# Public-private contracting under limited commitment[*]

Daniel Danau[†]        Annalisa Vinella[‡]

July 2012

## Abstract

A government delegates construction and operation of an essential facility to a private firm. When parties sit at the contracting table, they are uncertain about the operating cost. At the construction stage, the firm can improve its distribution by exerting some non-contractible effort. As soon as the facility is in place, the firm learns the realized cost privately. In case any of the parties breaks down the relationship and the firm is replaced during the operation phase, the government bears a cost that is more important the earlier the interruption, relative to the stipulated duration. We show that, under limited commitment, the optimal full-commitment allocation is implementable if and only if the firm holds some minimum amount of own funds that can be destined to the project, it is able to borrow funds for that specific project, and the replacement cost is sufficiently high. Implementation is made by instructing the firm to invest some intermediate amount of own and borrowed funds, by conditioning the loan guarantee (provided under the aegis of a third party not suffering from commitment problems) on the outcome of the potential renegotiation process between the government and the firm, and by setting duration neither too short nor too long. Making duration contingent on the realized operating cost helps the government lessen the more concerning between moral-hazard and commitment problems.

*Keywords*: Public-private contracting; limited commitment; duration; private funds; debt; guarantees; replacement cost

*J.E.L.* Classification Numbers: D82; H57; H81

# 1  Introduction

In contracts between governments and private firms for building and operating essential facilities, firms are often required to invest.[1] Concerned investments are generally huge, and largely financed with debt.[2] An important but still under-explored issue is whether it is desirable to involve private capital in large public projects and, if so, to what extent this capital is to be drawn from the own funds of the firms and/or borrowed on the credit market. Our study investigates this issue, nesting financial considerations into optimal public-contracting design.

We pursue our objective taking into account two major problems that typically plague the implementation of contracts awarded for the construction and operation of essential facilities. First, as these contracts last long (usually, some decades), there is often uncertainty about operating conditions when they are drawn up, and asymmetric information between the involved parties when they are executed. Second, especially (though not only) in developing countries, where institutions are weak, commitment is limited. Hence, contracts that fail to be self-enforcing are often reneged either by the government (*non-commitment*) or by the firm (*limited enforcement*), and possibly renegotiated.[3]

To capture these two problems, we adopt a model that, in its basic elements, is similar to Laffont [32] and the related studies of Guasch *et alii* [21] and [22]. We nonetheless innovate on a variety of aspects, which allow us to represent the situations that we have in mind more closely, and address the issue of our interest.

Specifically, following Laffont [32] and related studies, we assume that, when the government and the firm sign the contract for the construction and operation of an essential facility, they both face uncertainty about the cost that the firm will bear to provide the good during the operation phase. Furthermore, once the facility is in place, the firm observes the cost realization privately.[4] As an illustration, one can think about the realization of a tunnel. Prior

---

[1]Contracts for public projects in which firms are required to invest have first appeared in the UK under the denomination of *private finance initiative* (PFI).

[2]In June 2008, *The Economist* reported that infrastructure spending (representing a large share of world GDP, with $22 trillion allocated to projected investments over a ten-year horizon only in emerging economies) was mainly funded with corporate bonds issued by the private firms running the projects before the economic crisis, and with senior debt after the crisis. That leverage is core in large infrastructure projects is further witnessed by the circumstance that, in the UK, in March 2009, the Government created the Treasury Infrastructure Finance Unit with the task of lending funds to PFI projects for which it is difficult to borrow from commercial banks (House of Lords [25]). On top of that, in 2010, the Association for Consultancy and Engineering proposed the creation of a National Investment Bank, along the lines of the European Investment Bank that has lent €3-4bn. of funding on a not-for-profit basis since 2005 (ACE [1]). See also Flyvbjerg *et alii* [18] on debt financing of large public projects.

[3]We use the labels "non-commitment" and "limited enforcement" as reported in Estache and Wren-Lewis [17], who recall that non-commitment is explored in Chapter 9 of Laffont and Tirole (1993), and limited enforcement in Laffont [32] and, more widely, in Guasch *et alii* [21]. Lack of enforcement is referred to as a cause of pervasive renegotiations in Guasch [20], Engel *et alii* [15], Estache [16], Chong *et alii* [7], Saussier *et alii* [39].

[4]Laffont [32] makes this assumption to study monopoly regulation; Guasch *et alii* [21] - [22] use it with

to construction, operating costs are not perfectly predictable. The firm has the possibility of learning them as soon as it starts managing the facility. By contrast, the government, which does not perform the activity, does not observe them directly. In line with the same literature, we capture the limited-commitment problem by allowing the contract to be reneged during the operation phase.

First of all, our approach departs from previous work in terms of contract duration. The time length of the second phase of the project (operation) is not exogenously given. As we will illustrate at a later stage, the duration choice is of core relevance for the parties' achievements, especially under limited commitment.

Secondly, our approach departs from previous work in terms of information structure. In our model, at the construction stage, the firm decides whether or not to exert some non-contractible effort that can affect the operating conditions. This is in line with the studies about public-private partnerships (Bennett and Iossa [5], Hart [23], Iossa and Martimort [28] - [27], Martimort and Pouyet [35], among others), which evidence the presence of synergies between the phases of the project. Specifically, in our setting, effort provision in construction raises the probability of facing a low operating cost. Besides, in each period of operation, the firm decides whether or not to exert some non-contractible effort (say, to preserve the quality of the facility) that induces a systematic effect on the operating cost, which is yet unobservable to both the government and third parties.

Thirdly, rather than focusing on either firm-led or government-led renegotiation, we look at commitment problems more broadly, and allow for any of the contractual parties to renege on the contract. This helps us capture the far-from-abstract possibility of institutional weaknesses making contract enforcement difficult with regards to either party. Moreover, the benefits/penalties accruing to parties after the contract is reneged are strictly related to its time length, which is endogenous in our study, as we said. If the relationship is stopped and the firm replaced with a new operator, then the government bears a cost that is more important the earlier the interruption, relative to the deadline of the contract. This modelling device is meant to reflect the acknowledged circumstance that, by being unable to keep the relationship in place, governments lose reputation *vis-à-vis* current and prospective partners, customers and voters, and that the loss is lower when the end of the contract is closer.[5] The presence of this cost is of crucial importance as, even in the event of renegotiation, parties' payoffs do depend upon that cost, hence, indirectly, upon the time when reneging occurs.

Financial aspects, which are the very interest of this study, matter in our model because

_____

regards to concession contracts.

[5]Irwin [29] stresses that, in real world, government-firm games that involve infrastructure investments are repeated games in a double sense. Not only the government is concerned with its reputation *vis-à-vis* the firm involved in the concerned project. It also cares about the information that its behaviour and achievements convey to third parties with whom it can potentially interact in the future.

the realization of the project can be financed with a combination of public and private funds. The former can be provided through an up-front transfer from the government to the firm. The latter can be drawn by the firm from two sources, namely its own resource endowment and lenders (such as banks). This representation is coherent with the real-world evidence that, during construction, expenses are generally financed with own funds of the concerned firms and bank loans, sometimes complemented with governmental subsidies.[6] Among the studies aforementioned, financial considerations appear in Guasch *et alii* [21] in a framework of firm-led renegotiation. However, the focus is there on outside financing only. That is, the firm does not provide its own resources up-front to fund the investment, which is covered with a bank loan and, possibly, governmental funds.

The duration aspect emerges as a companion interest of this study as the design of an appropriate financial structure cannot spare reference to the time length of the contract. The reason for this is twofold. First, the profit that the firm obtains at each instant of the operation phase depends upon how much it invests up-front and how long the contract lasts. Second, the instantaneous total payment that the government makes during the operation phase depends upon how much the firm invests, as just said, and how much the firm needs to reimburse to the lender at each instant, which is in turn determined according to the size of the loan and to the contract duration. As the outcome of the renegotiation process depends upon the instantaneous returns that parties attain under the contract, and those returns depend upon the amount of private resources injected into the project, one cannot study the optimal financial structure making abstraction from duration concerns.

We begin by considering the benchmark situation in which parties fully commit to the contract. As usual with *ex-ante* contracting, full commitment and no liability concerns, the original agreement stipulates the efficient allocation and remains in place for the whole duration agreed upon. The mix of sources used to fund the investment is irrelevant in this context. However, a restriction in terms of duration is imposed by the moral-hazard problem that arises at the construction stage. That is, in line with the findings of the literature on public-private partnerships (see Iossa and Martimort [28]), the contract must be sufficiently long to allow the firm to profit from effort provision. By contrast, moral hazard in operation is not an issue as the incentives to shirk are handily removed by making the firm residual claimant over the benefit generated by the effort exerted in each production period.

We then move to study the situation of our interest, that in which commitment is limited and some party may have incentives to renege during the operation phase. Preventing this behaviour casts additional constraints on contract design. Taking the original contract to be the one that stipulates the optimal full-commitment allocation, we investigate whether such

---

[6]Engel *et alii* [12] stress that this is a major financial characteristics of private-public partnerships.

constraints translate into restrictions in terms of financial structure and duration and, if so, how the resources and the termination date should be chosen and combined for the contract to be still effected under limited commitment. In so doing, we address a number of specific questions. Should the firm be instructed to use its own funds and, if so, in large or in small amount? Should it be induced to borrow money on the credit market, and should the loan be guaranteed somehow (say, by involving a third party, such as an international institution, that does not suffer from commitment problems)? How much private funds should be injected, overall, into the project? How should duration be picked to ease contract enforcement?

We obtain a number of results that we illustrate hereafter.

To begin with, three conditions are necessary for the contract that stipulates the optimal full-commitment allocation to be implementable under limited commitment.

First, replacing the firm during the operation phase must be sufficiently costly to the government. As long as replacement does not yield a sufficiently large penalty, the government finds it convenient to interrupt the relationship before the termination date originally stipulated, and thus expropriate the investment made by the firm and the lender at the outset of the project.

Additionally, it is necessary that both own funds of the firm and funds borrowed on the credit market be invested in the project. To see why injection of own funds is essential, consider that, to induce information release at no cost as well as effort provision in construction, the government offers an incentive scheme under which, in each period of operation, the firm makes a profit in the event that a low cost is realized, and bears a loss otherwise. The firm might be unwilling to produce if it finds out that the true cost is high. To prevent the firm from reneging on the contract in the latter case, it must be required to contribute a sufficiently important amount of own funds up-front, and be entitled to recover those funds during the operation phase by making its instantaneous return sufficiently large. This theoretical prediction has a non-negligible practical implication. That is, projects of the kind that we refer to should be delegated only to firms that do have some endowment to fund investments up-front.

In turn, the loan is essential in that it allows the government to harden the firm's budget constraint, which is another way to lessen the firm's incentives to abandon the project. This is made by offering the firm a guarantee in favour of the lender, conditional on the relationship between the government and the firm going on till the termination date. It means that the guarantee should be provided in the event that the contract is either executed or renegotiated, and denied otherwise. Of course, in a limited-commitment framework, guarantees are only feasible under the aegis of a third party (say, an international institution) that does not suffer from commitment problems. By conditioning the guarantee on the prosecution of the relationship, contract break-down followed by firm replacement becomes less costly to the government than renegotiation. The incentive of the firm to renege on the contract in the hope for a profitable

renegotiation is thus removed.

Once it is ensured that these requirements are all met, the subsequent step is to set the amount of resources and the duration such that the contract stipulating the optimal allocation is effected, indeed. The next contribution of our study consists in predicting how this should be done.

Let us first consider duration. As under full commitment, the contract should not last too little due to the moral-hazard problem that arises in construction. At the same time, under limited commitment, the contract should not last very long either, unless the replacement cost is very high. This is explained as follows. To tackle asymmetric-information problems (namely, moral hazard in construction and misrepresentation of operating cost), the government needs to condition the cumulated operating profits of the firm on the realized state of nature *i.e.*, on whether the operating cost is low or high. The longer the duration of the contract, the lower the per-period profit accruing to the firm, especially when the cost is high, the stronger the latter's incentives to renege in that case. To lessen this problem, the firm should be required to invest more own funds, which would involve raising its operating profit. However, when the firm contributes too important an amount of own funds, it is the government that might be willing to renege and expropriate the firm's investment, especially if the operating cost is low. When replacement is sufficiently (even though not very) costly, the incentives of the government to renege cannot be removed unless, rather than insisting on the firm's contribution, the duration of the contract is shortened. The less costly replacing the firm is, the shorter the contract should be made to persuade the government to abide. By contrast, when the cost of replacement is not sufficiently large, it becomes impossible to find an "intermediate" duration that both complies with the moral-hazard requirement and discourages the government from reneging.

We now move to the optimal financial structure of the project. Not only the firm ought to invest an intermediate amount of own funds, as just explained. It should also borrow some intermediate amount of funds on the credit market. The reason why a loan is to be taken to run the project has been already illustrated. We still need to explain why the loan should not be too large, in turn. Again, in that case, the government would be tempted to renege on the contract so as to appropriate the facility without reimbursing the lender (through the firm). This points to the conclusion that, in environments where contractual parties suffer from commitment problems, reliance on private capital in public projects should not be massive, even when private partners have deep pockets and/or unconstrained access to financial markets.

As a final step of the work, we extend the model to investigate how results are affected if, rather than offering a contract in which the termination date is the same no matter the realized state of nature, the government designs a contract in which it chooses a different duration for each possible state. In the literature, contracts with these characteristics are referred

6

to as fixed- and flexible-term contract, respectively. Our study predicts that making dura-
tion state-dependent can be beneficial in two ways, depending upon how the compensation
scheme is structured. First of all, it can help the government lessen commitment problems.
Actually, the contract that stipulates the optimal allocation becomes implementable in the
limited-commitment framework even when the cost of replacing the firm is not sufficiently
large. This is because the wedge between the profits accruing to the firm in the different states
can be *narrowed* enough to remove both the government's temptation to renege when a low cost
is realized and the firm's temptation to renege when a high cost is realized. On the other hand,
making duration state-dependent can help the government address the moral-hazard problem
in construction. By differentiating duration between states, indeed, the profit-wedge can be
*enlarged* to the point that the return is sufficiently uncertain to trigger effort provision even
when the associated disutility is very high.

## 1.1 Mainly related literature

Our work is related to the literature about reliance on private resources for the realization
of public projects. Engel *et alii* [11] argue that requiring the private firm to fund the ini-
tial investment entirely and recover costs directly from user-fees, rather than receiving public
transfers, is a desirable option in situations where the budgetary authority that monitors the
governmental agency in charge of shifting funds from the public budget to the firm faces agency
problems. Indeed, private financing helps remove incentive issues between different tiers of the
governmental hierarchy, which can plague the performance of the project. Our results suggest a
different motivation for involving private funds in public projects. In environments with limited
commitment, private resources represent a useful device to induce the firm to remain in the
contract when an unfavorable state of nature is realized.

Engel *et alii* [11] further suggest that, when the project has an uncertain outcome, unde-
sirable involvement of public funds can be avoided by offering a flexible-term contract. This
contract is such that, once uncertainty is resolved, the duration and, indirectly, the profits of
the firm are adjusted to the realized state of nature. Hence, it is not necessary to employ public
transfers to make the project financially viable. However, the main motivation for relying on
flexible-term contracts seems to come from the authors' previous works (Engel *et alii* [13] and
[14]), in which such contracts are put forward as a useful tool to tackle limited-enforcement
problems. The authors argue that, by adjusting duration to let the firm obtain exactly the
same payoff whatever the state realization, the government removes the temptation of the firm
to renege when the state is unfavorable. In our model, the incentive of the firm to renege on the
contract is not removed by simply eliminating uncertainty from the payoff scheme. Whether
the return is uncertain or not, the firm might be able to extract some extra-benefit through

renegotiation because replacing it is costly to the government. Moreover, asymmetric information requires that payoffs be, indeed, differentiated between states. Yet, in our framework, state-dependent duration is still a useful incentive tool, and in a broader sense than emerged in previous studies. With both asymmetric-information and limited-commitment concerns, making duration contingent on the state enables the government to adjust the payoff distribution by choosing whether to make it more or less spread, depending upon the relative importance of the incentive problems.[7]

According to de Bettignies and Ross [10], private investment is beneficial because private firms credibly commit to early termination of socially inefficient projects when the latter generate low cash flows. By contrast, a public authority would not do so for political reasons. Indeed, the termination of any project (whether it generates high or low cash flow) provides a bad signal to society about the activity of the government. While de Bettignies and Ross [10] focus on projects for which early termination is socially desirable, we explore situations in which this is not the case. From this standpoint, our analysis is related to that of Dewatripont and Maskin [9]. They show that, under decentralized financing, borrowing money from a small financier provides the firm with good incentives to avoid default. In our model, in which the firm runs a public (rather than a private) project, incentives are provided by the government (rather than by the lender) also by instructing the firm on how much to borrow.

Our study is further related to the literature about capital structure in agency problems. Spiegel [41] and Spiegel and Spulber [42] - [43] investigate the effects of the capital structure chosen by the agent/firm on the contractual relationship with the principal/regulator. They assume that the regulated firm exercises discretion in its choice of a capital structure as this accords with what they observe to occur, in practice, for the U.S. regulated utilities. By contrast, we are interested in identifying the mix of financing sources, including debt, that allows the government to decentralize the optimal full-commitment allocation through the contract offered to the firm.[8] From this standpoint, our approach is closer to that of Lewis and Sappington [33]. However, in the latter's framework, renegotiation issues are ruled out as parties are taken to fully commit to the initial agreement.

Lastly, our paper is related to the literature about contract renegotiation after an investment cost is sunk. Hart and Moore [24] consider a credit contract for a project, the outcome of which is observable by all parties but not verifiable. Based on the observed cash-flow, the firm and

---

[7]Flexible-term contracts have not only advantages but also drawbacks. Danau [8] shows that, when uncertainty is not resolved at the outset of the operation phase, the expected duration of a flexible-term contract yielding to the firm its reservation payoff is larger than the (certain) duration of a fixed-term contract yielding to the firm that same payoff (in expectation), and that this effect becomes more pronounced as uncertainty increases over time.

[8]This seems to be in line with the attitude, displayed by U.S. regulators before the Eighties, to control utility company debt, as detected in Taggart [44].

the creditor either renegotiate or break down the agreement. In the event of break-down, the firm does not share the cash-flow with the creditor. The latter liquidates the project and obtains some benefit out of this. In our model, the revenues of the firm are endogenous and thus verifiable, provided the firm receives transfers from the government and collects market revenues during the operation phase. However, the firm does not commit to return money to the lender. Moreover, the creditor is not in a position to liquidate assets, which belong to the government and have no other potential use than the public project for which they were created. Under these circumstances, a credit contract can be drawn up not because the creditor can exercise residual control rights on the assets, as in Hart and Moore [24]. Rather, it can be signed because the government pledges a guarantee (under the aegis of a third party) in favour of the private firm for the latter to be able to raise funds from external sources.

## 1.2 Outline

The remainder of the paper is organized as follows. In section 2, we describe the model. In section 3, we illustrate how the contract between the government and the firm and that between the firm and the lender are structured. In section 4, we present the benchmark situation in which parties fully commit and characterize the optimal contract in that environment. In section 5, we introduce limited commitment into the picture, explaining how it is approached formally. In section 6, we present the renegotiation game. In section 7, we describe the whole set of conditions under which the contract that stipulates the optimal full-commitment allocation is implemented in the limited-commitment framework. Section 8 concludes. Some of the mathematical details are relegated to an appendix.

## 2 The model

We consider the contractual relationship between a government (denoted G) and a private firm (denoted F) for the provision of a service of general interest. The project unfolds over two stages. The first stage, which takes place at instant $\tau = 0$, represents the *construction phase*, during which the facility that is needed to provide the service is financed and built. The second stage, which begins as soon as the facility is in place and lasts till time $T > 0$, represents the *operation phase*, during which the service is provided. As frequent in recent decades, the private party F is delegated both stages of the project. At time $T$, when the contract ends, the infrastructure is transferred to G, which manages the activity thereafter.

**Technology, production, consumer surplus, demand** At time 0, F builds the facility bearing the sunk cost $I$ and exerting some unobservable and non-contractible effort $a \in \{0, 1\}$

with disutility $\psi(a)$, where $\psi(0) = 0$ and $\psi(1) = \psi > 0$. At each instant $\tau \in (0,T)$, F provides the service incurring the cost $\gamma q + K$, with $\gamma$ the marginal cost, $q \geq 0$ the production level, and $K > 0$ the fixed cost. At $\tau \in (0,T)$, $\gamma = \theta > 0$ if, at that time, F exerts some unobservable and non-contractible effort $b = 1$ with disutility $\phi(1) = \phi > 0$, whereas $\gamma = \theta + c$, $c > 0$, if it does not $(b = 0, \phi(0) = 0)$. $\theta$ represents the inner unit variable cost of production associated with a brand-new facility; it is stochastic and its distribution depends upon the construction effort $a$ (see Information structure below). In turn, $b$ captures the maintenance activity performed in each period to preserve the quality of the facility, and $c$ the extra-cost that F faces when it does not maintain the facility.

F receives a transfer $t$ from G and collects revenues $p(q)q$ on the market. Allowing the private firm to receive a combination of subsidies and fees warrants that a variety of real-world situations be encompassed, ranging from conventional infrastructure provision, in which the government pays for the activity and the firm earns no money from consumers, to traditional concession, in which the firm only relies upon market revenues.[9]

Consumption of $q$ units of the service yields instantaneous gross surplus $S(q)$, such that $S' > 0$, $S'' < 0$, $S(0) = 0$, and the Inada conditions are satisfied. Consumers cannot store the service and transfer consumption to future periods so that the output produced at some given $\tau$ is entirely consumed at that same time and sold on the market at price $p(q) \equiv S'(q)$. This defines the inverse demand function.

Once the investment is made, both technology and demand parameters remain constant for the whole duration of the project, including the period in which the activity is run by G (say, through a public firm). Only the marginal cost can change over time, as we said, depending upon whether effort is or not exerted in operation.

**Information structure** The contract between G and F is signed, the investment $I$ made, the effort $a$ exerted, and the disutility $\psi(a)$ borne *ex ante i.e.*, when the value of $\theta$ is unknown to either contractual party. The assumption of *ex-ante* contracting is meant to mirror situations in which the contours of the activity are designed before the firm receives any specific piece of information on productivity. At the contracting stage, it is commonly known that $\theta$ will be either low $(\theta_l)$ or high $(\theta_h > \theta_l)$ with probabilities $\nu_1$ and $1 - \nu_1$ if $a = 1$, and $\nu_0$ and $1 - \nu_0$ otherwise. Let $\Delta\nu = \nu_1 - \nu_0 > 0$, meaning that exerting effort at the construction stage propitiates a lower inner cost, stochastically.[10] We denote $\Delta\theta = \theta_h - \theta_l$ the degree of

---

[9] As an example, in the EU it is required that BOT (*B*uild-*O*perate-*T*ransfer) concession holders rely upon revenues from market sales only, in order to ensure that they bear possible operation and demand risks entirely (as an illustration, Auriol and Picard [2] mention the Channel Tunnel project).

[10] Synergies between project phases are pervasively represented in models on public-private partnerships that take them to provide a rationale for bundling. As an illustration, in Iossa and Martimort [28] effort provision at the construction phase reduces costs at the operation stage.

uncertainty about the value of $\theta$. Once the facility is in place, and production is about to begin, F observes privately the state of nature $i \in \{h, l\}$ *i.e.*, whether $\theta_l$ or $\theta_h$ has materialized. Hence, at that point, F enjoys an information advantage on the operating cost *vis-à-vis* the contractual partner. Once state $i$ is realized, it is commonly known that, at each instant thereafter, the unit variable cost will be equal to $\theta_i$ if $b = 1$, and to $\theta_i + c$ otherwise. However, neither G nor a third party (say, an auditor or a court of justice) can observe the true cost value.

**Project financing** Three financing sources can be used to fund the investment. First, F injects an amount of own funds $M \in [0, E]$, with $E > 0$ its resource endowment. Second, F borrows an amount of funds $C \geq 0$ on the competitive credit market. Third, G makes an up-front payment $t_0 \in \mathbb{R}$ to F so that the three financing sources cover the monetary investment that is made initially:

$$t_0 + M + C = I. \tag{1}$$

The transfer $t_0$ is positive when the project is partially financed with public funds. It is negative when the project is financed only with private funds and F makes a payment $[I - (M + C)]$ to G to be awarded the contract.

## 2.1  Payoffs under symmetric information

**The payoff of F** Let $d \geq 0$ denote the repayment that, at each instant $\tau \in (0, T)$, F should make to the lender in return for the amount $C$ received initially. The operating profit of F at $\tau$ is given by

$$\pi = t + pq - (\gamma q + K + \phi(b)) - d, \tag{2}$$

with $p \equiv p(q)$. Further denoting $r$ the discount factor, the present value at $\tau$ of the whole stream of profits from $\tau$ to $T$ is written as

$$\Pi_\tau = \int_\tau^T \pi e^{-r(x-\tau)} dx. \tag{3}$$

The payoff of F, to be denoted $\widetilde{\Pi}$, includes $\Pi_0$ together with the resources initially used to fund the investment, namely $t_0$, $M$ and $C$, net of the investment cost $I$, of the amount $M$ injected by the firm itself into the project, and of the disutility of effort exerted in construction $\psi(a)$. That is, $\widetilde{\Pi} = \Pi_0 + t_0 + M + C - I - M - \psi(a)$. Using (1), the payoff of F further becomes

$$\widetilde{\Pi} = (\Pi_0 - M) - \psi(a), \tag{4}$$

and is thus the difference between the net present value of the project, as measured by the

11

discounted operating profits net of the firm's investment of own funds, and the additional contribution that the firm makes by providing effort in construction.

**The payoff of G**   G is a benevolent government that aims at maximizing the discounted consumer surplus, net of the market expenditure and of the social cost of transferring resources from taxpayers to producers, over the whole time horizon. This includes not only the surplus generated under the contract, while F runs the activity, but also the surplus generated after the end of the contract, under public management.

Whatever the regime, to finance the transfers G needs to raise distortionary taxes. Each transferred euro requires that $1 + \lambda$ euros be collected from taxpayers, with $\lambda > 0$.[11] The imperfections of the taxation system are taken not to vary over time so that $\lambda$ remains constant.

The discounted benefit of G over the period $(\tau, T)$ is given by

$$V_\tau \equiv \int_\tau^T \left[ S(q) - pq - (1 + \lambda) t \right] e^{-r(x-\tau)} dx.$$

Using (2) and (3), defining

$$w(q) \equiv S(q) + \lambda p(q)q - (1 + \lambda)(\theta q + K + \phi(b)),$$

and denoting

$$D_\tau = \int_\tau^T de^{-r(x-\tau)} d\tau$$

the value of the debt of F at instant $\tau$, we rewrite

$$V_\tau = \int_\tau^T w(q) e^{-r(x-\tau)} dx - (1 + \lambda)(\Pi_\tau + D_\tau). \tag{5}$$

The benefit of G, net of the social cost of the up-front payment $t_0$, is expressed as

$$
\begin{aligned}
U &= V_0 - (1 + \lambda) t_0 \\
&= \int_0^T w(q) e^{-rx} dx - (1 + \lambda)(\Pi_0 + I - M).
\end{aligned}
$$

Assuming that no further investment is necessary to continue the activity after the end of the contract, the optimized return of G under the public regime at time $T$ is equal to $\int_T^\infty w^* e^{-r(y-T)} dy$, where $w^* \equiv w(q^*)$ and $q^*$ is the quantity that maximizes $w(q)$. This is characterized by the

---

[11] According to Snow and Warren [40], the shadow cost of public funds is around 0.3 in developed economies. The World Bank [45] provides a figure of 0.9 with regards to developing countries.

Ramsey-Boiteaux condition

$$\frac{p(q^*) - \gamma}{p(q^*)} = \frac{\lambda}{1 + \lambda} \frac{1}{\varepsilon(q^*)}, \tag{6}$$

with $\varepsilon(q) \equiv -\left(dp(q)/dq\right)q/p(q)$ the absolute value of the price elasticity of market demand.

Overall, given that the project lasts forever, although the contract with F can be terminated at some finite $T$, the payoff of G is given by

$$
\begin{aligned}
W &= U + \int_T^\infty w^* e^{-ry} dy \\
&= \int_0^T w(q) e^{-rx} dx - (1 + \lambda)\left(\Pi_0 + I - M\right) + \int_T^\infty w^* e^{-ry} dy. \tag{7}
\end{aligned}
$$

# 3  Contracts

**The contract between G and F**  G designs for F an incentive scheme that is structured as follows.

First, it specifies the triplet $(M, C, t_0)$ *i.e.*, how much private and public resources should be devoted to fund the investment during the construction phase.

Second, to address the adverse-selection problem (and, as explained below, the moral-hazard problem in construction), the Revelation Principle can be invoked and attention restricted to direct revelation mechanisms under which F does release private information. At this aim, the menu of allocations $\{(q_l, t_l); (q_h, t_h)\}$ is included in the scheme, with $q_i$ the quantity to be produced and $t_i$ the transfer to be made at each instant $\tau \in (0, T)$ in the event that the realized inner cost is $\theta_i$, $i \in \{l, h\}$. That is, G conditions the instantaneous allocation on the state of nature, to be publicly revealed by the report that F makes at the outset of the operation phase, according to which a quantity-transfer pair is picked within the menu offered by G. From now on, the subscript $i$ will be appended to all functions and variables that are contingent on the realized state.

Lastly, the contract stipulates for how long F should run the project *i.e.*, the overall duration $T$.[12]

One might wonder why the allocation is made contingent on the realized value of the inner marginal cost $(\theta_i)$, rather than on that of the actual marginal cost $(\gamma_i)$, despite that G faces concerns not only about possible (mis)representation of information, but also about moral hazard in both construction and operation. Let us clarify why this is not an issue, in fact. On the one hand, as effort $a$ has a stochastic impact on $\theta_i$, conditioning the allocation on the state does suffice to trigger effort provision at the construction stage. Effort $a$ being desirable as long

---

[12]See Section 8 for the case in which duration is made contingent on the realized state of nature.

as

$$\int_0^\infty \left( \mathbb{E}_i \left[ w_i^* \right] - \widetilde{\mathbb{E}}_i \left[ w_i^* \right] \right) e^{-rx} dx > \psi, \tag{8}$$

where $\mathbb{E}_i$ is the expectation operator over the two states corresponding to $a = 1$, $\widetilde{\mathbb{E}}_i$ that corresponding to $a = 0$, and $w_i^* \equiv w_i(q_i^*)$, we assume that (8) is indeed satisfied.[13] On the other hand, as effort $b$ has a deterministic effect on $\gamma_i$ through the extra-cost $c$ (if any), conditioning the allocation on the state enables G to make F residual claimant over the benefit from that effort. Then, F does have incentives to choose $b = 1$ in each period, and faces the cost $\theta_i$, rather than $\theta_i + c$, all along the operation phase. Effort $b$ being desirable as long as, for any given $q_i$, $i \in \{l, h\}$,

$$cq_i > \phi, \tag{9}$$

we take this to hold at the optimal output in state $h$ $(q_h^*)$, which means that it holds *a fortiori* at the optimal output in state $l$ $(q_l^*)$.

**The credit contract**  First of all, the credit contract includes the amount of money $C$ that F should receive from the lender. Additionally, it fixes the instantaneous repayment $d_i$ that F should make to the lender during the operation phase. Observe that, as the quantity-transfer pair in the contract between G and F, the repayment is conditioned on the realized state of nature, which becomes common knowledge as soon as it is revealed to G, whereas the loan is not because money is transferred to F before the inner marginal cost is realized.[14]

Assuming, for simplicity, that there is a large number of lenders in the market, each facing zero outside opportunity, $d_i$ is set to yield neither a surplus nor a loss *i.e.*, $\mathbb{E}_i[D_{i,0}] = C$, with $D_{i,0} = \int_0^T d_i e^{-rx} dx$, or, equivalently,

$$\mathbb{E}_i[d_i] = \frac{rC}{1 - e^{-rT}}. \tag{10}$$

# 4  Full commitment

Suppose that both G and F commit to the contract that they stipulate, and that F also commits to abide by its obligations to the lender.

Referring to the pair of discounted cumulated profits $\{\Pi_{l,0}, \Pi_{h,0}\}$, rather than to the pair of instantaneous transfers $\{t_l, t_h\}$, with a standard change of variables, G solves the following

---

[13]To be more precise, (8) is a sufficient condition for effort in construction to be desirable as long as the allocation that is optimal under full commitment is implemented, which is the situation that we look at in the whole study.

[14]Kartasheva [31] takes the delegated firm to observe the project profitability (its type) privately at the first stage of the relationship. In that context, unlike in our setting, the debt contract is used to screen projects.

programme:

$$\underset{\left\{(M,C);\left(q_l,q_h,\Pi_{l,0},\Pi_{h,0}\right);T\right\}}{Max} \mathbb{E}_i\left[W_i\right]$$

subject to

$$\Pi_{l,0} \geq \Pi_{h,0} + \int_0^T \Delta\theta q_h e^{-rx} dx \tag{11a}$$

$$\Pi_{h,0} \geq \Pi_{l,0} - \int_0^T \Delta\theta q_l e^{-rx} dx \tag{11b}$$

$$\Pi_{l,0} - \Pi_{h,0} \geq \frac{\psi}{\Delta\nu} \tag{11c}$$

$$\mathbb{E}_i\left[\Pi_{i,0}\right] \geq M + \psi \tag{11d}$$

and $(10)$.

In the programme, (11a) and (11b) are the incentive-compatibility constraints whereby the $\theta_i-$firm not be tempted to choose the quantity-profit pair designed for the $\theta_j-$firm, $i \neq j \in \{l,h\}$, (11c) is the moral-hazard constraint whereby F not be tempted to shirk at the construction stage, (11d) is the participation constraint, implicitly taking the best outside opportunity of F to be zero, and (10) is the condition on external funding previously introduced.

At optimum, production is fixed at the level $q_i^*$ for all $i \in \{l,h\}$. Moreover, the expected profit $\mathbb{E}_i\left[\Pi_{i,0}^*\right]$ is such that (11d) is binding, namely $\mathbb{E}_i\left[\Pi_{i,0}^*\right] = M + \psi$, meaning that no information rent is given up to F. Neglecting (11c) for a moment, the pair of optimal profits satisfying (11a) and (11b) is any pair

$$\Pi_{l,0}^*(z) \equiv M + \psi + (1-\nu)\int_0^T \Delta\theta z e^{-rx} dx \tag{12a}$$

$$\Pi_{h,0}^*(z) \equiv M + \psi - \nu\int_0^T \Delta\theta z e^{-rx} dx \tag{12b}$$

that is determined by picking the "sharing rule" $z$ within $Z \equiv [q_h^*, q_l^*]$ for any given duration $T$ (see Appendix A for details). For (12a) and (12b) to satisfy (11c) as well, it is necessary that

$$\frac{\Delta\theta z}{r} > \frac{\psi}{\Delta\nu} \tag{13}$$

$$T \geq \underline{T}(z) \equiv \frac{1}{r} \ln \frac{\Delta\nu\Delta\theta z}{\Delta\nu\Delta\theta z - r\psi}. \tag{14}$$

---

[14]Recall from (10) that also the lender is assumed to have zero outside opportunity. Overall, it means that at same risk for debt and equity financing in the project (the latter reflected by injection of own funds $M$), their rate of return is the same.

This means that the moral-hazard problem that arises in construction imposes two restrictions. First, the sharing rule ought to be such that the discounted return that F would get in state $h$ if the contract were to last forever $(\Delta\theta z/r)$ is larger than that it would get if the contract were to last the minimum number of periods $\underline{T}(z)$ corresponding to that sharing rule $(\psi/\Delta\nu)$. Of course, if (13) is satisfied for $z = q_h^*$, then it is for all $z \in Z$, and G has full flexibility at choosing the rule. More generally, as long as some sharing rule can be found, among those that are incentive-compatible, under which (13) holds, there exists a range of contract durations $[\underline{T}(z),\infty)$ for which the return accruing to F in state $h$ is sufficiently low (hence, the risk transferred to the firm sufficiently important) to induce effort provision at the construction stage. Condition (14) precisely requires that $T$ be drawn from that range so that the contract lasts long enough to ensure that the firm is repaid for that effort. Observe that (14) is most relaxed when $z = q_l^*$ as $\underline{T}(z)$ is smaller the larger $z$. This is easily explained. By raising $z$, G introduces more risk in the rent distribution and, thus, reinforces the incentives to effort provision.

Consider now the payoff that G obtains from the project. At optimum, with $\mathbb{E}_i\left[\Pi_{i,0}^*\right] = \psi + M$, this is written as

$$\mathbb{E}_i\left[W_i^*\right] = \int_0^\infty \mathbb{E}_i\left[w_i^*\right]e^{-rx}dx - (1+\lambda)(I+\psi).\tag{15}$$

This evidences that, in expectation, G reaps the same net benefit that it would obtain if F were not to observe the realization of $\theta$ privately when it starts operating. Importantly, this result is attained no matter the way in which $M$ and $C$ are mixed to fund the project.

**Proposition 1** *(Benchmark) Under full commitment, the payoff $\mathbb{E}_i\left[W_i^*\right]$ is achievable if and only if $\exists z \in Z$ such that (13) holds. This result is attained by setting $T$ according to (14). The mix of financing sources used in the project is irrelevant.*

For later use, we let

$$\Pi_{l,\tau}^*(z) = \left(\frac{(M+\psi)r}{1-e^{-rT}} + (1-\nu)\Delta\theta z\right)\frac{1-e^{-r(T-\tau)}}{r}\tag{16a}$$

$$\Pi_{h,\tau}^*(z) = \left(\frac{(M+\psi)r}{1-e^{-rT}} - \nu\Delta\theta z\right)\frac{1-e^{-r(T-\tau)}}{r}\tag{16b}$$

denote the discounted streams of optimized profits of F from $\tau$ to $T$, respectively in the good and in the bad state, given the sharing rule $z$. Correspondingly, evaluated at the optimal quantity $q_i^*$ and profit $\Pi_{i,\tau}^*(z)$, the expression in (5), which represents the discounted payoff of G from

$\tau$ to $T$ in state $i = l, h$, becomes

$$V_{i,\tau}^*(z) = w_i^* \frac{1 - e^{-r(T-\tau)}}{r} - (1 + \lambda) (\Pi_{i,\tau}^*(z) + D_{i,\tau}). \tag{17}$$

In what follows, we consider frameworks where commitment is limited and new constraints appear in the programme of G. We investigate whether there exist values of $M$, $C$ and $T$ satisfying those constraints such that the contract that stipulates the full-commitment allocation can still be effected.

# 5  Limited commitment

Under limited commitment, the contract between G and F may end earlier than originally agreed upon. Two scenarios are possible. First, F induces G to come back to the contracting table (the case of *limited enforcement* according to the terminology adopted in previous works). Second, G breaks the initial agreement during the contract execution (the case of *non-commitment*). Both situations lead either to the revision of the contract between G and F, if renegotiation is successful, or to the replacement of F with another operator, if renegotiation fails. Moreover, the execution of the contract between F and the lender is affected as well, under both scenarios. Before presenting the renegotiation game that unfolds between G and F, we describe these issues from a practical viewpoint and motivate the way in which they are approached formally thereafter.

## 5.1  Limited enforcement

Once it is informed about the realized value of $\theta$, F may credibly threat G to stop operating and quit the activity unless the contract is revised. F takes this initiative in two cases. First, conditional on parties both knowing the realized state (which we will show to be the case, at equilibrium, provided (22a) and (22b) below are satisfied), F would like to renege on the initial agreement if, under the latter, it obtains a low return *ex post*. For instance, according to (16b), this would occur when $M$ is small and F has a high operating cost, in which case the operating profit under the full-commitment compensation scheme is negative. Second, F may threat to abandon the project as a deliberate strategy to retain more surplus in the relationship with G, when it is aware that replacement with another firm would be costly to G. Examples of limited enforcement leading to firm-led renegotiation are pervasive in public contracting. In institutionally weak contexts (developing countries, in general), strong rules of law seldom exist and renegotiation is likely to take place. For instance, Estache and Wren-Lewis [17] recall that, in Ghana, the incumbent monopoly for fixed telephony entered the mobile business despite the

17

explicit interdiction. In Tanzania, the regulator failed to enforce regional mobile license and the dominant operator began to expand at the national level. Guasch [20] and Guasch *et alii* [21] - [22] provide further examples in Latin America and in the Caribbean regions. Although less often, firms renege on contracts also in frameworks where institutions are solid (typically, developed countries) and contracts should be, in principle, more easily enforced, say, by fining firms that are reluctant to produce. For instance, Gagnepain *et alii* [19] detect a progressive increase in the subsidies paid to French urban transport concessionaires all over the contract execution, suggesting that governments are weak and/or not prone to engage in costly and time-consuming litigations to enforce contracts.

### 5.1.1 Consequences for the credit contract

In a limited-enforcement framework, not only the execution of the contract between G and F is problematic. It is also that of the credit contract, provided F cannot be compelled to return money to the lender. In turn, this involves that F may be unable to borrow on the credit market in the first place.[15] Anticipating this, G can induce financiers' participation by stipulating that, as long as the relationship with F is not broken down (meaning that either the original contract or a renegotiated contract is on-going), it will pay some guaranteed amount directly to the creditor by abating the instantaneous transfer to F of that same amount. By contrast, G is not responsible for the residual debt in the event that F quits the project and is replaced.[16]

Reliance on *conditional* guarantees of this sort (*i.e.*, on guarantees that remain in force as long as the firm-government relationship is in place) is coherent, in particular, with the project finance technique. The latter requires making the project legally and economically self-

---

[15]F would credibly commit to repay the lender if it had reputation concerns. However, a reasonable conjecture is that reputation losses are smaller for private firms than for governments, especially when firms have the possibility of diversifying activities and locations and/or disguising themselves behind subsidiaries with different denominations. A very simple way to formalize this circumstance is to assume that the government bearS a loss, whereas the firm does not. This is how we proceed in our model. More precisely, and consistently with the issues most seriously plaguing public-private contracts in real world, we assume that, when the project is not entirely executed, all involved players incur a cost but of a different nature: a reputation/credibility loss accrues to the government, an expropriation cost to the firm and the lender that are not repaid for their investments.

[16]Guasch *et alii* [21] assume that the assets of the firm can be used to pledge debt collateral. By contrast, we do not consider this possibility, for the following reason. In the private sector, when the debt is not repaid, the creditor undertakes and liquidates or reorganizes the activity. By contrast, in the situations that we represent, the government undertakes the activity, which goes on even when it is no longer run by the initial firm. Hence, in our model, if the relationship between G and F breaks down and F stops repaying the debt, the assets of F that are sunk in the project cannot be liquidated to reimburse the creditor. The latter is paid or not according to the guarantee that the government provides indirectly, through its contract with F. Yet, even if there were assets that could be liquidated without compromising the project execution under the new management, relevant in our model would only be the residual debt *i.e.*, the part of the debt not protected by the collateral. Therefore, allowing for the firm to use its assets to pledge some debt collateral would bring no qualitative change in our analysis and results.

contained, an outcome that is attained in two ways. First, a stand-alone firm, the so-called *Special Purpose Vehicle* (SPV), is created to undertake no other business than building and operating the concerned project, and endowed with the sole assets pertaining to it, which are kept separated from the assets of the parent firm.[17] Second, lenders are provided no guarantees beyond the right to be paid out of the resources generated within the project (namely, user fees and governmental transfers, if provided for, as represented in our model), which means that any repayment guarantee is foregone in the event that the firm abandons the activity.[18]

In practice, however, it is often the case that, while firms remain responsible for their debts as long as they earn profits from the project, governments bail out the activity as difficulties arise, and debt responsibilities are passed onto taxpayers. This happened, for instance, with the 2002-03 London Underground maintaining-and-upgrading project. The public sector was uncertain over whether Metronet, the consortium in charge of the project, could borrow enough funds on the credit market. To boost the banks' appetite, during the bidding stage, Transport for London guaranteed 95% of Metronet's debt obligations. Eventually, Metronet failed and the Department for Transport had to make a £1.7 billion payment to help Transport for London meet the guarantee (House of Lords [25] - [26]). According to the National Audit Office [36], taxpayers incurred a direct loss of between £170 million and £410 million. Therefore, two events, both negative for taxpayers, took place: the contract between public administration and private investors broke down (involving a cost of replacement) and the risk of the debt was transferred to taxpayers. This epilogue points to the conclusion that it is not desirable that public guarantees for debt repayment be provided in the event that the relationship is interrupted. To stimulate investors' participation, governments should rather offer a contract under which the relationship is preserved at equilibrium. Resting on this, in our model, we take G to guarantee the debt only as long as F does remain in the project, whether the original contract is maintained or revised. Formally, we let $g_{i,\tau}^{rn}$ denote the instantaneous amount guaranteed to the creditor, from date $\tau$ on, in the event that the contract is renegotiated in state $i = l, h$ at date $\tau$. By contrast, if the contract is reneged and the relationship between G and F ends at $\tau$, starting from that moment, no amount is guaranteed to the creditor.

## 5.2 Non-commitment

As previously mentioned, non-commitment means that G can break the initial agreement during its execution, despite that this may be detrimental to F. Specifically, once the investment cost has been sunk and G has received a report from F, it may wish to modify the allocation

---

[17]Actually, in project finance initiatives, it is sometimes the case that a plurality of firms forming a consortium (rather than a single firm) concur to create the SPV.

[18]See Engel *et alii* [12], who further refer to Yescombe (2002) and (2007), for details about financial arrangements in public-private partnerships.

designed *ex ante*, in case it proves inefficient *ex post*. For instance, according to (16a), this would happen when F (correctly) declares $\theta_l$, in which case the firm should be rewarded. In developing countries, government failure to honor contractual terms is even bigger a concern than limited enforcement, because large-scale investments, which are there desperately needed, especially in utilities, may not take place if governments cannot warrant investors' remuneration. That this may occur is suggested by the result, which Banarjee *et alii* [4] draw from a cross-country analysis (see also Estache and Wren-Lewis [17]), that governments' opportunistic behaviour does not propitiate private investment. It would also be in line with the observation that political risk has challenged public contracting in Central and Eastern Europe in various occasions over the last decades.[19]

### 5.2.1   Consequences for the credit contract

As under limited enforcement, execution of the credit contract is problematic also in a framework where the government may not comply with the contractual obligation to make transfers. The guaranteed debt may remain unpaid. To avoid this, G can use "external" means to commit itself to this payment. For instance, one can think of G as depositing resources with a third party, which should then be released to the creditor, in the event that it would not receive money directly from G. In practice, strategies of the kind just described are adopted when governments mandate an Investment Insurance Agency (IIA) to act as an intermediary, providing insurance and/or direct cover in the event of any default in payment by a borrower (or its guarantor) under some loan agreement. Originally created as government entities to promote, facilitate and support the exports of goods and services, starting from the Nineties, IIAs have began to operate in project financing as well, and are now widely spread across countries.[20] Moreover, in developing countries, the World Bank and other multilateral development banks (such as the Inter-American Development Bank) provide guarantees that are less subject to project and country limits, as compared to insurance, and are intended to cover debt up to 100% of principal and interest. According to Irwin *et alii* [30], if appropriately managed, these guarantees are essential at reinforcing governments' resolve to abide by their commitments.

---

[19]Brench *et alii* [6] evidence that repeated changes in political attitude towards partnerships with private firms have slowed down the development of transportation projects in Hungary.

[20]Most European governments have set up IIAs for the purposes described in the text. All countries that have official IIAs, alternatively labelled Export Credit Agencies, are now party to the "Arrangement on Guidelines for Officially Supported Export Credits," which provides specific rules for project finance, derogating from the usual Consensus Rules to allow, among other things, for a longer repayment term (of up to 14 years). Examples of European IIAs are Compagnie Française d'Assurance pour le Commerce Extérieur (Coface), Euler Hermes Kreditversicherungs (Hermes), Istituto per i Servizi Assicurativi del Credito all'Esportazione (SACE), Office National du Ducroire (ONDD), to mention only a few. See Sader [37] on the core role that both bilateral and multilateral Export Credit Agencies play in developing countries at providing political risk insurance by pledging guarantees on major parts of the debt package in the realization of BOT-type infrastructure projects (debt covering the three-quarters of the costs of a typical such project).

# 6 The renegotiation game

We now come back to the formal analysis and consider the possibility that, at some instant $\tau \in (0, T)$, either F or G wishes to renege on the initial agreement. The former might threat to abandon the project, the latter to stop making payments during the operation phase, unless the agreement is modified. In either scenario, execution of the initial contract is suspended and parties come back to the contracting table. If renegotiation fails, then F is relieved of the activity and a new firm, denoted F', steps in.

Break-down of the relationship and resort to a new firm yields a "replacement cost" to G. Under limited enforcement, this cost reflects the reputation loss that a government bears for not being sufficiently authoritative to have a contract executed by the partner with which it was signed, despite that the partner invested (own and borrowed) funds in the concerned project (see Guasch *et alii* [21]). Under non-commitment, the cost mirrors the loss of credibility that a government incurs by not keeping its promises *vis-à-vis*, *in primis*, the private financiers of the project (here, F and the lender) and, additionally, other potential investors and customers.[21] Naturally enough, the magnitude of the replacement cost depends upon how much time is left till the end of the contract when the latter is reneged. We thus denote it $R_\delta$, with $\delta = T - \tau$. Specifically, the cost is larger the earlier the replacement, and diminishes as the time at which replacement occurs comes closer to the termination date. Formally, taking $R_\delta$ to be continuously differentiable on $(0, T)$, $R'_\delta \equiv (dR/d\delta) > 0 \ \forall \delta \in (0, T)$. The cost is nonetheless positive even in the event that the contract is broken down just before the date originally stipulated *i.e.*, $R_\delta > 0 \ \forall \delta \in (0, T)$, with $\lim_{\delta \to 0} R_\delta = \varepsilon > 0$. It only vanishes when $\tau = T$ so that $R_0 = 0$.

We begin by considering the situation in which one party proposes the other to renegotiate, without specifying which party takes the initiative. This occurs in some state $i \in \{l, h\}$, which is commonly known at this stage of the relationship.[22]

## 6.1 Replacement payoffs

When renegotiation fails and F is replaced, it no longer receives any compensation. That is, starting from the instant $\tau$ at which replacement occurs, its instantaneous profit becomes $\pi_i^{rp} = 0$, the superscript $rp$ being appended to denote the *re*placement scenario. Thus, the payoff of F at instant $\tau$ is simply given by

$$\Pi_{i,\tau}^{rp} = \int_\tau^T \pi_i^{rp} e^{-r(x-\tau)} dx = 0.$$

---

[21] See Irwin [29]. See also Martimort [34] on the information value that contractual deviations by the government have *vis-à-vis* third parties, and on the negative consequences for its credibility.

[22] As aforementioned, at a later stage, we will show that, in fact, the realized state of nature is revealed before any party might renege on the initial contract (compare (22a) and (22b) below).

Let us next come to G. As the production technology is related to the inner characteristics of the facility, once the latter is in place, the inner marginal cost of production remains the same whatever the firm.[23] G is thus aware that F' operates at marginal cost $\theta_i$ if it chooses effort $b = 1$, and $\theta_i + c$ otherwise. Accordingly, G offers to F' the quantity-transfer pair $(q_i^{rp}, t_i^{rp})$ that maximizes its own payoff. In particular, G offers the transfer that makes the operating profit of F', when it exerts effort, equal to zero at all instants between $\tau$ and $T$ i.e., $t_i^{rp} = \theta_i q_i^{rp} + K + \phi - p(q_i^{rp}) q_i^{rp}$, together with the output $q_i^{rp} = q_i^*$.[24] As a result, G obtains

$$V_{i,\tau}^{rp} = \int_{\tau}^{T} w_i^* e^{-r(x-\tau)} dx - R_{\delta}. \tag{18}$$

## 6.2  Renegotiation payoffs

When renegotiation succeeds, the relationship between G and F proceeds under the revised contract. We now describe the renegotiation process and present parties' payoffs following to renegotiation.

In principle, parties could renegotiate all variables concerning the operation phase agreed upon in the original contract, namely quantity, transfer (or profit), duration. However, in this context, neither party has something to gain from a change in the termination date (see Appendix B). The reason is that any benefit from renegotiation is reaped through a variation in the transfer. Because of this, we hereafter concentrate on quantity-transfer proposals.

First consider G making a take-it-or-leave-it offer to F at date $\tau$. This occurs with probability $\alpha \in [0, 1]$. According to the offer, at each instant between $\tau$ and $T$, F should produce the quantity $q_i^G$ and receive the transfer $t_i^G$, including the amount $g_{i,\tau}^{rn}$ destined to the lender.[25] The offer of G is such that F is just indifferent between renegotiating and abandoning the project i.e., $\Pi_\tau^G = \Pi_\tau^{rp}$. This requires setting $t_i^G = \theta_i q_i^G + K + \phi - p(q_i^G)q_i^G + g_{i,\tau}^{rn}$. Furthermore, G offers

---

[23]The circumstance that the inner marginal cost results as a characteristics of the facility, rather than being specific to the technology of the firm in charge of the project, makes it unnecessary, in our framework, to run an auction in the first place in order to select the firm that would provide the good most efficiently. If the marginal cost were to reflect the very productivity of the executing firm, in which case the most efficient firm would be selected by tendering out the project, then replacing it during the operation phase would mean to switch to a higher marginal cost. In that case, the replacement cost could be viewed as embodying the penalty that G would bear, in terms of higher operating cost and lower optimal output, over the residual period.

[24]One can easily deduce that neither F' nor G would have an interest in reneging on the contract that they sign when F is replaced. On the one hand, under that contract, the instantaneous profit accruing to F' is just equal to its best outside opportunity (which is zero). Moreover, as F' does not invest in the project, G would bear no reputation loss if it were to replace F', hence F' cannot extract any benefit from G in view of a renegotiation. On the other hand, G already retains the whole surplus from F' and could not do better.

[25]Rather than assuming that the transfer $t_i^G$ includes the guaranteed amount $g_i^{rn}$, meaning that F receives $t_i^G$ and then transfers $g_i^{rn}$ to the lender, one could alternatively think of G as making a payment $t_i^G - g_i^{rn}$ to F and a payment $g_i^{rn}$ directly to the lender. The two alternatives are formally equivalent.

the quantity $q_i^G = q_i^*$ that maximizes its own payoff at $\tau$, which is then written as

$$V_{i,\tau}^G = \int_\tau^T w_i^* e^{-r(x-\tau)} dx - (1+\lambda) D_{i,\tau}^{rn},$$

where

$$D_{i,\tau}^{rn} = \int_\tau^T g_{i,\tau}^{rn} e^{-r(x-\tau)} dx$$

is the value of the guaranteed debt at instant $\tau$ for a contract that is renegotiated precisely at $\tau$.

With probability $(1-\alpha)$, F makes a take-it-or-leave-it-offer to G. The offer consists in producing the quantity $q_i^F$ and receiving the transfer $t_i^F$, which again includes the amount $g_{i,\tau}^{rn}$ destined to the lender. The transfer $t_i^F$ is set to ensure that G gets the payoff $V_\tau^G = V_\tau^{rp}$ i.e.,

$$t_i^F = \frac{1}{1+\lambda} \left( S(q_i^F) - p\left(q_i^F\right) q_i^F - w_i^* + \frac{rR_\delta}{1 - e^{-r(T^F-\tau)}} \right).$$

Observe that this payment is deflated by one plus the shadow cost of public funds. That is, all else equal, the larger the cost of collecting resources from taxpayers and/or distorting production away from the efficient level, the smaller the surplus that F can extract from G at the renegotiation stage. F further chooses the output $q_i^F = q_i^*$ that maximizes its own payoff at $\tau$, which is then written as

$$\Pi_{i,\tau}^F = \frac{R_\delta}{1+\lambda} - D_{i,\tau}^{rn}.$$

Overall, in state $i \in \{l, h\}$, the payoffs that F and G obtain from renegotiation at $\tau$ are respectively given by

$$\Pi_{i,\tau}^{rn} = (1-\alpha) \left( \frac{R_\delta}{1+\lambda} - D_{i,\tau}^{rn} \right) \tag{19a}$$

$$V_{i,\tau}^{rn} = \int_\tau^T w_i^* e^{-r(x-\tau)} dx - (1-\alpha) R_\delta - \alpha (1+\lambda) D_{i,\tau}^{rn} \tag{19b}$$

Noticeably, while a larger replacement cost $R_\delta$ benefits F, it penalizes G in the renegotiation process. By contrast, the larger the guaranteed debt $D_{i,\tau}^{rn}$, the lower the payoff for either party, the weaker the incentives to renegotiate the contract.

# 7 Implementation of the full-commitment allocation under limited commitment

We now suppose that, at time 0, G and F sign a contract that precisely stipulates the quantity-and-transfer (or profit) allocation that would be optimal in the full-commitment framework. We denote this contract $\Psi$, for the sake of shortness. We look for the conditions under which $\Psi$ is implemented in the limited-commitment environment.

## 7.1 Additional constraints

We begin by identifying the additional constraints to be met under limited commitment. First of all, given the realized state $i \in \{l, h\}$, and conditional on having renegotiated $\Psi$ at some instant $\tau \in (0, T)$, neither F nor G should be willing to renegotiate again at some date $\tau' \in (\tau, T)$. This is the case if and only if, for all $\tau' \in (\tau, T)$:

$$\Pi_{i,\tau}^{rn} \geq e^{-r(\tau'-\tau)}\Pi_{i,\tau'}^{rn} \tag{20a}$$

$$V_{i,\tau}^{rn} \geq e^{-r(\tau'-\tau)}V_{i,\tau'}^{rn}. \tag{20b}$$

Second, conditional on F truthtelling about the observed state $i \in \{l, h\}$, no party should have any incentive to either renegotiate or stop the relationship, once the execution of $\Psi$ has begun. In formal terms, this requires that, for all $\tau \in (0, T)$:

$$\Pi_{i,\tau}^{*}(z) \geq \max\left\{0; \Pi_{i,\tau}^{rn}\right\} \tag{21a}$$

$$V_{i,\tau}^{*}(z) \geq \max\left\{V_{i,\tau}^{rp}; V_{i,\tau}^{rn}\right\}. \tag{21b}$$

Third, F should not be tempted to lie about the inner marginal cost $\theta_i$, $i \in \{l, h\}$, immediately after building the facility at time 0, even in the event that some party would renege at date $\tau \in (0, T)$. Denote $\Pi_{i,\tau}^{RN}$ the stream of profits that F obtains in state $i$, discounted at time $\tau$, in the event that it misrepresents information at time 0 and the initial contract is reneged at $\tau$. Then, truthtelling in either state requires that, for all $\tau \in (0, T)$:

$$\Pi_{l,0}^{*}(z) \geq \int_{0}^{\tau}\left(\pi_{h,x}^{*} + \Delta\theta q_{h}^{*}\right)e^{-rx}dx + \max\left\{0; \Pi_{l,\tau}^{RN}\right\} \tag{22a}$$

$$\Pi_{h,0}^{*}(z) \geq \int_{0}^{\tau}\left(\pi_{l,x}^{*} - \Delta\theta q_{l}^{*}\right)e^{-rx}dx + \max\left\{0; \Pi_{h,\tau}^{RN}\right\}. \tag{22b}$$

It turns out that (22a) and (22b) are satisfied as long as (21a) holds true jointly with (11a) and (11b), respectively (see Appendix C.1). We can thus neglect (22a) and (22b) and concentrate

on the remaining constraints.

We first investigate the circumstances under which (20a) and (20b) are satisfied. Let

$$\widehat{D}_{i,\tau/\tau'}^{rn} \equiv \int_{\tau'}^{T} g_{i,\tau}^{rn} e^{-r(x-\tau')} dx$$

denote the value, at instant $\tau'$, of the debt guaranteed in state $i$ under the contract renegotiated at $\tau < \tau'$.

**Lemma 1** *Suppose that, in state* $i \in \{l,h\}$, $\Psi$ *is renegotiated at some time* $\tau \in (0,T)$. *Then,* (20a) *and* (20b) *are jointly satisfied if and only if*

$$D_{i,\tau'}^{rn} - \widehat{D}_{i,\tau/\tau'}^{rn} \geq \frac{1}{1+\lambda} \max \left\{ \left( R_{\delta'} - R_\delta \frac{1-e^{-r\delta'}}{1-e^{-r\delta}} \right) ; \left( R_\delta \frac{1-e^{-r\delta'}}{1-e^{-r\delta}} - R_{\delta'} \right) \frac{1-\alpha}{\alpha} \right\} \quad (23)$$

$$\forall \tau' \in (\tau,T),$$

*where* $\delta = T - \tau$ *and* $\delta' = T - \tau'$.

**Proof.** See Appendix C.2. ∎

Provided parties do renegotiate at instant $\tau$, to prevent subsequent renegotiations, it is necessary to guarantee, in the hypothetical event of a later renegotiation, a debt that is at least as large as that guaranteed at the earlier renegotiation. In particular, (23) implies that the later debt guarantee must strictly exceed the earlier one whenever the benefit that F would obtain if a new renegotiation were to take place at instant $\tau' > \tau$, without changing the guarantee, either increases or decreases *i.e.*, $\frac{d}{d\delta}\left(\frac{R_\delta}{1-e^{-r\delta}}\right) \neq 0$. To interpret this finding, consider that, by renegotiating at $\tau$, F obtains a lump-sum return proportional to $R_\delta$. Receiving an amount $R_\delta$ at $\tau$ is equivalent to receiving an instantaneous amount $rR_\delta$ all over the period $(\tau,\infty)$, as one can deduce from the standard relation $R_\delta = \int_\tau^\infty rR_\delta e^{-(x-\tau)}dx$. Besides, receiving an amount $R_\delta$ at $\tau$ is further equivalent to obtaining an instantaneous return of $rR_\delta/\left(1-e^{-r\delta}\right)$ all over the finite period $(\tau,T)$, provided it is also $R_\delta = \int_\tau^T \frac{rR_\delta}{1-e^{-r\delta}} e^{-r(T-x)}dx$. Therefore, renegotiation occurring at $\tau$, F earns an instantaneous return that is proportional to $rR_\delta/\left(1-e^{-r\delta}\right)$ all over the period $(\tau,T)$. If renegotiation is repeated at $\tau' > \tau$, then its instantaneous return from $\tau'$ to $T$ becomes proportional to $rR_{\delta'}/(1-e^{-r\delta'})$. It follows that, when $\frac{d}{d\delta}\left(\frac{R_\delta}{1-e^{-r\delta}}\right) > 0$, F would have something to gain from the contract being renegotiated in each subsequent period, unless the guarantee of the debt were adjusted accordingly. Conversely, when $\frac{d}{d\delta}\left(\frac{R_\delta}{1-e^{-r\delta}}\right) < 0$, it would be G to benefit from the guarantee being kept constant because replacement would become less costly, in discounted terms, over the residual period. In either situation, raising enough the debt guarantee at each later renegotiation discourages parties from reneging. This can be viewed from (19a) and (19b), according to which the return from renegotiation decreases with

25

the debt guarantee for both F and G.

We next come to constraints (21a) and (21b). To investigate the circumstances under which they are satisfied, one should consider that, after some party reneges on $\Psi$, two alternative situations are possible. First, there is no room for renegotiation and G replaces F. Second, there is room for renegotiation and a new agreement is achieved. The following lemma specifies the condition under which the former situation arises, rather than the latter.

**Lemma 2** *Suppose that, in state $i \in \{l, h\}$, some party reneges on $\Psi$ at some $\tau \in (0, T)$ and that (23) holds. Then, replacement takes place, rather than renegotiation, if and only if*

$$D_{i,\tau}^{rn} \geq \frac{R_\delta}{1 + \lambda}. \tag{24}$$

Anticipating that, conditional on some party reneging on $\Psi$, renegotiation will take place if and only if (24) is violated, the incentives to renege on $\Psi$ will ultimately depend upon how much debt is guaranteed in the event of renegotiation.

One can show that preventing replacement imposes milder requirements, in terms of private funds, as compared to preventing renegotiation. This means that $\Psi$ is more easily effected if replacement, rather than renegotiation, is anticipated after any contractual renege. Because of this, in the next lemma, we provide the conditions under which reneging on $\Psi$ is prevented when the guarantee is large enough that replacement is anticipated. These conditions will then enable us to draw conclusions on the implementability of $\Psi$.

**Lemma 3** *Suppose that, in state $i \in \{l, h\}$, at some given $\tau \in (0, T)$, (23) and (24) hold. Then, when $i = l$, (21a) is satisfied; when $i = h$, it is if and only if*

$$M \geq \nu_1 \Delta\theta z \frac{1 - e^{-rT}}{r} - \psi. \tag{25}$$

*Moreover, (21b) is satisfied for $i = l$ and $i = h$ if and only if, respectively,*

$$D_{l,\tau} \leq \frac{R_\delta}{1 + \lambda} - \left(\frac{(M + \psi)\, r}{1 - e^{-rT}} + (1 - \nu_1)\, \Delta\theta z\right) \frac{1 - e^{-r\delta}}{r} \tag{26a}$$

$$D_{h,\tau} \leq \frac{R_\delta}{1 + \lambda} - \left(\frac{(M + \psi)\, r}{1 - e^{-rT}} - \nu_1 \Delta\theta z\right) \frac{1 - e^{-r\delta}}{r}, \tag{26b}$$

*together with*

$$M \leq \left(\frac{r}{1 - e^{-r\delta}} \frac{R_\delta}{1 + \lambda} - (1 - \nu_1)\, \Delta\theta z\right) \frac{1 - e^{-rT}}{r} - \psi. \tag{27}$$

**Proof.** See Appendix C.3. ∎

26

Condition (25) means that F cannot be prevented from reneging on $\Psi$ at some time $\tau$, unless it is required to invest an amount of own funds sufficiently large to ensure that the profit to be obtained at each instant by operating till the termination date $T$, given the sharing rule $z$, is non-negative even in the bad state. In other words, $M$ is to be raised enough *ex ante* to prevent losses *ex post* and thus lessen the incentive of F to renege. While this is necessary to remove the temptation of F to renege on $\Psi$, it is however not sufficient. Provided that the replacement cost that G incurs is sufficiently large, there might still be more to reap by breaking down the agreement and threatening G to abandon the project. To make this strategy unattractive, G is to pledge sufficiently high guarantees for the debt of F, in the event that $\Psi$ is renegotiated (*i.e.*, (24) must hold). This result, which might look counter-intuitive, is in fact easily explained. Guaranteeing (a part of) the debt involves abating the compensation of F by the amount that is to accrue to the creditor. Given the replacement cost of G, the more debt is guaranteed, the less the benefit for F from reneging on $\Psi$. Therefore, when the guarantee is fixed larger than the replacement cost (deflated by the shadow cost of public funds), renegotiation would yield a loss in either state, hence F would prefer to quit the project.

Condition (27) means that G cannot be prevented from reneging on $\Psi$ at some time $\tau$, unless F is required to invest an amount of own funds sufficiently small to ensure that, even in the good state, the profit that G awards to F at each instant till the termination date $T$, as inflated by the shadow cost of public funds, does not exceed the penalty that G would incur if it were to replace F. In other words, $M$ should be kept low enough *ex ante* to prevent comparative benefits *ex post* and, thus, to weaken the incentive of G to renege. However, just as for F, the requirement on $M$ is necessary but not sufficient to remove the temptation of G to renege on $\Psi$. Indeed, provided that the replacement cost is not too important, G can still gain from either renegotiating or replacing F. To make these options unattractive, G should tie its hands (through the third party) on a double ground. First, it should guarantee a sufficiently large amount of debt, in the event that $\Psi$ is renegotiated (*i.e.*, (24) should hold). Second, it should guarantee sufficiently little debt, in the event that $\Psi$ is fully executed (as from (26a) and (26b)). This result is explained as follows. On the one hand, while replacing F yields a loss of reputation, it allows G to save on the debt, which is guaranteed only as long as F remains in the project. Given the loss, the more debt is guaranteed when renegotiating, the larger the benefit from replacing F, the less attractive renegotiation as compared to replacement. On the other hand, the larger the debt guaranteed in $\Psi$, the more costly $\Psi$ is to G (as from (17), $i = l, h$), hence the less convenient with respect to the alternative options. Therefore, when the guarantee in the renegotiation scenario is fixed larger than the replacement cost (deflated by the shadow cost of public funds), renegotiation would be so costly to G that its payoff would be negative whatever the operating cost of the firm. G would definitely prefer to replace F. For $\Psi$

to be more attractive than replacement, the debt burden under $\Psi$ should be not too important, in turn.

## 7.2   Implementation of $\Psi$

In Lemma 3 we enlisted the weakest conditions under which break-down of the relationship between G and F is prevented. We can now be based on that list to investigate the requirements that $\Psi$ is to satisfy to be effected in the limited-commitment environment. As a first step, we identify the set of exogenous conditions under which this outcome is achieved.

**Proposition 2** $\Psi$ *is implementable if and only if* $\exists z \in Z$, $T \in [\underline{T}(z), +\infty)$ *such that* (13) *holds together with*

$$R_\delta \geq (1 + \lambda) \Delta \theta z \frac{1 - e^{-r\delta}}{r}, \ \forall \delta \in (0, T), \tag{28}$$

*and, additionally,*

$$E \ \geq \ \nu_1 \frac{\psi}{\Delta \nu} \tag{29}$$
$$C \ > \ 0. \tag{30}$$

**Proof.** See Appendix C.4.  ■

According to (28), $\Psi$ is beyond reach under limited commitment unless the replacement cost that G would bear at time $\tau$ is big enough. More precisely, the cost must be at least as large as the present value of the *ex-post* cumulated return (as inflated by the shadow cost of public funds) that F would obtain in state $h$ if it were to remain in $\Psi$ for the subsequent $(T - \tau)$ periods. This is explained as follows. For any given $T$, when $R_\delta$ is low, G has an interest in reneging in the state in which F would be rewarded (state $l$). In so doing, G would avoid to give large operating profits to F and, at the same time, it would appropriate a facility that can be operated at a low cost. To remove this temptation, F should be required to put fewer own funds on the table up-front, so that the good-state operating profit in $\Psi$ is reduced. However, setting $M$ small is not desirable either because, in that case, F has an incentive to renege in the bad state, in turn, so as not to incur a low operating profit. Unless $R_\delta$ is big enough, the own funds of the firm cannot be used to reconcile these two purposes that they serve, and $\Psi$ is not enforceable. We have illustrated this point taking $T$ as given. In a moment, we will investigate how duration should be chosen for (28) to be satisfied, provided both the replacement cost and the firm's operating profit depend upon this choice.

For $\Psi$ to be enforceable, (29) and (30) must hold as well. For F to be able to invest as much own funds as it is necessary to effect $\Psi$, its pockets must be sufficiently deep to begin with. Specifically, (29) shows that, if the duration is contained to the lowest admissible number

of periods $(T = \underline{T}(z))$, then the endowment of F should be large enough to satisfy (25) for $M = E$. Otherwise, there does not exist any contract duration for which the operating profit in the bad state is neither too high to address the moral-hazard problem nor too low to remove the incentives of F to renege on the initial contract. Lastly, (30) evidences that F needs to have access to the credit market and be able to get a loan within the specific relationship with G, under the conditions stipulated in $\Psi$.

Overall, Proposition 2 conveys a strong message. For the contract that stipulates the full-commitment allocation to be effected, private capital must be available to run the project, both in the form of own funds of the firm that is delegated to perform the activity, and in the form of outside financing. The reason is that involving private funds of a diverse nature provides two different commitment devices, which are both functional to refrain F from reneging. On one side, involving own funds reinforces the firm's willingness to remain in the relationship as at least a part of them would no longer be recovered by abandoning the project in favour of the replacement payoff. On the other side, forcing F to take a loan hardens its budget constraint through the guarantees offered to the creditor, as illustrated previously. Indeed, by reducing the benefit that G would obtain from renegotiation, these guarantees lower the surplus that F can extract from G in the event of renegotiation. As a result, F finds it less attractive to renege on the contract opportunistically anticipating profitable renegotiation.

Let us now discuss the impact that the need to satisfy condition (28) has on the choice of contract duration. Depending upon the properties of the function $R_\delta$, it may impose restrictions on $T$, in addition to (14). The next corollary identifies those restrictions and assesses when they can be met together with (14). After explaining the content of the corollary, an example will be provided to better illustrate results.

**Corollary 1** *Take* (29) *and* (30) *to hold. First suppose that*

$$R'_\delta \geq (1 + \lambda) \Delta\theta z e^{-r\delta}, \ \forall \delta \in (0, T), \ \forall T \in [\underline{T}(z), \infty). \tag{31}$$

*Then, there exist values of $T$ for which $\Psi$ is implemented:*

$$
\begin{aligned}
T &\in \ [\underline{T}(z), \widetilde{T}(z, E)] \quad when \ E \in \left[\nu_1 \frac{\psi}{\Delta\nu}, \nu_1 \frac{\Delta\theta z}{r} - \psi\right) \\
T &\in \ [\underline{T}(z), \infty) \quad when \ E \geq \nu_1 \frac{\Delta\theta z}{r} - \psi,
\end{aligned}
$$

*where*

$$\widetilde{T}(z, E) \equiv \frac{1}{r} \ln \frac{\nu_1 \Delta\theta z}{\nu_1 \Delta\theta z - r(E + \psi)} < \infty. \tag{32}$$

*Next suppose that*

$$R'_\delta < (1 + \lambda) \Delta\theta z e^{-r\delta}, \ \forall \delta \in (0, T), \ \forall T \in [\underline{T}(z), \infty). \tag{33}$$

*Then, there exist values of $T$ for which $\Psi$ is implemented if and only if $R_{\overline{T}(z)} \geq (1 + \lambda)\psi/\Delta\nu$ :*

$$T \in \left[\underline{T}(z), \min\left\{\widetilde{T}(z, E); \overline{T}(z)\right\}\right] \quad when \ E \in \left[\nu_1 \frac{\psi}{\Delta\nu}, \nu_1 \frac{\Delta\theta z}{r} - \psi\right)$$

$$T \in [\underline{T}(z), \overline{T}(z)] \quad when \ E \geq \nu_1 \frac{\Delta\theta z}{r} - \psi,$$

*where*

$$\overline{T}(z) \equiv \frac{1}{r} \ln \frac{\Delta\theta z}{\Delta\theta z - r R_{\overline{T}(z)}/(1 + \lambda)} < \infty. \tag{34}$$

**Proof.** See Appendix C.5. ∎

The corollary identifies two possible scenarios.

The first scenario arises when $R_\delta$ increases fast enough with $\delta$ to satisfy (31) for all $\delta \in (0, T)$, $T \in [\underline{T}(z), \infty)$, given the sharing rule $z$. Then, $R_\delta$ is very large so that it meets (28) for all feasible $T$. Under this circumstance, the sole restriction that can appear in terms of contract duration, apart from the one imposed by the moral-hazard problem, follows from (25). Hence, it depends upon "how rich" the firm is. When the own funds that the private partner could devote to the project do not even cover the expected return that it would get in the bad state if $T = \infty$, the contract cannot last more than $\widetilde{T}(z, E)$ periods, a threshold that is tighter the lower the endowment. To see why this is the case, recall that, the larger $M$, the higher the operating profit that F obtains in $\Psi$ at each instant over the execution period, the weaker the incentive of F to quit the project. When $E$ is low, G can no longer rely on $M$ to discourage F from reneging on the contract. The alternative option is to shorten the relationship. This hardens the budget constraint of F and its instantaneous payoff is to be raised. As a result, F finds it less attractive to renege and the contract remains in place. By contrast, for a sufficiently rich firm, this restriction does not arise and $T$ is to be chosen in compliance with (14) only.

The second scenario arises when, conversely, the rate of increase of $R_\delta$ is not very large so that it satisfies (33) for all $\delta \in (0, T)$, $T \in [\underline{T}(z), \infty)$. In this case, (28) induces further restrictions on the choice of duration. Implementation of $\Psi$ requires that the replacement cost be "sufficiently large," meaning that $R_{\overline{T}(z)}$, with $\overline{T}(z)$ as defined by (34), does not fall below the return (inflated by the shadow cost of public funds) that is assigned in state $h$ when $\Psi$ lasts forever. Once this is ensured, the restrictions on $T$ reflect how the replacement cost compares with the firm's endowment. With a poor firm, the relevant threshold is $\widetilde{T}(z, E)$ as long as $R_{\overline{T}(z)} \geq (1 + \lambda) E$, in which case the maximum admissible duration is still driven by the

endowment effect triggered by (29). The relevant threshold becomes $\overline{T}(z)$ when, conversely, $R_{\overline{T}(z)} < (1 + \lambda) E$. Then, (28) is so hard to satisfy that the maximum admissible duration is decreased even further than the poorness of the firm would require. Of course, as the firm becomes wealthier, $\overline{T}(z)$ remains the relevant threshold, provided the endowment effect is then lessened. By contrast, when replacing F is not enough costly to meet the aforementioned requirement on the replacement cost, it is impossible to pick a contractual duration such that both the incentives of F to shirk and those of G to renege are removed.

Importantly, in the first scenario, restrictions on the choice of $T$, apart from (14), solely reflect the magnitude of the firm's endowment. It follows that the sharing rule that facilitates mostly the implementation of $\Psi$ is the one that is associated with the largest range of feasible durations $i.e.$, $z = q_h^*$. In the second scenario, admissible durations are determined not only by the endowment effect but also by the behaviour of the function $R_\delta$. Because of this, the most convenient sharing rule cannot be univocally identified, unless specific cases are considered. Example 1 below illustrates a situation in which the best sharing rule in the second scenario is $q_l^*$.

**Example 1** *Take $R_\delta = \left(1 - ae^{-r\delta}\right)/r$, where*

$$
\begin{align}
a &> (1 + \lambda) \Delta\theta q_h^* \tag{35}\\
a &< 1 < (1 + \lambda) \Delta\theta q_l^*. \tag{36}
\end{align}
$$

*As $\lim_{\delta \to 0} \left(1 - ae^{-r\delta}\right) = 1 - a > 0$, $R_\delta$ satisfied the condition that $\lim_{\delta \to 0} R_\delta > 0$. Let $\widehat{\delta} \equiv \frac{1}{r} \ln \frac{(1+\lambda)\Delta\theta z - a}{(1+\lambda)\Delta\theta z - 1}$ denote the unique value of $\delta$ for which (28) holds as an equality. Taking first $z = q_h^*$, $\widehat{\delta} < 0$ and (28) is strictly satisfied for all $\delta \in (0, T)$ and $T \in (0, \infty)$. Taking next $z = q_l^*$, $\widehat{\delta} > 0$ and (28) is satisfied if and only if $\delta \in \left(0, \min\{\widehat{\delta}, T\}\right)$, for any given $T \in (0, \infty)$. In this case, the largest feasible interval for $\delta$ is attained when $T = \widehat{\delta}$ i.e., when the upper extreme of the interval is $\overline{T}(q_l^*)$ as defined in (34). From (14), it must be $T \geq \underline{T}(q_l^*)$. Then, $\Psi$ is implementable only if the interval $\left[\underline{T}(q_l^*), \overline{T}(q_l^*)\right]$ exists and $T$ is drawn from this interval. Plugging $R_\delta = \left(1 - ae^{-r\delta}\right)/r$ into (34), we find that the interval $\left[\underline{T}(q_l^*), \overline{T}(q_l^*)\right]$ exists if and only if $a \leq \Delta\theta q_l^* \frac{\Delta\nu - r(1+\lambda)\psi}{\Delta\nu \Delta\theta q_l^* - r\psi}$, which means that $R_{\overline{T}(q_l^*)}$ is sufficiently high to satisfy the condition required in the second scenario of Corollary 1. Overall, (28) is not an issue when $z$ is chosen equal to $q_h^*$. When $z = q_l^*$, instead, (28) is satisfied only if the interval $\left[\underline{T}(q_l^*), \overline{T}(q_l^*)\right]$ exists. Actually, (35) and (36) mean that (31) holds for $z = q_h^*$ and (33) for $z = q_l^*$. Results are thus explained in the light of Corollary 1.*

An important point emerges from the finding in Corollary 1. When the replacement cost is not large enough in all periods till the termination date, contractual parties anticipate that,

at some point, it will become convenient for one of them to interrupt the relationship and $\Psi$ will not survive. Then, the only way to make the agreement viable is to reduce the length of the contract. That is, duration becomes the very instrument through which G can persuade the partner of its intention not to renege and, at the same time, keep the partner in the initial agreement.

Once the conditions that are necessary and sufficient for $\Psi$ to be implementable are identified, we can establish how the mix of private funds must exactly be set in $\Psi$, given the duration agreed upon, for the contract to be enforced in the limited-commitment framework.

**Corollary 2** *Suppose that* $(28)$, $(29)$ *and* $(30)$ *hold. Then,* $\Psi$ *is implemented by choosing M and C such that, for all* $\delta \in (0, T)$,

$$\nu_1 \Delta \theta z \frac{1 - e^{-rT}}{r} - \psi \leq M \leq \frac{R_\delta}{1 + \lambda} \frac{1 - e^{-rT}}{1 - e^{-r\delta}} - (1 - \nu_1) \Delta \theta z \frac{1 - e^{-rT}}{r} - \psi, \qquad (37)$$

*together with*

$$M + C \leq \frac{R_T}{1 + \lambda} - \psi. \qquad (38)$$

**Proof.** See Appendix C.6. ∎

This corollary channels two main lessons. First, the amount of own funds that F must be required to inject up-front should be neither too small nor too large. These restrictions reflect the need to remove the temptation of F and G, respectively, to renege on $\Psi$, for the reasons previously explained. Second, the total amount of private resources (own funds and loan) to be invested in the project cannot be too large, in turn. This requirement is again dictated by the need to prevent G from reneging. To see this, consider that, by replacing F, G would escape reimbursing private financiers, whether in total (if the contract is reneged immediately after operation has begun) or in part (if the contract is reneged at a later stage). Condition (38) evidences that this option is unattractive to G as long as the maximum amount that it could expropriate by breaking down the relationship with F (namely, $M + C + \psi$) does not exceed the largest cost that it would incur by replacing F (namely, $R_T / (1 + \lambda)$), which is attained if replacement is made at the outset of the operation phase. Importantly, this result implies that it might be desirable not to fund the investment entirely with private resources $(M + C < I)$. Then, G ought to provide an up-front payment $t_0 > 0$ to complete the initial investment. That is, the project should be run with a mix of private and public funds. Observe however that, as private resources must be such that $M + C > \left[\nu_1 \Delta \theta z \left(1 - e^{-rT}\right) / r\right] - \psi$, it might even be desirable that F makes an up-front transfer to G $(t_0 < 0)$, which is to be interpreted as a fee for being awarded the contract.

Two final points are worth making. The first concerns the impact of the shadow cost of public funds on results. As $\lambda$ increases, the upper bounds on private resources (in (37) and

32

(38)), the lower bounds on $R_\delta$ (in (28)) and that on the rate of increase of $R_\delta$ (in (33)), become all more severe. Furthermore, the longest duration that is admissible under (33), namely $\overline{T}(z)$, reduces. This means that a raise in $\lambda$ tightens all restrictions on the implementability of $\Psi$ that ensue from the necessity to remove the incentives of G to renege. The reason is rather intuitive. The more costly it is to collect/transfer public funds (and to distort price), the less prone G is to compensate F (and, indirectly, the lender) during the operation phase, the harder it becomes to motivate G to remain in the relationship with F. By contrast, the requirements that reflect the incentives of F are not affected by the magnitude of $\lambda$, provided the payoff of F would depend upon it only in the event of renegotiation.[26]

# 8 State-dependent duration

So far we have focused on a contract that has the same duration whatever the inner operating cost (*i.e.*, on a fixed-term contract). Given the crucial role that, according to our results, the length of the contract plays in the implementation of the optimal allocation, we now investigate whether there can be any improvement in the achievements of the government when duration is allowed to vary with the state of nature (*i.e.*, when a flexible-term contract is designed).

Denote $T_l > 0$ the duration in state $l$ and $T_h > 0$ that in state $h$. Further let $\delta(i) \equiv T_i - \tau$, $\forall i \in \{l, h\}$, $\forall \tau \in (0, T_i)$. Then, we can rewrite the profits at time $\tau$ in the good and bad state as

$$\Pi_{l,\tau} = \left( M + \psi + (1 - \nu_1) \int_0^{T_i} \Delta\theta\alpha_i e^{-rx} dx \right) \frac{1 - e^{-r\delta(l)}}{1 - e^{-rT_l}} \tag{39}$$

$$\Pi_{h,\tau} = \left( M + \psi - \nu_1 \int_0^{T_i} \Delta\theta\alpha_i e^{-rx} dx \right) \frac{1 - e^{-r\delta(h)}}{1 - e^{-rT_h}}, \tag{40}$$

where $\alpha_i > 0$ indicates the sharing rule in this framework, such that (11a) and (11b) are satisfied and (11d) is binding. This formulation of the profits helps us emphasize that, with state-dependent duration, the range of feasible values of the sharing rule is possibly enlarged, as compared to the fixed-term setting. Indeed, the feasible range of $\alpha_l$ is $A_l \equiv \left[ q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}}, q_l^* \right]$ and that of $\alpha_h$ is $A_h \equiv \left[ q_h^*, q_l^* \frac{1-e^{-rT_l}}{1-e^{-rT_h}} \right]$. Hence, $A_i \supseteq Z$ for all $i$, as long as $T_h < T_l$.[27] Fixing $T_h$ below $T_l$ has an important implication: the wedge between good- and bad-state profit is either increased or decreased with respect to the compensation scheme in $\Psi$. This is beneficial to G in that G can either raise or reduce the wedge at its best convenience, depending upon the difficulties that it faces at implementing the optimal allocation.

---

[26]This is immediately checked by looking at (29), (30), (32), and at the lower bound in (37).

[27]With $T_h < T_l$, $q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}} < q_h^*$ and $q_l^* \frac{1-e^{-rT_l}}{1-e^{-rT_h}} > q_l^*$. See Appendix 8 for details about the construction of the sharing rule $\alpha_i$.

Prior to stating the new results, it is however useful to devote a few words to the contract with state-dependent duration in the full-commitment framework. Not surprisingly, the highest payoff, which is still $E_i[W_i^*]$, is achievable if and only if, for any given $\alpha_i \in A_i$, duration is chosen such that $T_i \geq \underline{T}(\alpha_i)$. Again, the mix of financing sources used in the project does not matter. Further remark that, once the sharing rule $\alpha_i$ is adopted, the profit distribution is independent of the contract duration associated with the state of nature $j \neq i$ that the sharing rule does not refer to. From now on, we denote $\Psi_i$, $i = l, h$, the contract with state-dependent duration that stipulates the optimal (full-commitment) allocation with sharing rule $\alpha_i$.

We begin by identifying the conditions under which $\Psi_l$ is implementable, whereas $\Psi$ is not, in the limited-commitment framework.

**Proposition 3** *Suppose that* (28) *does not hold for* $z = q_h^*$ *and* $T \geq \underline{T}(q_h^*)$. *Then,* $\exists \alpha_l \in \left[ q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}}, q_h^* \right)$ *for which* $\Psi_l$ *is implementable if and only if* $T_l > T_h$, *together with*

$$\frac{\Delta\theta}{r}\alpha_l > \frac{\psi}{\Delta\nu} \tag{41}$$

$$R_{\delta(l)} \geq (1+\lambda)\Delta\theta\alpha_l \frac{1-e^{-r\delta(l)}}{r}, \ \forall\delta(l) \in (0, T_l), \tag{42}$$

*and with* (29) *and* (30).

**Proof.** See Appendix D. ∎

The sharing rule $\alpha_l$ is useful to G when the replacement cost is not sufficiently large. As we know, in that case, under $\Psi$, it is not possible to remove at once the temptation of G to renege in state $l$ to expropriate the private investment and the temptation of F to renege in state $h$ to escape low returns. This might be feasible under $\Psi_l$, instead, as condition (42) is less stringent than (28). The compensation scheme associated with $\alpha_l$ is such that, at each instant of the operation phase, the profit distribution is less spread than in $\Psi$. This facilitates the composition of the two commitment problems so that the requirement on the replacement cost is relaxed. Example 2 below illustrates this finding.

**Example 2** *Take again the case of Example* 1, *except that now*

$$a < (1+\lambda)\Delta\theta q_h^* \tag{43}$$

*so that, when* $z = q_h^*$, *we are in the second scenario of Corollary* 1. *Then, with fixed duration, implementation of* $\Psi$ *requires existence of the interval* $\left[\underline{T}(q_h^*), \overline{T}(q_h^*)\right]$, *which is tantamount to having* $a \leq \Delta\theta q_h^* \frac{\Delta\nu - r(1+\lambda)\psi}{\Delta\nu\Delta\theta q_h^* - r\psi}$. *With state-dependent duration, if* $T_l$ *and* $T_h$ *are chosen such that* $T_l \geq \underline{T}(\alpha_l)$ *and* $q_h^* \frac{1-e^{-rT_h}}{1-e^{-rT_l}} \leq \alpha_l$ *for* $\alpha_l = a/(1+\lambda)\Delta\theta$, *then* $\Psi_l$ *is implemented without requiring that the previous condition on* $a$ *be satisfied.*

While the sharing rule $\alpha_l$ helps G circumvent the difficulties that arise in terms of commitment when the replacement cost is not sufficiently large, the sharing rule $\alpha_h$ helps G lessen the moral-hazard problem that arises in construction. More precisely, when inducing F to exert effort $a = 1$ is especially hard ($\psi$ is large), G is to exacerbate (rather than contain) the spread in the profit distribution so as to transfer more risk to the firm. The compensation scheme associated with $\alpha_h$ enables G to reach this outcome.

**Proposition 4** *Suppose that* (14) *does not hold for $z = q_l^*$ and $T$ chosen so as to satisfy* (28). *Then,* $\exists \alpha_h \in \left( q_h^*, \frac{1-e^{-rT_l}}{1-e^{-rT_h}} q_l^* \right]$ *for which $\Psi_h$ is implementable if and only if $T_l > T_h$, together with*

$$\frac{\Delta\theta}{r}\alpha_h \;>\; \frac{\psi}{\Delta\nu} \tag{44}$$

$$R_{\delta(l)} \;\geq\; (1+\lambda)\,\Delta\theta\alpha_h\frac{1-e^{-rT_h}}{r}\frac{1-e^{-r\delta(l)}}{1-e^{-rT_l}}, \;\; \forall\delta\,(l) \in (0,T_l)\,, \tag{45}$$

*and with* (29) *and* (30)*.*

**Proof.** See Appendix D. ■

The requirement expressed by (45), which is the counterpart of (28) with sharing rule $z$, and of (42) with sharing rule $\alpha_l$, follows from the way in which the sharing rule $\alpha_i$, $i \in \{l, h\}$, is constructed. Recall that, in the fixed-term framework, (28) is necessary to make sure that neither G reneges in the good state nor F reneges in the bad state and that, whether this is the case or not, it also depends upon how $T$ is set. Analogous role (42) plays in the state-dependent framework with sharing rule $\alpha_l$. When the contract length is differentiated across states $(T_l \neq T_h)$ and the sharing rule $\alpha_h$ (rather than $\alpha_l$) is adopted, the low return that is inflicted to F in the bad state is contingent on $T_h$ (rather than $T_l$), whereas the gain that G would obtain by reneging in the good state is contingent on the contract duration in the renege state $T_l$. Condition (45), which contains both $T_l$ and $T_h$ (rather than $T_l$ only), reflects precisely this circumstance.

Proposition 3 and 4 clarify the conditions under which $\Psi_i$, $i \in \{l, h\}$, is enforceable in a limited-commitment environment. We are now left with specifying how $M$ and $C$ should be set to actually effect $\Psi_i$. This final result is the counterpart of Corollary 2 in a context of state-dependent duration.

**Corollary 3** *Suppose that $\exists \alpha_i \in A_i$, $i \in \{l, h\}$, for which $\Psi_i$ is implementable. Then, imple-*

*mentation is made by setting $M$ and $C$ are such that, $\forall \delta\left(i\right) \in \left(0, T_i\right),$*

$$\nu_1 \Delta \theta \alpha_i \frac{1 - e^{-rT_i}}{r} - \psi \;\; \le \;\; M \le \min \left\{ \frac{R_{\delta(h)}}{1 + \lambda} \frac{1 - e^{-rT_h}}{1 - e^{-r(T_h - \tau)}} + \nu_1 \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx; \right. \quad (46)$$

$$\left. \frac{R_{\delta(l)}}{1 + \lambda} \frac{1 - e^{-rT_l}}{1 - e^{-r(T_l - \tau)}} - \left(1 - \nu_1\right) \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx \right\} - \psi$$

*together with*

$$M + C \le \frac{E_i\left[R_{T_i}\right]}{1 + \lambda} - \psi. \quad (47)$$

# 9  Concluding remarks

Resting on a relatively simple model, we drew a few important predictions about public-private contracting in limited-commitment environments. While it is rather intuitive that, under limited commitment, the financing structure of the project can be used as a commitment device to implement the contract that stipulates the optimal (full-commitment) allocation, we assessed how exactly private funds, drawn from different sources, should be mixed to achieve this objective, given the exogenous conditions that come to matter in the (potential) renegotiation process. More than that, the very contribution of our study was to show that an appropriate financing structure of the project cannot be designed sparing reference to the duration of the contract, provided early break-down of the relationship with the private firm is not as costly for the public partner as late break-down is.

One important prediction of our model is that, for the optimal allocation to be implemented at equilibrium under limited commitment, it is essential that the debt guarantees be set in an appropriate manner, despite that their value is only relevant for the out-of-equilibrium payoffs so that one would not expect these guarantees to be actually used in practice. In particular, we established that the funds to be borrowed by the firm on the credit market should be guaranteed conditionally on the government-firm relationship not being interrupted in advance, whether the contract is maintained as originally signed till the termination date or renegotiated during the operation phase. Concerning this prediction, two observations are in order. First, in several practical instances, loans are guaranteed even in that event that renegotiation fails and the relationship is broken down (as an illustration, recall the Metronet case). This behaviour is based on the argument that it would be impossible to attract outside financiers otherwise. Nonetheless, the result that, at equilibrium, the relationship is preserved and the loan reimbursed, casts doubts on the validity of this argument, and rather suggests that providing conditional guarantees does not compromise the ability to raise external funds. Second, under limited commitment, it is not desirable to have lenders actively involved in

renegotiation. In our model, if the lender could say a word when parties come back to the contracting table, it would accept a partial repayment. Obviously, this would be preferable to being denied any reimbursement in the event of early interruption of the relationship between the government and the firm. But then out-of-equilibrium guarantees could no longer be used as a commitment device to keep that relationship in place. Hence, for the committing role of guarantees to be preserved, lenders should remain "passive." Importantly, this is beneficial to lenders themselves insofar as credits are entirely recovered.

# References

[1] ACE (2010), *A UK Infrastructure Bank*, ACE Publications, January 2010

[2] Auriol, E., and P.M. Picard (2011), "A Theory of BOT Concession Contracts," *Journal of Economic Behavior and Organization*, forthcoming

[3] Bajari, P., S. Houghton and S. Tadelis (2006), "Bidding for Incomplete Contracts: An Empirical Analysis," *NBER Working Paper No. 12051*

[4] Banerjee, S.G., J.M. Oetzel and R. Ranganathan (2006), "Private Provision of Infrastructure in Emerging Markets: Do Institutions Matter?," *Development Policy Review*, 24(2), 175–202

[5] Bennett, J., and E. Iossa (2006), "Building and Managing Facilities for Public Services," *Journal of Public Economics*, 90, 2143-2160

[6] Brench, A., T. Beckers, M. Heinrich and C. von Hirschhausen (2005), "Public-Private Partnerships in New EU Member Countries of Central and Eastern Europe," *European Investment Bank*, 10(2)

[7] Chong, E., F. Huet, S. Saussier and F. Steiner (2006), "Public-Private Partnerships and Prices: Evidence from Water Distribution in France," *Review of Industrial Organization*, 29(1), 149-169

[8] Danau, D. (2009), "A note on fixed and flexible-term contracts," *Economics Bulletin*, 29(2), 964-975

[9] Dewatripont, M., and E. Maskin (1995), "Credit and efficiency in centralized and decentralized economies," *Review of Economic Studies*, 62(4), 541–555

[10] de Bettignies, J.E., and T.W. Ross (2009), "Public–private partnerships and the privatization of financing: An incomplete contracts approach," *International Journal of Industrial Organization*, 27(3), 358-368

[11] Engel, E., R. Fischer and A. Galetovic (2011), "The Basic Public Finance of Public-Private Partnerships," *Journal of the European Economic Association*, forthcoming

[12] Engel, E., R. Fischer and A. Galetovic (2010), "The economics of infrastructure finance: Public-Private Partnerships *versus* public provision," *EIB Papers* 15(1)

[13] Engel, E., R. Fischer and A. Galetovic (2001), "Least-Present-Value-of-Revenue Auctions and Highway Franchising," *Journal of Political Economy*, 109(5), 993-1020

[14] Engel, E., R. Fischer and A. Galetovic (1997), "Highway Franchising: Pitfalls and Opportunities," *American Economic Review*, vol. 87(2), 68-72

[15] Engel, E., R. Fischer, A. Galetovic and M. Hermosilla (2008), "Renegociación de concesiones en Chile," *Estudios Públicos*, 113, 151-205

[16] Estache, A. (2006), "PPI Partnerships vs. PPI Divorces in LDCs," *Review of Industrial Organization*, 29(1), 3-26

[17] Estache, A., and L. Wren-Lewis (2009), "Towards a Theory of Regulation for Developing Countries: Following Laffont's Lead," *Journal of Economic Literature*, 47(3), 729–770

[18] Flyvbjerg, B., M. Garbuio and D. Lovallo (2009), "Delusion and Deception in Large Infrastructure Projects: Two Models for Explaining and Preventing Executive Disaster," *California Management Review*, 51(2), 170-193

[19] Gagnepain, P., M. Ivaldi and D. Martimort (2009), "Renégociation de contrats dans l'industrie du transport urbain en France," *Revue Economique*, 60(4), 927-947

[20] Guasch, J. L. (2004), *Granting and Renegotiating Infrastructure Concessions: Doing it Right*, The World Bank Institute

[21] Guasch, J. L., J.J. Laffont and S. Straub (2006), "Renegotiation of concession contracts: A theoretical approach," *Review of Industrial Organization*, 29, 55-73

[22] Guasch, J. L., J.J. Laffont and S. Straub (2008), "Renegotiation of Concession Contracts in Latin America, Evidence from the Water and Transport Sectors," *International Journal of Industrial Organization*, 26(2), 421-442

[23] Hart, O. (2003), "Incomplete Contracts and Public Ownership: Remarks, and an Application to Public-Private Partnerships," *The Economic Journal*, 113(486), C69-C76

[24] Hart, O., and J. Moore (1998), "Default And Renegotiation: A Dynamic Model Of Debt," *Quarterly Journal of Economics*, 113(1), 1-41

[25] House of Lords (2010), *Private Finance Projects and off-balance sheet debt*, First Report of Session 2009-10, Vokume I: Report, HL Paper 63-I

[26] House of Lords (2010), *Private Finance Projects and off-balance sheet debt*, First Report of Session 2009-10, Volume II: Evidence, HL Paper 63-II

[27] Iossa, E., and D. Martimort (2009), ""The Theory of Incentives Applied to PPPs in the Transport Sector," in *The Handbook of Transportation Economics*, ed. A. de Palma, R. Lindsey, E. Quinet and R. Vickerman, Edwar Elgar

[28] Iossa, E., and D. Martimort (2008), "The simple microeconomics of public-private partnerships", *CEIS Research Paper Series, 6(12), No.139*

[29] Irwin, T.C. (2007), *Government Guarantees. Allocationg and Valuing Risk in Privately Financed Infrastructure Projects*, The World Bank

[30] Irwin, T., M. Klein, G.E. Perry and M. Thobani (1997), *Dealing with Public Risk in Private Infrastructure*, World Bank Latin American and Carribean Studies, Washington D.C.

[31] Kartasheva, A.V. (2011), "Design of investment promotion policies," *International Journal of Industrial Organization*, article in press

[32] Laffont, J.J. (2003), "Enforcement, regulation and development," *Journal of African Economies*, 12, 193-211

[33] Lewis, T.R., and D.E.M. Sappington (1995), "Optimal Capital Structure in Agency Relationships," *The RAND Journal of Economics*, 26(3), 343-361

[34] Martimort, D. (2006), "An Agency Perspective on the Costs and Benefits of Privatization," *Journal of Regulatory Economics*, 30(1), 5-44

[35] Martimort, D., and J. Pouyet (2008), "To build or not to build: normative and positive theories of Public-Private Partnerships," *International Journal of Industrial Organization*, 26(2), 393-411

[36] National Audit Office, *The failure of Metronet*, HC 512, 2008–09

[37] Sader, F. (1999), *Attracting Foreign Direct Investment into Infrastructure: Why Is It So Difficult?*, World Bank Publications, Washington DC

[38] Sappington, D., and J. Stiglitz (1987), "Privatization, Information and Incentives," *Journal of Policy Analysis and Management*, 6, 567-582.

[39] Saussier, S., C. Staropoli and A. Yvrande-Billon (2009), "Public-Private Agreements, Institutions, and Competition: When Economic Theory Meets Facts," *Review of Industrial Organization*, 35(1-2), 1-18

[40] Snow, A., and R. Warren Jr. (1996), "The marginal welfare cost of public funds: Theory and estimates," *Journal of Public Economics*, 61(2), 289-305

[41] Spiegel, Y. (1996), "The role of debt in procurement contracts," *Journal of Economics and Management Strategy*, 5(3), 379-407

[42] Spiegel, Y., and D. Spulber (1994), "The capital structure of a regulated firm," *The RAND Journal of Economics*, 25(3), 424-440

[43] Spiegel, Y., and D. Spulber (1997), "Capital structure with countervailing incentives," *The RAND Journal of Economics*, 28(1), 1-24

[44] Taggart, R.A. (1985), "Effects of Regulation on Utility Financing: Theory and Evidence," *Journal of Industrial Economics*, 33, 257-276

[45] World Bank (1998), *World Development Indicators*, Washington D.C. World Bank

# A Proof of (12a) and (12b)

Based on (11a) and (11b), we see that $\exists \varepsilon_1 \geq 0, \varepsilon_2 \geq 0$ such that

$$\Pi_{l,0} = \Pi_{h,0} + \int_0^T \Delta\theta \left( q_h^* + \varepsilon_1 \right) e^{-rx} dx \tag{48}$$

$$\Pi_{h,0} = \Pi_{l,0} - \int_0^T \Delta\theta \left( q_l^* - \varepsilon_2 \right) e^{-rx} dx. \tag{49}$$

Using (11d), they are rewritten as

$$\Pi_{l,0} = M + \psi + (1 - \nu_1) \int_0^T \Delta\theta \left(q_h^* + \varepsilon_1\right) e^{-rx} dx \qquad (50)$$

$$\Pi_{h,0} = M + \psi - \nu_1 \int_0^T \Delta\theta \left(q_h^* + \varepsilon_1\right) e^{-rx} dx. \qquad (51)$$

Replacing (50) into (48), we further obtain

$$\Pi_{h,0} = M + \psi + (1 - \nu_1) \int_0^T \Delta\theta \left(q_h^* + \varepsilon_1\right) e^{-rx} dx - \int_0^T \Delta\theta \left(q_l^* - \varepsilon_2\right) e^{-rx} dx. \qquad (52)$$

From (51) and (52) we get

$$\int_0^T \Delta\theta \left(q_h^* + \varepsilon_1\right) e^{-rx} dx = \int_0^T \Delta\theta \left(q_l^* - \varepsilon_2\right) e^{-rx} dx,$$

which is equivalent to

$$q_h^* + \varepsilon_1 = q_l^* - \varepsilon_2.$$

Setting $z \equiv q_h^* + \varepsilon_1 = q_l^* - \varepsilon_2$, we obtain the profits in the main text. Further using $\varepsilon_1 \geq 0$ and $\varepsilon_2 \geq 0$ in the above equality, we also derive the range of values $[q_h^*, q_l^*]$ from which $z$ is to be drawn.

Consider now (11c). From (50), (51) and $z \equiv q_h^* + \varepsilon_1 = q_l^* - \varepsilon_2$, we obtain

$$\Pi_{l,0}\left(z\right) - \Pi_{h,0}\left(z\right) = \int_0^T \Delta\theta z e^{-rx} dx$$

so that (11c) is satisfied if and only if $\int_0^T \Delta\theta z e^{-rx} dx \geq \frac{\psi}{\Delta\nu}$. This is equivalent to $T \geq \underline{T}\left(z\right)$, with $\underline{T}\left(z\right)$ as defined in (14).

# B    No incentive to renegotiate $T$

Suppose that G makes the offer and that it proposes to end the contract at some $T^G > \tau$, $T^G \neq T$. If $T^G > T$, then G proposes the quantity-transfer pair $(q_i^*, t_i^{G,1})$ for all $x \in [\tau, T)$, as well as the quantity-transfer pair $(q_i^*, t_i^{G,2})$ for all $x \in [T, T^G)$. To make F indifferent between renegotiation and replacement, the two transfers are to be set such that the instantaneous profits $\pi_{i,\tau}^{G,1}$ and $\pi_{i,\tau}^{G,2}$ over the periods $[\tau, T)$ and $[T, T^G)$ are equal to $\pi_i^{rp}$, which is zero. Using $\gamma_i = \theta_i$, $\phi(1) = \phi$ and $q_i = q_i^*$ in (2) to rewrite $\pi_{i,\tau}^{G,1}$ and $\pi_{i,\tau}^{G,2}$, and then denoting $t_i = t_i^{G,1}$ in $\pi_{i,\tau}^{G,1}$ and $t_i^{G,2}$ in $\pi_{i,\tau}^{G,2}$, these equalities yield

$$t_i^{G,1} = \theta_i q_i^* + K + \phi - p(q_i^*)q_i^* + g_{i,\tau}^{rn}, \ \forall x \in [\tau, T)$$
$$t_i^{G,2} = \theta_i q_i^* + K + \phi - p(q_i^*)q_i^*, \ \forall x \in [T, T^G).$$

Further using these transfers in (5), together with $q_i = q_i^*$, the payoff of G under the renegotiated contract is given by

$$\widehat{V}_{i,\tau}^G = \int_\tau^{T^G} w(q_i^*)e^{-r(x-\tau)}dx - (1+\lambda)\int_\tau^T g_{i,\tau}^{rn}e^{-r(x-\tau)}dx.$$

If $T^G < T$, then, apart from $T^G$, the proposal of G includes the quantity-transfer pair $(q_i^*, t_i^{G,1})$ for all $x \in \left[\tau, T^G\right]$. Hence, G obtains the payoff $\widehat{V}_{i,\tau}^G$. The termination date $T^G$ is chosen so as to maximize

$$\begin{aligned}
W_{i,\tau} &= \widehat{V}_{i,\tau}^G + \int_{T^G}^{+\infty} w(q_i^*)e^{-r(y-T^G)}dy \\
&= \int_\tau^{+\infty} w(q_i^*)e^{-r(x-\tau)}dx - (1+\lambda)\int_\tau^T g_{i,\tau}^{rn}e^{-r(x-\tau)}dx,
\end{aligned}$$

which is independent of $T^G$.

Next suppose that F makes the offer and that it proposes to end the contract at some instant $T^F > \tau$, $T^F \neq T$. The offer includes the quantity-transfer pair $(q_i^*, t_i^{F,1})$ for all $x \in \left[\tau, T^F\right]$. To make G indifferent between the payoff under the renegotiated contract all over the period thereafter and the return from immediate replacement, $t_i^{F,1}$ and $T^F$ must be set such that

$$\begin{aligned}
&\int_\tau^{T^F} \left(S(q_i^*) - p\left(q_i^*\right)q_i^* - (1+\lambda)t_i^*\right)e^{-r(x-\tau)}dx + \int_{T^F}^\infty w(q_i^*)e^{-r\left(y-T^F\right)}dy \\
&= \int_\tau^\infty w(q_i^*)e^{-r(y-\tau)}dx - R_\delta.
\end{aligned}$$

Then, the transfer is written as

$$t_i^{F,1} = \frac{1}{1+\lambda}\left(S(q_i^F) - p\left(q_i^F\right)q_i^F - w(q_i^*) + \frac{rR_\delta}{1 - e^{-r(T^F-\tau)}}\right).$$

The payoff of F at $\tau$ is given by

$$\Pi_{i,\tau}^F = \frac{R_\delta}{1+\lambda} - \int_\tau^T g_{i,\tau}^{rn}e^{-r(x-\tau)}dx,$$

and is thus independent of $T^F$.

# C Implementation of $\Psi$

## C.1 (22a) and (22b) are satisfied

Suppose first that, in state $l$, F reports $h$ at time 0 and then the contract is renegotiated at some instant $\tau \in (0, T)$. The instantaneous profit of F is given by

$$\pi_{l,\tau}^{RN} = t_h^{rn} + p\left(q_h^*\right) q_h^* - \left(\theta_l q_h^* + K + \phi\right) - g_{h,\tau}^{rn},$$

where $t_h^{rn} = \alpha t_h^G + (1 - \alpha) t_h^F$ denotes the expected transfer that results from renegotiating at $\tau$, given the report $h$. Using the formulas of $t_h^G$ and $t_h^F$ from the main text, $t_h^{rn}$ is rewritten as

$$t_h^{rn} = \alpha \left(\theta_h q_h^* + K + \phi + g_{h,\tau}^{rn}\right) + \frac{1-\alpha}{1+\lambda} \left(S\left(q_h^*\right) - w_h^* + \frac{rR_\delta}{1 - e^{-r\delta}}\right) - \frac{1 + \alpha\lambda}{1 + \lambda} p\left(q_h^*\right) q_h^*.$$

Replacing into the expression of $\pi_{l,\tau}^{RN}$, the latter is rewritten as

$$\pi_{l,\tau}^{RN} = (1 - \alpha)\left(\frac{R_\delta}{1+\lambda}\frac{r}{1 - e^{-r\delta}} - g_{h,\tau}^{rn}\right) + \Delta\theta q_h^*.$$

In discounted terms, the profit is given by

$$\begin{aligned}
\Pi_{l,\tau}^{RN} &= \int_\tau^T \left[(1 - \alpha)\left(\frac{R_\delta}{1+\lambda}\frac{r}{1 - e^{-r\delta}} - g_{h,\tau}^{rn}\right) + \Delta\theta q_h^*\right] e^{-r(x-\tau)} d\tau \\
&= \Pi_{h,\tau}^{rn} + \int_\tau^T \Delta\theta q_h^* e^{-r(x-\tau)} d\tau.
\end{aligned}$$

Replacing this into (22b), the latter is rewritten as

$$\Pi_{l,0}^* \geq \int_0^\tau \left(\pi_{h,x}^* + \Delta\theta q_h^*\right) e^{-rx} dx + e^{-r\tau} \max\left\{0; \Pi_{h,\tau}^{rn} + \int_\tau^T \Delta\theta q_h^* e^{-r(x-\tau)} d\tau\right\},$$

which is equivalent to

$$\begin{aligned}
\Pi_{l,0}^* \geq \ & \Pi_{h,0}^* + \int_0^T \Delta\theta q_h^* e^{-rx} d\tau \\
& + e^{-r\tau}\left(\max\left\{0; \Pi_{h,\tau}^{rn} + \int_\tau^T \Delta\theta q_h^* e^{-r(x-\tau)} d\tau\right\} - \left(\Pi_{h,\tau}^* + \int_\tau^T \Delta\theta q_h^* e^{-r(x-\tau)} d\tau\right)\right).
\end{aligned}$$

This is implied by (11a) and (21a). Hence, (22a) does hold.

The proof that (22b) is satisfied proceeds analogously, leading to the conclusion that (22b) is implied by (11b) and (21a).

## C.2  Proof of Lemma 1

We first rewrite the incentive constraints (20a) and (20b). Suppose that the current period is $\tau' \in (0,T)$ and that the contract has been renegotiated at $\tau < \tau'$, $\tau \in (0,T)$. The payoff of F and that of G over the period $[\tau', T)$ under the contract renegotiated at $\tau$ and never again thereafter, are respectively written as

$$
\widehat{\Pi}_{i,\tau'/\tau}^{rn} = (1-\alpha) \int_{\tau'}^{T} \frac{R_\delta}{1+\lambda} \frac{r}{1-e^{-r\delta}} e^{-r(x-\tau')} dx - \widehat{D}_{i,\tau/\tau'}^{rn}
$$

$$
\widehat{V}_{i,\tau'/\tau}^{rn} = \int_{\tau'}^{T} \left( w(q_i^{sb}) - (1-\alpha) \frac{rR_\delta}{1-e^{-r\delta}} \right) e^{-r(x-\tau')} dx - \alpha(1+\lambda) \widehat{D}_{i,\tau/\tau'}^{rn}.
$$

If the contract is renegotiated at $\tau'$, then the discounted payoff of F is $\Pi_{i,\tau'}^{rn}$ and that of G is $V_{i,\tau'}^{rn}$. F does not wish to renegotiate at $\tau'$ if and only if $\widehat{\Pi}_{i,\tau'/\tau}^{rn} \geq \Pi_{i,\tau'}^{rn}$, which is equivalent to (20a). G does not wish to renegotiate at $\tau'$ if and only if $\widehat{V}_{i,\tau'/\tau}^{rn} \geq V_{i,\tau'}^{rn}$, which is equivalent to (20b).

Using the definitions of $\widehat{\Pi}_{i,\tau'/\tau}^{rn}$ and $\Pi_{i,\tau'}^{rn}$, $\widehat{\Pi}_{i,\tau'/\tau}^{rn} \geq \Pi_{i,\tau'}^{rn}$ is also equivalent to

$$
D_{i,\tau'}^{rn} - \widehat{D}_{i,\tau/\tau'}^{rn} \geq \left( R_{\delta'} - R_\delta \frac{1-e^{-r\delta'}}{1-e^{-r\delta}} \right) \frac{1}{1+\lambda}. \tag{53}
$$

Using the definitions of $\widehat{V}_{i,\tau'/\tau}^{rn}$ and $V_{i,\tau'}^{rn}$, $\widehat{V}_{i,\tau'/\tau}^{rn} \geq V_{i,\tau'}^{rn}$ is also equivalent to

$$
D_{i,\tau'}^{rn} - \widehat{D}_{i,\tau/\tau'}^{rn} \geq -\left( R_{\delta'} - R_\delta \frac{1-e^{-r\delta'}}{1-e^{-r\delta}} \right) \frac{1-\alpha}{\alpha(1+\lambda)}. \tag{54}
$$

(23) follows after combining (53) with (54).


## C.3  Proof of Lemma 3

Using the expressions of $\Pi_{l,\tau}^*(z)$ and $\Pi_{h,\tau}^*(z)$ in (16a) and (16b), together with (19a) and $\Pi_{i,\tau}^{rp} = 0$, (21a) is rewritten in state $l$ and $h$ as follows:

$$
(M+\psi)\frac{1-e^{-r\delta}}{1-e^{-rT}} + (1-\nu_1)\Delta\theta z \frac{1-e^{-r\delta}}{r} \geq (1-\alpha)\left( \frac{R_\delta}{1+\lambda} - D_{l,\tau}^{rn} \right) \tag{55a}
$$

$$
(M+\psi)\frac{1-e^{-r\delta}}{1-e^{-rT}} - \nu_1\Delta\theta z \frac{1-e^{-r\delta}}{r} \geq \max\left\{ (1-\alpha)\left( \frac{R_\delta}{1+\lambda} - D_{h,\tau}^{rn} \right); 0 \right\}, \tag{55b}
$$

With (24) and (55b) satisfied, (55a) is satisfied as well, and can thus be ignored. (55b) is rewritten as (25) in the lemma.

With (24) satisfied, (21b) is equivalent to $V_{i,\tau}^* \geq V_{i,\tau}^{rp}$, $i \in \{l,h\}$. Using (17) together with (18) for all $i$, this inequality is rewritten as

$$
D_{i,\tau} \leq \frac{R_\delta}{1+\lambda} - \Pi_{i,\tau}^*(M,T).
$$

Replacing the expression of $\Pi_{i,\tau}^*(z)$ for $i = l, h$, this yields (26a) and (26b) in the text. (26a) and (26b) hold only if their right-hand sides are positive *i.e.*,

$$\frac{R_\delta}{1+\lambda} \geq \left(\frac{(M+\psi)\,r}{1-e^{-rT}} + (1-\nu_1)\,\Delta\theta z\right)\frac{1-e^{-r\delta}}{r}$$

$$\frac{R_\delta}{1+\lambda} \geq \left(\frac{(M+\psi)\,r}{1-e^{-rT}} - \nu_1\Delta\theta z\right)\frac{1-e^{-r\delta}}{r},$$

where the former imply the latter and is equivalent to (27) in the lemma.

## C.4 Proof of Proposition 2

The condition $T \geq \underline{T}(z)$ is necessary for (11c) to be satisfied.

Recall that (21a) and (21b) are tighter when (24) is not satisfied than they are when it is. From the proof of Lemma 3 further recall that, when (24) is satisfied, (21a) and (21b) are rewritten as (25), (26a), (26b) and (27). These represent the weakest conditions on $M$ and $D_{i,\tau}$, $i = l, h$, under which $\Psi$ is implementable.

Now take (24) to hold for either type. (25) and (27) hold jointly only if (28) is satisfied. Further, from (25) and $M \leq E$, we deduce

$$E \geq \nu_1\Delta\theta z\frac{1-e^{-rT}}{r} - \psi.$$

If $E \geq (\nu_1\Delta\theta z/r) - \psi$, then this condition is satisfied for all $T > 0$. Otherwise, it is rewritten as $T \leq \widetilde{T}(z, E)$, with $\widetilde{T}(z, E)$ as defined in (32). Then, as $T \geq \underline{T}(z)$, it is necessary that $\underline{T}(z) \leq \widetilde{T}(z, E)$, which is equivalent to $E \geq \nu_1\psi/\Delta\nu$. Overall, the above condition is satisfied together with $T \geq \underline{T}(z)$ if and only if (29) holds.

Suppose that $C = 0$ and $\Psi$ is implemented. Then, $D_{i,\tau'}^{rn} = \widehat{D}_{i,\tau/\tau'}^{rn} = 0$ for all $\tau, \tau' \in (0, T)$, $\tau' > \tau$. Hence, we cannot have $D_{i,\tau'}^{rn} - \widehat{D}_{i,\tau/\tau'}^{rn} > 0$, as it is required for (23) to be met, which is a contradiction. It must be the case that (30) holds.

With (28) and (29) satisfied, G can find values of $M$ such that (25) and (27) both hold. With (30) satisfied, G can find out-of-equilibrium guarantees such that the out-of-equilibrium conditions hold. Hence, the three conditions are both necessary and sufficient for implementation of $\Psi$.

## C.5 Proof of Corollary 1

By assumption, $\lim_{\delta \to 0} R_\delta$ is positive and finite. Hence, (28) holds as $\delta$ tends to zero.

First suppose that, for some given $z$, (31) holds. It means that, as $\delta$ is raised (*i.e.*, $\tau$ is decreased and/or $T$ increased), (28) is relaxed. Then, provided it is satisfied as $\delta$ tends to zero, it is as for all $\delta \in (0, T)$, $T \in [\underline{T}(z), \infty)$.

Next suppose that (33) holds. Then, for any given $T \geq \underline{T}(z)$, (28) is tightest as $\tau$ tends to zero. Then, replacing $\tau = 0$ into (28), the latter is satisfied if and only if $T \leq \overline{T}(z)$. As $T$ must be at least as large as $\underline{T}(z)$, for $\Psi$ to be implementable it must be the case that the interval $[\underline{T}(z), \overline{T}(z)]$ exists and that $T$ belongs to this interval.

The remaining conditions in the corollary are the restrictions on $T$ that were shown in the

proof of Proposition 2, namely $T \geq \underline{T}(z)$ when $E \geq (\nu_1 \Delta \theta z / r) - \psi$ and $T \in \left[\underline{T}(z), \widetilde{T}(z, E)\right]$ when $E < (\nu_1 \Delta \theta z / r) - \psi$.

## C.6 Proof of Corollary 2

Recall that, when (24) is satisfied, (21b) is rewritten as (25), (26a), (26b) and (27). (25) and 27) are rewritten as (37). Using the definition of $E_i[D_{i,\tau}]$ in (26a) and (26b, we obtain

$$E_i[D_{i,\tau}] \leq \frac{R_\delta}{1+\lambda} - (M + \psi) \frac{1 - e^{-r\delta}}{1 - e^{-rT}}. \tag{56}$$

Recalling that $E_i[D_{i,0}] = C$, (56) together with $C > 0$ collapses onto (38).

# D Proof of Proposition 3 and 4

Replacing $z$ with $\alpha_i$ and $T$ with $T_i$ in (12a) and (12b), $i \in \{l, h\}$, we find (39) and (40), from which

$$\Pi_{l,0} - \Pi_{h,0} = \int_0^{T_l} \Delta \theta \alpha_l e^{-rx} dx = \int_0^{T_h} \Delta \theta \alpha_h e^{-rx} dx. \tag{57}$$

$\alpha_i$ attains the lowest feasible value when the sharing rule is that indicated by $\alpha_l$. This value is found by replacing $\alpha_h$ with $q_h^*$ in (57) and solving for $\alpha_l$, which yields $\alpha_l = q_h^* \frac{1 - e^{-rT_h}}{1 - e^{-rT_l}}$. $\alpha_i$ attains the highest feasible value when the sharing rule is that indicated by $\alpha_h$. This value is found by replacing $\alpha_l$ with $q_l^*$ in (57) and solving for $\alpha_h$, which yields $\alpha_h = q_l^* \frac{1 - e^{-rT_l}}{1 - e^{-rT_h}}$. The feasible set of $\alpha_i$ is thus determined for $i \in \{h, l\}$.

We now check the conditions under which $\Psi_i$ is implemented. Using (57) with reference to the rule $\alpha_i$, (11c) is rewritten as

$$\int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx \geq \frac{\psi}{\Delta \nu},$$

which requires that (41) be satisfied for $i = l$ and (44) for $i = h$, together with $T_i \geq \underline{T}(\alpha_i)$. (25) is equivalent to $\Pi_{h,\tau} \geq 0$, which is in turn written as

$$M \geq \nu_1 \Delta \theta \alpha_i \frac{1 - e^{-rT_i}}{r} - \psi. \tag{58}$$

Using $T_i \geq \underline{T}(\alpha_i)$, $E \geq M$ and (58) altogether, (29) follows.

Recall from the proof of Lemma 3 that, in the framework with $T$ fixed, the constraint $V_{i,\tau}^* \geq V_{i,\tau}^{rp}$ is rewritten as (26a) and (26b). Their counterparts in the state-dependent context are given by

$$D_{h,\tau} \leq \frac{R_{\delta(h)}}{1+\lambda} - \left(M + \psi - \nu_1 \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx\right) \frac{1 - e^{-r(T_h - \tau)}}{1 - e^{-rT_h}} \tag{59}$$

$$D_{l,\tau} \leq \frac{R_{\delta(l)}}{1+\lambda} - \left(M + \psi + (1 - \nu_1) \int_0^{T_i} \Delta \theta \alpha_i e^{-rx} dx\right) \frac{1 - e^{-r(T_l - \tau)}}{1 - e^{-rT_l}}, \tag{60}$$

where $R_{\delta(i)}$ denotes the replacement cost in state $i = h, l$ when the residual contractual period is $(T_i - \tau)$. Further recall that, in Lemma 3, (27) is obtained by considering that the right-hand side of either of the conditions on debt must be non-negative. Using (59) and (60), it is $D_{i,\tau} \geq 0$ for $i = h, l$ only if

$$M \quad \leq \quad \frac{R_{\delta(h)}}{1 + \lambda} \frac{1 - e^{-rT_h}}{1 - e^{-r(T_h - \tau)}} + \nu_1 \int_0^{T_i} \Delta\theta\alpha_i e^{-rx} dx - \psi \tag{61}$$

$$M \quad \leq \quad \frac{R_{\delta(l)}}{1 + \lambda} \frac{1 - e^{-rT_l}}{1 - e^{-r(T_l - \tau)}} - (1 - \nu_1) \int_0^{T_i} \Delta\theta\alpha_i e^{-rx} dx - \psi. \tag{62}$$

In Proposition 2, (28) is obtained from (25) and (27), which should hold jointly. In the state-dependent setting, (58) holds jointly with (61) in state $h$ only if $R_{\delta(h)}/(1 + \lambda) \geq 0$, which is indeed satisfied by the assumptions made on $R_\delta$. In state $l$, two situations can arise. If the sharing rule is $\alpha_l$, then (58) and (62) hold jointly only if (42) is satisfied. If the sharing rule is $\alpha_h$, then they hold jointly only if (45) is satisfied.

The proof of (30) is similar to that reported with regards to Proposition 2.


## D.1  Proof of Corollary 3

Condition (46) is obtained from (58), (61) and (62). Using the definition of $E_i[D_{i,\tau}]$ in (60) and (59), we get

$$E_i[D_{i,\tau}] \leq \frac{E_i[R_{\delta(i)}]}{1 + \lambda} - (M + \psi) E_i \left[ \frac{1 - e^{-r\delta(i)}}{1 - e^{-rT_i}} \right].$$

Then, recalling that $E_i[D_{i,0}] = C$, this condition together with $C > 0$ collapses onto (47).