so that as x increases, $\overline{\phi}$, $\underline{\phi}$, and Δw all increase. It is crucial to note that as x increases, also $\overline{\phi}^{\max}$ increases, according to

$$\frac{d\overline{\phi}^{\max}}{d\underline{\phi}}\frac{d\underline{\phi}}{dx} = const\frac{d\underline{\phi}}{dx} \tag{B.30}$$

Hence, in principle, we do not know whether the economy will stay in a TSE or switch to TPE as x increases. We know that (spiegare perche'), $const > 1$. Then,

$$\frac{d\overline{\phi}}{dx} > const\frac{d\underline{\phi}}{dt} \tag{B.31}$$

is a sufficient condition for the existence of a critical value of x, call it $\widehat{x}$, such that the equilibrium is TSE is $x \equiv \widehat{x}$ and TPE otherwise, where

$$\widehat{x} : \overline{\phi}(\widehat{x}) = \overline{\phi}^{\max}(\underline{\phi}(\widehat{x})) \tag{B.32}$$

That is, the prevailing equilibrium can be also characterized for values of x, other things equal, equivalently to proposition 1 .

Furthermore, since Δw is strictly increasing in x, the equilibrium can be also characterized in terms of wage dispersion values. If the wage dispersion associated with a candidate TSE is less than $\Delta w(\widehat{x})$ the equilibrium is TSE, otherwise is TPE.

Cose da fare

1. Studiare il modello con n livelli di ricchezza omega e due livelli di talento 2. Sviluppare versione del modello con: a. Exogenous technical progress (Acemoglu) b. N livelli di wealth 3. Statica comparata con technological progess e con crescita economica 4. Dati

Thoughts on motivation and empirical trends

1. Endogenous SBTP hypothesis (acemoglu, 2002) suggests that an increase in the supply of skilled workers can promote an even larger increase in demand of skilled workers thereby resulting in an increase of the wage skill premium

So, how do we reconcile this with the fact that (a) wage skill premium has grown larger for PGs than for CGs while supply of PGs has increased less than the supply of CGs

2. We should look at the education composition by occupation: how many professionals are PGs and how many white collars are CGs, and how many blue collars.....? (It is our figure 6). Maybe if occupation is a measure of skills....then the fact that CGs have increased more than PGs among blue collars, while the opposite has occurred among white collars and especially professional is a signal that the quality of the signal provided by CG is becoming less precise over time compared to the signal provided by PG education

3. Undefeated equilibrium per studiare cosa succede a quelli che stanno sotto wi

B.2 Decreasing marginal returns to labor

Example with

$$y = \frac{\tau}{1-\alpha} l^{1-\alpha} \tag{B.33}$$

and

$$g_\phi(\phi, x) = \phi \tag{B.34}$$

so that, given $\tau = g_\phi(\phi, x)$,

$$y = \frac{\phi}{1-\alpha} l^{1-\alpha} \tag{B.35}$$

Given this production function, the equilibrium wage paid by a firm hiring an amount l of labor whose expected level of talent equals $E(\phi)$, will be equal to:

$$w_{E(\phi)} = E(\phi) l^{-\alpha} \tag{B.36}$$

Consider a separating equilibrium in which, all talented individuals get a level n_{13} of education, while untalented make no investment in education ($n_{24} = 0$). Assuming that the population consists of N individuals and N firms, the population of talented individuals amounts to πN and π measures also the quantity of talented individuals hired by each firm (we consider symmetric equilibria). Similarly, the population of untalented individuals amounts to $(1-\pi)N$ so that $1-\pi$ measures the quantity of untalented labor hired by each firm. Salaries paid by firms to talented and untalented individuals, respectively are equal to:

$$w_{13} = \overline{\phi} \pi^{-\alpha} \tag{B.37}$$

$$w_{24} = \underline{\phi} (1-\pi)^{-\alpha} \tag{B.38}$$

Consider now an equilibrium where type 1 separates and types 2,3, and 4, pool together. Salaries will be:

$$w_1 = \overline{\phi} (\delta\pi)^{-\alpha} \tag{B.39}$$

$$w_{234} = \phi_{234} \left[(1-\pi) + (1-\delta)\pi\right]^{-\alpha} \tag{B.40}$$

where

$$\phi_{234} = \left[\frac{(1-\delta)\pi}{(1-\delta)+(1-\pi)\delta}\right] \overline{\phi} + \left[\frac{(1-\pi)}{(1-\delta)+(1-\pi)\delta}\right] \underline{\phi} \tag{B.41}$$

Note that $\phi_{234} = \tau_{234}$.

Finally, consider the equilibrium wehere types 1 and 4 separate, and types 2 and 3 pool together. Salaries will be:

$$w_1 = \overline{\phi}(\delta\pi)^{-\alpha} \tag{B.42}$$

$$w_4 = \underline{\phi}\left[(1-\delta)(1-\pi)\right]^{-\alpha} \tag{B.43}$$

$$w_{23} = \phi_{23}\left[(\delta(1-\pi)+(1-\delta)\pi\right]^{-\alpha} \tag{B.44}$$

where

$$\phi_{23} \equiv \left[\frac{\pi(1-\delta)}{\pi(1-\delta)+(1-\pi)\delta}\right]\overline{\phi} + \left[\frac{(1-\pi)\delta}{\pi(1-\delta)+(1-\pi)\delta}\right]\underline{\phi} \tag{B.45}$$

note that π_{23} is the same as τ_{23}.

Consider our microfoundation (da aggiungere in appendice). Then, condition (22), which refers to type 2 individuals, becomes:

$$\mu\int_{n_{34}^{\max}}^{N} w_{13}^{\max}e^{-rt}dt + (1-\mu)\int_{n_{34}^{\max}}^{N} w_{24}e^{-rt}dt - c(n_{34}^{\max}) + \overline{\omega} = \int_{0}^{N} w_{24}e^{-rt}dt + \overline{\omega} \tag{B.46}$$

Substituting for $c)n_{34}^{\max} = \underline{\omega}$ and solving for $w_{13}^{\max}$, we get

$$w_{13}^{\max} = \frac{r\underline{\omega}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} + w_{24}\left[\frac{1-e^{-rN}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} - \frac{1-\mu}{\mu}\right] \tag{B.47}$$

Substituting for w_{24}, we obtain:

$$w_{13}^{\max} = \frac{r\underline{\omega}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} + \underline{\phi}\left(1-\pi\right)^{-\alpha}\left[\frac{1-e^{-rN}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} - \frac{1-\mu}{\mu}\right] \tag{B.48}$$

The condition for existence of a separating equilibrium is

$$w_{13} \leq w_{13}^{\max} \tag{B.49}$$

$$\Leftrightarrow \tag{B.50}$$

$$\overline{\phi}\pi^{-\alpha} \leq \frac{r\underline{\omega}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} + \underline{\phi}\left(1-\pi\right)^{-\alpha}\left[\frac{1-e^{-rN}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} - \frac{1-\mu}{\mu}\right] \tag{B.51}$$

Dividing both sides by $\pi^{-\alpha}$

$$\overline{\phi} \leq \frac{r\underline{\omega}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)}\pi^{\alpha} + \underline{\phi}\left(\frac{1}{\pi^{-1}-1}\right)^{\alpha}\frac{1}{\mu}\left[\frac{1-e^{-rN}}{\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} - (1-\mu)\right] \tag{B.52}$$

As π becomes smaller, the RHS of the above inequality gets smaller so that it is more difficult that the condition is satisfied. Define

$$RHS = \frac{r\underline{\omega}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)}\pi^{\alpha} + \underline{\phi}\left(\frac{1}{\pi^{-1}-1}\right)^{\alpha}\frac{1}{\mu}\left[\frac{1-e^{-rN}}{\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} - (1-\mu)\right] \tag{B.53}$$

In the limit:

$$\lim_{\pi\to 0} RHS = \frac{r\underline{\omega}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} \tag{B.54}$$

Hence, if

$$\overline{\phi} > \frac{r\underline{\omega}}{\mu\left(e^{-rn_{13}^{\max}} - e^{-rN}\right)} \tag{B.55}$$

then there exist a threshold value for π, call it $\pi^{\min} > 0$, such that the separating equilibrium does not exist for $\pi < \pi^{\min}$.

Therefore, we can conclude that –with decreasing marginal returns to labor, conditional on talent– if the fraction of talented individuals becomes too low –other things equal– the separating equilibrium might disappear.

(look also at δ)

****

Definition 4. *Consider a candidate equilibrium E and suppose that players deviates to some education n' that is never announced under E. Suppose that there existes another equilibrium E′ in which a non emplty set of types, $\Theta_1 \subset \Theta$, announce n and assume that Θ_1 is precisely the set of types who prefer E′ to E. According to the undefeated criterion, upon observing n′, firms' beliefs n′ must satisfy the following: all types Θ_1 whose E'-equilibrium payoff is striclty greater than their E-equilibrium payoff are belived to deviate to n′ with probability one (see Adriani and Deidda, 2009).*

B.3 Generalization to any wealth distribution

Suppose that, instead of two levels of wealth, the wealth distribution is characterized by N levels of wealth: $\omega_1 < ...... < \omega_N$. Let The δ_i be the fraction of individuals with wealth equal to ω_i, so that

$$\sum_{i=1}^{N} \delta_i = 1 \tag{B.56}$$

For each level of wealth, the fraction of talented individuals is still equal to π. Equivalently to the case of two levels of wealth (see equation (3)), we define

$$n_i^{\max} : c(n_i^{\max}) = \omega_i \tag{B.57}$$

the maximum level of education that an individual endowed with an amount of wealth equal to ω_i can finance.

B.3.1 Characterization of the prevailing equilibrium

Let us define $\mathcal{N}_p = \{n_1, ..., n_p\}$ the set of values of n played with positive probability in a given equilibrium by individuals heterogeneous in talent. Similarly, define $\overline{\mathcal{N}}$, $(\underline{\mathcal{N}})$, the set of values of n such that if in a given equilibrium an individual plays $n \in \overline{\mathcal{N}}$, $(n \in \underline{\mathcal{N}})$, that individual is talented (untalented).

We note that, by construction, in a candidate equilibrium where two out of three of the above sets are empty, $\mathcal{N}_p$ must be non-empty. Also, if $\mathcal{N}_p$ is empty, then the other sets must be non-empty.

Rich and talented always separate. We show that in any robust equilibrium (to the intuitive criterion), $\overline{\mathcal{N}}$ must be non-empty. The proof is by contraction. Consider a candidate equilibrium E such that $\overline{\mathcal{N}} = \emptyset$ and $\mathcal{N}^E \equiv \mathcal{N}_p \cup \underline{\mathcal{N}}$ is the non-empty set of values of n played with positive probability. Define $n_p^{\max} \in \mathcal{N}^E$ such that:

$$V_{\underline{\theta}}(n_p^{\max}, w(n_p^{\max}), \underline{\phi}, c(n_p^{\max})) \geq V_{\underline{\theta}}(n, w(n), \underline{\phi}, c(n)) \qquad \forall n \in \mathcal{N}^E \tag{B.58}$$

Corrispondingly, we define n_s as the maximum value of n that untalented individuals are willing to play if –by doing so– they would be believed to be talented:

$$n_s : V_{\underline{\theta}}(n_s, \overline{\phi}, \underline{\phi}, c(n_s)) = V_{\underline{\theta}}(n_p^{\max}, w(n_p^{\max}), \underline{\phi}, c(n_p^{\max})) \tag{B.59}$$

Then, by our sorting condition

$$V_{\overline{\theta}}(n_s, \overline{\phi}, \underline{\phi}, c(n_s)) > V_{\overline{\theta}}(n_p^{\max}, w(n_p^{\max}), \underline{\phi}, c(n_p^{\max})) \tag{B.60}$$

Hence, there always exists a value $n' > n_s$ that enables talented and rich individuals to signal themselves, which makes them better off. [Scrivere meglio]

(Robust) talent separating equilibrium (TSE). By definition in a TSE, $\mathcal{N}_p = \emptyset$, all talented (untalented) individuals play $n \in \overline{\mathcal{N}}$, ($n \in \underline{\mathcal{N}}$) and $\underline{\mathcal{N}} \cap \overline{\mathcal{N}} = \emptyset$. In any TSE, $n \in \underline{\mathcal{N}}$ satisfies $n = 0$. The equilibrium wage schedule satisfies $w(0) = \underline{\phi}$, and $w(n) = \overline{\phi}$ for any $n \in \overline{\mathcal{N}}$. Define,

$$\underline{n_{\overline{\theta}}} : V_{\underline{\theta}}(\underline{n_{\overline{\theta}}}, \overline{\phi}, \underline{\phi}, c(\underline{\phi})) = V_{\underline{\theta}}(0, \underline{\phi}, \underline{\phi}, c(0)) \tag{B.61}$$

Then,

1. In any TSE, $n \in \overline{\mathcal{N}}$ satisfies $n \geq \underline{n_{\overline{\theta}}}$;

2. In any robust (to Intuitive criterion) TSE, $n \in \overline{\mathcal{N}}$ satisfies $n = \underline{n_{\overline{\theta}}}$;

3. A TSE exists if and only if $\underline{n_{\overline{\theta}}} \leq n_1^{\max}$, where $n_1^{\max}$ is defined by equation (B.57).

Definition of D1. Intuitively, the D1 refinement makes the following restriction: when firms observe a deviation , they believe it comes from those agents who are more likely to benefit from the deviation given the set of all possible off equilibrium wages associated with such deviation given the set of all possible systems of off equilibrium beliefs for firms.

Definition 5. *Consider a candidate equilibrium E such that $\mathcal{N}^E$ is the set of equilibrium values of education, n^E, played with positive probability. Let for any deviation n, i.e. for any $n \notin \mathcal{N}^E$, let $\mathcal{W}(n)$ the set of all possible off equilibrium wages associated with n. Given $\mathcal{W}(n)$, for any type $\theta \in \Theta$, where Θ is the set of possible types, let $\mathcal{W}(n, \theta)$ be the set of off equilibrium wages such that type θ strictly benefits from the deviation, and $W^0(n, \theta)$ be the set of off equilibrium wages such that type θ is indifferent between playing the equilibrium or deviating. Assume that, if for a deviation $n \notin N^E$, $\mathcal{W}^0(n, \theta) \cup \mathcal{W}(n, \theta) \subset \mathcal{W}(n, \theta')$, for some $\theta, \theta' \in \Theta$, then, firms assign probability zero to the event that the worker playing n is of type θ. Then, if - given such refined firms' beliefs - playing n makes type some θ' better off, then E is not robust to D1.*

D1-robust Talent Pooling Equilibria (TPE) In any robust TPE:

1. There exist $i \leq N$, such that talented individuals endowed with wealth $\omega_j \geq \omega_i$ perfectly separate playing $n_{\overline{\theta}}$, and get a salary $w(n_{\overline{\theta}}) = \overline{\phi}$.[7] Hence, in any TPE, $\overline{\mathcal{N}} \neq \emptyset$.

2. For any $n_j, n_h \in \mathcal{N}_p$, with $j > h$ the equilibrium wage function $w(n)$ satisfies the following properties:

$$V_{\overline{\theta}}(n_j, w(n_j)) \geq V_{\overline{\theta}}(n_h, w(n_h)) \tag{B.62}$$
$$V_{\underline{\theta}}(n_j, w(n_j)) = V_{\underline{\theta}}(n_h, w(n_h)) \tag{B.63}$$

3. Furthermore, in D1-robust TPEs, for any $n_j \in \mathcal{N}_p$,

$$n_j = n_j^{max} \tag{B.64}$$

4. If $\underline{\mathcal{N}} \neq \emptyset$, then $n \in \underline{\mathcal{N}}$, satisfies $n = 0$. Furthermore,

$$V_{\underline{\theta}}(0, \underline{\phi}) = V_{\underline{\theta}}(n_h, w(n_h)) \tag{B.65}$$

5. Given point 4, in any TPE all talented individuals either separate themselves (if sufficiently rich) or play $n_j \in \mathcal{N}_p$, i. e. they never play $n = 0$ (questo ci serve per dimostrare che un equilibrio con separazione degli untalented puo' esistere).

Point 1, follows directly from the sorting condition: richest talented benefit strictly more from education than other individuals, hence they are always able and willing to separate (intuitive criterion is enough). Equation (B.62) (point 2) follows from the fact that the set of feasible levels of education for richer talented individuals includes that for poorer talented individuals. Equation (B.63) (Point 2) follows from the fact that, in order for equation (B.62) to be satisfied, the average level of talent associated with n_j should be strictly decreasing in j. This implies that the proportion of untalented individuals relative to talented individuals playing n_j is strictly increasing in j. Let $n_j \in \mathcal{N}_p$, the maximum pooling level of education that an untalented with wealth w_j can afford. Then, the fact that the proportion of untalented individuals relative to talented individuals playing n_h is strictly increasing in h means that for some j untalented individuals with wealth j are playing $n_h < n_j$, that is they choose a level of education lower than the maximum pooling level of education that they could afford.

Point 2 implies that in any TPE wages are increasing in education: the expected level of skills increases with education. (verificare dispersion for education level, in generale...anche prima della generalizzazione)

Wealth distribution, supply of labor by education, expected skills levels, and wage dispersion

[7] Note that, this definition encompasses TSE, for $i = 1$.