Sraffa and Keynes on the definition of commodity-rate of interest

== first draft == please do not quote ==

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I. Introduction

In *General Theory*'s Chapter 17 (*The Essential Properties of Interest and Money*), after arguments he had already developed in previous chapters, Keynes proposed to provide a further and presumably deeper explanation of how investments could settle at a level too low to allow for full employment. As a first step towards this end he stated that «an analogue of the rate of interest on money» may exist «for every kind of capital asset» (GT p.222) and referred to Sraffa's 1932 review of Hayek's *Prices and Production* as the locus where the relationship between that magnitude (named *own-rate of interest* or *commodity-rate of interest*) and the money-rate of interest «was first pointed out» (GT p.223, n.1).

In general, economists interested in discussions of commodity-rates of interest have not noticed that the definitions given by Sraffa and Keynes were not the same. But two letters sent by Keynes to Sraffa in December 1931 give us a hint to explore the differences between their approaches to the subject.

Considering these letters it becomes apparent that Sraffa's definition presents two important peculiarities. On the one hand, taken literally, it contains an important ambiguity in the sense that it does not make clear if the commodity-rate of interest must be expressed in terms of *spot* or *forward* quantities of the given commodity. On the other hand, it may acquire precision if, considered in the light of Keynes's extant comments, an *implicit* statement that the commodity-rate of interest must be expressed in terms of *spot* quantity of commodity is attributed to Sraffa. On the contrary, Keynes provided a very clear formal presentation of his definition of commodity-rate of interest, and he consistently adhered to that definition both in the 1931 discussion with Sraffa and in the *General Theory*.

However, once it emerges that the concept of commodity-rate of interest may be defined in more than one way, it also becomes important to understand the grounds of Sraffa's and Keynes's respective choices. Our answer to these questions, as far as the *General Theory* is concerned, will connect Keynes's choice to his aim of comparing properties of money and of capital assets in general. But, considering that his position had been essentially the same also in 1931, a deeper root might be found in an essentially *pre-monetary* or *non-monetary* conception of what commodity-rates of interest are. Furthermore, Keynes might also have been influenced by the reflections on the relationship between interest rates and spot and forward foreign exchanges he had developed in the early 1920s. In the case of Sraffa, on the other hand, in order to explain both the peculiarities of his text and his determination not to follow the definition of commodity rates of interest proposed by Keynes, we will put forward some conjectures, essentially based upon the distinction between non-monetary and monetary economies – i.e., upon the distinction between the context to which

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Sraffa meant to apply his discussion of the concept of commodity-rate of interest and the context in which he defined and studied it. Sraffa's approach will turn out to be based upon a *monetary conception* of what commodity-rates of interest and interest costs are.

II. Sraffa's definition of commodity-rate of interest

To start with it is important to note that Sraffa, in his 1932 review of Hayek's *Prices and Production*, was not pursuing the aim of presenting a particular theory, as that of criticizing the theoretical analysis of the relationship between money, prices and levels of production developed by Hayek. In particular, he introduced the concept of commodity-rate of interest to criticize the way Hayek had distinguished between monetary and non-monetary economies and his proposal of fixing the money-rate of interest at the level of the *natural* rate. Sraffa did not even explicitly put forward a specific name for the concept he was discussing; he simply extended Hayek's terminology (which, in turn, had been mutuated and adapted from Wicksell) to his own analysis, unassumingly adding the denomination of *commodity rate of interest*.

Sraffa stressed that Hayek had distinguished between monetary and non-monetary economies as if in a nonmonetary economy the equilibrium, or *natural*, rate of interest would be unique and directly determined by demand and supply for capital, while, in a monetary economy, capital demand and supply being expressed in terms of money, the action of banks could cause the money rate of interest to diverge from the equilibrium, or *natural*, level (Sraffa 1932 p.49).² Sraffa was convinced that such a distinction was wrong, because also in a non-monetary economy more than one *natural* rate exist and, out of equilibrium, these rates would diverge from one another:

«An essential confusion, is the belief that the divergence of rates is characteristic of a money economy [...] If money did not exist, and loans were made in terms of all sorts of commodities, there would be a single rate which satisfies the conditions of equilibrium, but there might be at any one moment as many "natural" rates of interest as there are commodities, though they would not be "equilibrium" rates. [...] if loans were made in wheat and farmers [...] "arbitrarily changed" the quantity of wheat produced, the actual rate of interest on loans in terms of wheat would diverge from the rate on other commodities and there would be no single equilibrium rate» (Sraffa 1932 p.49).

But while we might have expected that, in order to develop his critique, Sraffa would have pursued the description and analysis of *natural* rates (or commodity-rates), within a non-monetary economy, what he actually did was considering the way their existence and their variations may be recognised in a monetary economy. Then, he completed his discussion reverting to his original point and reasserting that divergences among rates is an ordinary feature of any non-monetary economy.

Sraffa unfolds his argumentative strategy indicating how commodity-rates of interest may be recognised in nowadays economic activity:

«When a cotton spinner borrows a sum of money for three months and uses the proceeds to purchase spot, a quantity of raw cotton which he simultanously sells three months forward, he is actually "borrowing cotton" for that period» (Sraffa 1932 pp.49-50).³

² In Hayek's words: «In a money economy, the actual or money rate of interest ("Geldzins") may differ from the equilibrium or natural rate, because the demand for and the supply of capital do not meet in their natural form but in the form of money, the quantity of which available for capital purposes may be arbitrarily changed by the banks» (Hayek 1931 pp.20-21).

³ Incidentally, we may note that the reason why cotton spinners should follow this line of conduct may not be as obvious as Sraffa seems to imply: their ordinary activity leads them to buy cotton to use and

Having shown that an operation amounting to *borrowing cotton* is ordinarily performed in a modern economy, Sraffa indicates how the rate of interest paid on that operation may be calculated *in terms of cotton*:

«The rate of interest which [the cotton spinner] pays, per hundred bales of cotton, is the number of bales that can be purchased with the following sum of money: the interest on the money required to buy spot 100 bales, plus the excess (or minus the deficiency) of the spot over the forward prices of the 100 bales» (Sraffa 1932 p.50).

He then continues by arguing that divergences between spot and forward prices for a commodity reveal divergences between the *natural* rate of interest on that commodity and the *natural* rates on other commodities and the *equilibrium* rate of interest. When equilibrium will be restored, spot and forward prices will coincide and commodity-rates will have all the same value (equal to the money-rate of interest).⁴ On this basis, following Sraffa's argument, we may take for granted that, also in a non-monetary economy, a disequilibrium in which production of some commodity-rates.⁵ But also in this case, «for the date on which equilibrium is expected to be restored» (1932 p.50) the relevant rates will be all at the same level (although, we may add, given that no unique standard would exist, it could not be said that they would be equal to a specific commodity-rate).

To probe into the meaning of Sraffa's description of the relation between spot and forward prices, moneyrate of interest and commodity-rate of interest we must, first of all, note that Sraffa is considering a monetary economy, and that the operation he examines is only *equivalent* to borrowing 100 bales of cotton: in a monetary economy only money is directly borrowed – cotton is not. The task that Sraffa explicitly sets himself,

transform it, not to keep it for a period and return it afterwards. An explanation for their purchasing spot and simultaneously selling forward may be based on the observation that cotton spinners, assuming that prices of raw and wrought cotton move in the same direction, could sell forward the same amount of raw cotton bought spot in order to hedge against future variations in the price of wrought cotton. If prices go down, the profits obtained on the forward contracts may compensate for the loss on the price of wrought cotton, which has to be sold at a price lower than expected. If prices go up, the extra profits obtained selling wrought cotton will be cancelled out by the loss on the forward contracts. This explanation is closely mirrored in a passage from Marshall's Industry and Trade: «A British miller may bring shiploads of wheat of various descriptions alongside his own elevator, and mix them by automatic flow in various proportions to make different sorts of flour. He often buys direct from far-off farmers or local elevators, through agents on the spot, who know his requirements exactly: but he can, if need be, send special instructions in return to telegraphic reports; basing himself on the last records that have been received in Liverpool, or other centre of the wheat trade, of the prices of standard grades in all the chief markets. Having ordered the purchase of a certain quantity of what he needs, he "hedges," by selling at once in a central market an equal quantity of standard wheat for delivery at about the time at which he expects that the wheat, which he has just bought, will be in his elevator ready to be made quickly into flour. If wheat falls in the interval, his flour has to compete with that made from cheaper wheat; but, what he loses through that fall, is returned to him almost exactly by his gain on the "future" which he has sold. Conversely, if wheat rises in the interval, he has to pay on the sale of his "future" about as much as he gains from the corresponding upward movement of his flour. By buying a future he does not speculate; he throws on the shoulders of the general market the risks and the chances of the gain that would otherwise have come to him through general movements external to his own business» (Marshall 1923, pp.259-60). I wish to thank Luca Fantacci and Cristina Marcuzzo, who, respectively, pointed out to me this explanation and Marshall's passage.

⁴ We may add that being money, in a monetary economy, by definition, the only commodity for which spot and forward prices cannot diverge (or do not exist, in the sense that it is the only commodity for which futures markets operate in quantities and not in prices), its rate of interest always turns its demand and supply into a state of equality; and it necessarily sets the equilibrium level for all the other commodity-rates.

⁵ To mark this second part of his reasoning Sraffa uses the following words: «It is only one step to pass from this to the case of a non-money economy, and to see that [...] there may be as many "natural" rates as there are commodities» (Sraffa 1932 p.50).

nevertheless, consists of calculating a number of bales of cotton which, turned into a rate, would give us what could be called *cotton-rate of interest*.

In this context, in order to *borrow cotton* we must first of all borrow the quantity of money we need to buy 100 bales spot; the cost of this operation is $100_{bales}P_c^S i_m$. Then we must also consider the difference between the spot price we pay to buy the 100 bales and the price we earn when we sell them forward (at the time corresponding to the end of the *borrowing period*): $100_{bales}(P_c^S - P_c^F)$. If the spot price is higher than the forward price, this is an additional cost of the operation; if the spot price is lower than the forward price this is an earning which reduces the total cost. Given that cotton is not *returned*, at the end of the period, *augmented* according to the value of the cotton-rate of interest, such a rate cannot emerge from a direct comparison between quantity of cotton initially borrowed and quantity of cotton finally returned. In fact, the crucial magnitude in Sraffa's definition is the total monetary cost of the operation:

[1]
$$(P_c^S i_m + (P_c^S - P_c^F))100_{bales}$$

By calculating how many bales of cotton this amount of money could buy, as hinted at by Sraffa, we can figure out the cost of the operation in terms of bales of cotton. Sraffa, however, as noted by Potestio (1989 p.261),⁶ does not explicitly state if this number of bales must be calculated by dividing the total cost by P_c^S or by P_c^F . His argument has often been understood as pointing to the latter magnitude (see, for instance, Barens and Caspari 1997; Bonifati 1991; Eatwell 1987; Kregel 1982; Kurz 1995, 2010; Musu 1984; Naldi 2001; Panico 1988; Ranchetti 1998, 2001),⁷ but the correspondence between Sraffa and Keynes on the content of Sraffa's article gives the impression that Sraffa did not favour this solution.

III. Keynes on Sraffa's definition of commodity-rate of interest

The extant correspondence between Keynes and Sraffa has been studied by Fabio Ranchetti (2005) and the most interesting point which emerges with regard to Sraffa's 1932 article relates precisely to the definition of the interest rate on loans of bales of cotton. Keynes, reading the final typescript of Sraffa's article (SP D3/9==MI/6/2i-xx), seems to have believed that Sraffa did not express himself with sufficient clarity:

«I have pencilled some small verbal changes, either for the sake of the English or for the sake of clearness. The only material point is the rewording which I suggest on page 16. But here I think I must be giving the meaning which you intend» (letter from Keynes to Sraffa, 18.12.1931, SP D3/11/65/56; quoted in Ranchetti 2005 p.130).

Accordingly, he suggested to substitute Sraffa's original formulation with an alternative sentence which read as follows:

The rate of interest which he pays, per hundred bales of cotton, is the number of forward bales that can be purchased with the interest on the money required to buy spot 100 bales, plus the excess over 100 (or

⁶ Potestio quotes an unpublished manuscript by Ian Steedman entitled *Own Interest Rates and Concepts of Equilibrium* where the same point had already been raised.

⁷ As far as I know, only Deleplace (1986) and Oka (2010 p.2 n.1) have opted for the alternative view (see also Majewski 1988).

minus the deficency) of the number of forward bales which can be purchased for the same price as 100 spot bales (SP D3/9== MI/6/2xvi; quoted in Ranchetti 2005 p.131).

Keynes's suggestion, however, was not accepted by Sraffa: the formulation at page 16 of his typescript was reproduced with no important differences also in the published article.

If it is clear that Sraffa was not convinced by Keynes's proposal, the letter just considered does not illuminate the source of their disagreement. But a second letter, written by Keynes while sending to the printer the typescript of the article, seems to touch exactly on that point:

«As regards the forward bales, I am sending the first of your alternatives to the printer, but will you in proof again consider my alternative, since I am not yet persuaded that it is wrong? It is a characteristic of interest to be payable in arrear, and not in advance. If it is payable in advance we call it discount. Thus it seems to me to be of the essence of the case that the amount of interest be calculated in <u>forward</u> bales; that it is to say we have to find how many forward bales can be obtained by parting with a given number of spot bales» (letter from Keynes to Sraffa, 21.12.1931, emphasis in the original, SP D3/11/65/52; quoted in Ranchetti 2005 pp.131-2).⁸

These remarks suggest that Sraffa did not share Keynes's view that the result had to be expressed in terms of *forward* bales and, by implication, they lead us to think that Sraffa believed that the cotton-rate of interest had to be expressed by a number of bales contracted for *spot* delivery. Indeed, Keynes also hints at what he seems to consider the criterion apt to be applied to solve the controversy: does the *borrowing cost*, in the case at hand, emerge as payable in advance (in which case it would be named *discount*) or in arrear (in which case it would be named *interest*)?⁹

Once the question is put in these terms, Keynes's view seems to be unassailable: the *cost of borrowing* 100 bales of cotton, understood as the amount of money indicated by Sraffa, actually emerges only at the end of the period; how could it be treated as if it could buy a number of bales of cotton spot (i.e., at the beginning of the period)? If Sraffa had a different opinion, Keynes's letter and Sraffa's own article do not seem to provide any hint as to its grounds. The question, however, may be further considered and, as we shall see, there will be reasons to believe that the divergence hinges upon Keynes's idea that «we have to find how many forward bales can be obtained by parting with a given number of spot bales» (letter from Keynes to Sraffa, 21.12.1931, SP D3/11/65/52), which is not necessarily consistent with the idea that in a monetary economy cotton is not directly borrowed and returned augmented according to an interest.

IV. The grounds for Sraffa's definition: some hypotheses

First of all, we may stress that the fact that Sraffa believed that the cotton-rate of interest had to be expressed by a number of bales contracted for *spot* delivery is not as obvious at it may appear from Keynes's second letter. The text published by Sraffa, just like the typescript he had submitted to Keynes and all the preparatory notes known to us, does not state that the cotton-rate of interest should be defined in terms of *spot* bales. This possibility emerges from Keynes's comments and may be recognized in the text of Sraffa's article only if we assume that when Sraffa mentions *the number of bales that can be purchased* with the monetary cost of borrowing cotton he *implicitly* refers to *spot bales*. To probe into the question it may then be appropriate to

⁸ Sraffa's second alternative was not significantly different.

⁹ This remark seems to rule out the possibility that the price referred to by Sraffa could have been the spot price currently quoted at the time of delivery referred to in the relevant forward contract, which would give rise to a payment *in arrear* just like the forward price.

consider two different directions of enquiry. On the one hand, we may consider which reasons might have led Sraffa to refrain from providing a full specification of an algorithm apt to calculate the value of the commodity-rate of interest. On the other hand, we may consider which reasons might have led Sraffa to favour a definition of the commodity-rate of interest in terms of *spot* bales.

Our view, as we shall argue, is that both cases may be interpreted as reflecting the argumentative strategy followed by Sraffa in this part of his paper, his opinions on how economic science should approach the analysis of economic phenomena by including in its schemes only objectively observable and measurable magnitudes,¹⁰ and his view of what, in a monetary economy, may be described as commodity-rate of interest.

Let us start from the question of the absence, in Sraffa's 1932 article, of a full specification of an algorithm apt to calculate the value of a commodity-rate of interest.

To explore this direction of research, we must take as a fact that when Sraffa refers to *the number of bales that the cotton spinner can purchase* his wording of the definition of commodity-rate of interest is genuinely ambiguous and that Sraffa was aware of that ambiguity (the remarks contained in Keynes's two letters could not have failed to alert him). Granting this, we may move to consider the context of Sraffa's analysis – i.e., the fact that Sraffa does not discuss commodity-rates of interest in order to develop a theory of those magnitudes, but only to show the inconsistency of Hayek's approach - and the fact that, although his aim is that of showing how that concept articulates in the case of a non-monetary economy, he studies it within a monetary economy.

Given that non-monetary economies are not observable, we may conjecture that Sraffa, consistently with his own views on how economic theory should be developed, considered that a direct examination of a non-observable case was inappropriate.¹¹ In addition, we may conjecture that he believed that the analogy between the operation of *borrowing cotton* in a monetary economy and commodity loans in a non-monetary economy could indeed illuminate the existence of commodity-rates in non-monetary economies, but that that analogy should not be carried to the point of applying to a non-monetary economy an algorithm devised to calculate those rates in the context of a monetary economy. This could explain why Sraffa had not directly developed his case within a non-monetary economy and why he had not presented a complete algorithm designed to calculate the value of the commodity-rate of interest. However, if this explanation has the advantage of addressing Sraffa's definition exactly as it emerges from the text of his article, it cannot account for Keynes's remarks (contained in his second December 1931 letter) as far as they imply that Sraffa – either in private conversation or in non-extant documents - had been arguing that Keynes's alternative definition *was wrong* and that a different definition was to be preferred.

This brings us to consider the possibility that Sraffa favoured a definition of commodity-rate of interest in terms of *spot quantities* and that such a definition is *implicit* in the text he published in 1932.

To this end, we may start by illustrating the formal differences ensuing from expressing the cotton-rate of interest by dividing the monetary cost of borrowing cotton by spot or forward prices. The result, in the two

¹⁰ See Kurz 2003 and Kurz-Salvadori 2004, 2005.

¹¹ Indeed Sraffa almost ridiculed such a perspective: «In order to realise this [i.e., how in non-monetary economies commodity-rates may diverge from one another and from their equilibrium level] we need not to strech our imagination and think of an organised loan market amongst savages bartering deer for beavers. Loans are currently made in the present world in terms of every commodity for which there is a forward market» (Sraffa 1932 pp.49-50; my insertion).

cases, is approximately equal, but not exactly equal,¹² even though the relationships between commodity-rate, money-rate and spot and forward prices in the two cases go in the same directions.

Sraffa's expression, if interpreted as implying the division of the cost of borrowing cotton by the spot price of cotton, would be:

$$[2] \quad \frac{i_m P_c^{S} 100_{bales}}{P_c^{S}} + \frac{(P_c^{S} - P_c^{F}) 100_{bales}}{P_c^{S}} = (i_m + \frac{P_c^{S} - P_c^{F}}{P_c^{S}}) 100_{bales} = (i_m + 1 - \frac{P_c^{F}}{P_c^{S}}) 100_{bales} = \alpha_{bales}$$

and from this number of bales of cotton we would get the relevant rate: $\frac{\alpha_{bales}}{100_{bales}} = \alpha$.

On the other hand, Keynes's contention that the cost of the operation should be expressed in terms of *forward quantities* would lead to a different equation:

$$[3] \quad \frac{i_m P_c^S 100_{bales}}{P_c^F} + \frac{(P_c^S - P_c^F) 100_{bales}}{P_c^F} = (i_m \frac{P_c^S}{P_c^F} + \frac{P_c^S}{P_c^F} - 1) 100_{bales} = \beta_{bales}$$

and to a different relevant rate: $\frac{\beta_{bales}}{100_{bales}} = \beta$.¹³

In this case, arguing that the object of Sraffa's reasoning concerns the existence of commodity-rates in a nonmonetary economy cannot explain his preference for equation [2] against equation [3] – in fact both equations incorporate reference to money and monetary rate of interest. But when it is clear that we are developing our reasoning within the context of a monetary economy, we may consider that, given the monetary cost of borrowing something, *in order to express it as a rate of interest*, it is obvious to divide it by the amount of money we have borrowed. If we borrow money in order to borrow 100 bales of cotton, we may find the rate of interest paid on such an operation by dividing its monetary cost (equation [1]) by $100_{bales}P_c^S$; or, if we wish to proceed by stages, by dividing that cost first by P_c^S (in which way we would get a number of bales of cotton, as required by Sraffa), then by 100_{bales} (as would be necessary in order to obtain a proper *rate*). The result may then be described as the money-rate of interest paid on an operation *equivalent to borrowing cotton* and, in this sense, we may call it *cotton-rate of interest*.

This seems to offer a sound explanation for Sraffa's implicit or supposed preference for dividing the monetary cost of borrowing cotton by P_c^S . And Keynes would be wrong in attributing to Sraffa's approach the characteristic of depicting a reward paid *in advance* rather than *in arrear* - i.e., *a discount*, not *an interest* (letter from Keynes to Sraffa, 21.12.1931, SP D3/11/65/52; quoted above). The real point separating Sraffa's and Keynes's approaches seems to be that the operation depicted by Sraffa may be described as borrowing cotton, but it does not actually imply the equivalent of returning an augmented amount of cotton at the end of the borrowing period: it only implies buying cotton today and selling that very quantity of cotton at the end of the

¹² The difference between the two increases with the difference between spot and forward prices, but it tends to zero when the length of the time interval considered for the payment of interest approaches to zero (Fisher 1896 pp.360-1; Oka 2010 p.2 n.1).

¹³ As we shall see, this corresponds to the definition of the commodity-rate of interest presented by Keynes in Chapter 17 of his *General Theory*

period. Keynes, on the contrary, conceived the operation as implying, at the end of the period, a sale of cotton in the quantity necessary to repay the amount of money initially borrowed and the corresponding monetary interest. But, if this interpretation is correct, it must also be acknowledged that Sraffa's reference to *the number of bales that can be purchased with the monetary cost of borrowing cotton* (the intermediate step in a process divided in two stages) does not help to clear the matter.

To put it in other words, Keynes's scheme is the one which would come as close as possible to what we may imagine would be the logic of an operation of borrowing cotton in a non-monetary economy. But such a scheme would be totally unconnected to the logic of economic behaviour in a monetary economy, where money is the standard of value and, in ordinary circumstances, no one is interested in calculating returns in terms of cotton or any other commodity.

Hence, we may say that if we want to follow *strictly* the logic of the analogy with a non-monetary economy, we must calculate the cotton-rate of interest by dividing the monetary cost of borrowing cotton by P_c^F . On the contrary, if we wish to concentrate on a monetary economy and if we accept the idea that the monetary cost of borrowing money, of buying cotton spot and of selling it forward may be described as the monetary cost of borrowing cotton, then we may choose to calculate the relevant *commodity-rate of interest* by dividing the latter quantity by P_c^S . Indeed, if we are only interested in recognizing the existence of commodity-rates of interest in a monetary economy in order to illuminate their existence in a non-monetary economy, we may be content with the observation of the actual existence of spot and forward prices and the description of the mechanics of the operation. On this basis, we would not even need to wonder if such an operation as that attributed by Sraffa to *a cotton spinner* is actually performed or not, nor to calculate *a number of bales* – whether spot or forward – corresponding to the cotton-rate of interest. In this sense, we are brought back to the fact that Sraffa's text does not actually specify whether the calculation should be completed in terms of spot or forward bales.

V. Keynes's definitions of the own-rates of interest in Chapter 17 of the General Theory

Keynes's definition of commodity-rate of interest in the *General Theory* corresponds exactly to rate β as identified by equation [3], and reflects the point of view Keynes had expressed in the discussion with Sraffa in December 1931. Indeed, the route followed by Keynes in the *General Theory* to reach his definition of commodity-rate of interest does not move from Sraffa's identification of the monetary cost of borrowing a commodity (which now, in Keynes's instances, is *wheat*) but from the idea of finding the amount of forward wheat that can be obtained by parting with a given quantity of spot wheat. This means that Keynes assumes as starting point the definition of what a commodity-rate of interest would be in a non-monetary economy and *adapt* it to the case of a monetary economy. The contingent reason why Keynes, in the *General Theory*, follows this route seems to be that, in order to show that the rate of interest on money may pose particular obstacles to the attainment of a volume of investment sufficient to allow for full employment, he proposes to develop a comparison between some properties of money and of commodities in general.¹⁴ But, given that the same

¹⁴ It might also be argued that a discussion based on the concept of commodity-rate of interest was not necessary to the argument developed by Keynes in *General Theory*'s Chapter 17, which could have been pursued in terms of the concept of marginal efficiency of capital assets, already introduced in Chapter 11 of the *General Theory*. We will not enter into this discussion, which would lead us beyond the

approach had already been outlined by Keynes in his 1931 letters to Sraffa,¹⁵ we may presume that the real roots of his position lie in a pre-monetary or non-monetary conception of what commodity-rates (or *own-rates*) of interest are.¹⁶

Be it as it may, in order to pursue his contingent aims, Keynes argues that what are generally regarded as essential features of money are common, although in different degrees, to any commodity or capital asset and that money and other commodities may be compared as if, in principle, any commodity could be borrowed just like money.¹⁷ Indeed, we read that if «the money-rate of interest [...] is nothing more than the percentage excess of a sum of money contracted for forward delivery [...] over what we may call the 'spot' or cash price of the sum thus contracted for forward delivery [...] for every kind of capital asset there must be an analogue of the rate of interest on money», so that, for every commodity we would have «a rate of interest in terms of itself, - a wheat-rate of interest, a copper-rate of interest, a house-rate of interest, even a steel-plant-rate of interest» (GT pp.222-3). All this may be expressed by calculating the rate of interest in the case of wheat (equation [4b] where *w* stands for *wheat*), just like we calculate it in the case of money (equation [4a]):

[4a]
$$i_m \equiv \frac{Q_m^F - Q_m^S}{Q_m^S}$$
 [4b] $i_w \equiv \frac{Q_w^F - Q_w^S}{Q_w^S}$

Obviously, however, while equation [4a] clearly describes what may be directly observed in a monetary economy, given that in such an econcomy commodities are not directly borrowed, equation [4b] does not describes something which may be directly observed in a monetary economy. In order to compare properties of money and of other commodities within the context of a monetary economy and by applying the concept of commodity-rate of interest, Keynes has to adapt his definition of commodity-rates to the fact that in a monetary economy only money is directly borrowed, while commodities are not: if, in the case of ordinary commodities, Q^F cannot be directly known as in the case of money, then to calculate it we must go through a roundabout way based on knowledge of spot and forward prices and money-rate of interest. Hence, if 100 quarters of wheat spot may be contracted for 100£, and if for the intertemporal equivalent of 100£ (i.e., the sum of money which turns out to be equal to 100£ once discounted by applying the monetary rate of interest for the relevant period) we may contract 105 quarters of wheat for forward delivery, then we may deduce that the wheat-rate of interest is 5% [5] (GT p.222):

boundaries of a discussion of the definition of the concept of commodity-rate of interest. We would only note that it should consider subsequent parts of the text of Chapter 17 and Sraffa's manuscript notes to that chapter (SP I 100).

¹⁵ «We have to find how many forward bales can be obtained by parting with a given number of spot bales» (letter from Keynes to Sraffa, 21.12.1931, SP D3/11/65/52).

¹⁶ The rate of interest that an asset may command, measured in terms of itself, is called by Keynes *own-rate of interest*. *Money-rate of interest* and *commodity-rate of interest* are other names for *own-rate of interest* when the latter refers, respectively, to money or to a commodity.

¹⁷ It comes as no surprise, having reached this conclusion, that Keynes's definition of commodity-rate of interest is equivalent to Fisher's formula for determining the values of a given rate of interest when expressed in terms of different measurement units. Fisher's approach presupposes that every commodity may be directly borrowed and command an interest just like money.

[5]
$$i_w = \frac{Q_w^F - Q_w^S}{Q_w^S} = \frac{105 - 100}{100} = 0,05$$

The equation connecting spot and forward prices of wheat (P_w^S and P_w^F respectively), money-rate of interest and Q_w^S and Q_w^F which underlies this instance is

$$[6] \quad \frac{Q_w^F P_w^F}{1+i_m} = Q_w^S P_w^S.$$

But the complete relationship is illustrated by Keynes in greater details with an example whose data are the following:

100£ = price of 100 quarters of wheat for spot delivery ($Q_w^S P_w^S$);

107£ (to be paid a year hence) = price of 100 quarters of wheat for forward (a year hence) delivery $(Q_w^F P_w^F)$; 5% = money-rate of interest (i_m) ;

Given these data, if today we possess 100£, we can alternatively turn them into 100 quarters of wheat spot, or in 105£ for forward delivery (a year hence) which can be used to buy 98.13 quarters of wheat for delivery a year hence $(\frac{105}{107}100 = 98.13)$. This means that 100 quarters of wheat spot are equivalent to 98.13 quarters of wheat for forward delivery and that the wheat-rate of interest is -1.87%. This example may be translated in the following formula (where *q* stands for *quarters of wheat*) for the wheat-rate of interest:

[7]
$$i_w = \frac{Q_w^F - Q_w^S}{Q_w^S} = \frac{\frac{105\pounds}{107\pounds}100q - 100q}{100q} = \frac{98.13q - 100q}{100q} = -0,0187$$

which reflects Keynes's words¹⁸ and corresponds to the following equation:

$$[8] \quad i_w = \frac{Q_w^F - Q_w^S}{Q_w^S} = \frac{\frac{(1 + i_m)100qP_w^S}{100qP_w^F}100q - 100q}{100q} = \frac{(1 + i_m)P_w^S}{P_w^F} - 1 = \frac{P_w^S}{P_w^F} + i_m\frac{P_w^S}{P_w^F} - 1$$

¹⁸ Keynes's words run as follows: «£100 spot will buy £105 for forward delivery, and £105 for forward delivery will buy 105_{107}^{-1} 100 [...] quarters for forward delivery. Alternatively £100 spot will buy 100 quarters for spot delivery» (GT p.223). As a matter of fact, however, Keynes's numerical conclusion is somewhat at variance with the data and the algorithm of equations [7] and [8]: «Thus 100 quarters of wheat for spot delivery will buy 98 quarters of wheat for forward delivery. It follows that the wheat-rate of interest is *minus* 2 per cent per annum» (GT p.223; emphasis in the original). We may conjecture that this result, if not the product of an implicit numerical approximation, reflected Keynes's implicit adoption of Fisher's formula j = i - a - ai mutilated of the term -ai - indeed, such a formula would give $i_w = 5\% - 7\% = -2\%$ (the same Keynes will actually do, again without mentioning Fisher's works, in the following page 224: «[if wheat] is expected to appreciate at a steady rate of *a* per cent per annum in terms of money; the marginal efficiency of an asset, which is *x* per cent in terms of money, will then be x - a per cent in terms of wheat») – (see Fisher 1896 p.9; Fisher 1907 p.359; Fisher 1930 p.39 and also Naldi 2011 pp.==). It may also be mentioned that it is rather obscure why Keynes wrote " 105_{107} 100" instead of just " 105_{107} ".

Concluding his illustration of the definition of commodity-rates of interest, Keynes appended the already mentioned footnote recalling Sraffa's 1932 review of Hayek's book:

«This relationship was first pointed out by Mr Sraffa, *Economic Journal*, March 1932, p.50» (GT p.223, n.1).

Knowing that in December 1931 Keynes had disagreed with Sraffa precisely on the definition of the commodity-rate of interest that Sraffa was to include in his 1932 article, also this statement is somewhat surprising. We may only accept it if we consider that both ways of defining the concept of commodity-rate lead to the same kind of relationship between money-rate of interest, spot and forward prices, and commodity-rate of interest. Be it as it may, having defined the concept of commodity-rate of interest, Keynes goes on arguing that there is no reason why we should expect that at any moment in time these rates will be the same for every commodity: «For the relation between 'spot' and 'future' contracts, as quoted in the market, is notoriously different for different commodities» (GT p.223). This, indeed, is the same result reached by Sraffa.

Then, parenthetically, Keynes refers to the fact that a similar variety of rates may be easily observed in markets for currencies (GT p.224). The latter point is interesting because an antecedent to Keynes's attention to commodity-rates of interest may be recognised in his April 1922 article *The Forward Market in Foreign Exchanges* (reprinted the following year as Section 4 of Chapter 3 of his *Tract on Monetary Reform*).¹⁹ In that paper a concept of own rate of interest was not explicitly defined and discussed, but Keynes's later algorithm may be understood to be implicit in his 1922 analysis of the relations between spot and forward exchange rates and the levels of national and foreign short term interest rates, which he describes as "a mathematical calculation of interest rates" (Keynes 1923 p.105).²⁰

Indeed, equations [8] correspond to the definition of a state of indifference between investing 1£ in money or in wheat [9], or between investing 1£ in the UK or in the USA [10]:

[9]
$$i_w = i_m \frac{P_w^S}{P_w^F} - 1 + \frac{P_w^S}{P_w^F}$$
 implies $\frac{1\pounds}{P_w^S}(1+i_w)P_w^F = 1\pounds(1+i_m)$

[10]
$$i_{\$} = i_{\pounds} \frac{e^{\$}}{e^{F}} - 1 + \frac{e^{\$}}{e^{F}}$$
 implies $\frac{1\pounds}{e^{\$}}(1 + i_{\$})e^{F} = 1\pounds(1 + i_{\pounds})$ where $e = \frac{x\pounds}{1\$}$

In his April 1922 article Keynes takes as given the rates of interests in the UK and in the USA, and determines the difference between spot and forward exchanges, but the same scheme could also determine the

¹⁹ In this sense see Kregel 1982 pp.452-5.

²⁰ In his April 1922 article Keynes, discussing these relationships, also used data on spot and forward exchanges in Milan he had been provided by Sraffa (letter from Keynes to Sraffa, 14.2.1923, SP Add.ms.a.427/7; Keynes 1923 p.108). The latter circumstance is consistent with testimonies that, on the occasion of their first meeting (most probably in August 1921), Keynes was particularly impressed by Sraffa's considerations on the forward markets for currencies (Ingrao and Ranchetti 1996 p.520; Ranchetti 2005 p.119, p.136 n.1; Naldi 2001 p.26, p.37 n.16). A similar reconstruction is contained in one of the obituaries which appeared after Sraffa's death: "on a visit to England in 1921 he met Keynes and took his fancy with a discussion of hedging on the forward exchanges" (*The Times*, 6 September 1983).==

dollar-rate of interest taking as given the rate on interest in the UK and the values of spot and forward exchanges - i.e., it could lead to equation [8]. It then comes as no surprise that in the *General Theory* the definition of commodity-rates is illustrated with a reference to foreign exchange.

But the analogy between commodity-rates of interest and currency-rates of interest conceals a difference which illuminates an important characteristic of the commodity-rates as Keynes had defined them - although it is not clear to what extent it was understood by Keynes himself. The point is the following. As we have already noted, commodity-rates can only be *constructed* through observation of money-rate of interest and spot and forward prices of the commodities in question. Accordingly, as it appears from equation [9], when, taking as given money-rate of interest and spot and forward prices, we apply Keynes's formula for the determination of commodity-rates, we automatically obtain the equilibrium value for the commodity-rate we are considering (i.e., the value which marks the indifference between investing in money or in the commodity in question).²¹ But, given that in a monetary economy commodity-rates do not have an autonomous and directly observable existence, their values, as we may construct them, cannot be compared with any actual observable rate and cannot guide our behaviour in any way. In this sense, we may say that the concept of commodity-rate of interest as defined by Keynes is void of operative content – and (although this point was not mentioned neither by Sraffa nor by Keynes in any extant manuscript or published document) we may say that such a result adds additional support to Sraffa's decision not to follow Keynes in his definition of commodity-rates in terms of forward quantities. In the case of currencies, on the contrary, own-rates can be directly observed and have an operative content. If, for instance, taking as given current sterling-rate of interest and spot and forward dollar-sterling exchange rates, we apply Keynes's formula to the determination of the dollar-rate of interest, we obtain a value which, also in this case, is an equilibrium value, and which may be compared to the actual dollar-rate of interest. If the two rates turn out to be different, we know that profits or losses can be obtained by embarking in an international arbitrage operation; if the two rates turn out to be equal no such opportunity would arise (unless our expectations on the future exchange rate are different from its quoted forward value).²²

VI. Conclusions

To sum up, it seems undeniable that the way Sraffa defined the concept of commodity-rate of interest in his 1932 article was not sufficiently clear.

On the one hand, Sraffa's definition was ambiguous in the sense that he clearly implied that to obtain a *commodity-rate of interest* the cost of borrowing cotton should be divided by the price of cotton, but he did not specify whether it should be divided by the spot price or by the forward price, and he did not explain the reasons why he was convinced that such an ambiguity, or incompleteness, was to be preferred to a definition of the concept of commodity-rate showing a full statement of its algorithm (and the remarks we know Keynes submitted to Sraffa prove that the latter could not have been unaware of this feature).

²¹ See Barens and Caspari 1997 p.293.

²² Let $x \pounds /1$ \$ be the price of \$ measured in terms of £, $i_{\$}$ be the *equilibrium* dollar-rate of interest, $i_{\* be the *actual* dollar-rate of interest, and $i_{\$} > i_{\* . Then, if we own 1£, we may earn a profit by turning it spot into x\$, buying USA treasury bills, and turning the amount of dollars we will get back at the end of the period into forward pounds.

On the other hand, if, as it appears from Keynes's letters, Sraffa, in private discussions, had maintained that the cost of borrowing cotton should be divided by the spot price of cotton, we know of no extant document showing how Sraffa grounded such a view.

In the first case, Sraffa's position may be understood by stressing that it might reflect Sraffa's methodological stance, which amounts to distinguish between observable and non-observable magnitudes and to state that economic science should be built only upon data belonging to the first category.

In the second case, we have argued that the same methodological stance might have led Sraffa to exert his circumspection by avoiding a formulation based upon an almost mechanical transposition of a relationship conceived with regard to a non-monetary economy (the concept of commodity-rate of interest as implying the delivery of an agreed amount of cotton at the end of the borrowing period) into the context of a monetary economy.

But in order to understand the divergences between Sraffa's and Keynes's approaches we must also consider the aims they were pursuing.

Sraffa only meant to highlight the existence of commodity-rates in monetary and non-monetary economies and to illustrate their most essential properties. Keynes meant to establish a thorough comparison between the properties of the money-rate of interest and commodity-rates of interest. To this purpose he defined commodityrates as analogues of money-rate, which means that he defined them in the context of a monetary economy but following a pattern which mirrored (or mimicd) the way they would appear in a non-monetary economy. This was essential to his purpose, because, paradoxically, from this point of view, money, in a monetary economy, is the analogous of an ordinary commodity in a non monetary economy – i.e., it is the only commodity which may be directly borrowed through signing a contract implying that an agreed amount of itself (money) will be returned at the end of the borrowing period.

Unfortunately, we know of no extant document proving that Sraffa explicitly elaborated arguments similar to those that we have ventured to put forward as grounds for his positions. Moreover, the matter is further obscured by the fact that in his notes on Keynes's book Sraffa himself wrote that Keynes's definition was *OK*. However, and this we would count as a point in favour of our interpretation, the qualification that Sraffa added to his own *OK* suggests that in any case he maintained a critical attitude towards Keynes's definition and that that attitude might have had something in common with the interpretative hypotheses we have put forward - Sraffa's qualification run as follows: «Sect.I. Commodity rates. OK. *as far as it goes, but irrelevant*» (SP I 100/6, emphasis added).

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