Quantity competition vs. price competition under optimal subsidy in a mixed duopoly

Marcella Scrimitore
University of Salento (Italy)

Abstract. This paper reconsiders the literature on the irrelevance of privatization on mixed markets, addressing competition in a duopoly with both quantity and price competition. We assume partial privatization under different time structures and derive the conditions on firm ownership under which an optimal subsidy allows to achieve maximum efficiency. We find that, independently of the mode of competition, the firms' ownership structure is irrelevant when firms play simultaneously and conversely matters when they compete sequentially, requiring the state-controlled firm to be publicly-owned in order to let the optimal subsidy restore first best. The paper also focuses on the extent to which a subsidy is necessary to attain the social optimum, highlighting the equivalence between a price (quantity) game with public leadership or simultaneous moves and a quantity (price) game with private leadership.

JEL codes: H21, H44, L13

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Dipartimento di Scienze Economiche e Matematico-Statistiche, Università del Salento, Lecce (Italia). Tel. +39 0832298772; fax +39 0832298757. E-mail: marcella.scrimitore@unisalento.it.

1 Introduction

This paper contributes to a growing literature which advocates the use of subsidies in mixed markets. A number of papers discuss the effectiveness of production subsidies, chosen by a government on a welfare-maximizing basis, in restoring the first-best allocation, and focus on the absence of consequences from privatization when governments undertake such subsidization policies. This irrelevance result has been first highlighted by White (1996) in a context of simultaneous actions in which the optimal subsidy, all the equilibrium market variables and social welfare are shown to be identical before and after privatization.[1] By addressing sequential quantity competition, Poyago-Theotoky (2001) and Myles (2002)[2] prove that the irrelevance result is not conditional on the order of firms' moves and holds even when the public firm moves as a leader. Their analysis, however, relies on the assumption that firms compete simultaneously in the privatized market, so that their result derives from the equivalence between the simultaneous and the sequential game caused by the operating of an effective subsidy in a mixed market. Indeed, by assuming that the public firm keeps the leadership after privatization, the irrelevance result does not hold anymore, as shown by Fjell and Heywood (2004) in a quantity setting. An explanation for this result is that while an optimal subsidy succeeds in implementing the first best in a mixed market irrespective of whether the public firm plays simultaneously against the private rivals or rather acts as a leader in the market, it fails to do it in a private market à la Stackelberg.

The above literature clearly shows that an irrelevance result applies when the same amount of subsidy is necessary to recover the social optimum, despite a different ownership regime or a different order of firms' moves. By addressing both quantity and price competition in a differentiated duopoly, and by assuming different timing structures including simultaneous moves and sequential moves with public or private leadership, the present paper focuses on the extent to which an optimal subsidy ensures the social optimum in the considered scenarios. Among these, it identifies those which determine an irrelevance result with respect to both the optimal privatization policy and the game timing. Our analysis, indeed, on the one hand allows us to relate the optimal subsidy's amount to the toughness of market competition, defined by higher or lower firms' aggressiveness under different timings, and to capture the frictions which may prevent subsidies from achieving efficiency objectives. On the other hand the model, by defining the optimal degree of firm's ownership through partial privatization, sheds light on those competitive scenarios in which an irrelevant result with respect a privatization policy applies, for any given assumed order of firms' moves.[3] The comparison across the different frameworks also allows

[1] The analysis has been also extended to a price competition framework with differentiated products by Hashimzade et al. (2007).

[2] Myles extends Poyago-Theotoky's analysis to a setting with general demand and costs.

[3] Partial privatization was first addressed by Matsumura (1998) and then extended to a number of competitive settings, included the product differentiation framewok by Fujiwara (2007). Under partial privatization the government is allowed to decide upon the optimal own-

us to highlight the cases in which the optimal subsidies and the outcomes at equilibrium coincide regardless of the timing or the mode of competition.

The results obtained are the following. For both the Cournot and the Bertrand model, we demonstrate that the irrelevance of partial or full privatization holds in the presence of simultaneous moves. By contrast, when sequential moves are assumed, public ownership of the state controlled firm is required for an optimal subsidy to restore efficiency. Moreover, the paper establishes an equivalence between games with public leadership and games with simultaneous moves, irrespective of the mode of competition, and an equivalence between Cournot (Bertrand) public leadership and Bertrand (Cournot) private leadership under optimal subsidy.

The paper is organized as follows: Section 2 presents the model, while Section 3 discusses the main results and draws some conclusions.

2 The model

Two technologically identical firms are assumed to compete in quantities or prices facing a linear demand on a market with differentiated products. One firm is private, we denote it as firm 2, while the ownership structure of the other one, the *ex-ante* public firm denoted as firm 1, is defined following the decision upon its optimal ownership structure by a welfare-maximizing government. As standard in the literature on partial privatization, the government optimally chooses whether to retain full ownership of a firm or rather to share ownership with the private sector or, finally, to fully privatize it. The different alternatives are captured by the parameter α attached to public firms' profit, with $\alpha \in (0.1)$, ranging from full nationalization ($\alpha = 0$) to full privatization ($\alpha = 1$), and entailing partial privatization in all the intermediate cases. The government selects the optimal degree of privatization for its firm at the first stage of a game which describes, at the last stage(s), simultaneous or sequential competition against the private firm on the product market. In our model a further stage is added to this game, since the hypothesis on firms' subsidization is introduced. Reasonably, we assume that the government first decides upon firm 1's optimal ownership structure, then chooses the optimal subsidy to give both firms which finally compete in prices or quantities.

2.0.1 Quantity competition

We assume the inverse linear demand $p_i = 1 - q_i - \gamma q_j$ $(i = 1, 2)$ deriving from a quadratic utility function, where the parameter γ, with $\gamma \in (0, 1)$, captures the degree of product substitutability (goods are independent, weakly substitute or perfect substitute according to whether $\gamma = 0$, $0 < \gamma < 1$ or $\gamma = 1$). We assume

ership structure of the controlled firm, namely it chooses to retain full ownership of that firm or rather to fully/partially privatize it. An irrelevance result emerges in such a context when a given market outcome and the underlying optimal subsidy are sustained as an equilibrium, irrespective of public firm's ownership.

that constant marginal costs c and null fixed costs are sustained by firm 1 and firm 2 and that both firms receive an undifferentiated subsidy on production. We first address simultaneous competition, then we consider sequential competition with the public firm in the role of the leader (public leadership indexed by PL) and finally solve a sequential game with the private firm in the role of the leader (private leadership indexed by $\Pr L$).[4]

Simultaneous moves

Given the following profit functions of the two firms:
$\pi_1(q_1, q_2, s) = (1 - q_1 - \gamma q_2 - c) q_1 + s q_1$
$\pi_2(q_1, q_2, s) = (1 - q_2 - \gamma q_1 - c) q_2 + s q_2$

and the consumers' surplus $CS(q_1, q_2) = (1 - \gamma)\left(q_1^2 + q_2^2\right) + \gamma (q_1 + q_2)^2 / 2$,

we define the social welfare function as the sum of consumers' surplus and the aggregate profits of subsidized firms, net of the social cost of subsidies:[5]

$$W(q_1, q_2) = CS(q_1, q_2) + \sum_{i=1}^{2} \pi_i(q_1, q_2, s) - s(q_1 + q_2) \tag{1}$$

At the last stage of the game, firm 1 maximizes a weighted average of social welfare and its own profits $G_1(q_1, q_2, s, \alpha) = \alpha W(q_1, q_2) + (1 - \alpha)\pi_1(q_1, q_2, s)$.

The first order condition (FOC) $\partial G_1(q_1, q_2, s, \alpha) / \partial q_1 = 0$ is satisfied at the following firm 1's quantity:

$$q_1^*(q_2, s, \alpha) = \frac{1 - c + s(1 - \alpha) - \gamma q_2}{2 - \alpha} \tag{2}$$

At the same game stage, firm 2 maximizes its own profits by choosing that quantity which satisfies the FOC $\partial \pi_2(q_1, q_2, s) / \partial q_2 = 0$. The following reaction function is obtained:

$$q_2^*(q_1, s) = \frac{1 - c + s - \gamma q_1}{2} \tag{3}$$

The solution of the system of the two reaction functions in $q_1^*(q_2, s, \alpha) = R_1(q_2, s, \alpha)$ and $q_2^*(q_1, s) = R_2(q_1, s)$ yields the following optimal quantities:

[4] In the model we take as given the order of firms' moves. As well-known in the microeconomic literature, the role played by firms (leader, follower or simultaneous player) can be optimally determined by solving an observable delay game. See Pal (1998), Bàrcena-Ruiz (2007) and Tomaru and Saito (2010) as examples in the mixed oligopoly literature. Among these, Tomaru and Saito (2010) tackle the issue of endogenous timing in a market with subsidized firms.

[5] Notice that social welfare is not directly affected by the subsidy s which conversely impacts on both firms' profits.

$$
\begin{aligned}
q_1^*(s,\alpha) &= \frac{(2-\gamma)(1-c)+s(2(1-\alpha)-\gamma)}{(2-\gamma)(2+\gamma)-2\alpha} && (4)\\
q_1^*(s,\alpha) &= \frac{(2-\gamma-\alpha)(1-c)+s(2-\alpha-\gamma(1-\alpha))}{(2-\gamma)(2+\gamma)-2\alpha} && (5)
\end{aligned}
$$

By substituting (4) and (5) in the social welfare function in (1) and by maximizing it with respect to s, we obtain the optimal subsidy chosen by the government, denoted by s^C in this simultaneous Cournot game:

$$
s^C = \frac{1-c}{1+\gamma} \tag{6}
$$

The following remark, extending the privatization neutrality theorem proved by White (1996, Poyago-Theotoky (2001) and Myles (2002) and confirming the result Tomaru (2006),[6] is straightforward.

Remark 1 In a quantity game with simultaneous moves, the optimal subsidy is independent of α and succeeds in restoring the first best solution $q_i^C = q_i^{SO} = (1-c)/(1+\gamma)$ (SO denoting the social optimum) and $p_i^C = p_i^{SO} = c$ $(i = 1, 2)$. Social welfare is maximum whatever α and is equal to $W^{SO} = (1-c)^2/(1+\gamma)$. Indeed, the optimal subsidy and social welfare are not affected by firm 1's ownership structure, which confirms the irrelevance of privatization and partial privatization in a simultaneous quantity setting .

Sequential moves with the public firm in the role of leader (Public Leadership - PL)

Under quantity public leadership, the public firm takes as given the reaction function $R_2(q_1, s, \alpha)$ of the private firm in (3) which moves at the last stage of the game. The objective function of the government is therefore expressed as a function of q_1 only and is:
$G_1(q_1, s, \alpha) = \alpha W(q_1, R_2(q_1, s, \alpha)) + (1-\alpha)\pi_1(q_1, R_2(q_1, s, \alpha))$.

By maximizing $G_1(q_1, s, \alpha)$ and substituting the optimal public firm's quantity in firm 1's reaction function, we obtain the following solutions:

$$
\begin{aligned}
q_1^*(s,\alpha) &= \frac{(1-c)(2\gamma-4+\alpha\gamma)+s(2\gamma-4+\alpha(4-\gamma))}{4(\gamma^2-2)+\alpha(2-\gamma)(\gamma+2)}\\
q_2^*(s,\alpha) &= \frac{(1-c)\left(\gamma(2+\gamma)-4+\alpha\left(2-\gamma^2\right)\right)+s(\gamma(2+\gamma)-4+2\alpha(1-\gamma))}{4(\gamma^2-2)+\alpha(2-\gamma)(\gamma+2)}
\end{aligned}
$$

The solution of the FOC $\partial W(s,\alpha)/\partial s = 0$ yields the optimal subsidy $s_{PL}^C(\alpha,\gamma)$ which is reported in the Appendix.

[6] Tomaru (2006) first demonstrated in a quantity setting with simultaneous-moves game that the irrelevance result survives the introduction of partial privatization.

Solving for the optimal degree of privatization at the first stage of the game, we obtain $\alpha^* = 1$ as a solution. At the Subgame Perfect Nash Equilibrium (SPNE) the optimal subsidy is:

$$s^C_{PL} = \frac{1-c}{1+\gamma} \tag{7}$$

with the market variables coinciding with the efficient outcomes and the social welfare achieving its maximum.

Sequential moves with the public firm in the role of follower (Private Leadership - $\Pr L$)

Under quantity private leadership, the private firm takes as given the reaction function $R_1(q_2, s, \alpha)$ of the public firm in (2) which moves at the last stage of the game. By maximizing $\pi_2(R_1(q_2, s, \alpha), q_2)$ and by substituting the solution in the firm 1's reaction function, we obtain the following quantities:

$$\begin{aligned} q_1^*(s,\alpha) &= \tfrac{\left(4-\gamma^2-2\alpha-\gamma(2-\alpha)\right)(1-c)+s\left(4-\gamma^2-2\alpha(3-\alpha)-\gamma(2-\alpha(1+\gamma))\right)}{2(2-\gamma^2-\alpha)(2-\alpha)} \\ q_2^*(s,\alpha) &= \tfrac{(1-c)(2-(\alpha+\gamma))+s(\alpha\gamma+2-(\alpha+\gamma))}{2(2-\gamma^2-\alpha)} \end{aligned}$$

The solution of the FOC $\partial W(s,\alpha)/\partial s = 0$ to the welfare-maximization problem is given by the optimal subsidy $s^C_{\Pr L}(\alpha,\gamma)$ which is reported in the Appendix.

We substitute $s^C_{\Pr L}(\alpha,\gamma)$ in the social welfare function and solve its maximization problem with respect to α, thus obtaining the optimal degree of privatization $\alpha^* = 1$. At the SPNE, the welfare-maximizing subsidy is:

$$s^C_{\Pr L} = (1-c)(1-\gamma) \tag{8}$$

yielding first best quantities and prices, and the highest social welfare.

2.0.2 Price competition

We keep the assumptions on demand and costs of the quantity framework, addressing price competition in simultaneous and sequential moves as in the previous model.

Simultaneous moves

Given the direct demand function $q_i = \dfrac{(1-\gamma)-p_i+\gamma p_j}{(1-\gamma^2)}$ $(i = 1.2)$, the profit functions of the two firms are:

$\pi_1(p_1,p_2,s) = (p_1-c)\left(\frac{(1-\gamma)-p_1+\gamma p_2}{(1-\gamma^2)}\right) + s\left(\frac{(1-\gamma)-p_1+\gamma p_2}{(1-\gamma^2)}\right)$
$\pi_2(p_1,p_2,s) = (p_2-c)\left(\frac{(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)}\right) + s\left(\frac{(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)}\right)$

The consumers' surplus is $CS(p_1,p_2) = \frac{p_1^2+p_2^2+2(1-p_1-p_2)-2\gamma(1-p_1)(1-p_2)}{2(1-\gamma^2)}$, so that the social welfare is:

$$W(p_1,p_2) = CS(p_1,p_2) + \sum_{i=1}^{2} \pi_i(p_1,p_2,s) - s\left(\frac{2-p_1-p_2}{\gamma+1}\right) \tag{9}$$

Given $G_1(p_1,p_2,s,\alpha) = \alpha W(p_1,p_2) + (1-\alpha)\pi_1(p_1,p_2,s)$, at the last stage the public firm maximizes this function choosing the following price:

$$p_1^*(p_2,s,\alpha) = \frac{(1-\alpha)(1-\gamma) + c(1-\alpha\gamma) - s(1-\alpha) + \gamma p_2}{2-\alpha} \tag{10}$$

At the same stage the private firm's maximizes its own profits by choosing the following price which satisfies the FOC $\partial\pi_2(p_1,p_2,s)/\partial p_2 = 0$:

$$p_2^*(p_1,s) = \frac{1+c-\gamma(1-p_1)-s}{2} \tag{11}$$

By solving the system of the reaction functions in (10) and (11), we obtain the following optimal prices:

$$p_1^*(s,\alpha) = \frac{2c(1-\alpha\gamma) + \gamma c + (2(1-\alpha)+\gamma)(1-\gamma-s)}{(2-\gamma)(\gamma+2)-2\alpha} \tag{12}$$

$$p_2^*(s,\alpha) = \frac{c(2-\alpha) + \gamma c(1-\alpha\gamma) + (2-\alpha+(1-\alpha)\gamma)(1-\gamma-s)}{(2-\gamma)(\gamma+2)-2\alpha} \tag{13}$$

The subsidy maximizing $W(s,\alpha)$, obtained by substituting (12) and (13) in the welfare function in (9), is denoted by s^B in this simultaneous Bertrand game and is:

$$s^B = (1-c)(1-\gamma) \tag{14}$$

We can now establish the following remark.

Remark 2 The irrelevance of the ownership structure.of a state controlled firm in a market with subsidized firms also applies to a price simultaneous-moves setting . Indeed, the optimal subsidy in (14) is independend of α and succedes in restoring the first best quantities $q_i^B = q_i^{SO} = (1-c)/(1+\gamma)$, with clearing market prices equal to $p_i^B = p_i^{SO} = c$ $(i = 1,2)$. Social welfare is maximum whatever α and is equal to $W^{SO} = (1-c)^2/(1+\gamma)$.

Sequential moves with the public firm in the role of leader (Public Leadership - PL)

Under price public leadership, the public firm takes as given the reaction function $R_2(p_1, s)$ of the private firm in (11) which moves at the last stage. The objective function of the government is:
$G_1(p_1, s, \alpha) = (\alpha W(p_1, R_2(p_1, s)) + (1-\alpha)\pi_1(p_1, R_2(p_1, s)))$.

By maximizing $G_1(p_1, s, \alpha)$ with respect to p_1 and by substituting the optimal public firm's price in the rival's reaction function, we obtain the following solutions:

$$p_1^*(s,\alpha) = \frac{(1-\gamma)(2(2+\gamma)-\alpha(4+\gamma))+c(2(2+\gamma)-\gamma(2\gamma+3\alpha))-s(2(1+\gamma)(2-\gamma)-\alpha(4+\gamma(1-2\gamma)))}{4(2-\gamma^2)-\alpha(2-\gamma)(2+\gamma)}$$
$$p_2^*(s,\alpha) = \frac{(1-\gamma)\left(4-\gamma^2-2\alpha\gamma+2\gamma-2\alpha\right)+c\left(2(2+\gamma)-\gamma^2(1+\gamma)-\alpha\left(2+\gamma^2\right)\right)}{4(2-\gamma^2)-\alpha(2-\gamma)(2+\gamma)} - \\ - \frac{s\left(4+\gamma(2+\gamma)(1-\gamma)+\alpha\left(\gamma^3-2(1+\gamma)\right)\right)}{4(2-\gamma^2)-\alpha(2-\gamma)(2+\gamma)}$$

The solution to the FOC $\partial W(s,\alpha)/\partial s = 0$ yields the optimal subsidy $s_{PL}^B(\alpha,\gamma)$, which is reported in the Appendix.

The search for the optimal degree of privatization, gives $\alpha^* = 1$ as optimal solution. At this equilibrium the optimal subsidy is:

$$s_{PL}^B = (1-c)(1-\gamma) \tag{15}$$

and the first best is achieved.

Sequential moves with the public firm in the role of follower (Private Leadership - $\Pr L$)

Under price private leadership, the private firm takes as given the reaction function of the public firm $R_1(p_2, s, \alpha)$ which moves at the last stage of the game. By maximizing $\pi_2(R_1(p_2, s, \alpha), p_2)$ and by substituting the optimal private firm's quantity in firm 1's reaction function, we obtain the following solutions:

$$p_1^*(s,\alpha) = \frac{(1-\gamma)\left(4-\gamma^2+2\gamma\right)+\alpha\gamma(5-2\alpha+\gamma(2-\gamma))-2\alpha(3-\alpha)+c(4+\gamma(2+\gamma)(1-\gamma))}{2(2-\alpha-\gamma^2)(2-\alpha)} - \\ - \frac{\alpha c\left(2+\gamma\left(5-2\alpha-\gamma^2\right)\right)-s(6\alpha-\alpha(2\alpha-\gamma(1-\gamma))-4-\gamma(2+\gamma)(1-\gamma))}{2(2-\alpha-\gamma^2)(2-\alpha)}$$
$$p_2^*(s,\alpha) = \frac{(1-\gamma)(2+\gamma-\alpha(1+\gamma))-\alpha c\left(1+\gamma^2\right)+c(1+\gamma)(2-\gamma)+s(1+\gamma)(\alpha+\gamma-2)}{2(2-\alpha-\gamma^2)}$$

The optimal subsidy chosen by the welfare-maximizing government, denoted by $s_{\Pr L}^B(\alpha,\gamma)$, satisfies the FOC $\partial W(s,\alpha)/\partial s = 0$ and is reported in the Appendix.

Finally, the search for the optimal α reveals that $\alpha^* = 1$. At the SPNE the folllowing subsidy maximizes sociale welfare and restores the first best:

$$s^{B}_{\Pr L} = \frac{1-c}{\gamma+1} \tag{16}$$

3 The main results

In this section we discuss the results presented in the previous section. By endogeneizing the optimal ownership structure of the state controlled firm, we have identified the conditions under which a welfare-maximizing subsidy succedes in maximizing allocative efficiency. These conditions are established in the following proposition.

Proposition 1 *A welfare-maximizing subsidy operates effectively independently of firm* 1*'s ownership structure when firms compete simultaneously on the market. This confirms that an ownership irrelevance result only applies to simultaneous games, irrespective of the mode of competition. By contrast, an irrelevance result does not apply to sequential games for which the optimal subsidy requires firm* 1 *to be entirely owned by the public sector, namely to maximize pure welfare, in order to achieve the first best.*

Proof: It follows from Remark 1 and Remark 2.

A subsidy evaluated on a welfare-maximizing basis succeeds in maximizing economic efficiency when it induces all competing firms to produce the efficient quantity and set the efficient price. In this paper we demonstrate that a subsidy has a potential to enhance the efficiency of private resource allocation, provided that firms value equally subsidies at the margin. This is the case in which firms act simultaneously and the amount of the subsidy is optimally defined on the basis of the incentive of identical firms to behave efficiently. When sequential actions are assumed, the incentives of a private leader and a private follower towards a socially optimal conduct differ, so that a unique subsidy fails to induce a socially optimal behavior by both firms. In contrast, the presence of a welfare maximizing firm on the market allows a government to weight the subsidy according to the private firm's incentives only, since the public firm's incentive is already direct to maximize welfare, thus aligning the objectives of different firms. Thus, in a mixed market an optimal subsidy becomes effective even in a context of sequential competition where a subsidy regulates the private leader/follower's conduct in line with the public firm's. A subsidy corrective of allocative distortions, however, is found to be independent on the order of firms' moves only in a pure mixed market. In a partially privatized market, as in a private market, the need to incentivate towards efficiency both the private firm and the rival concerned to some extend with profits cannot be met by applying a unique subsidy. Tackling this question, our analysis reveals how the effectiveness of a subsidy policy does not depend on the ability to restore firms' cost efficiency, as stated by Fjell and Heywood (2004), rather on the possibility to align firms' conduct on the efficient outcome through an undifferentiated subsidy.

By comparing the settings with simultaneous moves and public leadership, and by focusing on the extent to which a subsidy is provided, we state the first equivalence result in the following proposition.

Proposition 2 *Under both quantity and price competition, the optimal subsidy under simultaneous moves coincides with the subsidy under public leadership. Indeed:* $s^C = s^C_{PL} = (1-c)\,/\,(1+\gamma)$ *and* $s^B = s^B_{PL} = (1-c)\,(1-\gamma)$.

Proof: It descends from (6-7) and (14-15).

The above proposition demonstrates that the same amount of the corrective subsidy is required in markets with simultaneous moves and public leadership, for any given mode of competition. Indeed, through the subsidy the welfare-maximizing government drives the private firm towards efficiency, shifting its reaction function in these frameworks. The latter represents the optimal reply of a private simultaneous player to any given quantity of the public firm in the first case and the private follower's reaction embedded in the public leader's reaction function in the second case. As shown in Figure 1a and Figure 2a where the reaction functions of the two firms R_1 and R_2 are depicted for $\alpha = 1$ and generic values of s, the equilibrium under simultaneous moves C and B and the equilibrium under public leadership PL, are identified by a point on the R_2 curve. Efficiency is achieved by shifting the R_2 curve until it crosses the R_1 curve in SO. The measure of this shift, namely the higher production or the lower price induced by the optimal subsidy, is clearly independent of the *ex-ante* public firm's choice, which can differ depending on whether that firm acts as simultaneous player or leader in the market, but does not affect the optimal private firm's behavior.

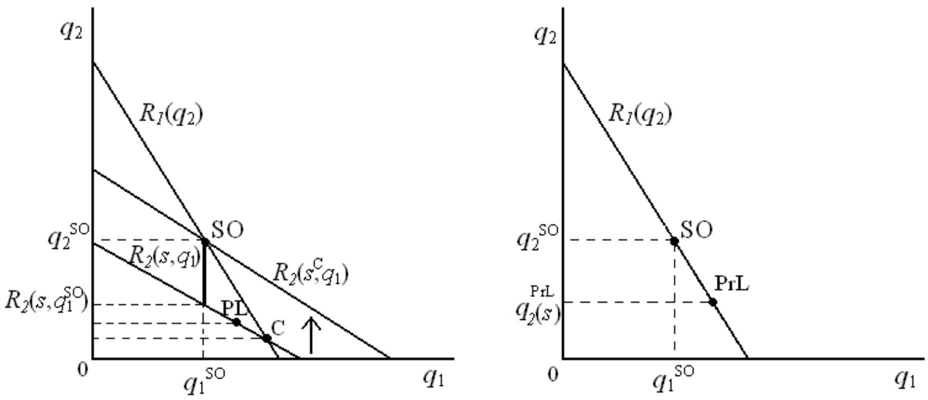

Figure 1 The firms' reaction functions and the equilibria under simultaneous moves and public leadership (case a) and under private leadership (case b) in the quantity competition game.

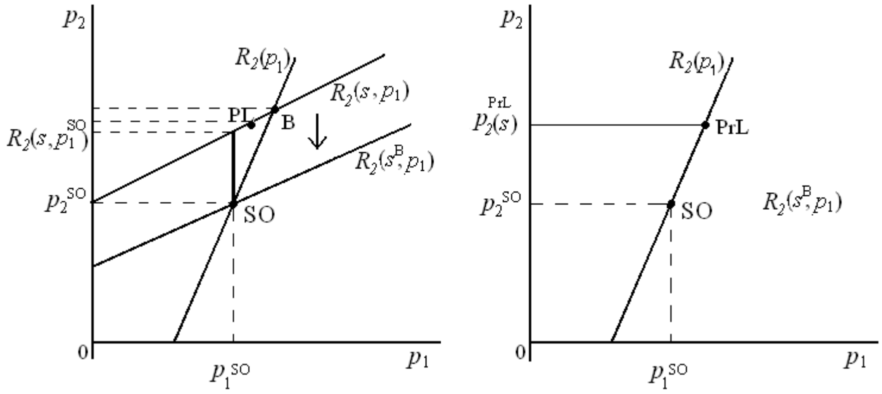

Figure 2 The firms' reaction functions and the equilibria under simultaneous moves and public leadership (case a) and under private leadership (case b) in the price competition game.

A comparison across all settings introduces the following proposition.

Proposition 3 *With respect to the optimal subsidy , a sequential quantity (price) game with private leadership is equivalent to a simultaneous or a sequential price (quantity) game with public leadership or sequential moves. Indeed:* $s^C = s^C_{PL} = s^B_{\Pr L} = (1-c)\,/\,(1+\gamma)$ *and* $s^B = s^B_{PL} = s^C_{\Pr L} = (1-c)\,(1-\gamma)$.

Proof: It descends from (6-7-8) and (14-15-16).

When quantiy competition is assumed, while the same subsidy is provided at equilibrium under simultaneous moves and public leadership, a subsidy of smaller magnitude is required under private leadership $((1-c)\,(1-\gamma) < (1-c)\,/\,(1+\gamma))$. In the latter case the provision of a subsidy is finalized to discipline the behavior of a private firm which anticipates the more aggressive reaction of a welfare-maximizing public firm and exploits its position of first-mover to expand its production, consistently with the aim of maximizing profits under strategic substitutability. This increased aggressiveness reduces the behavioral differences between the two firms and thus the distortion from the social optimum with respect to the games with simultaneous moves or public leadership. A similar argument applies to the price competition case to demonstrate that the optimal subsidy under private leadership is of a greater magnitude than the equivalent subsidy under simultaneous moves or public leadership. The subsidy regulates in this case the behavior of a private leader that anticipates the aggressive reaction of a welfare-maximizing follower and under strategic complementarity takes the advantage of being the first-mover setting a price that is higher than in the two other cases. This choice widens the behavioral firms' differences and the distortion from the social optimum, thus requiring a higher subsidy in order to achieve the first best solution. Proposition 3, indeed, states that the lower (greater) subsidy required under private leadership in a quantity (price) setting coincides with the subsidies characterizing the more (less) efficient price (quantity) competition in sequential moves and public leadership. These equivalence results are demonstrated as follows.

The equivalence between a price game with private leadership and a quantity game with public leadership.

In this paragraph we show how the same optimal subsidy $s^* = (1-c)/(1+\gamma)$ restores the first best under both price private leadership and quantity public leadership/simultaneous moves, since it induces the same price reduction by the private firm.

In a price game with private leadership, the reaction function of the public firm evaluated at $\alpha^* = 1$ is $R_1(p_2) = c(1-\gamma) + \gamma p_2$, which is clearly independent of s. For a generic value of s, the equilibrium with private leadership is represented in Figure 2a by point $\Pr L$ on the $R_1(p_2)$ curve at which the private firm sets the price $p_2^{\Pr L}(s) = \frac{1+c(1+2\gamma)-s(1+\gamma)}{2(1+\gamma)}$. A subsidy on production disciplines firm 2's behavior incentivating it to set lower prices through an output expansion. The price reduction needed for firm 2's to behave efficiently and set $p_2^{SO} = c$ is therefore measured by the difference $\Phi_1 = p_2^{\Pr L}(s) - p_2^{SO} = \frac{1-c-s(1+\gamma)}{2(1+\gamma)}$ which shrinks to zero at the optimal subsidy $s^B_{\Pr L} = (1-c)/(1+\gamma)$: the latter, indeed, allows the first best to be achieved with point $\Pr L$ coinciding with the social optimum SO. By setting the efficient price at the first stage, the private firm also enables the public firm to react at the second stage to the rival's decision by setting the efficient price.

We turn now to consider a quantity game with simultaneous moves or public leadership. By evaluating the reaction function of the private firm at $\alpha^* = 1$ i, we obtain $R_2(s, q_1) = (1-c+s-\gamma q_1)/2$, which clearly depends on s and defines the set of all possible firm 2's reaction functions. In Figure 1a, $R_2(s, q_1)$ is depicted for a given value of s, while $R_2\left(s^C, q_1\right)$ is the reaction function evaluated at the optimal subsidy. The ordinates of point C and point PL represent the quantities produced by the private firm respectively in a simultaneous game and in a game with public leadership. As known from Proposition 2, in both these games the same optimal subsidy $s^C = s^C_{PL} = (1-c)/(1+\gamma)$ regulates firm 2's behavior moving its reaction curve upwards until firm 2 replies to the public firm's efficient quantity $q_1^{SO} = (1-c)/(1+\gamma)$ by producing the same efficient quantity given by $q_2^{SO} = R_2\left(s^C, q_1^{SO}\right) = (1-c)/(1+\gamma)$. The quantity expansion needed for firm 2's to be efficient is therefore measured by the difference $\Phi_2(s, q_1, q_2) = q_2^{SO} - R_2\left(s, q_1^{SO}\right)$ where $R_2\left(s, q_1^{SO}\right) = \frac{1-c+s(1+\gamma)}{2(1+\gamma)}$. At the optimal subsidy the following equality holds $\Phi_2\left(s^C, q_1, q_2\right) = 0$, which implies that point C, as well as point PL, coincides with the social optimum SO. By evaluating the difference $\Phi_2(s, q_1, q_2)$ in terms of prices, we demonstrate that the quantity expansion which ensures that firm 2 behaves efficiently in a quantity game with simultaneous moves or public leadership is associated to the same price reduction needed for firm 2 to behave efficiently in a price game with private leadership. Indeed, by substituting $q_2 = R_2\left(s, q_1^{SO}\right)$ and $q_1 = q_1^{SO}$ in the inverse demand function $p_2 = 1 - q_2 - \gamma q_1$, we find the price $p_2(s) = \frac{1+c(1+2\gamma)-s(1+\gamma)}{2(1+\gamma)}$ set by the private firm when the public firm behaves efficiently. By denoting with $\Phi_2^{'}(s, p_1, p_2)$ the price difference $p_2(s) - p_2^{SO} = \frac{1-c-s(1+\gamma)}{2(1+\gamma)}$, we find $\Phi_2^{'}(s, p_1, p_2) = \Phi_1(s, p_1, p_2)$.

The equivalence between a quantity game with private leadership and a price game with public leadership.

In this paragraph we show how the same subsidy $s^* = (1-c)(1-\gamma)$ restores the first best under both quantity private leadership and price public leadership/simultaneous moves, since it induces the same output increase by the private firm.

In a quantity game with private leadership, the reaction function of the public firm evaluated at $\alpha^* = 1$ is $R_1(q_2) = 1 - c - \gamma q_2$, which is independend of s. For a generic value of s, the equilibrium with private leadership is represented in Figure 1b by point $\Pr L$ on the $R_1(q_2)$ curve at which the private firm produces the quantity $q_2^{\Pr L}(s) = \frac{1-\gamma-c(1-\gamma)+s}{2(1-\gamma^2)}$. The subsidy s regulates firm 2's behavior inducing it towards a greater production. The output expansion needed for firm 2's to behave efficiently and produce $q_2^{SO} = (1-c)/(1+\gamma)$ is therefore measured by the difference $\Psi_1(s, q_1, q_2) = q_2^{SO} - q_2^{\Pr L}(s) = \frac{1-\gamma-c(1-\gamma)-s}{2(1-\gamma^2)}$ which shrinks to zero at the subsidy $s_{\Pr L}^{C} = (1-c)(1-\gamma)$. The optimal subsidy, indeed, allows to achieve the first best leading the point $\Pr L$ to coincide with the social optimum SO: the efficient production by the private firm at the first stage also induces an efficient output to be produced by the public firm at the second stage.

Turning to examine a price game with simultaneous moves or public leadership, we calculate the reaction function of the private firm at $\alpha^* = 1$ and find it is $R_2(s, p_1) = (1 + c - s - \gamma(1 - p_1))/2$, which defines the set of the possible firm 2's reaction functions when s varies. In Figure 2a we depict $R_2(s, p_1)$ for a generic value of s, and $R_2\left(s^B, p_1\right)$ which is evaluated at the optimal subsidy. The ordinates of point B and point PL represent the prices set by the private firm respectively in a simultaneous game and in a game with public leadership. We know that in both these games the optimal subsidy $s^B = s_{PL}^B = (1-c)(1-\gamma)$ disciplines firm 2's behavior moving its reaction curve downwards until firm 2 replies to the public firm's efficient price $p_1^{SO} = c$ by setting the efficient price $p_2^{SO} = R_2\left(s^B, p_1^{SO}\right) = c$. The price reduction needed for firm 2's behavior to be efficient is therefore measured by the difference $\Psi_2(s, p_1, p_2) = R_2\left(s, p_1^{SO}\right) - p_2^{SO} = (1 + c - s - \gamma(1-c))/2 - c = (1 - \gamma - c(1-\gamma) - s)/2$. At the optimal subsidy the following equality holds $\Psi_2\left(s^B, p_1, p_2\right) = 0$, implying that point B and point PL coincide with the social optimum SO. We now evaluate the difference $\Psi_2(s, p_1, p_2)$ in terms of quantities in order to demonstrate that the price reduction ensuring firm 2's efficient behavior in a price game with simultaneous moves or public leadership is associated to the same output expansion needed for firm 2 to be efficient in a quantity game with private leadership. Indeed, by substituting $p_2 = R_2\left(s, p_1^{SO}\right)$ and $p_1 = p_1^{SO}$ in the direct demand function $q_2 = \frac{(1-\gamma)-p_2+\gamma p_1}{(1-\gamma^2)}$, we find the quantity $q_2(s) = \frac{1-\gamma-c(1-\gamma)+s}{2(1-\gamma^2)}$ produced by the private firm when the public firm behaves efficiently. By denoting with $\Psi_2^{'}(s, q_1, q_2)$ the difference $q_2^{SO} - q_2(s) = \frac{1-\gamma-c(1-\gamma)-s}{2(1-\gamma^2)}$, we find $\Psi_2^{'}(s, q_1, q_2) = \Psi_1(s, q_1, q_2)$.

3.1 Concluding remarks

This paper examines simultaneous and sequential competition between a state-controlled firm and a private firm under optimal subsidies. Our findings contribute to the existing literature on Cournot and Bertrand mixed markets highlighting on the one hand the firms' ownership structures required for a subsidy to maximize allocative efficiency, on the other hand the circumstances under which the operating of an optimal subsidy causes some equivalence results. By exploring the latter, our analysis has driven attention to the nature of competition and the order of firms' moves as relevant variables to be considered in the design a subsidy policy. The analysis under more general demand assumptions is left to future research.

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Appendix

The optimal subsidy under Cournot public leadership:

$$s_{PL}^{C}(\alpha)=\frac{(1-c)\left(\alpha\gamma\left(\alpha\gamma\left(\gamma^2+3-7\gamma\right)-2\gamma\left(\gamma^2+4-8\gamma\right)+20\alpha-48\right)+4\alpha(12-5\alpha)+\gamma^2\left(\gamma^2-12\gamma+8\right)-32(1-\gamma)\right)}{\gamma^2(32-4\gamma-5\gamma^2)-32+4\alpha(12-5\alpha)+\alpha\gamma^2(2\gamma(\gamma+4)+\alpha(15-4\gamma)-40)}$$

The optimal subsidy under Cournot private leadership:

$$s_{\mathrm{Pr}\,L}^{C}(\alpha)=\frac{(1-c)\left(32-\gamma\left(32+8\gamma+\gamma^3-12\gamma^2\right)-\alpha\left(80-76\alpha+32\alpha^2-5\alpha^3\right)+\alpha\gamma\left(72+\gamma\left(\gamma^2+16-20\gamma\right)-\alpha\left(60+3\alpha^2-22\alpha\right)-\gamma\alpha(12-3\alpha+2\alpha\gamma-11\gamma)\right)\right)}{32+\gamma^2(4\gamma-32+5\gamma^2)-\alpha(80-76\alpha+32\alpha^2-5\alpha^3)-\alpha\gamma(+8-2\gamma(38-3\gamma-6\gamma^2)-2\alpha(8-5\alpha+\alpha^2)+\alpha\gamma(66+2\alpha\gamma^2-\gamma(2+9\gamma)-3\alpha(8-\alpha)))}$$

The optimal subsidy under Bertrand public leadership:

$$s_{PL}^{B}(\alpha)=\frac{(1-\gamma)(1-c)\left(32(1+\gamma)-\gamma^2\left(\gamma^2+12\gamma+8\right)-4\alpha(12-5\alpha)-\alpha\gamma\left(4(12-5\alpha)+\alpha\gamma\left(\gamma^2+7\gamma+3\right)-2\gamma\left(\gamma^2+8\gamma+4\right)\right)\right)}{32-4\alpha(12-5\alpha)+\gamma(1-\gamma)(32+\gamma(16-3\gamma^2-4\gamma))+\alpha\gamma(2\gamma(2+\gamma)(6-3\gamma^2+5\gamma)+\alpha\gamma(\gamma(\gamma+3\gamma^2-15)-11)-4(12-5\alpha))}$$

The optimal subsidy under Bertrand private leadership:

$$s_{\mathrm{Pr}\,L}^{B}(\alpha)=\frac{(1-c)(1-\gamma)\left(32(1+\gamma)-\gamma^2\left(\gamma^2+12\gamma+8\right)+\alpha\left(5\alpha^3+76\alpha-80-32\alpha^2\right)+\alpha\gamma\left(\gamma\left(\gamma^2+20\gamma+16\right)+\gamma\alpha(2\alpha\gamma+3\alpha-11\gamma-12)+\alpha\left(3\alpha^2-22\alpha+60\right)\right.\right.}{32(1+\gamma)+\gamma^2(\gamma(3\gamma^2+\gamma-20)-16)+\alpha(5\alpha^3+76\alpha-32\alpha^2-80)-\alpha\gamma(72+2\gamma(\gamma+1)(\gamma^2-\gamma-14)-\alpha(3\alpha^2-22\alpha+60)-2\gamma\alpha(\alpha(2+\gamma)-9-7}$$