Informal labour incentive contracts and product market competition

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Abstract

This paper studies the dynamic interaction between product market competition and shirking incentives in a context where workers’ effort is unverifiable and the probability of the unemployed workers getting a job can depend on their employment histories according to the degree of product market competition. It is shown that efficiency wages paid by each firm can decrease when competition becomes fiercer. With discretionary bonuses, instead, wages are generally uncorrelated with competition, but there exists an upper threshold for the number of competing firms, over which profits collapse to zero. Moreover, if information about firms’ misbehaviour in paying bonuses flows in the labour market at a low rate, there is the possibility for firms to make higher profits by paying efficiency wages instead of bonuses.

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1 Introduction

As well known, the difficulty of measuring individual performance in an objective way limits the use of legally enforceable incentive contracts and implies that firms often rely on informal, or implicit, agreements to motivate employees. In order to provide parties with incentives to fulfil informal agreements, labour contracts must be designed so that the value of continuing the relationship in the future is sufficiently large that neither party wishes to renege on the contract (e.g. Bull 1987). In this regard, efficiency wages (Shapiro and Stiglitz 1984) and contracts with discretionary bonuses (MacLeod and Malcomson 1989) have been largely studied and compared by the literature. Most notably, MacLeod and Malcomson (1998) model the choice between such incentive schemes as a function of labour market conditions (i.e. presence of unemployed workers or unfilled vacancies). However, by concentrating on the labour market, previous analysis implicitly assumes that product markets, where firms operate, are perfectly competitive.

By introducing imperfect (Cournot) competition in the product market, this paper aims to extend previous literature on informal labour incentive contracts in order to analyze the dynamic interaction between product market competition and shirking incentives, as well as to compare different outcomes that efficiency wages and discretionary bonuses attain in such a context. The questions addressed here include the following: does product market competition affect wage profiles and how does this relate to the incentive scheme used by firms? Which relationship we should expect to find between product market competition and industry profits, according to alternative incentive contracts? Since, as known, efficiency wages imply firms must pay a rent to motivate their workers while discretionary bonuses do not, should we expect that profits are always higher with the latter? Or, does the degree of product market competition play a role, possibly, by affecting the relative profitability of alternative schemes? All such issues are obviously relevant to the concerns of labour economics and industrial organization.
We also introduce an important departure with respect to standard hypothesis about workers’ reputation, by assuming that the probability of an unemployed worker finding a job can depend on his/her past employment history. More exactly, workers who have been previously fired as the result of shirking may have a lower probability of finding a new job with respect to other workers. Furthermore, and more importantly, we relate such a possibility to the number of firms competing in the product market. As we will discuss, this can be motivated assuming that gathering information about workers’ previous employment histories becomes more difficult as the number of firms in the market increases.

Our main results can be summarized as follows. When firms use efficiency wages and the number of firms competing in the product market is low, which implies that workers’ reputation matters, the wage rent paid, in equilibrium, by each firm decreases if the probability of unemployed workers finding a job increases. This result, which is in contrast with the Shapiro and Stiglitz’s (1984) shirking version of efficiency wages (where workers’ reputation can never be established), is due to the fact that an increase in the probability of finding a job also increases the “opportunity-cost” of shirking and permits firms to elicit effort from workers even with a lower wage. Moreover, since the “matching” probability for the unemployed increases with competition in the product market (i.e. with employment), if the number of competing firms is sufficiently low and the (positive) effect on the wage that derives from vanishing workers’ reputation is not excessively strong, the efficiency wage paid by each single firm decreases as competition becomes fiercer.

When firms adopt discretionary bonuses, instead, they do not need to provide any rent to their workers to motivate them. Thus workers’ wages do not depend on unemployment in the labour market and, as a consequence, wages are generally uncorrelated with the number of firms competing in the product market. However, this holds true only if the number of firms is no higher than a given threshold, which is related to product market as well as labour market parameters. Indeed, since profits decrease as the number
of firms increases, there exists a critical threshold for the number of firms competing in the market, over which each single firm’s profit is too low to make its promise to pay the bonus credible. Hence workers shirk on the job and profits collapse to zero.

The above results also open up the possibility of comparative analysis of the relation between the two incentive schemes considered and (industry) profits. Although efficiency wages imply firms pay a rent to motivate their workers while discretionary bonuses do not, there remains a possibility for profits to be higher when firms adopt efficiency wages. This could happen if profits with efficiency wages are still positive when the threshold related to the number of firms competing in the market is approached (that is, when profits collapse to zero when firms pay discretionary bonuses). In particular, in such a case, while profits are always higher with discretionary bonuses for relatively low numbers of competing firms, there exists a range, over and above the threshold, for which firms make higher (positive) profits by paying efficiency wages. We show that this applies when there is a relatively low rate at which information about firms’ misbehaviour in paying bonuses flows in the labour market.

Our work relates (and largely draws) on the informal (self-enforcing) contracts literature. Most notably, as already mentioned, in “anonymous” labour markets, that is, in a context where establishing an external reputation is impossible for both workers and firms, MacLeod and Malcomson (1998) model the choice between efficiency wages and performance pay with discretionary bonuses as a function of labour market conditions and show that, when there are more workers than jobs (i.e. unemployed workers), at an efficient equilibrium the rent required to make the compensation agreement self-enforcing must go to the worker in the form of a high (efficiency) wage, with the threat of job loss to provide motivation. Conversely, if there are more jobs than workers (i.e. unfilled vacancies), efficient market equilibrium has performance pay and, since firms have an incentive to renege on the bonus, performance pay can be enforced only if the firm receives a future rent
from continued employment. In contrast to MacLeod and Malcomson (1998), the aim of this paper is to compare efficiency wages and performance pay in relation to product (instead of labour) market conditions. This requires that we consider a situation in which efficiency wages and discretionary bonuses are together sustainable in the labour market and this will lead to a framework, in which, on the one hand, there are unemployed workers and, on the other, some sort of firms’ (and workers’) reputation must play a role.¹

Due to the emphasis we place on incentives for workers and imperfect competition in the product market, our paper could also be in some way related to the growing literature that investigates managerial delegation (see the seminal works of Fershtman (1985), Vickers (1985), Fershtman and Judd (1987) and Sklivas (1987)) and incentive contracts (see, in particular, Schmidt (1997) and Raith (2003)) in oligopolistic markets.² This literature, however, differs from our work mainly because it considers principal-agent problems, in which formal incentive contracts that link workers’ (managers’) pay to firms’ performance measures (e.g. profit and revenue) are feasible, and studies changes in the optimal “shape” of incentive contracts following changes in product market competition. By contrast, we consider the interaction between labour incentive contracts and imperfect product market competition in a context in which formal incentive contracts are not feasible, thus parties must rely on other contractual schemes, such as termination contracts or informal (implicit) incentive contracts.

The relationship between the number of firms competing in the (oligopolistic) product market, wages and/or (industry) profits with endogenous production costs is also studied by Dowrick (1989), Naylor (2002) and Matsushima (2006). In particular, Dowrick (1989) shows that the effect of an

¹The role of workers’ reputation is discussed in Malcomson (1999), but it is not related to the degree of product market competition, as will be effected in this paper.

²See Cuñat and Guadalupe (2005) for an empirical study on the effect of product market competition on the explicit compensation packages that firms offer their CEOs, executives and workers.
increase in competition on wages is ambiguous but, generally, wages decrease as the number of competing firms increases. However, in Dowrick (1989) the effects of competition on wages operate by affecting rents over which unions bargain, while, in our framework, they relate to changes produced in the optimal incentive wage contract. Naylor (2002), instead, considers a bilateral oligopoly model, in which firms’ wages are determined through (Nash) bargaining with labour unions, and shows that the relationship between industry profits and the number of firms in the downstream sector depends on the relative bargaining power of agents. Finally, Matsushima (2006) shows that, under free entry into input markets, the relationship between industry profits and the number of firms competing in the (downstream) market depends on fixed costs (the ease of entry) in the input markets. Our paper differs in that, to the best of our knowledge, it is the first to study the connections between workers’ motivation concerns, the number of firms in the (downstream) market and profits.

The rest of the paper is organized as follows: in Section 2 the basic framework, unemployment values and flows, and the hypothesis concerning workers’ reputation are described. The competition game in the product market and the design of incentive labour schemes, as well as the wage profiles they produce according to the degree of product market competition, are studied in Section 3. Section 4 compares alternative incentive schemes’ outcomes in relation to their effects on industry profits via the number of competing firms. Finally, Section 5 concludes, while further details and technical proofs are relegated to the Appendix.
2 Model

2.1 Economic environment

Time is discrete, \( t = 1, 2, ... \)\(^3\) There is a number \( n \geq 1 \) of identical firms competing à la Cournot repeatedly over time in a homogeneous good market, with inverse demand function given by:

\[
p = a - cQ
\]

where \( Q = \sum_{i=1}^{n} q_i \). There is also a pool of \( \ell \) identical workers, with \( \ell > n \). Each employment relationship consists of a repeated game played between a firm and a pool worker who form an employment relationship in a certain period and interact until their relationship is severed. Let us suppose that, at the end of each period, each relationship becomes unprofitable at the rate \( s \) for exogenous reasons and in such a case firm and employee separate. Firms and workers have infinite life, they are risk-neutral and discount the future with the same rate \( r \). For simplicity, we concentrate on a situation in which each firm employs one single worker (e.g. a manager) and all marginal costs, other than the worker’s wage, are constant and normalized at zero.

With regard to labour contracts and workers’ effort, we follow MacLeod and Malcomson (1989, 1998) by assuming that: i) firms perfectly observe their workers’ decision about effort, but the only legally verifiable pieces of information that can be included in a labour contract are money payments and whether or not a person is employed by a firm;\(^4\) and ii) in relation to the

\(^3\)Since in this environment the technology, the preferences and any other variable are stationary, that is, they remain unchanged over time, we do not need to denote variables by a time index.

\(^4\)In other words, effort is unverifiable by a court, hence it cannot be included in a legally enforceable contract. Furthermore, other measures of performance, such as output and profits, are not verifiable too (see, e.g., Williamson et al. 1975; Carmichael 1989). Alternatively, it could be assumed that, for reasons explained by Holmström and Milgrom (1991) and Baker (1992), it is not in firms’ interests to use the latter to motivate workers.
worker employed by firm \( i \), the decision about effort consists in each period either to work \( (e_i = 1) \) or to shirk \( (e_i = 0) \), obtaining an utility given by:

\[
  u_i = w_i - ve_i
\]

where \( w_i \) is the wage paid by firm \( i \) and \( v > 0 \) is the disutility of work, while we normalize to zero the utility of the worker when unemployed.

Obviously, the worker’s decision is essential for production and, in particular, we assume that:

\[
  q_i = \begin{cases} 
  0 & \text{if } e_i = 0 \\
  \arg \max \pi_i & \text{if } e_i = 1 
  \end{cases}
\]

where \( \pi_i \) is the firm \( i \)’s per-period profit. That is, while worker’s decision to work ensures producing the level of output that maximizes the firm’s profit (which will be derived below in detail), there is no firm’s production (hence, profits) when the worker shirks (recall that neither output nor profits are assumed to be verifiable by courts in this context).

### 2.2 Unemployment values and flows

In relation to labour market functioning, it is important to define first the general aspects connected with unemployment values and flows, since we introduce an important departure with respect to standard assumptions.\(^5\)

We admit that the probability of an unemployed worker finding a job in any period can depend on his/her past employment history. In particular, workers who have been previously fired by a firm as the result of shirking can be characterized by a different (i.e. lower) probability of finding a new job compared with other workers. This is in contrast with the standard shirking version of efficiency wage models, stemming from Shapiro and

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\(^5\)It is worth remarking that our following discussion and analysis hinge on the presence of a labour market, which is specifically linked to the product market. For instance, this can be rationalized with the presence of product market-specific skills.
Stiglitz (1984), where a bad reputation for shirking workers cannot be established in the labour market, hence unemployed values for “shirkers” and “non-shirkers” are always the same. As emphasized by Malcomson (1999, p. 2340), the Shapiro-Stiglitz’s “anonymous” labour market assumption is plausible when acquiring information about workers’ previous employment experience is costly for firms.

In this paper, we admit that the possibility (or the cost) to acquire information about workers’ previous employment histories is related to the number of firms operating in the market, that is, the larger the number of firms, the lower the possibility (the higher the cost that firms must bear) to acquire information about workers. In particular, we hypothesize that once a match between an unemployed worker and a firm has occurred, the latter can try to search for information about the former’s previous employment history. Only if the firm finds out that the worker is a shirker (i.e. she has been previously fired for shirking), it decides not to hire her/him. However, the possibility that such search reaches a successful is inversely related to the number of firms in the product market. Consider, from one hand, the extreme case of a monopolistic market. Since a worker who has been fired for shirking could find another job (in the same market) only with the same firm, the possibility for the latter to get information about the worker’s previous misbehaviour is complete (or, in other words, the cost the firm must bear to acquire information on that worker is negligible). On the other hand, if the product market is extremely competitive, that is, there are many firms in the market, it is very complex to discover workers’ previous employment experience, thus the Shapiro-Stiglitz assumption holds. For instance, a very large number of potential previous employers can discourage, in the first place, the firm to search (possibly, because too costly). Finally, also notice that, since

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6 We believe this is consistent with MacLeod and Malcomson’s (1998, pp. 392-3) argument that “in an anonymous market [...] it is hard to keep track of participants, something that may well be true of workers from poor areas of large cities”. “Large markets” (i.e. markets with a relatively high number of competing firms), similarly to large cities, make
a firm decides not to hire a worker only if acquires evidence about her/his previous misbehaviour, if this is not the case, also a shirker will be hired just as other workers.

To sum up, the timing of events for each period is as follows: i) at the beginning of the period, matching occurs between unmatched firm and worker and, unless the firm finds out that the worker has been previously fired for shirking, an employment relationship is formed; ii) the firm designs a proper labour incentive contract that can be either efficiency wage-type or bonus-type; iii) the firm makes production decision (the product market game) and the worker decides to work or to shirk; iv) the firm pays the (contractual) efficiency wage or, if instead the incentive contract provides for a discretionary bonus, decides whether or not to pay the bonus, hence payoffs (i.e. wage and profit) realize; v) finally in the period, separation may occur either for exogenous reasons or because the firm decides to fire the worker (in this regard, notice that, since effort is perfectly observable by the firm, the latter always fires a shirker at the end of the period).

According to the above discussion, by using $U^S$ and $U^{NS}$ to indicate the expected discounted lifetime utility of an unemployed worker who has and has not been previously fired for shirking, respectively, starting from a generic period of time, we have (from here onwards, in order to streamline the notation, we omit the index $i$ whenever it is unnecessary):

$$rU^k = Jm\left(E^k - U^k\right) \implies U^k = \frac{JmE^k}{r + Jm} \quad (4)$$

where $k \in \{S, NS\}$, $E^k$ indicates the expected discounted lifetime utility, from a generic period of time, of an employed worker of type $k$, $m$ is the per-period probability for an unemployed worker to match with a firm and, finally, $J$ is an index function, such that $J = 1$ if $k = NS$ and $J = \theta$ if $k = S$.

Hence, the term $\theta$ relates to the possibility for a firm to find out that a worker is a shirker and, as discussed above, we assume that it depends on keeping track of workers more difficult and costly for firms.
the number of firms in the market.

**Assumption 1** The function $\theta = \theta(n) \in [0, 1)$, with $\theta(1) = 0$ and $\theta(n) \to 1$ for $n \to \infty$, with $\infty$ sufficiently large. Furthermore, for any $n$, $\theta(n)$ is continuously differentiable\(^7\) and non-decreasing.

According to Assumption 1, when the number of firms is sufficiently large ($n \to \infty$), hence the product market is (sufficiently) competitive, $\theta \to 1$ and the Shapiro-Stiglitz “anonymous” market hypothesis holds. Hence, the probability of getting a new job is the same and equal to $m$ for all workers. Instead, when the product market is a monopoly, a worker once fired for shirking is never employed again in this market. Obviously, for intermediate $n$’s values, “workers’ reputation” can be established to some extent (depending on $n$). Hence workers previously fired for shirking could get new jobs with lower (but positive) probability than other workers (i.e. $0 < \theta m < m$).

In a stationary equilibrium, all employed workers do not shirk (i.e. $e_i = 1, \forall i$) and lose their jobs only for exogenous reasons. Furthermore, movements into and out of unemployment must balance. In each period, workers seeking a job consist of $\ell - n$, who were unemployed in the previous period, plus $sn$ who have just lost their jobs for exogenous reasons, while $sn$ jobs are created to replace those that have been lost. Hence, the matching probability for an unemployed worker is given by:

$$m = \frac{sn}{\ell - (1 - s)n}. \quad (5)$$

Instead, since $\ell$ is sufficiently large to satisfy whatever labour demand, and no search or matching frictions are assumed in this economic environment, in a stationary equilibrium, where all firms’ implicit promises or contracts are honoured, each firm promptly finds a new worker when an employment relationship is severed for exogenous reasons.\(^8\) Also note that in this

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\(^7\)For simplicity, we consider $n$ as a continuous variable.

\(^8\)Assumptions about firms’ reputation are described in greater detail in Section 3.2.2.
context, it is natural to assume that firms have all market power *vis-à-vis* their workers.

In what follows, following the backward induction logic, we first analyze competition *à la* Cournot in the product market, conceding that labour contracts have been previously designed adequately. Then, we study as firms must properly design labour contracts, that can be efficiency wage-type or discretionary bonus-type, to ensure that workers do not shirk.

3 Oligopolistic competition and informal labour incentive contracts

3.1 The product market game

According to the economic environment described above, per-period profit for the representative firm $i$ can be written as:

$$\pi_i = pq_i - w_i = [a - c(q_i + Q_{-i})]q_i - w_i$$  \hfill (6)

where $Q_{-i}$ is the sum of the quantities supplied by the other firms.$^9$

Under the Cournot-Nash assumption, differentiation of (6) with respect to $q_i$ yields the first-order condition for profit maximization by firm $i$, from which we can derive the firm $i$’s reaction function in the output space as:

$$q_i = \frac{a - cQ_{-i}}{2c}.$$  \hfill (7)

Solving all firms’ reaction functions simultaneously allows us to derive the stage-two symmetric equilibrium firm $i$’s output (with $q_i = q, \forall i$), as:

$$q = \frac{a}{(n + 1)c}.$$  \hfill (8)

By substituting (8) into (6), we get an expression for the firm $i$’s profit that, in symmetric equilibrium ($\pi_i = \pi, \forall i$), is given by:

$^9$Clearly, in the monopoly special case $Q_{-i} = 0$. 


\[ \pi = \frac{a^2}{(n+1)^2c} - w \]  

(9)

where \( w \) (\( = w_i, \forall i \)) is the outcome of the game determining the optimal incentive labour contract.\(^{10}\)

### 3.2 The labour incentive contract and wage profiles

#### 3.2.1 Efficiency wages

The best known model in shirking versions of efficiency wages is provided by Shapiro and Stiglitz (1984), where firms pay a contractual (non performance-contingent) wage and workers are discouraged from shirking by the threat that the contract will be terminated and fewer alternative employment opportunities will be available in the future.\(^{11}\) By incorporating into such model our hypothesis (described above) about workers’ probability of getting a job, standard analysis (see the final Appendix, Section A.1) leads to the following expression for the equilibrium efficiency wage:

\[ w_{EW} = v \left[ \frac{(m + r)(1 + \theta m + r)}{m + r - s(\theta m + r)} \right]. \]  

(10)

Define with \( \alpha \) the term in brackets of (10)’s r.h.s.. As usual, since \( \alpha > 1 \), firms must pay a rent to their workers in order to motivate them. Also note that, as is intuitive, \( \alpha \) positively depends on \( \theta \): when workers’ reputation matters (lower \( \theta \)’s values), firms are able to elicit effort by workers even by paying them lower wage rents. Furthermore, for \( n \to \pi \) (\( \theta \to 1 \)), \( \alpha \to \frac{1+m+r}{1-s} \) and standard (Shapiro-Stiglitz) results apply (i.e. the efficiency wage

\(^{10}\)Notice that in this environment, owing to the \( w \)’s nature of (quasi-)fixed cost, profits become negative for large \( n \)’s values. In what follows, however, we will generally assume that the product market parameter \( a \) is large enough to ensure that results are meaningful.

\(^{11}\)Indeed, as pointed out by MacLeod and Malcomson (1989, p. 448), also efficiency wage contracts have their own informal element, namely “that the employee will perform satisfactorily if employed and that the employer will continue the contract if performance is satisfactory, or terminate it if not”.

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increases with $m, r$ and $s$). By contrast, when $n$ (hence, $\theta$) is sufficiently low, a different result can be obtained in relation to $m$.

**Lemma 1**  For a sufficiently low $n$ (number of firms competing in the product market) the efficiency wage decreases when the matching probability $m$ increases.

**Proof.** See the Appendix (Section A.2). ■

The rationale behind Lemma 1 is straightforward. If workers’ reputation does not play any role, there is no difference for workers between losing a job due to shirking or for exogenous reasons. Thus, as highlighted by Shapiro and Stiglitz (1984), an increase in $m$ makes losing a job less severe for all workers, and forces firms to pay higher wages to motivate them. By contrast, when workers’ reputation matters (i.e. $n$ and $\theta$ are sufficiently low), an increase in $m$ increases the “opportunity-cost” of shirking (because losing a job due to shirking means that the probability of being re-employed becomes zero, or greatly decreases, for shirkers). This permits firms to elicit high effort from workers, even with a lower wage.\(^{12}\)

Since an increase in the number of firms competing in the product market increases the (steady-state) matching probability $m$ (see (5)) and, according to Lemma 1, there could be a negative relationship between $m$ and the equilibrium efficiency wage, it is interesting to investigate if also a negative relationship can exist between the latter and $n$. Notice that, in our framework, an increase of $n$ also implies a decrease of unemployment, thus a negative relationship between $w_{EW}$ and $n$ would imply a reversal of the efficiency wage.

\(^{12}\)From (10), it is straightforward to check that the Shapiro-Stiglitz result, as regards $r$, holds for any $n$ (i.e. any $\theta \in [0,1)$). The same applies for $s$, even if, when workers’ reputation matters, its role becomes more complex. This is because an increase in $s$, in turn, produces an increase in $m$ (see (5)), which actually reduces the efficiency wage. It may be shown, however, that, in a stationary equilibrium, the latter effect never outweighs the “traditional” one. Hence the efficiency wage always (i.e. for any $n$) increases with $s$ (formal proof is available from the authors upon request).
literature’s standard result that, in equilibrium, wages and unemployment
are always negatively correlated. The following result deals with this issue.

**Result 1** When competition increases, the efficiency wage paid by each single firm decreases if (and only if):

- $n$ is sufficiently low, and;
- the (positive) effect of increasing $\theta$ on the wage is relatively low.

However, the industry total wage bill (i.e. the sum of the firms’ wages) always increases with $n$.

**Proof.** See the Appendix (Section A.3). ■

While a complete formal proof of Result 1 is provided in the final Appendix, in order to understand the rationale behind it, we can differentiate the term $\alpha$, as defined above, with respect to $n$:

$$
\frac{\partial \alpha}{\partial n} = 
\frac{\theta'(n)m(m + r)(m + r + s)}{[m + r - s(\theta m + r)]^2} + \frac{\theta \frac{\partial m}{\partial n}(m + r)}{m + r - s(\theta m + r)} - \frac{(1 - \theta) \frac{\partial m}{\partial n}rs(1 + \theta m + r)}{[m + r - s(\theta m + r)]^2}.
$$

An increase in competition increases employment, thus leading to an increase in the matching probability $m$. In turn, the latter produces two effects on the efficiency wage, which operate against one another. The first is the standard Shapiro-Stiglitz (SS) effect, according to which reducing unemployment increases the efficiency wage. Instead, the second effect (labelled in (11) as “reputation” effect) reflects the role played on the efficiency wage by the possibility for firms to acquire information about workers’ previous histories. This is negative because, when reputation matters, the higher is
n (hence, m), the higher the “opportunity cost” of shirking. Obviously, the relative importance of those effects depends on θ. We have previously showed (Lemma 1) that, if n, hence θ, is sufficiently low, the latter outweighs the former.

However, besides increasing m, an increase in n also produces another important effect, namely it increases θ, that is, it diminishes the possibility for firms to acquire information about workers’ previous employment histories. This also reduces the role played by workers’ reputation in the labour market, increasing the rent firms must pay to motivate their workers. Such effect is captured by the first term of (11), which clearly reinforces the standard SS effect against to the reputation effect. Finally, also note that Result 1 states that even if the “negative” m’s effect outweighs the “positive” m’s and θ’s effects combined, only wages paid by infra-marginal firms decrease, while the industry total wage bill increases. This is because the total wage reduction for infra-marginal firms is always lower than the wage paid by marginal firm.

\[ \theta = \frac{(n-1)^\gamma}{n^\gamma + \beta} \]

Figures 1 and 2 show, for the specific functional form \( \theta = \frac{(n-1)^\gamma}{n^\gamma + \beta} \), which is consistent with Assumption 1, and selected parameter values, different possible firm’s and corresponding industry’s (overall) wage profiles as a function of n, under an “optimal efficiency wage contract”.\(^{13}\)

\(^{13}\)Parameter values used for Figures 1 and 2 are: \( v = 100; s = r = 0.1; \ell = 50; \gamma = 10 \)
3.2.2 Discretionary bonuses

Let consider now an alternative incentive scheme that provides for a bonus payment conditional on the worker’s choice about effort (e.g. Bull 1987; MacLeod and Malcomson 1989). However, since effort is unverifiable by courts, the bonus cannot be made a legally binding contract tying pay to performance and, since paying the bonus is costly for the firm, the latter could always be tempted not to pay it even if the worker does not shirk. As known, this produces a classic Prisoners’ Dilemma distortion in this context: in a single period game, since the firm cannot commit to paying the bonus, the worker will shirk, and the firm will not produce at all. But, when the game is repeated infinitely (or indefinitely) a sort of “Folk Theorem” could apply. This, however, requires that the contract between parties be self-enforcing, i.e. it must always give both parties the incentive to fulfil their respective parts of the agreement, despite the fact that it is not enforceable by a court.

In the final Appendix (Section A.1), through standard analysis, we show that the equilibrium bonus chosen by firms is \( w_B = v \).\(^{14}\) Notice that, unlike the efficiency wages case, with discretionary bonuses: i) the wage does not depend on the number of firms competing in the product market; and ii) as well known (e.g. Malcomson 1999), firms can potentially motivate workers and \( \beta = 1000 \), for red solid lines; \( \gamma = 1 \) and \( \beta = 0 \), for blue dashed lines. Note that for \( \gamma = 1 \) and \( \beta = 0 \) the firm’s wage initially decreases in \( n \) since, in such a case (unlike that with \( \gamma = 10 \) and \( \beta = 1000 \)), workers’ reputation vanishes very slowly as \( n \) increases, i.e. for low \( n \) values, \( \theta'(n) \) is very small (see also condition (A16) of the Appendix, Section A.3).

\(^{14}\)Generally, together with the bonus that represents the implicit part of the contract, the latter also provides for a fixed salary, whose payment can be enforced by a court. Since in our framework firms have all the bargaining power \textit{vis-à-vis} workers, the former fix the salary such that, given the equilibrium bonus, workers exactly receive their opportunity cost. This implies that the salary equals the unemployment utility, which has been normalized at zero, permitting us to concentrate, without loss of generality, only on the bonus.
without providing them with a rent. Furthermore, $\theta$, hence differences in unemployment values of shirkers and non-shirkers, do not play any role in providing incentives. This is because, in the equilibrium with bonus, employed workers receive exactly the same utility as unemployed ones.

However, as discussed above, firms must be able to credibly commit themselves to paying the bonus. As pointed out in the literature (e.g. Carmichael 1984), in ongoing relationships, when agents learn past employment histories of partners, reputation can play a central role in ensuring that firms honor their promises, since losing one’s employee as the result of cheating on a promised bonus could produce worse future opportunities than when parties separate for other (exogenous) reasons. Labour unions, for instance, may contribute in this direction by monitoring the employment relationships between a firm and its workers and providing the workforce with valuable information regarding the firm’s adherence to implicit contracts, as formally studied in Hogan (2001). Furthermore, also firms themselves could have an interest in credibly fostering the transmission of such information to the market since, by committing themselves more strongly, they can offer a broader range of incentives (e.g. Kreps and Wilson 1982; Tirole 1996; Tadelis 1999; Levin 2002).

As emphasized by Malcomson (1999), even if there are reasons supporting the hypothesis that, in general, information on past employment behaviour flows in the labour market more widely in relation to firms than workers, it is implausible that each time a firm loses employees because of cheating on promised bonuses it is never able to recruit a new worker. Thus, in order to make our analysis more general, we consider a situation in which information on firms’ misbehaviour does not always flow in the labour market, but it does so only with a positive per-period probability $z$.\footnote{\textit{It could be argued that, since we have related $\theta$ (which captures how workers’ reputation flows in the market) to the number of competing firms, this could also be done for $z$. The latter argument, however, seems more problematic. For instance, in the monopoly case, it could be difficult (as well as in a more competitive case) for external agents to...}} Nevertheless, each
time this occurs, cheating behaviour by a firm is interpreted by the labour workforce as a whole as evidence that firm does not fulfil informal agreements with its workers. This means that no worker will be motivated to work for that firm in the future.\textsuperscript{16}

Previous hypothesis about firms’ reputation leads to the following condition, which derivation is provided in the Appendix (Section A.1), that makes implicit agreements on bonuses self-enforceable:

\[ \pi \geq \frac{rv}{z} \]  
which, by substituting for (9), i.e. the equilibrium value for the firm’s profit in the product market game, can be rewritten as:

\[ \frac{a^2}{(n + 1)^2c} - v \geq \frac{rv}{z}. \]  

Solving (13) for \( n \), we obtain a condition for the number of firms competing in the product market, which must be satisfied in a self-enforcing equilibrium:

\[ n \leq \tilde{n} \equiv \frac{a}{\sqrt{cv (\frac{r+z}{z})}} - 1. \]  

Since firms’ profits are decreasing in \( n \), (14) establishes an upper constraint for the number of firms competing in the product market, for which discretionary bonuses are sustainable as a self-enforcing equilibrium. In particular, such an upper constraint is related to both product market and labour market parameters. In detail, the higher \( a \) and the lower \( c \) (i.e. the higher the scale or size of the product market), the higher the upper constraint \( \tilde{n} \). Moreover, the lower the disutility of effort \( v \) and the higher the frequency with which information on firms’ misbehaviour flows in the labour verify whether the firm has promised to pay a bonus or if the latter was actually paid.

\textsuperscript{16}See Doering and Piore (1971) and Bewley (1999) for some evidence, which supports such a hypothesis.
market, $z$, the higher the upper constraint $\bar{n}$. Finally, for the usual reasons, it also negatively depends on the discount rate $r$. The following statement summarizes such findings.

**Result 2** There exists an upper threshold for the number of firms competing in the product market, over which firms’ profits collapse to zero when they use discretionary bonuses to motivate their workers. This threshold is positively related to $a$ and $z$ and negatively related to $c$, $v$ and $r$.

## 4 Informal labour incentive schemes, competition and industry profits

Using previous results, in this section, we will explore how competition in the product market affects industry profits according to the incentive scheme firms use to motivate their workers.\(^{18}\)

By substituting (10), the equilibrium efficiency wage, in the firm’s profit equation (9), we get:

$$\pi_{EW} = \frac{a^2}{(n + 1)^2c} - v\alpha. \quad (15)$$

\(^{17}\)In particular, note that if $z \to 0$ (i.e. a firm’s reputational mechanism does not work at all), the firm would never gain by sticking to the agreement even if the relationships were repeated over time. Hence there is no (positive) number of firms for which implicit self-enforcing contracts can be established.

\(^{18}\)As discussed in the Introduction, this issue is specifically studied, in different frameworks, by Naylor (2002) and Matsushima (2006), who find out the existence of a non-monotone relationship between the industry profits and the number of firms, with implications for merger, collusion and entry deterrence. As we will show, in our context such a non-monotone relationship does not apply, since, more standardly, industry profits and competition are always negatively correlated. Nevertheless, exploring as informal incentive contracts affect industry profits will prove to be particularly interesting in relation to the relative profitability of (hence, to the convenience for firms to adopt) alternative incentive schemes, according to the degree of product market competition.
Instead, with self-enforcing discretionary bonuses firm’s profit is given by:

\[
\pi_B = \begin{cases} 
\frac{a^2}{(n+1)^2} - v & \text{if } n \leq \tilde{n} \\
0 & \text{if } n > \tilde{n}.
\end{cases}
\]  

(16)

Furthermore, from (15) and (16), we can easily derive corresponding industry profits with the two alternative incentive schemes as, respectively:

\[
\sum \pi_{EW} = n\pi_{EW} = \frac{n a^2}{(n+1)^2} - n v \alpha 
\]

(17)

\[
\sum \pi_B = n\pi_B = \begin{cases} 
\frac{n a^2}{(n+1)^2} - n v & \text{if } n \leq \tilde{n} \\
0 & \text{if } n > \tilde{n}.
\end{cases}
\]  

(18)

Obviously, since both with efficiency wages and discretionary bonuses industry’s total wage bill increases (and total revenues decrease) with \( n \), industry profits always decrease when competition increases. Formally, by differentiating (17) and (18), respectively, with respect to \( n \) (and recalling from the proof of Result 1 that \( \alpha + n \frac{\partial \alpha}{\partial n} > 0 \); see the Appendix, Section A.3), it is easy to show that:

\[
\frac{\partial \left( \sum \pi_{EW} \right)}{\partial n} = \frac{(1 - n)a^2}{(n + 1)^3} - v(\alpha + n \frac{\partial \alpha}{\partial n}) < 0
\]

(19)

\[
\left. \frac{\partial \left( \sum \pi_B \right)}{\partial n} \right|_{n \leq \tilde{n}} = \frac{(1 - n)a^2}{(n + 1)^3} - v < 0
\]

(20)

and

\[
\left| \frac{\partial \left( \sum \pi_{EW} \right)}{\partial n} \right| > \left| \frac{\partial \left( \sum \pi_B \right)}{\partial n} \right|_{n \leq \tilde{n}}
\]

(21)

that is, as \( n \) increases, industry profits decrease more rapidly with efficiency wages than with discretionary bonuses.

At this point, according to above results, one could also be tempted to deduce that industry profits can never be greater with efficiency wages than with discretionary bonuses. Nevertheless, a further step is needed. This is because (industry) profits with discretionary bonuses collapse to zero.
when the number of competing firms exceeds a critical threshold. Hence, for relatively large numbers of firms, that is, for $n > \tilde{n}$, there could be the possibility that firms make greater profits with efficiency wages.

Figure 3 clarifies this issue in more detail: it describes industry profits behaviour, in relation to the number of firms competing in the market, with alternative incentive schemes (blue dashed lines for efficiency wages and red solid lines for discretionary bonuses) and for two alternative cases, both hypothetically plausible. In Case 1, industry profits with efficiency wages are already negative when $n$ approaches $\tilde{n}$, hence there is no possibility for them to be higher than with discretionary bonuses. By contrast, in Case 2, profits with efficiency wages are still positive when $n$ reaches $\tilde{n}$, hence there exists a range, over and above the threshold $\tilde{n}$, for which firms make higher (positive) profits by paying efficiency wages.

Result 3 If the rate $z$, with which firms’ reputation flows in the labour market, is lower than a critical threshold negatively related to the value of $\alpha$ for $n = \tilde{n}$, there exists a range over and above $\tilde{n}$, for which industry profits are higher with efficiency wages (i.e. Case 2 in Figure 3 applies). Otherwise, there is no $n$ for which industry profits are greater with efficiency wages than with discretionary bonuses (i.e. Case 1 in Figure 3 applies).

Proof. See the Appendix (Section A.4).
Industry profits can be higher with efficiency wages only if they are positive when \( n = \hat{n} \). Taking into account that \( \sum \pi_{EW} \) is (rapidly) decreasing in \( n \), this can happen only if \( \hat{n} \) is sufficiently low, which occurs also if \( z \) is (relatively) low. Moreover, when firms pay efficiency wages, industry profits decrease with \( \alpha \) (the term related to the wage rent). Hence, \( z \) should be relatively low with respect to a given threshold, negatively related to \( \alpha \) computed for \( n = \hat{n} \). In particular, in the final Appendix (Section A.4) we show that, for industry profits to be higher (or, in other words, to be positive when \( n = \hat{n} \)) with efficiency wages, the following condition must be satisfied:

\[
\frac{z}{\alpha} < \hat{\alpha} - 1 \tag{22}
\]

where “\( \hat{\cdot} \)” means that \( \alpha \) is evaluated in \( n = \hat{n} \). Notice that the condition defined in (22) is always satisfied when \( z \to 0 \). This is because discretionary bonuses, in such a case, cannot be made self-enforcing. Instead, it is never satisfied for \( z \to 1 \), because \( \hat{n} \) becomes too high for industry profits to be positive (for such a number of firms) with efficiency wages.\(^{19}\)

Before concluding, also note that when Case 1 in Figure 3 applies, that is, industry profits are never higher with efficiency wages, the critical threshold with discretionary bonuses, \( \hat{n} \), represents the largest number of firms for which industry profits can be positive. As already remarked, this threshold is related to product market (as well as labour market) parameters. In particular, the larger the size of the market, the larger the critical number of firms for which profits can be positive. Although this statement is hardly breaking new ground, it is important to stress that with respect to the standard rationale, according to which the number of firms operating (efficiently) in a market is directly related to its size simply due to the presence of “demand constraints”, we derived this result in quite a new fashion (which, in some sense, reinforces the standard rationale): when markets are thin (with low

\(^{19}\)In this regard, also note that the role of other parameters (particularly, of \( r \)) is not clear-cut, since their changes can generate both direct and indirect effects (e.g. increasing \( \alpha \) and decreasing \( \hat{n} \) at the same time) that can act against one another.
a/c), larger numbers of competing firms make implicit incentive contracts with discretionary bonus unsustainable as self-enforcing equilibria.

Instead, when Case 2 in Figure 3 applies, the threshold \( \hat{n} \) represents a critical degree of product market competition, above which firms find it worth modifying the incentive scheme adopted to motivate their workers. More exactly, when \( n = \hat{n} \) (and incumbent firms are making higher profits by using discretionary bonuses), a new firm can earn a positive profit by entering into the market, but only if it uses efficiency wages to elicit its worker’s effort. Furthermore, the entry of the new firm also forces those already present in the market to change their incentive scheme, since discretionary bonuses become no longer sustainable as a self-enforcing equilibrium. Hence, when \( n = \hat{n} \) and a new firm enters the market, the profits of incumbent firms decrease for two different reasons: first, as usual, because increasing competition reduces their revenues; secondly, because it also increases their wages, due to the fact that it forces them to switch from a less costly to a more costly (incentive) wage contract (i.e. from bonus to efficiency wage).\(^{20}\)

5 Conclusion

In this paper, the dynamic interaction between product market competition and shirking incentives was analyzed in a framework where workers’ effort is perfectly observable by firms, but is not verifiable by a third party (e.g. a court). Moreover, it was assumed that the probability of unemployed workers getting a job may depend on their employment histories and, more impor-

\(^{20}\)Notice that this finding opens up to non-trivial social welfare issues in relation to market entry by new firms which, however, fall outside the scope of this paper and are left for future research. Furthermore, it can also provide some important indications for testable hypotheses by empirical research on incentive contracts. For instance, it seems to suggest that, *ceteris paribus*, we would observe discretionary bonuses in industries with relatively low numbers of firms, while efficiency wages should emerge, in a time series view, when (in the same industries) competition becomes fiercer or, in a cross section view, in other industries characterized (at the same time) by a higher degree of competition.
tantly, that such a possibility relates to the degree of market competition. In this context, the effects of two well-known incentive schemes, namely, efficiency wages and contracts with discretionary bonuses, were studied and compared.

Differently from standard results, efficiency wages paid by each firm can decrease when competition (hence, employment) increases. At the same time, however, the industry total wage bill (i.e. the sum of firms’ wages) always increases (hence, industry profits always decrease) because, on increasing competition at the margin, the total wage reduction for infra-marginal firms is always lower than the wage paid by the marginal one. When firms adopt discretionary bonuses, instead, wages are uncorrelated with competition in the product market, but there exists an upper threshold for the number of competing firms, over which profits go to zero. This is because each single firm’s profit is too low to make its promise to pay the bonus credible. Moreover, although efficiency wages imply firms pay a rent to motivate their workers while discretionary bonuses do not, if the rate with which information about firms’ cheating behaviour flows in the labour market is relatively low, there exists a range for the number of firms, over and above the critical threshold with discretionary bonuses, for which firms can make positive profits only by paying efficiency wages.

Appendix

A.1 Asset value equations, incentive compatibility conditions and equilibrium wages with different schemes

Efficiency wages

Recalling that workers’ decision about effort is perfectly observable by firms, hence a shirker is always fired at the end of the period, we get that, with efficiency wages, the asset value equation or expected discounted lifetime
utility for a shirker, $E_{EW}^S$, is given by:

$$rE_{EW}^S = w_{EW} - (E_{EW}^S - U^S) \implies E_{EW}^S = \frac{w_{EW} + U^S}{1 + r}$$ (A1)

where $w_{EW}$ denotes the (efficiency) wage paid by the firm. Instead, the expected discounted lifetime utility for a non-shirker, $E_{EW}^{NS}$, is:

$$rE_{EW}^{NS} = w_{EW} - v - s(E_{EW}^{NS} - U^{NS}) \implies E_{EW}^{NS} = \frac{w_{EW} - v + sU^{NS}}{r + s}.$$  (A2)

The worker will certainly shirk unless $E_{EW}^{NS} \geq E_{EW}^S$. Substituting for $U^S$ and $U^{NS}$ from (4) in (A1) and (A2), respectively, rearranging and solving for $w_{EW}$, we get the following incentive-compatibility condition (or “no-shirking condition”) for the worker:

$$w_{EW} \geq v \left[ \left( \frac{m + r}{m + r - s(\theta m + r)} \right) \right].$$ (A3)

which, in equilibrium, holds with equality because profit-maximizing firms pay the lowest wages consistent with it.

**Discretionary bonuses**

Since effort is perfectly observable, when firms adopt discretionary bonuses to motivate workers, a shirker never receives the bonus payment and is always fired at the end of the period. Hence, we can represent the asset value equations or expected discounted lifetime utility of a shirker, $E_{B}^S$, as:

$$rE_{B}^S = - (E_{B}^S - U^S) \implies E_{B}^S = \frac{U^S}{1 + r}.$$ (A4)

Instead, if the worker chooses to work, his/her expected discounted lifetime utility $E_{B}^{NS}$ is:

$$rE_{B}^{NS} = w_{B} - v - s(E_{B}^{NS} - U^{NS}) \implies E_{B}^{NS} = \frac{w_{B} - v + sU^{NS}}{r + s}.$$ (A5)

where $w_{B}$ denotes the discretionary bonus paid by the firm.
Clearly, workers will shirk unless $E_B^{NS} \geq E_B^S$. Solving for the bonus, we get the following incentive-compatibility condition for the worker:

$$w_B \geq v.$$ \hspace{1cm} (A6)

Firms choose the lowest bonus compatible with (A6), which, in equilibrium, holds with equality. Firms, however, must be able to credibly commit themselves to paying the bonus. Formally, together with the incentive-compatibility condition for the worker, an incentive-compatibility condition for the firm must also be satisfied in equilibrium. According to the hypothesis about firms’ reputation, described in the main text, and considering that the firm’s profit is negative when workers shirk, hence it is always better for the firm to end an employment relationship than let it continue with no effort by the worker in the future, the asset value equation or expected discounted profit for a “cheating” firm, $\Pi^C$, is:

$$r\Pi^C = \pi + w_B - z\Pi^C \implies \Pi^C = \frac{\pi + w_B}{r + z}.$$ \hspace{1cm} (A7)

Instead, indicating with $\Pi^{NC}$ the expected discounted profit for a “non-cheating” firm, i.e. a firm that honestly pays the bonus to its worker, this is given by:

$$r\Pi^{NC} = \pi \implies \Pi^{NC} = \frac{\pi}{r}.$$ \hspace{1cm} (A8)

Hence, the firm cheats on the bonus payment unless $\Pi^{NC} \geq \Pi^C$. Solving for $\pi$, we obtain the following incentive-compatibility, or “no-cheating”, condition for the firm:

$$\pi \geq \frac{rw_B}{z}.$$ \hspace{1cm} (A9)

Finally, in order to define the aggregate condition that makes implicit agreements with bonus self-enforceable, we add the worker’s incentive-compatibility condition (A6) to the firm’s no-cheating condition (A9) and, taking into ac-
count that the firm makes the lowest (incentive-compatible) payments, we get:

$$\pi \geq \frac{rv}{z}. \quad (A10)$$

It follows directly from its derivation that the above equation, which corresponds to (12) in the main text, is a necessary and sufficient condition for cooperative payoffs to be supported, in such a case, as subgame perfect equilibria.

A.2 Proof of Lemma 1

Proof. By differentiating the efficiency wage $w_{EW} = v\alpha$ with respect to $m$ yields:

$$\frac{\partial w_{EW}}{\partial m} = v \frac{\partial \alpha}{\partial m} \geq 0 \Leftrightarrow \frac{\partial \alpha}{\partial m} \geq 0 \quad (A11)$$

where

$$\frac{\partial \alpha}{\partial m} = \frac{\theta(m + r)[m + r - s(\theta m + r)] - (1 - \theta)rs(1 + \theta m + r)}{[m + r - s(\theta m + r)]^2} \quad (A12)$$

whose sign depends on that of the r.h.s. numerator.

In particular, if $n \to \overline{n}$ (hence, $\theta \to 1$), $\frac{\partial \alpha}{\partial m}|_{n=\overline{n}} \to \frac{1}{1-s} > 0$, hence (in line with Shapiro and Stiglitz (1984)) $\frac{\partial w_{EW}}{\partial m}|_{n=\overline{n}} > 0$. Instead, if $n = 1$ and $\theta = 0$, we have that $\frac{\partial \alpha}{\partial m}|_{n=1} = -\frac{rs(1+r)}{(m+r-rs)^2} < 0$, hence $\frac{\partial w_{EW}}{\partial m}|_{n=1} < 0$.

Moreover, noting from (A12) that $\frac{\partial \alpha}{\partial m}$ is increasing in $\theta$ and taking into account, from Assumption 1, that $\theta$ is continuous and non-decreasing in $n$, there will be a number of firms $n^m \in (1, \overline{n})$ such that:

$$\frac{\partial w_{EW}}{\partial m} \triangleq 0 \Leftrightarrow n \geq n^m. \quad (A13)$$
A.3 Proof of Result 1

Proof. By differentiating the efficiency wage \( w_{EW} = v\alpha \) with respect to \( n \) yields:

\[
\frac{\partial w_{EW}}{\partial n} = v \frac{\partial \alpha}{\partial n} \geq 0 \iff \frac{\partial \alpha}{\partial n} \geq 0
\]  

(A14)

where \( \frac{\partial \alpha}{\partial m} \) can be written as:

\[
\frac{\partial \alpha}{\partial m} = \theta'(n) \frac{\partial \alpha}{\partial \theta} + \frac{\partial m}{\partial n} \frac{\partial \alpha}{\partial m}.
\]

(A15)

The changing \( \theta \) effect and the changing \( m \) effect have been specified in detail in the main text (Eq. (11)), where the latter has been also disentangled, distinguishing between the “Shapiro-Stiglitz effect” and the “reputation” effect.

First of all, notice that (A15) can be negative only if \( \frac{\partial \alpha}{\partial m} < 0 \). As shown in Section A.2, this can apply only if \( n \) is sufficiently low (\( n < n^m \)). Moreover, to be \( \frac{\partial \alpha}{\partial n} < 0 \), the following condition (with \( \frac{\partial \alpha}{\partial m} < 0 \)) also needs to be satisfied:

\[
\theta'(n) \frac{\partial \alpha}{\partial \theta} < \left| \frac{\partial m}{\partial n} \frac{\partial \alpha}{\partial m} \right|
\]

(A16)

that is, \( \frac{\partial \alpha}{\partial m} < 0 \) only if the (negative) effect operating via increasing \( m \) outweighs the (positive) effect operating via “vanishing workers’ reputation” (i.e. increasing \( \theta \)).

To proof that, with efficiency wages, the industry total wage bill, \( \sum w_{EW} \), always increases with \( n \) (even when \( \frac{\partial w_{EW}}{\partial m} < 0 \) for some \( n \)), recall that

\[
\frac{\partial (\sum w_{EW})}{\partial n} = v (\alpha + n \frac{\partial \alpha}{\partial m}),
\]

where \( v\alpha \) is the wage paid by the marginal firm, while \( nv\frac{\partial \alpha}{\partial m} \) is the total variation of wages paid by infra-marginal firms.

From (10) and (11), we know that:

\[
\alpha + n \frac{\partial \alpha}{\partial n} = \frac{(r + m)(1 + \theta m + r)}{m + r - s(\theta m + r)} - \frac{n(1 - \theta) \frac{\partial m}{\partial m} r s(1 + \theta m + r)}{[m + r - s(\theta m + r)]^2} + n\Psi
\]

(A17)
where \( \Psi \equiv \frac{\theta'(n)m(m+r)(m+r+s)}{[m+r-s(\theta m+r)]^2} + \frac{\theta \partial \theta m(m+r)}{m+r-s(\theta m+r)} > 0 \). Using (5) and defining \( \Omega \equiv \ell - (1-s)n > 0 \), the r.h.s. of (A17) can be rewritten as:

\[
\frac{1+\theta m+r}{[m+r-s(\theta m+r)]} \times \left[ \frac{r \Omega + sn}{\Omega} \left( \frac{r \Omega (1-s) + sn(1-\theta s)}{\Omega} - \frac{n(1-\theta)rs^2 \ell}{\Omega^2} \right) \right] + n\Psi
\]

which, with some tedious algebra (details available on request), becomes:

\[
\frac{1+\theta m+r}{[m+r-s(\theta m+r)]} \times \left\{ \frac{r \Omega [(r \Omega + sn)(1-s) + sn[r(1-s)(\ell - n)]] + s^2 n^2 [1 + \theta r(1-s) - \theta s]}{\Omega^2} \right\} + n\Psi > 0
\]

hence, for any \( n \), \( \frac{\partial (\sum w_{EW})}{\partial n} = v \left( \alpha + n \frac{\partial \alpha}{\partial n} \right) > 0 \). \( \blacksquare \)

### A.3 Proof of Result 3

**Proof.** As discussed in the main text, industry profits can be higher when firms elicit workers’ effort by adopting efficiency wages instead of discretionary bonuses if (and only if), under efficiency wages, they are positive for \( n = \tilde{n} \). By substituting for (14) in (17), and defining with \( \tilde{\alpha} \) the corresponding wage rent term, we get:

\[
\sum \pi_{EW}|_{n=\tilde{n}} = \tilde{n} \left\{ \frac{a^2}{\frac{a}{\sqrt{\alpha (\frac{r+z}{z})}} c} - v \tilde{\alpha} \right\}.
\]

(A20)

Using some algebra, (A20) becomes:

\[
\sum \pi_{EW}|_{n=\tilde{n}} = \tilde{n}v \left( \frac{r + z}{z} - \tilde{\alpha} \right).
\]

(A21)

which is strictly positive for:

\[
z < \frac{r}{\tilde{\alpha} - 1}
\]

(A22)
or, taking into account that \( \tilde{\alpha} \equiv (\tilde{m} + r)(1 + \tilde{\theta} \tilde{m} + r) \tilde{m} + r - s(\tilde{\theta} \tilde{m} + r) \), for:

\[
z < \frac{r \left( \tilde{m} + r - s(\tilde{\theta} \tilde{m} + r) \right)}{(\tilde{\theta} \tilde{m} + r)(\tilde{m} + r + s)}.
\]

(A23)

References


