Announcements as an equilibrium selection device*

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Abstract

We study the coordination failures that arise in models with multiple equilibria by considering the role of cheap talk communication as an equilibrium selection device. We introduce an outside option (which represents common-knowledge expected outcomes in the absence of coordination), and show that the player who sends a message may be forced to announce a sub-optimal outcome with respect to the commitment solution (a binding message) to keep the message credible. Furthermore, by focusing on some examples based mainly on macroeconomic modeling, we show how the existing tools of bargaining can be fruitfully applied to such models with multiple equilibria and produce sensible outcomes.

Keywords: Multiple Nash equilibria, coordination failures, cheap talk, complete information, self-committing actions, announcement equilibrium.

JEL classification : C72, C78, D61.

1. Introduction

Coordination failures often arise. A way to model these outcomes is to introduce game theoretical models with multiple equilibria (see e.g. Cooper and John, 1988).[1] The associated richness of possible strategic interactions fails, however, to predict the behavior of players. As David Kreps clearly states: "the theory is often of no help in sorting out whether any one is the solution and, if one is, which one is" (Kreps, 1990: 97).

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[1] Some further non-exhaustive examples taken from the economic and political science literature are Weil (1989), Durlauf (1991), Leeper (1991), Myerson (2004), Rohner and Frey (2009), Libich *et al*. (2012).

A natural solution to these problems is based on the idea of focal points (Schelling, 1960).[2] This approach is however not always appealing – in particular, when there is a conflict of interest between the agents.

Common sense tells us that agents make use of communication to *avoid* coordination failures. This has been modeled as a pre-play communication stage (Farrell, 1988).Although the promises made there are non-binding, if they are correctly interpreted and lead to advantageous outcomes, they are assumed to coordinate player's strategies.[3] We contribute to this debate by emphasizing that the success of communication depends on the beliefs of players about the results that would arise in the case of absence of (or unsuccessful) coordination.

We consider a model with pre-play communication in a game with multiple Nash equilibria. Specifically we assume that, in the absence of an explicit or implicit agreement, the parties will expect to carry out a given outcome: their outside option. An announcement is an offer to play a Nash equilibrium that Pareto-dominates the outside option's expected outcomes. The offer (announcement) must be accepted or declined by the receiving party, and there is a commitment that no further offers will be made or accepted after an offer has been accepted.

We define an announcement equilibrium as a set of credible and optimal messages from the point of view of the sender. Credibility requires that the announcement is both "self-committing" and "acceptable". Similar to Farrell (1988), an announcement is self-committing if the sender has no incentive to deviate from the promise and if he expects the receiver to believe the announcement. But, for an announcement to be acceptable, the outcome associated with it should dominate the outside option for both sender and receiver.

We show that the sender may be forced to announce a sub-optimal outcome with respect to the commitment (a binding message) in order to keep the message credible. The reason is that the announcement has to be better than *all* the expected outcomes in the absence of coordination (i.e. the outside option). Moreover, we discuss the relationship between the announcement equilibrium and other equilibrium concepts.

The rest of the paper is structured as follows. In Section 2, we formally define the announcement equilibrium. Section 3 illustrates our idea by introducing the coordination problem between fiscal and monetary authorities. Section 4 and 5 focus on linear-quadratic games and allow us to characterize the closed-form solution of the announcement equilibrium and its properties in such a context. Specifically, Section 4 considers one-sided announcements and compares them totraditional commitment (a Stackelberg equilibrium) and to cooperative solutions as alternative techniques for creating credible policy commitments. Section 5 introduces two-sided announcements and shows their equivalence with the non-cooperative bargaining game of alternating offers and to the Nash cooperative bargaining solution. Section 6 concludes. Formal proofs for our propositions are gathered in the appendices.

2. Announcement equilibrium

[2] Some refinements of the Nash equilibrium have also been proposed. The most relevant is that of Harsanyi and Selten (1988), which is based on the concept of risk dominance. Kohlberg and Mertens (1986) propose an alternative selection mechanism based on a forward-induction argument.

[3] Farrell's solution has been criticized by Aumann (1990). However, evolutionary models and experimental evidence seem to support Farrell's view (see, e.g., Kim and Sobel, 1995; and Charness, 2000), at least for simple games.

We consider a simple static, complete information game between two players. Each player, $k \in \{1,2\}$, chooses a strategy, $s_k \in S_k$, to maximize his payoff: $\Pi_k = \pi_k\left(s_k, s_{-k}\right)$. We designate this game $\Gamma = \left(k, S, \Pi\right)$, with $k \in \{1,2\}$, $S = S_1 \times S_2$ and $\Pi = \{\Pi_1, \Pi_2\}$.

The Nash equilibrium of $\Gamma = \left(k, S, \Pi\right)$ is a vector $s^* = \left(s_1^*, s_2^*\right)$ such that $s_k^* \in \arg\max \Pi_k\left(s_k, s_{-k}^*\right)$ for all $k \in \{1,2\}$. We assume that $\Gamma = \left(k, S, \Pi\right)$ has multiple Nash equilibria.

We allow one of the players (possibly chosen by Nature) to make an announcement before they play the underlying game described above. At the beginning, a player, *i* (*j* is the other player), makes a public announcement. Thereafter, without any binding commitment, the two players simultaneously set their instruments. Since the announcement is not binding, the players can take it into account or not as they prefer. If they do not take account of that announcement, an "outside option" is obtained.[4]

The **announcement** made by agent *i* is a vector of strategies of the underlying game upon which player *i* aims to coordinate. It is indicated by $A_i = \left(s_1^{A_i}, s_2^{A_i}\right) \in S$.

Note that the above definition implies that agents communicate by means of a common language since it associates the announcement to a unique vector of strategies that the players may choose to implement. The agent receiving the announcement correctly interprets it and unambiguously evaluates its effects.[5]

Like to Farrell (1988), we assume that, although announcements are modeled as cheap talk and do not restrict the set of Nash equilibria of the underlying game (babbling equilibria), some announcements can successfully communicate the real intentions of the sender and act as a coordination device.

If an agent decides to ignore the announcement, the payoff associated to the underlying game is common knowledge and is determined by an exogenously given probability distribution over the set of strategies, *g*(*S*):

$$\tilde{\pi}_k = \int \pi_k\left(S\right) g\left(S\right) dS \qquad \text{for } k = \{1,2\}.$$

These payoffs define the players' outside options. Without entering into a discussion about their possible derivation, we just point out that the distributions of these outcomes could have had their origin in a mixed strategy equilibrium; or alternatively in the players' prior beliefs which may be ultimately derived from institutional or historical experiences. However, a detailed analysis of the form of their joint probability distribution is beyond the scope of this paper.[6]

On the other hand, if the announcement is "credible", it will not be ignored and agents *will* act according to that announcement. The credibility of non-binding announcements is based on *self-committing* messages; and on an additional constraint, induced by the outside options, which ensures that the coordination phase is implemented.

[4] Outside option will be defined shortly.

[5] See Farrell (1993) for a detailed discussion of this point.

[6] In the next section we provide an example where outside options are derived from a mixed strategy argument. Alternatively, in sections 4 and 5 we assume that outside options are historically driven, as we consider continuous strategies in a LQ game.

We formally define the set of credible announcements (Ψ) as those satisfying two axioms.

Axiom 1 (*self-commitment*). An announcement, A_i, is self-committing if $\pi_k(A_i) \geq \pi_k(s_k, s_{-k}^{A_i})$ for all $s_k \in S_k$ and $k = \{1,2\}$.

Axiom 2 (*acceptability*). An announcement is acceptable if: $\pi_k(A_i) \geq \tilde{\pi}_k \quad k = \{1,2\}$.

Axiom 1 satisfies Farrell's requirements. It means that no agent has an incentive to deviate from the announcement equilibrium if he believes that the other will play it. It implies that a credible announcement should be a Nash equilibrium. Axiom 2 requires that both players do not prefer the outside option to the payoff they would achieve under the announced equilibrium. We assume that if a credible message is announced, it will be then implemented by both agents. Thus the "equilibrium" of the announcement game can be obtained by backward induction and the optimal announcement can be found as the solution to a Stackelberg problem (see e.g. Baliga and Morris, 2002: Section 2). This equilibrium concept is formally defined as:

An **announcement equilibrium** for the underlying game $\Gamma = (k, S, \Pi)$ is an announcement $\bar{A}_i \in \Psi$ such that $\pi_i(\bar{A}_i) \geq \pi_i(A_i)$ for all $A_i \in \Psi$.

Since credibility is assumed to be necessary and sufficient for an announcement to be implemented in the underlying game, the announcement equilibrium provides a complete description of the actions of the agents in the game. The outcome of the game is the one announced by player *i*, and corresponds to his most favorable outcome from among all those available in the set of credible announcements.

3. A simple problem of coordination: Sargent and Wallace's debate revisited

In order to illustrate our announcement equilibrium we first consider a simple game between the monetary (M) and fiscal (F) authorities. Their strategies are either an active or a passive policy. An active monetary policy means that the central bank aims at sustaining stable prices, whereas a passive one implies that it will reflate the economy – accommodating any fiscal spending plans in the process. An active fiscal policy means that the government spends excessively and/or avoids necessary fiscal reforms; instead, passive fiscal policy implies a balanced inter-temporal budget.

The payoffs are described by the normal form game (borrowed from Libich *et al*., 2012).

	P_F	A_F
A_M	0, –3	–4,–4
P_M	–4.05, –3.25	–3.8,0

In the matrix, A_M and P_M indicate the central bank's actions and P_F and A_F those of the government, where A and P stand for active or passive policy.

The matrix describes a Game of Chicken, which closely resembles the unpleasant monetarist arithmetic of Sargent and Wallace (1981). In that game, two pure strategy Nash equilibria arise, each preferred by a different policymaker. Both would like to pursue an active policy, when facing an opponent who chooses a passive one. In Leeper (2010)'s terms, the central bank prefers the *M*-regime (A_M, P_F), the socially optimal Ricardian outcome; whereas the ambitious government prefers the *F*-regime (P_M, A_F), where it spends excessively and would like the central bank to inflate some of the promises (debt) away. But if the policymakers do not coordinate their actions (by playing mixed strategies), they will engage in a tug-of-war leading to the off-diagonal outcomes which are inferior for both of them.

Let the outside option be defined by the expected outcome at the mixed strategy equilibrium. In such a case the central bank plays an active policy with probability 0.76, whereas the government plays an active policy with probability 0.96, with expected payoffs $\tilde{\pi}_M = -3.81$ and $\tilde{\pi}_F = -3.06$. By assuming, in addition, that one of the players, randomly chosen by nature, is allowed to make an announcement, we can compute our announcement equilibrium. Comparing the payoffs associated to the outside option with those corresponding to pure strategy Nash equilibria, it is clear that a policymaker will announce his preferred outcome and this way will obtain a better outcome from among the Nash equilibria. It is worth noticing that in this case the announcement equilibrium corresponds to the solution of a Stackelberg game, with the first policymaker to announce acting as the leader.

Now consider a variant of this game with richer policy interactions. Monetary and fiscal authorities can choose a third strategy, C, that is a compromise between being fully active or fully passive. The payoffs are as follows:

	P_F	C_F	A_F
A_M	0, –7	–6, –9	–9, –9
C_M	–2, –5	–4, –4	–9, –6
P_M	–3, –3	–5, –2	–7, 0

In this case, the payoffs associated with the mixed strategy equilibrium (defining the outside option) are $\tilde{\pi}_M = \tilde{\pi}_F = -5$.[7] This implies that if a policymaker announces his most preferred Nash equilibrium, he will fail to achieve it: the receiver would not react to a message that makes him worse-off than the outside option to which the receiver can always revert. Thus, whatever policy authorities are selected, the announcement will be a compromise (C_M, C_F). Announcements no longer correspond to the Stackelberg solutions [the *M*-regime for the monetary authorities and the *F*-regime for the fiscal one]. Here the announcements rule out fully active and passive policies since, in order to be in a position to support *credible* announcements, the only feasible option is to announce a compromise.

4. Announcement equilibrium in LQ games

The concept of the announcement equilibrium presented above can be better understood in a framework in which the set of Nash equilibria is compact. This allows us to evaluate the effects of small variations in the parameters defining the players' preferences and the outside option. A simple way to introduce the compactness assumption is to study the special case of linear-quadratic (LQ) games.

4.1 Coordination problems in LQ contexts

[7] The mixed strategies equilibrium of this game is one where each player plays a pure strategy with the same probability.

We consider a static LQ game between two players with complete information. Each player, *k*, sets an instrument, u_k, to minimize the following loss functions:

$$L_k(z) = \frac{1}{2}\left[(x - \bar{x}_k)^2 + \beta_k (y - \bar{y}_k)^2\right] \qquad k = \{1,2\} \tag{1}$$

defined over the deviations of two target variables, *x* and *y*, from their desired values $\bar{x}_k$ and $\bar{y}_k$. In vector notation $z = (x, y)'$, $\bar{z}_k = (\bar{x}_k, \bar{y}_k)'$); and $\beta_k \in (0, +\infty)$ are given preference weights (or relative priority) parameters.

The relationship between targets and instruments is summarized by a system of linear equations that describes the economy:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} c_x \\ c_y \end{bmatrix} \tag{2}$$

where a_{ji} are the policy multipliers of the instruments u_1 and u_2; and c_x and c_y are constants. In compact form, equation (2) can be rewritten as: $z = Au + c$.

We assume that the instruments of the players are linearly independent;[8] thus

$$Rank[A] = Rank\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = 2\,. \tag{3}$$

Proposition 1. *Necessary and sufficient conditions to obtain multiple Nash equilibria in this setting are given by:*

$$\text{(a)}\quad Rank\begin{bmatrix} a_{11} & \beta_1 a_{21} \\ a_{12} & \beta_2 a_{22} \end{bmatrix} = 1 \quad \textit{and} \qquad \text{(b)}\quad \begin{pmatrix} a_{11} \\ \beta_1 a_{21} \end{pmatrix} (\bar{z}_2 - \bar{z}_1) = 0 \tag{4}$$

or, more concisely, $\quad Rank\begin{pmatrix} a_{11} & \beta_1 a_{21} & a_{11}\bar{x}_1 + \beta_1 a_{21}\bar{y}_1 \\ a_{12} & \beta_2 a_{22} & a_{12}\bar{x}_2 + \beta_2 a_{22}\bar{y}_2 \end{pmatrix} = 1\,.$

A formal proof of the above proposition, and corollaries below, is provided e.g. in Acocella *et al*. (2012: Chapter 8).

The conditions in equation (4) can be derived easily from the first order conditions by introducing the *quasi-reaction functions*, i.e. reaction functions defined in the space of the targets. These reaction functions are given by the following pair of equations:

$$a_{1k}(x - \bar{x}_k) + \beta_k a_{2k}(y - \bar{y}_k) = 0 \qquad k = \{1,2\} \tag{5}$$

Clearly one can easily derive the conventional reaction functions from quasi-reaction functions, and vice versa, by using (2).

[8] If this assumption does not hold, there may be multiple Nash equilibria that support the same outcomes and, consequently, the same payoffs for the agents. In that case, the problem faced by each agent is to correctly anticipate the strategy played by the other player in order to get to one of these Nash equilibria. Since the agents are indifferent between these outcomes, the issue is of minor interest here.

Part (a) of (4) implies that the quasi reaction functions (and the reaction functions) of the two agents have the same slopes; and part (b) that they are overlapping (i.e., the first best outcomes for player 2 lies along the quasi reaction function of player 1, and vice versa).[9]

Thus, all outcomes satisfying:

$$\begin{pmatrix} a_{11} \\ \beta_1 a_{21} \end{pmatrix} (z - \overline{z}_1) = 0 \tag{6}$$

are the outcomes of a Nash equilibrium.[10]

Corollary 1. *If* $\overline{z}_1 \neq \overline{z}_2$*, each Nash equilibrium in the space of the outcomes,* z^N*, can be identified by a value of* $\lambda \in \mathbb{R}$ *such that*:

$$z^N = \lambda \overline{z}_1 + (1-\lambda)\overline{z}_2 \tag{7}$$

Within the above set, we can easily describe the subset that is efficient as follows:

Corollary 2. *If* $\overline{z}_1 \neq \overline{z}_2$*, the set of not Pareto dominated Nash equilibria (*Θ*) is identified by all the (linear) convex combinations of the first best outcomes of the players, i.e.* $\lambda \in [0,1]$.

A Nash equilibrium belongs to Θ if there are no other Nash equilibrium that Pareto dominate it. Note also that the set of not Pareto dominated Nash equilibria Θ does not represent the set of efficient outcomes; instead the set of efficient outcomes is described by the contract curve. All Nash equilibria that do not belong to Θ therefore represent coordination failures in the sense of Cooper and John (1988).

Of course, if $\overline{z}_1 = \overline{z}_2$, then all Nash equilibria that lead to different outcomes are Pareto inefficient.

4.2 Announcement equilibrium

Now we allow players to make announcements in the underlying game; that is, in the coordination game described in the previous section where condition (4) holds.

We use announcements about an outcome (z_i^A), instead of announcements about strategies, by assuming that the sender *forecasts* the optimal reaction of the receiver. If this is the case, announcing a couple of strategies, or even a policy u_i^A, is exactly equivalent to announcing the outcome of the game corresponding to that policy and the other agent's known (optimal) reaction to that policy: $z_i^A = (x_i^A, y_i^A) = A(u_i^A, r_j(u_i^A))' + c$, where $r_j(u_i^A)$ is the reaction function of agent *j* to the policy announcement by *i*.[11]

In this LQ context, an announcement, z_i^A, is thus a vector of outcomes of the underlying game. Its supporting strategies are a unique pair (u_i^S, u_j^S) such that $z_i^A = A(u_i^S, u_j^S)' + c$.

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[9] Thus, if only part (a) holds, there will be no equilibrium (Acocella and Di Bartolomeo, 2006; Acocella *et al*., 2009).

[10] The condition can be expressed equivalently in terms of $\overline{z}_2$.

[11] In this model, announcements about policies or outcomes can be used interchangeably. However, we prefer to use announcements about outcomes, since our focus is on policy games. We have in mind announcements of the kind implied by inflation targets.

The conditions required for the existence of a non-empty set of credible announcements are identified by the following lemma.

Lemma 1. There is at least one outcome on the quasi-reaction functions, equations (5), that Pareto dominates the outside option associated with $g(z)$ if and only if the following condition holds: $\sqrt{\frac{1}{\alpha_1}\tilde{L}_1}+\sqrt{\frac{1}{\alpha_2}\tilde{L}_2} \geq |\bar{x}_1-\bar{x}_2|$, where $\alpha_1=1+a_{11}^2/\beta_1 a_{21}^2$ and $\alpha_2=1+a_{12}^2/\beta_2 a_{22}^2$.

Proof. See Appendix A.

It is worth noticing that, to be credible, an announcement must lie on the quasi-reaction functions of both players (to avoid incentives to deviate) and that both players must prefer it to the expected loss under their outside option. Moreover, as the expected losses of the outside option increase, the more likely is this condition to be satisfied. In particular, since agents are risk averse (due to the quadratic form of the payoffs), their risk premium is an increasing function of the variance of $g(z)$. In addition, the condition above is always satisfied if there is no conflict between the agents (i.e., if $\bar{x}_1=\bar{x}_2$). Finally, if we restrict the set of possible random outcomes to the set of Nash equilibria, as Cooper and John (1988) do, then $g(z)$ will be defined with a positive probability only for those outcomes that satisfy (5) – with the result that the condition of Lemma 1 is always met. The intuition is not trivial: one has to notice that the expected outcome $z^e=\int zg(z)dz$ fulfills (5) – which means that it is always preferred to the outside option since players are risk averse.

Now we can state our theorem.

Theorem 1. *If there is at least one outcome on the quasi-reaction functions that Pareto dominates the outside option, the one-sided announcement game has a unique announcement equilibrium.*

Proof: Appendix B.

As the announcement equilibrium is always unique, and is always a Nash equilibrium, it acts as an equilibrium selection device. The idea is simply that player *i* can use his “announcement power” to eliminate the random outcomes generated by the density function $g(z)$ to obtain his most favourable credible outcome. However, if there is no outcome on the quasi-reaction functions that Pareto dominates the outside option, no credible announcement can be made and the outside option must be expected to prevail. The announcements therefore succeed in solving the equilibrium selection dilemma because they offer a way to signal a common strategy that will lead to an outcome which is better than the outside option for both players.

Finally notice, from Lemma 1, that announcements acting as an equilibrium selection device are more likely to be observed when there is a large degree of uncertainty, little disagreement between agents about the targets, and higher expected losses when players do not coordinate their actions.

Our theorem therefore has two key corollaries.[12]

Corollary 3. *No coordination failure can arise in our game if there is at least one outcome on the quasi-reaction functions that Pareto dominates the outside option (because credible announcements are available and can be used).*

[12] The proofs of these two corollaries follow directly from the proof of Theorem 1 (see Appendix B).

Corollary 3 derives from the simple fact that credibility requires that player *i* should announce a Nash equilibrium. Since he will choose the best among those available, this announcement cannot be Pareto dominated by other Nash equilibria. More generally, the selection device generated by these announcements gives us an explicit method for agents not only to coordinate on non-dominated Nash outcomes, but also to avoid the uncoordinated responses that follow from the impossibility of being able to forecast the rival's strategy to which they need to react optimally.

The next corollary shows another important normative consequence:

Corollary 4. *Player i will announce his first best* $\tilde{z}_i^A = \overline{z}_i$ *if and only if* $\overline{z}_i$ *satisfies player j's participation constraint, i.e.* $L_j\left(\overline{z}_i\right) \leq \tilde{L}_j$.

Put differently, agent *i* can announce his first best only under certain circumstances. If these are not satisfied, he can promise only second best outcomes. The participation constraint is the key element that distinguishes an announcement solution from commitment – as we show in the next sub-section.

4.3 Announcements, commitment and cooperative solutions

Agents or policymakers routinely promise future actions. These promises can be considered to be policy (Stackelberg) commitments or cheap talk announcements depending on whether they are supported by commitment technologies or not. Although in the real world it is sometimes difficult to distinguish between announcements and commitments, they are in fact quite different and will usually lead to different outcomes. This we show next. However, both may be used as equilibrium selection devices, as both transform the committed/announced outcome into a type of focal point.

By using promises based on commitments one always obtains a better outcome than by making announcements. However, these promises must be supported by an appropriate commitment technology, and that is not always available.[13] In the absence of such a technology, player *i* must resort to making an announcement.

If we look at an announcement as a substitute for a commitment for which an appropriate commitment technology is not available, we can understand why, in some cases, players do not always promise their first best option. Our results therefore give a rationale for the otherwise puzzling evidence of policymakers making promises leading to outcomes which are suboptimal from their own point of view – such as positive inflation targeting or incomplete reform strategies.

In summary, our results can be illustrated by the following diagram. In figure 1, points $\overline{z}_i$ and $\overline{z}_j$ represent the first best outcomes for the two players. Thus, assuming condition (4) holds, the segment $\overline{z}_i\overline{z}_j$ represents the un-dominated Nash equilibria that belong to the two overlapping quasi-reaction functions, i.e. straight line through $\overline{z}_i\overline{z}_j$ (see corollary 3).

[13] If this is not the case, the policy to which the player has committed may be time inconsistent and the promised outcomes never reached.

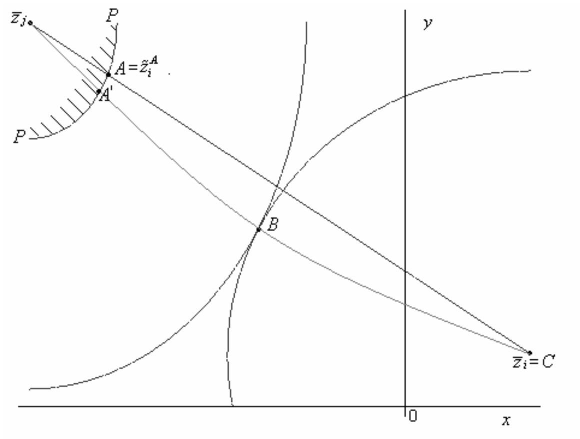

Figure 1 – Announcement equilibrium vs. commitment.

If player *i* owns a commitment technology, he will choose his best option along the reaction function of *j*. Hence the commitment solution is represented by point *C*, his first best outcome.

By contrast, if player *i* cannot use a commitment technology, but credible announcements are available, he will announce the best outcome given the participation constraint of *j*. This constraint is given by the North-West region of the *PP* line, which represents the iso-loss curve corresponding to $\tilde{L}_j$. The announcement solution is therefore described by point *A:* Player *i* announces the outcome closer to his first best (point *C*), conditional on the participation constraint of player *j*.

The figure also illustrates the locus of Pareto efficient outcomes; the contract curve, the curve passing through *B*. In contrast to the commitment, the announcement solution is always Pareto inefficient when the participation constraint of *j* is binding: point *A'* clearly Pareto dominates point *A* since it is possible to improve the satisfaction of player *i* while keeping that of *j* constant. However, player *i* cannot announce *A'* because it is not a credible proposition since it does not lie on either of the reaction functions.

The announcement equilibrium is however Pareto efficient only among the Nash equilibria of the underlying game. It could in fact be seen as the result of a bargaining model restricted to the Nash equilibria.[14]

4.4 An example: Inflation and fiscal announcements

We consider a stylized underlying game between the government and the central bank, which is characterized by multiple equilibria. We assume that the central bank sets the interest rate (*i*) and the government sets the average tax rate (*t*), measured as the ratio of fiscal revenues to potential GDP, and that these policymakers have different preferences for internal and external stabilization (i.e., different priorities for maintaining internal and external balances).

Formally, government (*G*) and central bank (*B*) losses are defined as follows:

$$L_k = \frac{1}{2}\left(b - b_k\right)^2 + \frac{1}{2}\beta_k\left(\pi - \pi_k\right)^2 \qquad k = \{G, B\} \tag{8}$$

[14] This will be more evident when the possibility of multiple (even infinite) announcements is introduced (Corollary 6).

where b represents the balance of payments (external stability), measured as the surplus or deficit as a proportion of potential output, and π is the inflation rate (internal stability). For the sake of simplicity, we assume that the economy is characterized by a simple Phillips curve relationship; thus different targets for inflation imply different opinions about how to solve the trade-off between unemployment and inflation.

We assume that the central bank aims to fully stabilize the balance of payments and that it has an inflation target of 2% (i.e., b_B=0 and π_B=2%). As regards fiscal targets, we assume that the government aims to reach a more inflationary equilibrium, and a balance of payments surplus for electoral reasons or for the satisfaction of political lobbies.[15] We set its inflation target at 4% and the surplus target at 6% (i.e., π_G=4% and b_G=6%). We also assume β_B=3 and β_G=2: the central bank is more conservative with respect to achieving low inflation than the government.

As said, monetary policy sets the interest rate and fiscal policy sets taxation. The effects of these policies are controversial (especially regarding the fiscal one), but that debate is not of interest in the present example. Thus, in Mundell fashion, we simply assume that monetary and fiscal are strategic complements:[16] an increase in the interest rate lowers inflation and improves the surplus (reduces the deficit) in the balance of payments (i.e., the income effect outweighs the relative price or exchange rate effect). The same occurs when taxes are raised. The reduced form of the model is then represented as follows:

$$\begin{bmatrix} b \\ \pi \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -2 & -3/2 \end{bmatrix} \begin{bmatrix} i \\ t \end{bmatrix} + \begin{bmatrix} -22 \\ 35 \end{bmatrix} \tag{9}$$

From (4), it is easy to verify that equations (8) and (9) imply multiple equilibria. In fact both quasi-reaction functions are given by:

$$\pi = \frac{1}{3} b + 2 \tag{10}$$

which means they are coincident.

Next we assume that before the underlying game begins, nature selects one player to make a non-binding declaration. We then compute the announcement equilibrium. Expected outcomes of the underlying game are common knowledge. We set them equal to E(π)=3% and E(b)=3%, with variances equal to σ^2 for both.[17] The expected losses for the central bank and the government are then E(L_B)=6+2σ^2, and E(L_G)=5.5+1.5 σ^2, respectively.

The equilibrium outcomes are then reported in the following table.

	Nature selection									
	Central bank (announces interest rate)					Government (announces taxation)				
	announcement	equilibrium				announcement	equilibrium			
Variance	(interest rate)	b	π	T	i	(taxation)	b	π	t	i
2	4.16	2.27	2.76	31.89	4.16	23.34	3.87	3.29	23.34	7.10

[15] As, for example, in Grossman and Helpman (1994).

[16] See Dixit and Lambertini (2003) for a discussion of several possible interactions between central banks and governments. Different assumptions about the effects of fiscal and monetary policy do not affect the qualitative results of our example.

[17] Assuming a common-knowledge asymmetric belief about means and variances does not affect our results.

5	2.54	1.38	2.46	36.60	2.54	17.87	4.90	3.63	17.87	8.98
10	0.38	0.21	2.07	42.88	0.38	12.00	6.00	4.00	12.00	11.00

Table 1 – Monetary and fiscal announcements (all values in %)

If the variance about the expected outcomes is 2, the central bank will announce an interest rate equal to 4.16% (or an inflation rate of 2.76%). As the variance increases the central bank will reduce its announcement by 162 basis points when σ^2=5; or by 378 basis points when σ^2=10, and so on. The higher the uncertainty, the more favorable is the equilibrium which the central bank can reach by using its announcements. This occurs because uncertainty increases the expected losses of the government in the underlying game. A symmetric result is obtained when nature selects the government for making the first announcement.

5. Multiple messages: An announcement war

5.1 A two-sided announcement game

Following Rubinstein (1982), we now model a process of alternating announcements that takes place between the two players over time. At the beginning of the game, at time 0, one player, player *i*, announces an outcome of $\left[z_i^A(0)\right]$. The other player, *j*, observes the announcement and either declares that he is ready to play; or waits a period to make his own announcement $\left[z_j^A(1)\right]$. If player *j* declares that he is ready, the two players play the underlying game and simultaneously set their instruments without any binding commitment being made. If, on the other hand, player *j* decides to wait, a new announcement round starts: player *j* makes his announcement and player *i* can either declare that he is ready to play or wait one more period to make a new announcement about his own policy. This way many (possibly infinite) alternating rounds can follow each other.

Notice that the receiver of a message neither accepts any offer, nor signs any binding agreement: the choice of closing the "announcement war" is just a strategic choice available to the player. This means that, as in the one-sided announcement game, this more complex pre-play communication phase does not restrict the Nash equilibria of the underlying games; and the babbling equilibria also remain untouched. We do however assume that agents are time impatient. The later the underlying game is played the higher the loss of a given outcome will be. Thus talk is not "completely" cheap in this case as announcements are time-costly, but they remain unable to restrict the set of Nash equilibria.

The loss functions can then be rewritten as:

$$L_k\left(z,t\right)=\gamma_k^t L_k\left(z\right)=\gamma_k^t\frac{1}{2}\left[\left(x-\overline{x}_k\right)^2+\beta_k\left(y-\overline{y}_k\right)^2\right] \qquad k=\{1,2\} \tag{11}$$

where $\gamma_k>1$ are the agents' loss "capitalization" rates, or patience factors, equal to the inverse of the discount factor. These patience factors are not necessarily equal among the players.[18]

[18] Patience factors can be also interpreted as factors that discount the risk of breakdown of the bargaining process (see Osborne and Rubinstein, 1990: Chapter 4).

The set of credible announcements is the same as those described by Lemma 1, because outcomes outside the quasi reaction functions are associated with incentives to deviate; and because any announcements made have to Pareto dominate the outside option.[19]

However we need to redefine the announcement equilibrium for a two-sided announcement game.

Definition (multiple announcement equilibrium). *An announcement equilibrium of the two-sided game is a sub-game perfect Nash equilibrium of the first stage of the game constrained to the credible announcement set.*

Again the game is solved by backward induction. The second stage constrains the agents to bargain only over the set of credible announcements. Clearly, if an announcement equilibrium exists, it will completely describe the outcome of the entire game.

We can now state the following theorem and corollary.

Theorem 2. *If there is at least one credible announcement to be made, the two-sided announcement game has a unique announcement equilibrium. The outcome of the game is the one announced by player i at time zero, but it will never be his most favorable outcome among the credible announcements unless there is only one credible announcement.*

Proof: Appendix C.

Again the announcement equilibrium is unique and is one of the Nash equilibria. We can therefore conclude that announcements can act as an equilibrium section device.

Corollary 5. *The outcome associated with the above announcement equilibrium is the value* z_i^A *that solves the following two equation system:*

$$\begin{cases} L_i\left(z_j^A,0\right)=\gamma_i L_i\left(z_i^A,0\right)+\left(1-\gamma_i\right)\hat{L}_i \\ L_j\left(z_i^A,0\right)=\gamma_j L_j\left(z_j^A,0\right)+\left(1-\gamma_j\right)\hat{L}_j \end{cases} \tag{12}$$

where $\hat{L}_i$ *and* $\hat{L}_j$ *are the players' losses relative to their worst announcement equilibrium.*

Allowing the agents to play a war of announcements in order to select the Nash equilibrium gives us a richer tool for the equilibrium selection procedure, and one which again solves the problem of coordination failure. The difference from the case of a one-sided announcement is that the announcing agent now has to take into account the fact that the other player might refuse the current announcement and make his own offer instead.

Theorem 2 guarantees the existence of a unique announcement equilibrium in the two-sided announcement game under exactly the same conditions as the one-sided announcements described in Theorem 1. Therefore, the same conclusions can be drawn about the likelihood of observing agents announcing their policies and using announcements to solve coordination problems.

In contrast to one-sided announcement games however, players will never announce their most favorable outcomes among those that are credible here (unless there is only one credible announcement to be made). This result may seem surprising and needs some explanation. In this section, player *i* has to propose an outcome that is not only better for *j* than the outside option, as required by Theorem 1, *but also* one at least as good as the outcome that player *j* could obtain by

[19] Discount factors do not affect this set because the comparison between announced outcomes and the outside option always refers to the same period of time.

waiting and then having his own announcement accepted. In informal terms, we should add a condition according to which player *j* is at least indifferent between accepting the proposal of *i* at time 0, $z_i^A(0)$, and making an acceptable announcement in the next period. However, in order to be sure that $z_j^A(1)$ will also be accepted, this inequality requires a similar condition to be applied to *i* who also needs to be indifferent between accepting *j's* proposal at time 1 or making another proposal at time 2; and so on for $z_i^A(2)$. As a result we obtain an infinite set of conditions that can be easily solved by obtaining the finite system described above.

Nevertheless, equation (12) does not give us a closed-form solution. However, not surprisingly, numerical simulations show that the higher the player's patience, the higher the payoff associated with the announcement equilibrium he achieves. To get the intuition behind this result, it is useful to study some limiting cases. If player *i* is patient and *j* is not, the game of two-sided announcements leads to the same solution as the one-sided game: player *i* announces his most favorable outcome among the credible announcements. The opposite case, where only *j* is patient, is only apparently surprising: from among the credible announcements, player *i* in fact chooses the most favorable outcome for the other agent. In other words, player *i* will offer player *j* player *j*'s most favorable outcome from among those that *j* would have been prepared to accept. Hence, if player *j* can wait forever, player *i* can only offer player *j* the outcome corresponding to the best available to *j* from among those that are credible.[20]

It is also interesting to study the limiting case where the cost of making additional announcements converges to zero. This arises when the loss capitalization rates of the agents goes to 1. The result is presented in the next corollary:

Corollary 6. *At the limit when* $\gamma_i = \gamma_j \to 1$, *the outcome of the announcement equilibrium of the two-sided announcement game converges to the Nash cooperative bargaining solution to* $\min_z \left(\hat{L}_i - L_i(z)\right)\left(\hat{L}_j - L_j(z)\right)$, *constrained by the set of credible announcements.*

Although the selected outcome is equivalent to a type of Nash bargaining solution, it is not Pareto efficient. Similar to the case discussed in Section 4.3, this apparent puzzle can be explained by the fact that the agents announce strategies that are consistent with Nash equilibria, which are generally not efficient. The Nash bargaining solution achieves inferior allocations here as it is constrained to the set of (inefficient) Nash equilibria. It is thus efficient only among those equilibria. It is also worth noting that the problem at hand brings us to solutions that are similar to bargaining over Nash equilibria, even if the announcement stage does not include any explicit acceptance of offers and does not force the players to play the moves they have announced.

The above result is in line with Binmore (1987), who establishes a connection between the bargaining games of alternating offers and the usual cooperative Nash solution in a more general framework.[21] In particular, he shows that the value of the unique subgame-perfect equilibrium in a non-cooperative bargaining game of alternate offers converges to the unique Nash bargaining solution of the corresponding bargaining problem.

5.2 An example: Advertising, R&D and firms competition

[20] Limit cases have closed-form solutions that are available upon request.

[21] See Osborne and Rubinstein (1990, Chapter 4) for a textbook exposition.

Next we illustrate our results with an example of multiple announcements. Consider a game between two firms that have to decide the amount of advertising. The underlying game lasts two periods. One firm, chosen by nature, can make an announcement about its expenditure for the advertising campaign. More specifically, each firm has to decide, in the first period, the amount in its exogenously fixed budget that it wishes to allocate to increasing or decreasing its advertising expenditure. The rest is used for R&D in order to influence the results in the second period.

R&D can affect the relative quality of the product and thus the market share, whereas advertising affects the perceived quality of the products. A high amount for advertising *ceteris paribus* increases current market share, and reduces that for the rival firm, but reduces future market share as it corresponds to lower investment in R&D (and increases the market share of the rival firm). The fixed budgets of the two firms are common knowledge. Thus announcing a budget for advertising or for R&D comes to the same thing; moreover, credible announcements about the market share target are also equivalent.

Formally each firm aims to maximize its market share, $q_k(s)$, at both stages. In other words, it aims to minimize the deviation of market share, $q_k(s)$, from 100% (unity) in both periods:

$$L_k(t) = \delta_k^{\Delta t} \sum_{s=1}^{2} \delta_k^{s-1} \left(q_k(s) - 1\right)^2 \qquad k = \{1, 2\} \tag{13}$$

where $\delta_1 = 1.1$ and $\delta_2 = 1.15$ are the capitalization factors of the firms. Note that, Δt being the time interval between one announcement and another, the firms discount later agreements by using the factor δ_k.

We define $m(s)$ to be the market share of firm 1 at stage s, $q_1(s)$, it follows that $q_2(s) = 1 - m(s)$ and we can write the economic system as:

$$\begin{bmatrix} m(1) \\ m(2) \end{bmatrix} = \begin{bmatrix} 1.1 & -1 \\ -1.32 & 1.2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \tag{14}$$

Clearly firm 1's advertising spending (p_1) increases *ceteris paribus* the current market share (and reduces that of firm 2), but reduces future market shares (and increases that of the rival firm) because firm 1's investment in R&D will be lower.

Using the condition at (4), it is now easy to verify that this game admits multiple equilibria. Assume that the expected outcome of the underlying game is characterized by an equal division of the market, i.e., $E_0(m(s)) = 0.5$, with $\mathrm{var}(m(1)) = 0.1$, $\mathrm{var}(m(2)) = 0.2$, so uncertainty increases with time (without loss of generality, we assume that the expected market share in the two stages are not correlated).

The equilibrium now depends on the length of the interval between one announcement and another. The following figure describe the relationship between the market share announced and $\Delta\tau$.

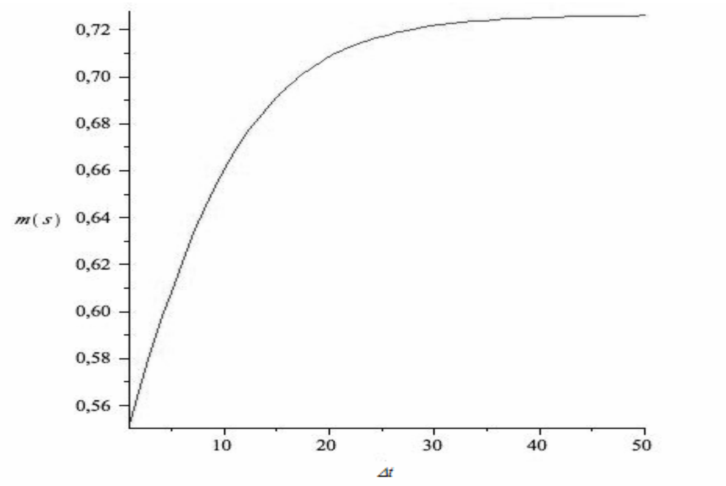

Figure 2 – Market shares and *Δt*.

The larger is the interval, the higher is the cost of a counter announcement. Thus, larger intervals give an advantage to the first mover. Indeed, the game converges to the unilateral announcement case when *Δt* becomes very large. Note that the market share announced will be the same for both stages.[22]

6. Conclusion

In models with multiple equilibria, without an equilibrium selection device, problems of coordination failure emerge. In such a case, a common strategy in the policy games literature has been to introduce binding commitments. This is very favorable to the committing agent, but needs an additional external enforcing mechanism to ensure credibility. Our paper explores an alternative mechanism.

Investigating the coordination problem in a model with multiple equilibria, we introduce a pre-game stage where cheap-talk promises (non-binding announcements) are available and the expected outcomes of the underlying game without announcements are considered as the natural (focal) outside option. Here announcements, while requiring some sort of commitment, are a "weaker" promise in comparison to full commitment to a specific action.

We find that announcements allow players to coordinate, and can thereby help reduce (but not entirely eliminate) inefficiencies even if the players are in conflict. Moreover, in order to assure some kind of credibility, announcements should be in general less favorable for the agent that makes the promise with respect to binding commitment.

The introduction of an outside option as a focal point strongly affects the cheap talk results. Specifically, it restricts the set of credible announcements to the set of feasible Nash equilibria as it requires some sort of commitment that depends on the outside option. Multiple announcements strengthen this result further as the sender has to take account also of the possibility that the receiver can dismiss the proposal and announce a new one of his own. Allowing for cheap talk announcements by both agents, we have also been able to derive additional insights into the outcomes without affecting the properties of the game. Indeed, using sequential announcements

[22] In fact, if firm 1 announces high expenditure for advertising, firm 2 (if it believes the announcement) will respond by increasing its expenditure on advertising. In the second stage, both will then invest less in R&D and the market shares will remain unchanged.

from both sides we always have a *unique* equilibrium that corresponds to implementing a Nash bargaining solution with appropriate weights representing bargaining powers: the “announcement war” outcome.

Finally, by focusing (mainly) on examples based on applied macroeconomic modeling, we also show how the existing tools of bargaining can be fruitfully employed also for macroeconomic issues (such as fiscal and monetary policy coordination) and produce sensible outcomes.

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Appendix A: Proof of Lemma 1

Agent k will prefer the outcome z^* on the quasi reaction function (5), to the outside option, if $L_k(z) = \alpha_k (x - \overline{x}_k)^2 \le \tilde{L}_k$ for $k = \{1,2\}$, where $\alpha_k = 1 + a_{1k}^2 / \beta_k a_{2k}^2$. We can rewrite this condition as $\left| x - \overline{x}_k \right| \le \sqrt{\frac{1}{\alpha_k} \tilde{L}_k}$, which must hold for both agents. Lemma 1 now follows.

Appendix B. Proof of Theorem 1.

As explained in Section 2, if an announcement is credible it will be implemented in the second stage of the game. Since the instruments are linearly independent, if there is a unique optimal announcement, the Nash equilibrium of the game will also be unique. Thus, to prove the theorem, we need only show that an optimal announcement for player *i* exists and is unique. We do this in the following three steps:

Step 1. *If the set of credible announcements (Ψ) is non empty, it contains at least one outcome of the set Θ (defined in Section 4.1, Corollary 2). Formally, $\Omega = \Theta \cap \Psi \ne \varnothing$. Moreover, Ω is a compact set.*

Let us assume, without loss of generality, that $\overline{x}_i < \overline{x}_j$. This implies the announcement set defined by Lemma 1 is not empty if and only if

(b.1) $\quad \overline{x}_i + \sqrt{\frac{1}{\alpha_i} \tilde{L}_i} \le \overline{x}_j - \sqrt{\frac{1}{\alpha_j} \tilde{L}_j}$.

We can rewrite this condition as a convex combination of the agents' first best. Using equation (7), we rewrite (b.1) as:

(b.2) $\quad \lambda_i \overline{x}_i + (1 - \lambda_i) \overline{x}_j \le \lambda_j \overline{x}_i + (1 - \lambda_j) \overline{x}_j$

where the pair (λ_i, λ_j) satisfies $\overline{x}_i + \sqrt{\frac{1}{\alpha_i} \tilde{L}_i} = \lambda_i \overline{x}_i + (1 - \lambda_i) \overline{x}_j$ and $\overline{x}_j + \sqrt{\frac{1}{\alpha_j} \tilde{L}_j} = \lambda_j \overline{x}_j + (1 - \lambda_j) \overline{x}_i$.

Equation (b.2) holds if and only if $\lambda_j \ge \lambda_i$. Moreover, $\lambda_i \le 1$ and $\lambda_j \ge 0$, since $\overline{x}_i + \sqrt{\frac{1}{\alpha_i} \tilde{L}_i} \ge \overline{x}_i$ and $\overline{x}_j - \sqrt{\frac{1}{\alpha_j} \tilde{L}_j} \le \overline{x}_j$. Then there exists a $\lambda^* \in [0,1]$ such that $\lambda_j \ge \lambda^* \ge \lambda_i$. The subset Ω is therefore non empty. It is moreover compact, since it is the intersection of two compact sets.

Step 2. *Agent i announces the outcome z^*, which is his most favorable outcome in the subset Ω.*

Agent *i* minimizes a continuous function on a compact set (Ψ). This guarantees the existence of at least one optimal announcement. Now assume agent *i* selects his most favorable outcome that does not belong to the subset Ω.There are then two cases in which a credible announcement could be Pareto dominated:

a) If $z' = \lambda' \overline{z}_i + (1 - \lambda') \overline{z}_j$ for some $\lambda' < 0$. Since z' belongs to the credible announcement set, $\overline{z}_i$ is also credible. Therefore agent *i* will never announce z' since it does not minimize his loss among the credible announcement set. This leads to a contradiction. b) If $z'' = \lambda'' \overline{z}_i + (1 - \lambda'') \overline{z}_j$ for some $\lambda'' > 1$, then by a similar argument it can be shown that $\overline{z}_j$ belongs to the credible announcement set that agent *i* would strictly prefer to z''. Again we find a contradiction. Hence step 2 follows.

Step 3. *The announced outcome is unique.*

Assume that the announced outcome is not unique. Then outcomes z' and z'' exist, such that $z' \neq z''$ and $L_i(z') = L_i(z'')$, that minimize player *i's* loss function. Next define $z''' = \lambda z' + (1-\lambda)z''$ with $\lambda \in (0,1)$. Then z''' is a) credible, since the credible set is convex; and b) such that $L_i(z''') < L_i(z') = L_i(z'')$, since the loss is strictly concave. Thus, the announced outcome is unique.

Appendix C. Proof of Theorem 2.

Following Step 1 of Appendix B, we can restrict ourselves to the announcements corresponding to outcomes in Ω. As these are uniquely defined by $\lambda \in \left[\underline{\lambda}, \overline{\lambda}\right]$ according to equation (7), with an small abuse of notation we can refer to them simply by using λ. This is legitimate because the subset Ω contains the outcomes satisfying (7) that can be expressed and ordered by $U_k(\lambda, t) = \gamma_k^t \left[\hat{L}_k - L_k(\lambda)\right]$.

Define the pair of losses associated to the player's worst outcomes, $\hat{L}_i = L_i(\underline{\lambda})$ and $\hat{L}_j = L_j(\overline{\lambda})$. Then, by an affine transformation, the preferences (8) on the set Ω can be rewritten, for convenience, as utility functions:

(c.1) $\quad U_k(\lambda, t) = \gamma_k^t \left[\hat{L}_k - L_k(\lambda)\right] \quad k = \{1,2\}$

These utility functions satisfy the following properties: 1) they imply a conflict between the two players (when λ increases, the utility of *i* rises and the utility of *j* falls); 2) player *k*'s utility strictly decreases in *t*, except for his worst outcome; 3) *k*'s utility for his worst outcome is zero, independently of time; 4) utilities are time stationary and continuous.

These conditions guarantee that we can apply Rubinstein (1982)'s theorems to obtain the result of Theorem 2 and Corollary 3, and Osborne and Rubinstein (1990: Section 4.4) to prove Corollary 4.